Towards Partial Compositeness on the Lattice: Baryons with Fermions in Multiple Representations

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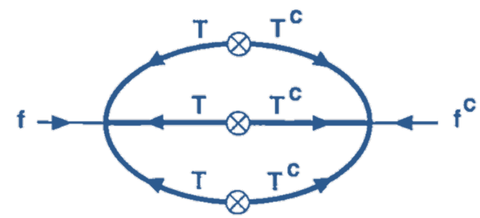
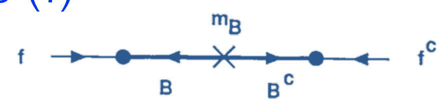
1. Introduction
   - Lightning review of partial compositeness
   - Our lattice model
   - Technical specifications
2. Lattice research program
   - Baryons in SU(3) and SU(4)
   - Non-relativistic quark models
   - Lattice results
3. Summary and Outlook

How does mass generation occur in strongly coupled BSM models?

- Classic "extended technicolor"
  - Chiral condensate breaks $SU(2)_L$
  - Higgs emerges from dynamics: dilaton (?)
- Composite Higgs -- Limited lattice investigation to date (!)
  - Chiral condensate preserves $SU(2)_L$
  - Higgs from SSB: exact Goldstone boson
  - SM loops generate potential for Higgs
- Fermion masses from 4-fermion interactions in both cases:
  - Partial compositeness means linear couplings to baryon operators





Trouble with FCNC constraints

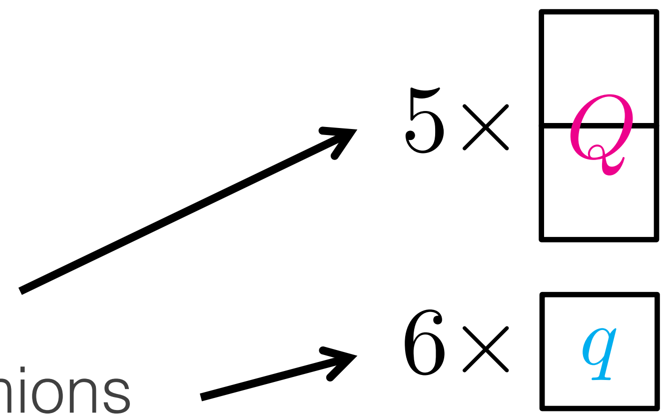
$$\bar{q}q\bar{\psi}\psi \sim \bar{q}q\mathcal{O}_{\rm ETC}$$

$$\bar{q}\psi\psi\psi \sim \bar{q}\mathcal{O}_{\rm PC}$$

- Better FCNC bounds
- Mass mixing
- Top quark partner(s)

Ferretti's Model (1404.7137)

A specific continuum UV theory for partial compositeness



- ✧ SU(4) gauge theory
- ✧ Fermions:
  - 5 sextet Majorana fermions
  - 6 fundamental Majorana fermions
  - Equivalent Dirac DOF: 2.5 sextet, 3 fundamental
- ✧ Symmetry breaking: **SU(5)/SO(5)** in the IR
  - Sextet SU(4) is a real representation
  - Symmetry breaking pattern is **different from QCD**

➢ Tough theory for lattice simulation

Our Lattice Deformation

(The model we actually simulate)

- Still SU(4) Gauge theory
- Modified matter content
  - $2.5 \mapsto 2$ sextet Dirac SU(4) fermions
  - $3 \mapsto 2$ fundamental Dirac SU(4) fermions
- Symmetry breaking: SU(4)/SO(4) in the IR
- Disclaimer 1: The deformation to SU(4)/SO(4) is *not* directly relevant for phenomenology.
- Disclaimer 2: Results today come from exploratory runs with partial quenching. Fully dynamical simulations are underway.

Technical Specifications

- "Multirep Milc" with "NDS action"
  - (DeGrand, Shamir, Svetitsky: 1407.4201)
- Wilson-Clover fermions
- SU(4) theory space parameterized by (β, κ_4 ,κ_6)
- Today
  - Exploratory study: partially quenched
  - Ensemble from DeGrand, Liu:1606.01277
  - $V=16^3 \times 32$
  - 2 x dynamical fundamental fermions

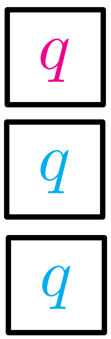
    ($\beta = 10.2$, $\kappa_4 = 0.1265$, $\kappa_{4;critical} = 0.1284$)

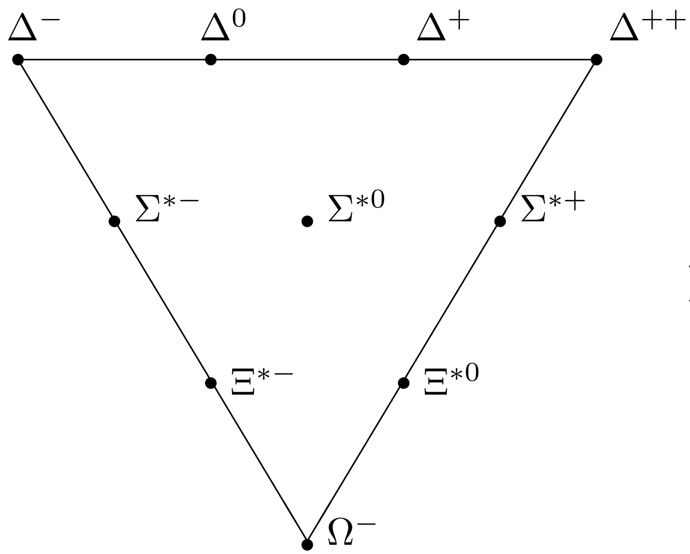
    $m_{PS}/m_V = 0.385(1)/0.560(3) = 0.688$
  - Quenched sextet propagators

