Viscosity of the pure SU(3) gauge theory revisited

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Introduction - linear response

Kubo formulas relevant for the shear viscosity

$$\eta(T) = \pi \lim_{\omega\to 0} \lim_{\mathbf{k}\to 0} \frac{\rho^{12,12}(\omega,\mathbf{k},T)}{\omega}$$

$$\frac{4}{3}\eta(T) + \zeta(T) = \pi \lim_{\omega\to 0} \lim_{\mathbf{k}\to 0} \frac{\rho^{11,11}(\omega,\mathbf{k},T)}{\omega}$$

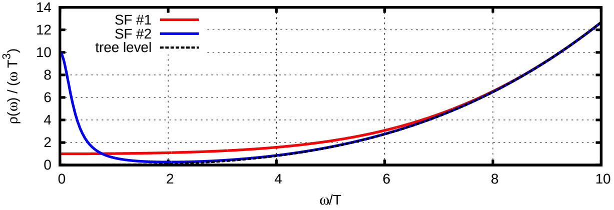
Deep in the deconfined phase, we expect $\zeta \ll \eta$. E.g. from kinetic theory one gets: $\zeta \approx \frac{1}{15}\left(\frac{1}{3} - c_s^2\right)^2 \eta$. See e.g. Arnold,Dogan,Moore hep-ph/0608012

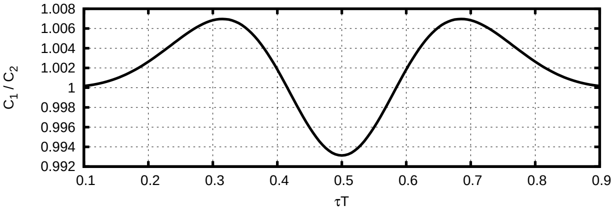
Sum rule → ill-posed problem

$$C_{\mu\nu,\rho\sigma}(\tau,\mathbf{p}) = \int_0^\infty \mathrm{d}\omega \rho_{\mu\nu,\rho\sigma}(\omega,\mathbf{p},\mathrm{T}) \frac{\cosh\left(\omega\left(\tau - 1/(2\mathrm{T})\right)\right)}{\sinh\left(\omega/(2\mathrm{T})\right)}$$

Why it is hard

Correlator insensitive to IR features and has a severe sign problem





Ways to proceed?

A few things that help (H. Meyer, 1104.3708)

- Using energy-momentum conservation:

$$-\omega^2 \rho_{01,01} = \mathbf{q}^2 \rho_{13,13}$$

  This means the UV behavior of $\rho_{01,01}$ is milder, only ω^2 unlike the ω^4 behaviour of $\rho_{13,13}$.
- But the thermodynamic identity $C_{01,01}(\tau, \mathbf{q}=0)/T^5 = s/T^3$ means we need nonzero momenta to obtain information about the viscosity.
- To proceed, we need an ansatz for the momentum dependence of the spectral function.
- But first things, first, what can we even calculate directly from the lattice.

Overview of the lattice simulations

Techniques

- Anisotropic lattices, with renormalized anisotropy: $\xi_R = 2$
- Multilevel algorithm: to reduce errors near $\tau T = 0.5$
- Tree level Symanzik improved gauge action: to reduce cut-off effects
- Clover discretization

Ensembles (for now)

- High statistics
- Two temperatures: $1.5T_c$ and $2T_c$
- $80 \times 20^2 \times 20$, $64 \times 16^2 \times 16$, $48 \times 12^2 \times 12$, $40 \times 10^2 \times 10$
- $N_t = 20$ only at $1.5T_c$ so far

Anisotropy tuning

We use an anisotropic gauge action with tree level Symanzik improvement. The bare anisotropy $\xi_0(\beta)$ is tuned so that $\chi_R \equiv 2$.
For the tuning we define a *spatial* and a *temporal* w_0 scale:

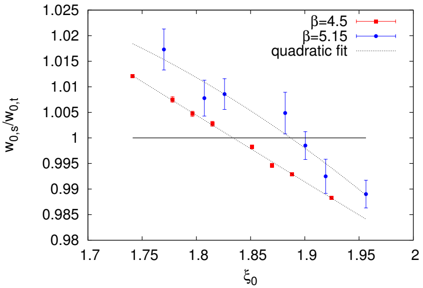
$$\left[\tau \frac{d}{d\tau} \tau^2 \langle E_{ss}(\tau) \rangle \right]_{\tau = w_{0,s}^2} = 0.15\,,$$

$$\left[\tau \frac{d}{d\tau} \tau^2 \langle E_{ts}(\tau) \rangle \right]_{\tau = w_{0,t}^2} = 0.15\,,$$

with

$$E_{ss}(\tau) = \frac{1}{4} \sum_{x, i \neq j} F_{ij}^2(x,\tau)\,,$$

$$E_{st}(\tau) = \xi_R^2 \frac{1}{2} \sum_{x,i} F_{i4}^2(x,\tau)\,.$$



For details, see arXiv:1205.0781.

The energy momentum tensor

$$T^R_{\mu\nu} = Z_6 T^{[6]}_{\mu\nu} + Z_3 T^{[3]}_{\mu\nu} + Z_1 (T^{[1]}_{\mu\nu} - T^{[1]}_{\mu\nu}(T=0))$$

with the definitions (no sum over μ and ν):

$$\begin{aligned}
T^{[6]}_{\mu\nu} &= \frac{1}{g_0^2} \sum_\sigma F^a_{\mu\sigma} F^a_{\nu\sigma} \\
T^{[3]}_{\mu\nu} &= \delta_{\mu\nu} \frac{1}{g_0^2} \left\{ \sum_\rho F^a_{\mu\rho} F^a_{\nu\rho} - \frac{1}{4} \sum_{\rho,\sigma} F^a_{\rho\sigma} F^a_{\rho\sigma} \right\} \\
T^{[1]}_{\mu\nu} &= \delta_{\mu\nu} \frac{1}{g_0^2} \sum_{\sigma,\rho} F^a_{\rho\sigma} F^a_{\rho\sigma}
\end{aligned}$$

We use the clover definition of $F^a_{\mu\nu}$ and define our correlators from the sextet (off-diagonal) components. In the presence of an anisotropy Z_6 splits into three different renormalization constants:

$$\begin{aligned}
T_{01} &= \frac{Z_6^{ts}}{g_0^2} F^a_{02} F^a_{12} + \frac{Z_6^{ts}}{g_0^2} F^a_{03} F^a_{13} \\
T_{12} &= \frac{Z_6^{tt}}{g_0^2} F^a_{01} F^a_{02} + \frac{Z_6^{ss}}{g_0^2} F^a_{13} F^a_{23}
\end{aligned}$$

Renormalization

Overall constant

The overall constant Z_6^{ts} can be determined form the thermodynamic identity:

$$C_{03,03}(\tau, \mathbf{q} = \mathbf{0})/T^5 = s/T^3$$

Renormalization of the sextet

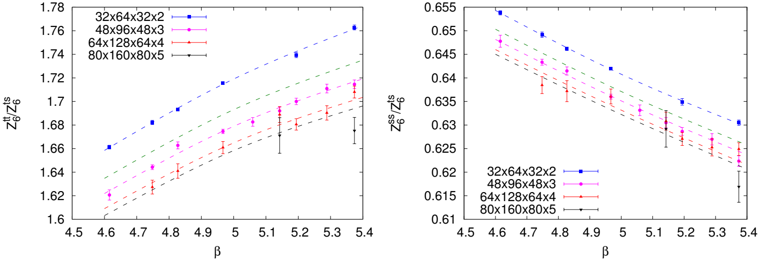
For an isotropic gauge action the renormalization constants have been worked out in 1503.07042 by Guisti et al. Using shifted boundary conditions $\vec{\xi} = (\xi_1, \xi_2, \xi_3) = (1, 1, 1)$ the off-diagonal T_{01} develops a non-vanishing expectation value ($T_{01} = T_{02} = T_{03}$):

$$2Z_6^{tt} \frac{1}{g_0^2} F_{02}^a F_{12}^a = 2Z_6^{ss} \frac{1}{g_0^2} F_{03}^a F_{13}^a = Z_6^{st} \frac{1}{g_0^2} (F_{01}^a F_{21}^a + F_{03}^a F_{23}^a)$$

Therefore the ratios Z_6^{ss}/Z_6^{ts} and Z_6^{tt}/Z_6^{ts} can be calculated from a single simulation with $L_0^{-1} = \sqrt{1 + |\vec{\xi}|^2} T = 2T$.