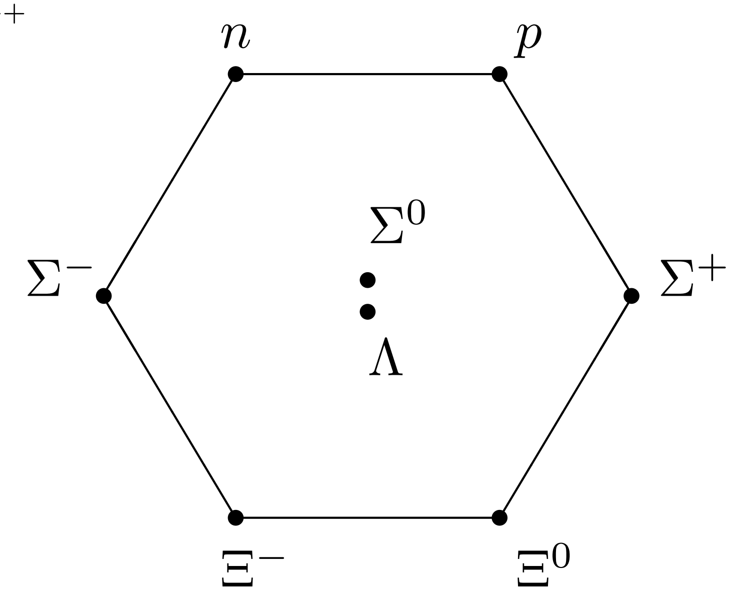
Warm-up for baryons in SU(4):

Hyperons in SU(3)

Baryons with (S=-1): uus, uds, dds







$$\Sigma^*(1390) : I(J^P) = 1(3/2^+)$$

$$\Sigma(1190) : I(J^P) = 1(1/2^+)$$

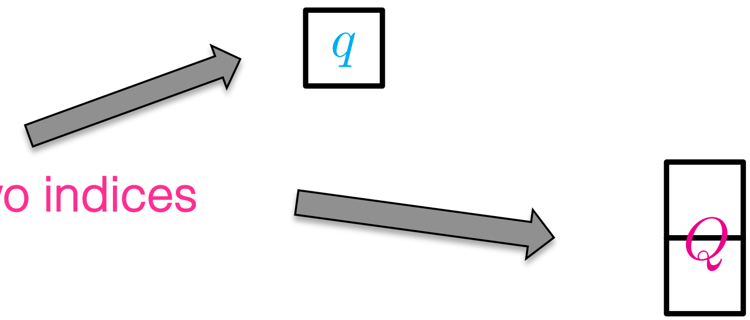
$$\Lambda(1120) : I(J^P) = 0(1/2^+)$$

Λ (isosinglet) =
lightest QCD hyperon

Baryons in SU(4)

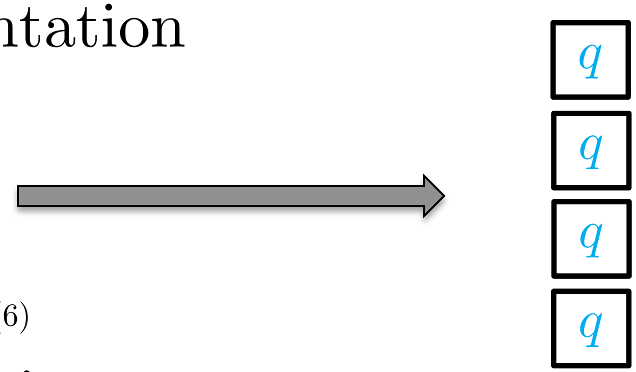
Building blocks

- Fundamental SU(4) fermion: q_a
- Sextet SU(4) fermion: Q_{ab} with two indices



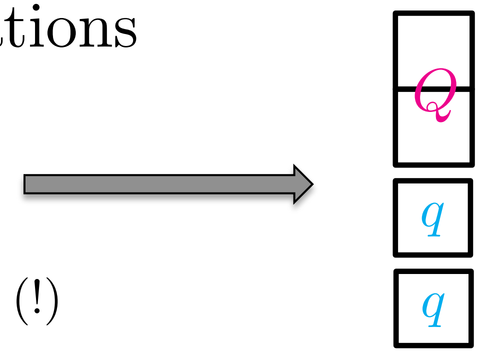
Quarks in a single representation

- Cousins of QCD nucleons
- Typical baryons: $(\mathbf{qqqq})_{\mathrm{SU}(4)}$
- 4 fermions: bosons
- Also appearing: $(\mathbf{QQQQQQ})_{\mathrm{SO}(6)}$



Quarks in both representations

- Cousins of QCD hyperons
- Chimera baryons $(\mathbf{Qqq})_{\mathrm{SU}(4)}$
- 3 fermions: fermions
- My code constructs these states (!)



Baryon Masses in SU(4)

Goal: qualitative understanding of baryon spectrum

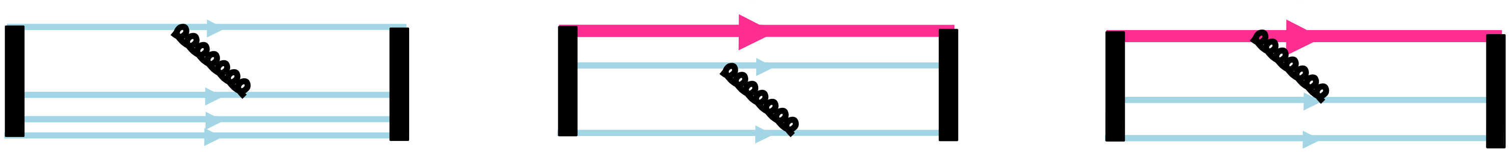
The tool: A non-relativistic quark model

Gell-Mann 1969

- "Constituent" quark masses with "color hyperfine" interactions
- A NR quark model also makes quantitative predictions for the entire spectrum of SU(4) baryons

$$m_{qqqq} = 4m_q + \frac{C}{m_q^2}\sum_{i<j}\vec{S}_i\cdot\vec{S}_j = 4m_q + \frac{C}{2m_q^2}\left(\vec{S}^2_{\rm tot} - 3\right)$$

$$m_{Qqq} = m_Q + 2m_q + \frac{C}{m_q^2}\left(\vec{S}_1\cdot\vec{S}_2 + 2\frac{m_q}{m_Q}\vec{S}_Q\cdot(\vec{S}_1+\vec{S}_2)\right)$$



Qqq Lattice Interpolating Fields

- Color Structure
  - Baryons are SU(4) color singlets
  - Code simulates six degrees of freedom for sextets
  - Must map indices SO(6) → SU(4) for correlation functions
- Spin Structure
  - Intuition from quark model as guide
  - Projection with $P_{\pm} = \frac{1}{2}(1 \pm \gamma_4)$ onto two-component NR basis
  - Clebsches $C^{\alpha\beta\gamma\delta}$ enact spin contraction

$$\begin{cases} 1 \mapsto 12 \\ 2 \mapsto 13 \\ \vdots \\ 6 \mapsto 34 \end{cases}$$