Renormalization factors

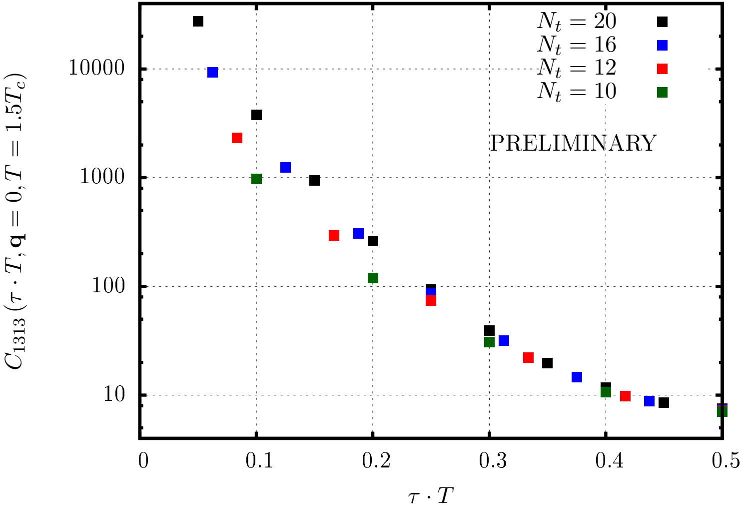
Thus, to renormalize $T_{\mu\nu}$ in a $N_\tau = 12$ simulation with $\xi_R = 2$, we make an auxilliary run on a $48 \times 96 \times 48 \times 3$ lattice with the same bare parameters. The resulting factors will depend on β and N_τ, and the method requires that $N_\tau/4$ is an integer.



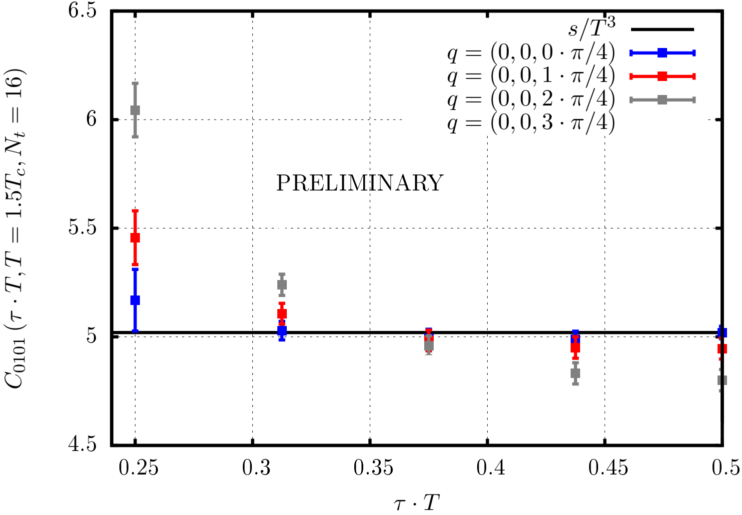
We observe an $1/N_\tau^2$ scaling. For $N_\tau = 10$ we can interpolate in N_τ.

In this talk we use the interpolated $Z_6^{tt}/Z_6^{ts}(\beta)$ and $Z_6^{ss}/Z_6^{ts}(\beta)$ ratios.