Color singlets

Spin contraction

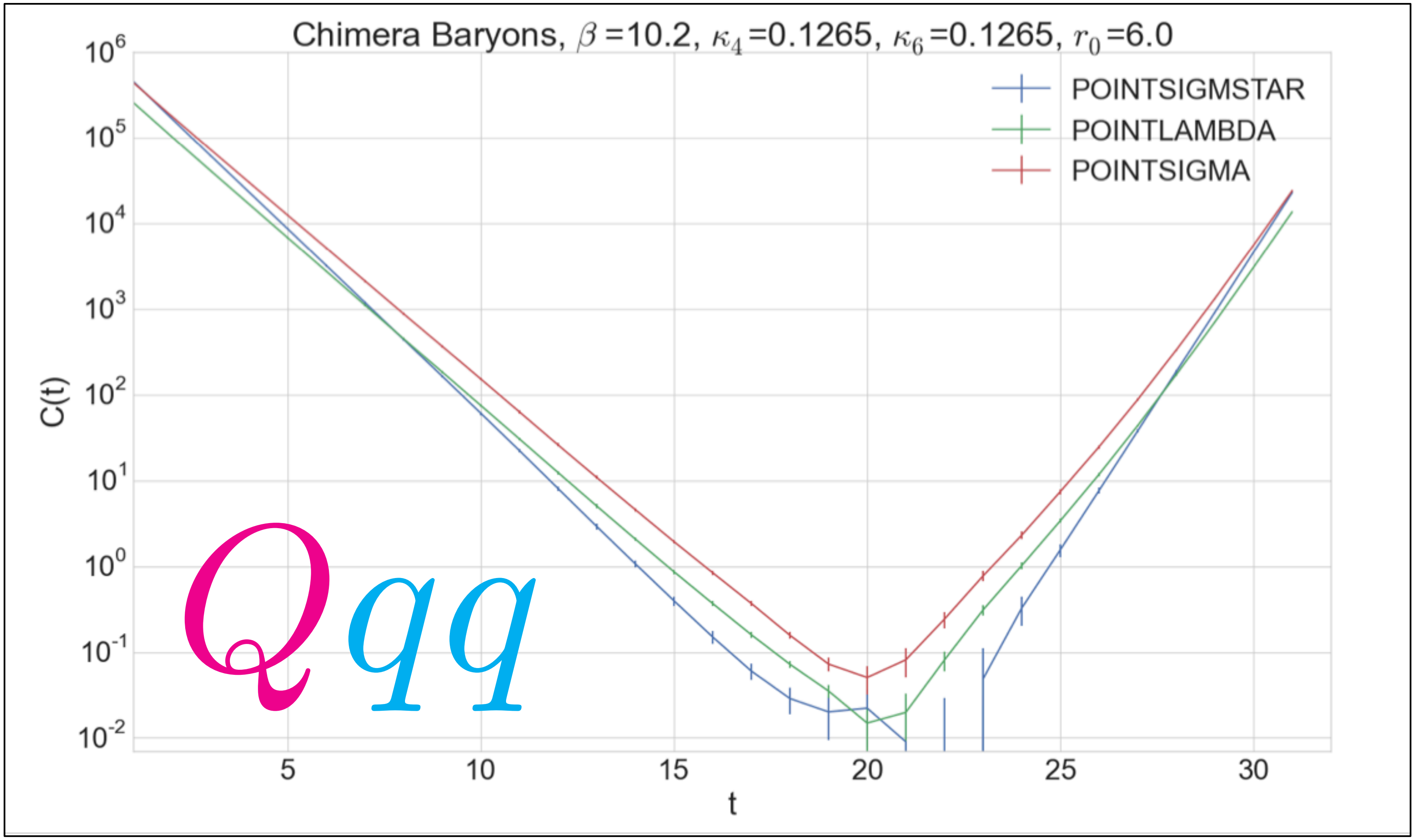
Sextet quark: two color indices

$$\langle \bar{\mathcal{O}}_B(m)\mathcal{O}_B(n)\rangle = \epsilon_{abcd}\epsilon_{efgh} C^{\alpha\gamma\delta\lambda} C^{\epsilon\phi\eta\zeta} D_Q^{-1}(m|n)^{ab,ef}_{\alpha,\epsilon}$$
$$\times \left[D_q^{-1}(m|n)^{c,g}_{\gamma,\phi} D_q^{-1}(m|n)^{d,h}_{\delta,h} - D_q^{-1}(m|n)^{c,h}_{\gamma,\eta} D_q^{-1}(m|n)^{d,g}_{\delta,\phi} \right]$$

Fundamental quarks: single color index

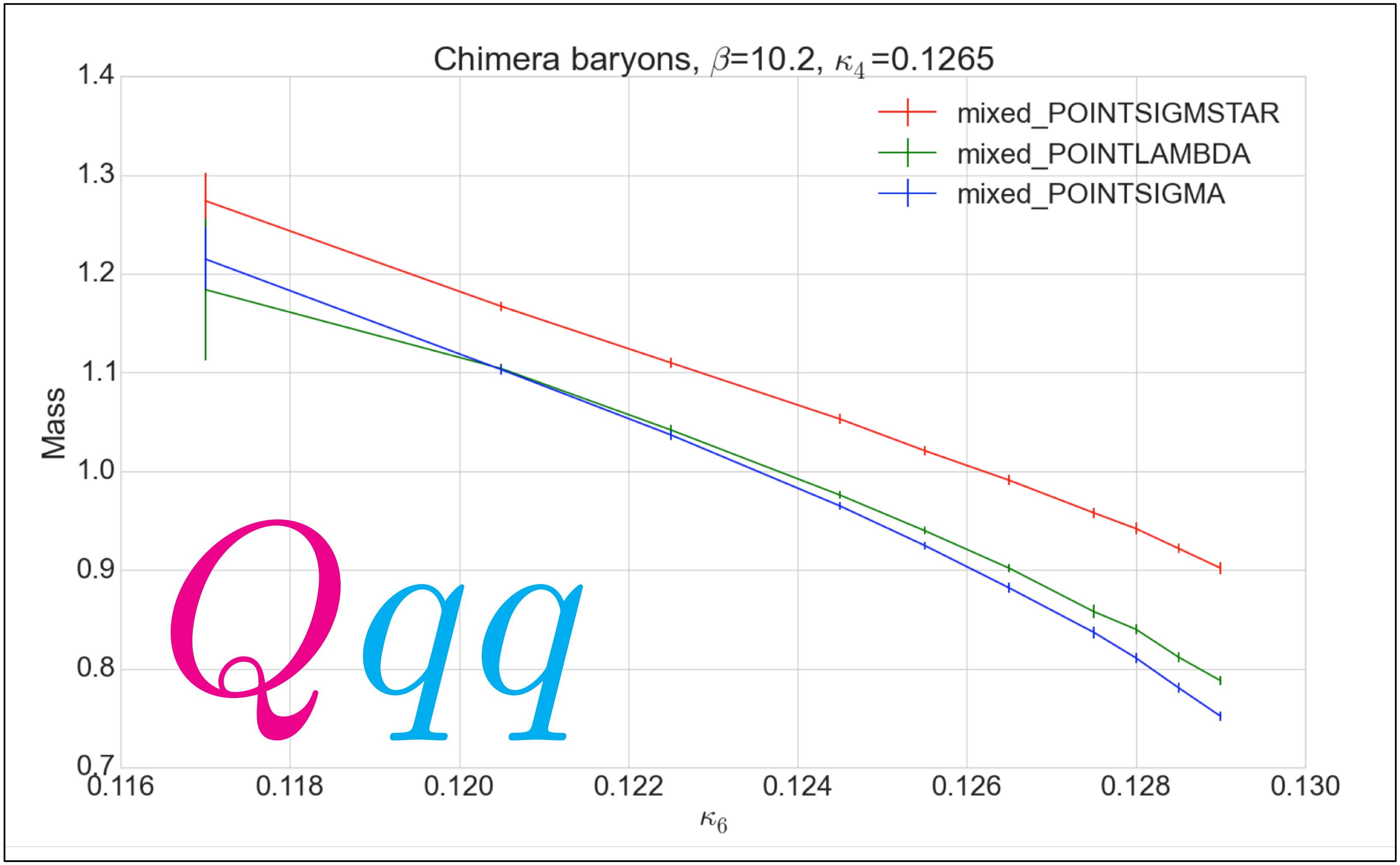
Minus sign from Wick's theorem

“Chimera” 2-point correlators



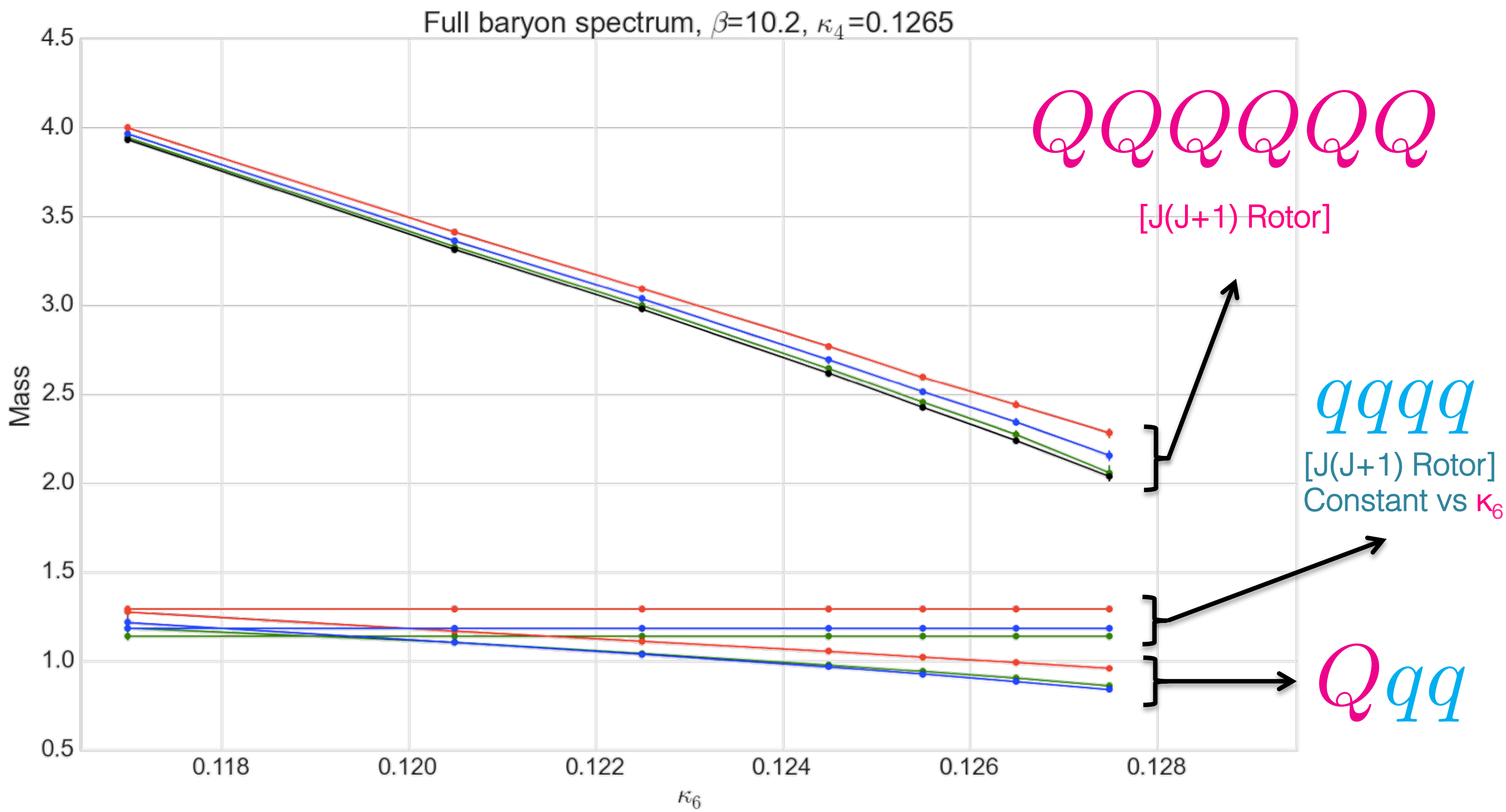
- Strong signals with 50 - 60 configurations
- Asymmetric correlators, as in QCD (cf. Leinweber 2005, nucl-th/0406032)

Chimera Spectrum vs κ_6 (fixed κ_4)



Isotriplet "Σ-like" state lighter than isosinglet "Λ-like" state at small sextet quark mass

SU(4) baryon spectrum vs κ$_6$



- Chimera Qqq baryons can be light particles in the heavy spectrum
- Will these features persist with both representations in the sea?