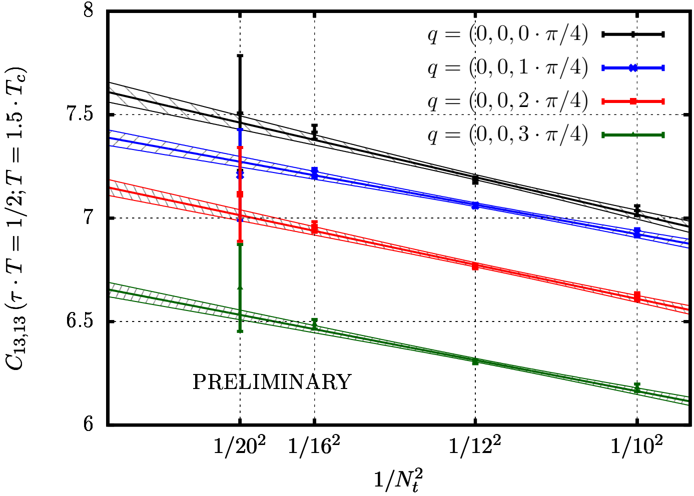
Renormalized shear correlators



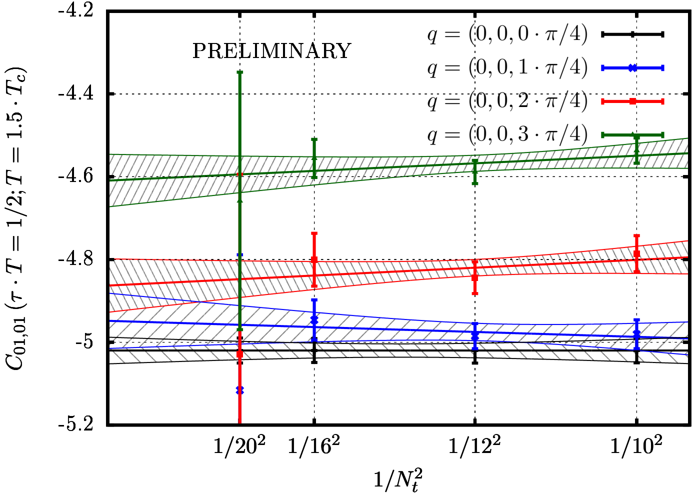
Renormalized shear correlators



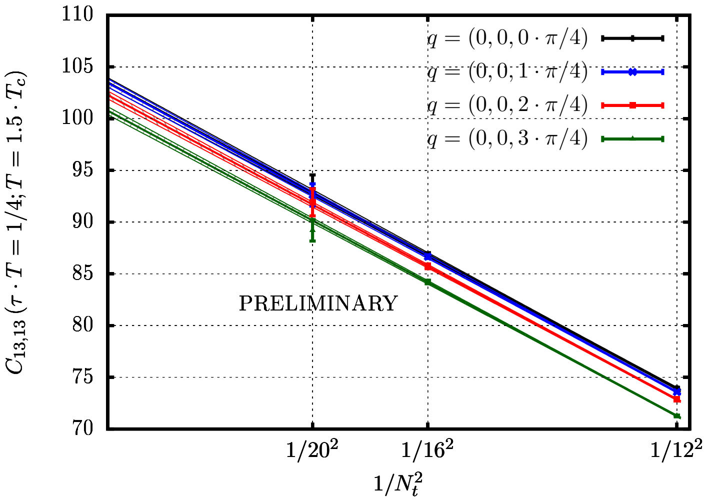
Continuum limit of: $C_{13,13}(\tau T = 1/2)$



Continuum limit of: $C_{01,01}(\tau T = 1/2)$



Continuum estimate of $C_{13,13}(\tau T = 1/4)$



The ansatz

Low and high frequency part

- We will use the prediction of hydrodynamics at low ω:

$$-\frac{\rho_{01,01}^{(hydro)}}{\omega} = \frac{\eta}{\pi}\frac{q^2}{\omega^2 + (\eta q^2/(sT))^2}$$

- And leading order perturbation theory at high frequency:

$$\begin{aligned}
-\rho_{01,01}^{(pert)} =& \frac{d_A}{8(4\pi)^2} q^2(\omega^2 - q^2)\mathcal{I}([1-z^4],\omega,q,T) \\
\mathcal{I}([P[z]],\omega,q,T) =& \theta(\omega - q)\int_0^1 dz \frac{P(z)\sinh(\omega/2T)}{\cosh(\omega/2T) - \cosh(qz/2T)} + \\
& + \theta(-\omega + q)\int_1^\infty \frac{-P(z)\sinh(\omega/2T)}{\cosh(\omega/2T) - \cosh(qz/2T)}
\end{aligned}$$

where we only take the part $\omega > q$.