Success with the Quark Model

(pending confirmation with both representations in the sea)

- This SU(4) system is not QCD
- But the quark model successfully predicts all the qualitative features of the low-lying hadron spectrum
  - Rotor splittings: $\delta m \sim J(J+1)$
  - Relative sizes of QQQQ, qqqq, Qqq
  - Presence of Σ-Λ inversion
- The chimera baryons are comparatively light → good for phenomenology

Summary and Outlook

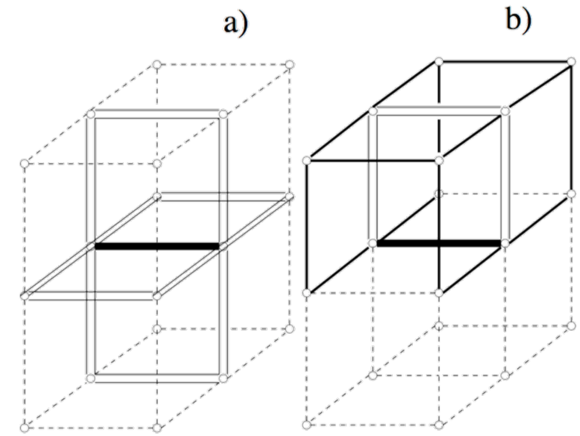
- We saw preliminary results for SU(4) gauge theory with fermions in mixed representations
  - A quark model plays a key role in our understanding the spectrum of this theory.
- Interesting related questions remain (in progress)
  - Pheno implications for the Σ-Λ inversion?
  - Calculation of the non-perturbative mixing of elementary fermions with composite operators
  - Calculation of anomalous dimensions for the four-fermion interactions
  - Extending Large-N results to mixed representations
  - …
- Other interesting questions we're actively pursuing
  - What does the thermodynamic phase diagram look like?
  - Do dynamically separated phases exist?
  - Do hierarchies of scales exist?

Thank you for your attention.

Back-up slides

The NDS Action

(Slide credit: E. Neil)



- HYP smearing: staple sum over "fat links" added to original. nHYP normalizes the smeared link W. $V = \Omega(\Omega^\dagger\Omega)^{-1/2}$
- $Q^{-1/2}$ appears in the fermion force, and small eigenvalues can cause spikes. "nHYP dislocation suppressing" action cancels these with additional marginal gauge terms $S_{\rm NDS}$:

$$Q^{-1/2} = (\Omega^\dagger\Omega)^{-1/2}$$

$$S_{\rm NDS} = \frac{1}{2N_c}\sum_x \mathrm{Tr}\left(\gamma_1 \sum_\mu \tilde{Q}^{-1}_{x,\mu} + \gamma_2 \sum_{\mu\neq\nu} \tilde{Q}^{-1}_{x,\mu;\nu} + \gamma_3 \sum_{\rho\neq\xi} Q^{-1}_{x,\rho;\xi}\right)$$

- Bare gauge coupling depends on β and γ. We fix the ratio and adjust β to move lattice spacing

More technical details 1/2

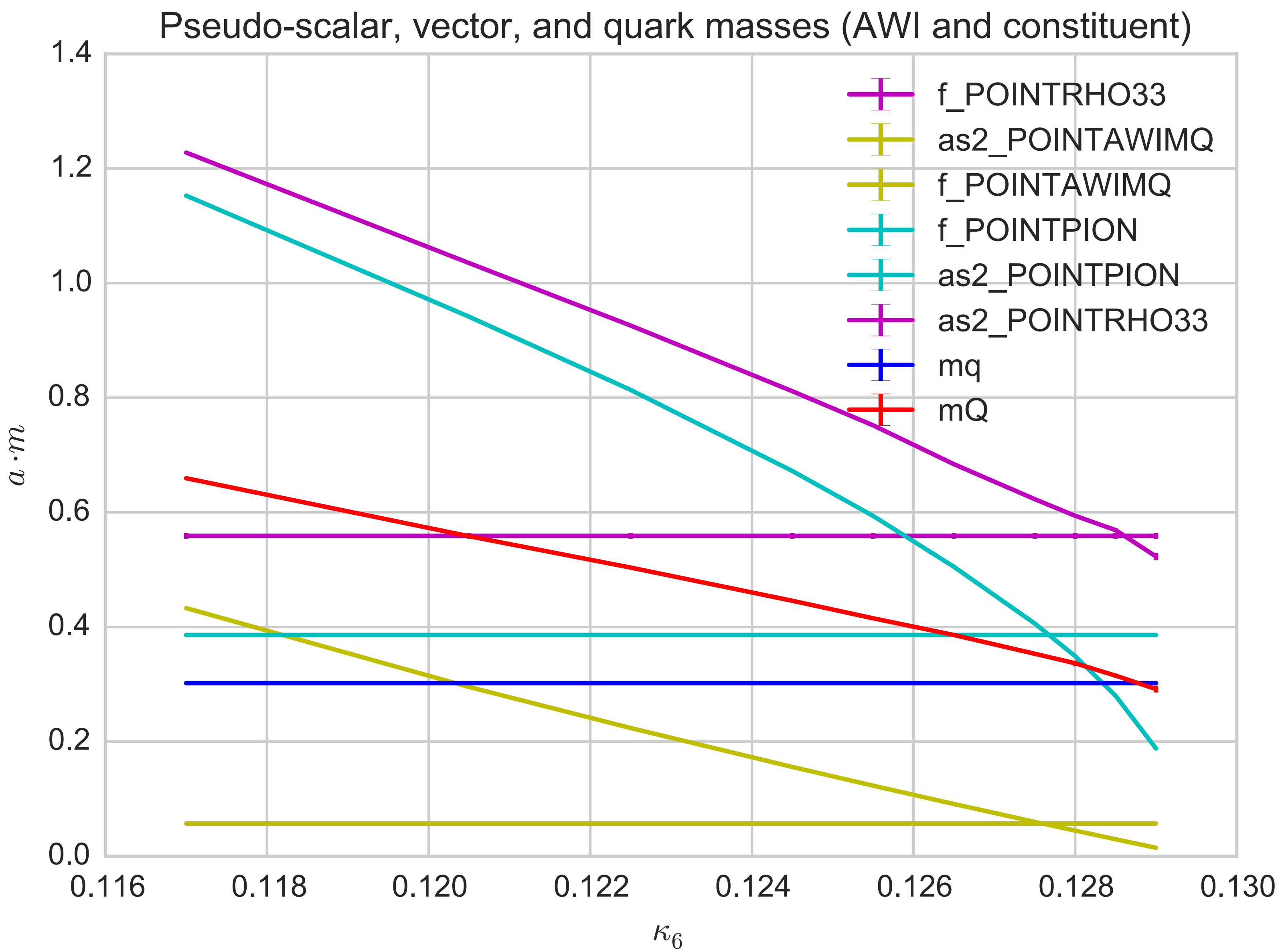
The "Multirep MILC" code…

- Runs SU(N_c) gauge theory with simultaneous dynamical fermions in multiple representations
- Is branched from the MILCv7 code, focusing on Wilson fermions
- Builds with dynamical code generation using Perl so that N_c and representation(s) are fixed during code generation, allowing the C compiler to produce optimized matrix operations
- Includes all the modern bells and whistles: Clover term, nHYP smearing, Hasenbusch preconditioning, multi-level integrators, dislocation-suppressing NDS action (DeGrand, Shamir, Svetitsky: 1407.4201)