The ansatz

Our ansatz is:

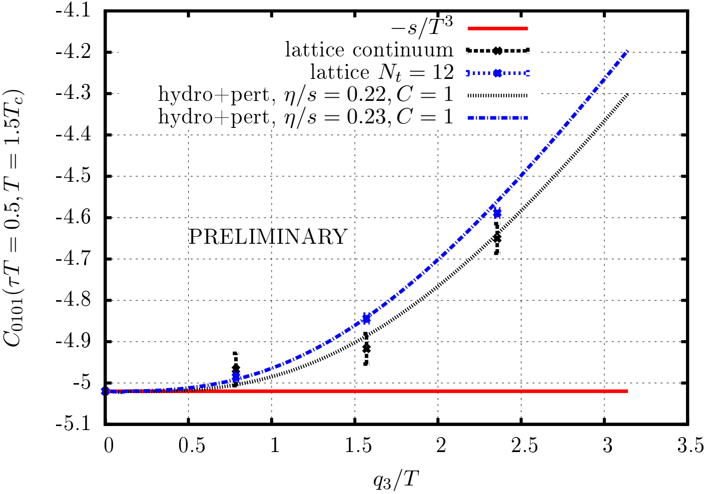
$$\rho_{01,01} = \rho_{0101}^{hydro} + C\rho_{0101}^{pert,\omega>q}$$

This has two parameters: C and η/s. With the available data we limit ourselves to these. The ansatz leaves out some of the physics. E.g.:

- The momentum dependence of the UV part might change if higher order corrections are included.
- The transport peak in $\rho_{13,13}$ should get narrower as we go to higher and higher temperatures. In perturbative estimates this leads to a rapid rise in the viscosity.

Using this ansatz, one can see that at least $C_{01,01}(\tau T = 0.5, \mathbf{q})$ is strongly dependent on the value of η/s.

η/s estimates in the continuum and on finite lattices ($C = 1$ for illustration)



Estimates of η/s from fits

From $N_t = 16$

- Data: C_{0101} as a function of τ and $\mathbf{q}$. Choice of channel is motivated by the smaller cut-off errors
- Preliminary results:

T	η/s	C
$1.5T_c$	$0.19(1)(2)(?)$	$0.62(3)(0)(?)$
$2.0T_c$	$0.16(2)(3)(?)$	$0.72(6)(0)(?)$

- The first error is statistical only. The second error is systematic error coming from the choice of τ_{min} and q_{max}. The $(?)$ is stand-in for unkown sytematic errors coming from the choice of the ansatz.

From the continuum

- Data: The $q_3/(\pi/4) = 0, 1, 2$ dependence of $C_{0101}(\tau T = 0.5)$ and $C_{1313}(\tau T = 0.5)$
- Preliminary results:

T	η/s	C
$1.5T_c$	$0.13(1)(?)$	$0.67(2)(?)$
$2.0T_c$	$0.11(2)(?)$	$0.72(3)(?)$

Summary and outlook

Summary

- We studied the continuum behavior of the energy-momentum tensor correlators in pure gauge theory, we found cut-off errors of approx. 3% for $N_t = 16$ for $C_{13,13}(\tau T = 0.5)$
- For some quanitites continuum extrapolation was possible
- We gave a model dependent estimate of η/s
- It is with previous estimates: Meyer '07, Meyer '09, Haas '13, Mages '14

Outlook, to do list

- more statistics for $N_t = 16, 20$
- higher temperatures
- tree level improvement for the improved action
- cross-checks