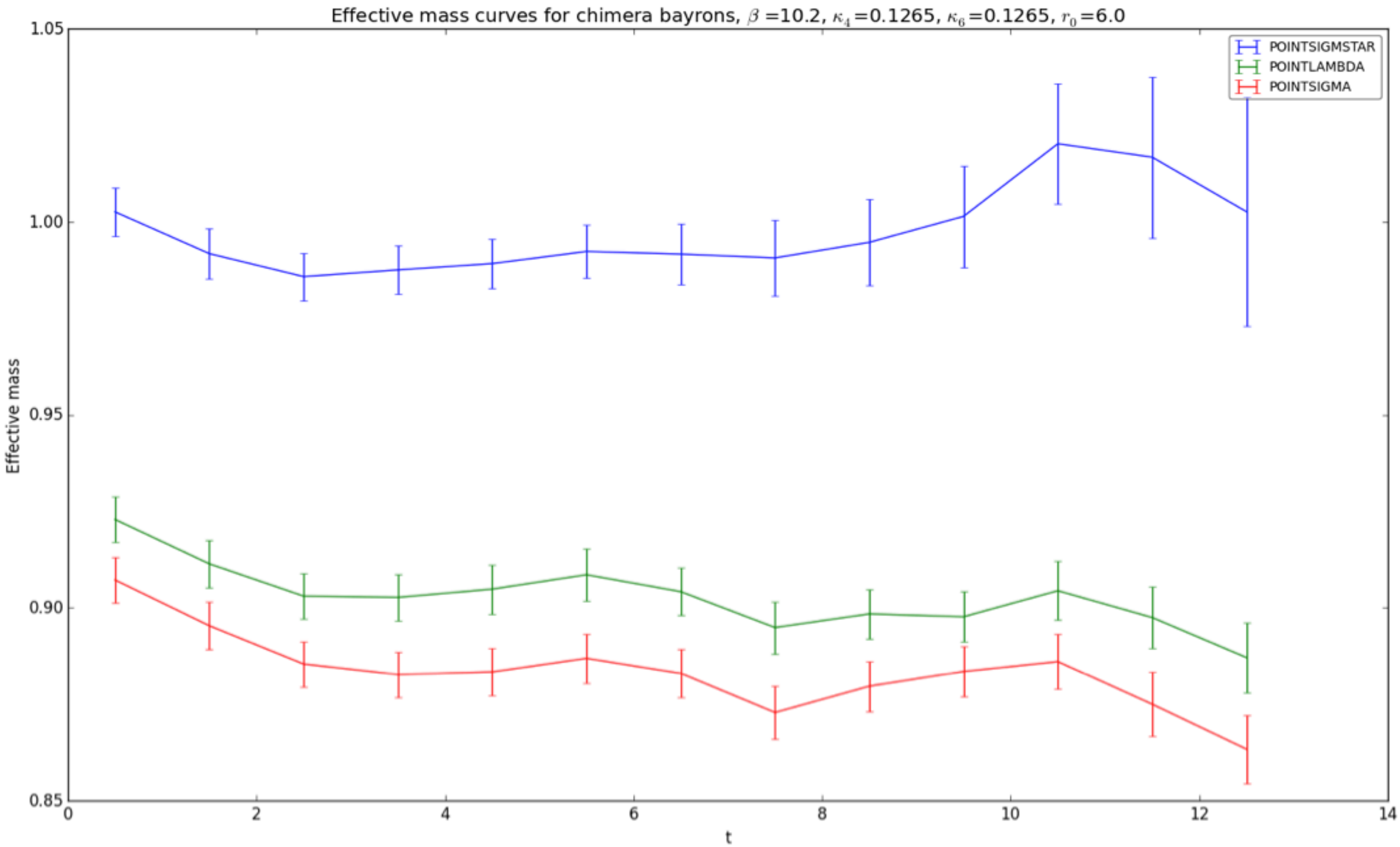
More technical details 2/2

Running parameters and results:

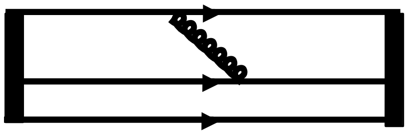
- 2 x Dynamical fundamental fermions
  - (β = 10.2, κ_4 = 0.1265, $\kappa_{4;critical}$ = 0.1284)
  - m_{PS}/m_V = 0.385(1) / 0.560(3) = 0.688
- Quenched sextet propagators
  - Range of kappa values: κ_6 = 0.1170 up to 0.1290, $\kappa_{6;critical}$ = 0.1295
  - m_{PS}/m_V ranging from 1.15 / 1.23 = 0.93 down to 0.19/0.52 = 0.36



Pseudo-scalar, vector, and quark masses (AWI and constituent)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$a \cdot m$
0.116
0.118
0.120
0.122
0.124
0.126
0.128
0.130
κ_6
f_POINTRHO33
as2_POINTAWIMQ
f_POINTAWIMQ
f_POINTPION
as2_POINTPION
as2_POINTRHO33
mq
mQ


Effective mass curves for chimera bayrons, β =10.2, κ_4=0.1265, κ_6=0.1265, r_0=6.0
POINTSIGMSTAR
POINTLAMBDA
POINTSIGMA
Effective mass
t
1.05
1.00
0.95
0.90
0.85
0
2
4
6
8
10
12
14

Baryons and the quark model



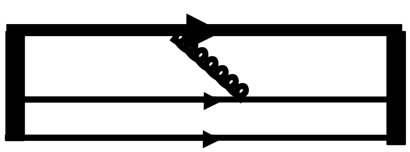
$$m_B = m_1 + m_2 + m_3 + \sum_{i<j} \frac{a\, c_{ij}}{m_i m_j} \vec{S}_i \cdot \vec{S}_j$$

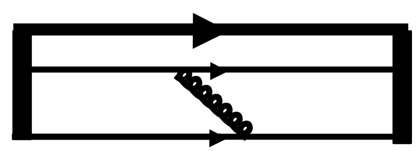
$$m_B = 3m + a_0 + a_1 J(J+1)$$

$$m_{\text{QCD hyperon}} = m_s + 2m_u + \frac{a}{m_u^2}\left(\vec{S}_1 \cdot \vec{S}_2 + \frac{m_u}{m_s}\vec{S}_Q \cdot (\vec{S}_1 + \vec{S}_2)\right)$$

$$m_{Qqq} = m_Q + 2m_q + \frac{a}{m_q^2}\left(\vec{S}_1 \cdot \vec{S}_2 + 2\frac{m_q}{m_Q}\vec{S}_Q \cdot (\vec{S}_1 + \vec{S}_2)\right)$$

Two distinct gluon exchanges: sextet quark feels twice as much color force.
Formally, this difference is a statement about relative sizes of Casimirs.





References

(A short and scandalously incomplete list)

- Composite Higgs
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  - Contino et al., On the effect of resonances in composite Higgs phenomenology, arXiv: 1109.1570
  - Contino and Salvarezza, One-loop effects from spin-1 resonances in Composite Higgs models, arXiv:1504.02750
- SU(4) models
  - Ferretti and Karateev, Fermionic UV completions of Composite Higgs Models, arXiv: 1312.5330
  - Ferretti, UV Completions of Partial Compositeness: The Case for a SU(4) Gauge Group, arXiv:1404.7137
  - Ferretti, Gauge theories of Partial Compositeness: Scenarios for Run-II of the LHChttp, arXiv:1604.06467
- Alternative perspectives
  - Luty and Okui, Confromal Technicolor, arXiv: hep-ph/0409274
  - Vecchi, A dangerous irrelevant UV-completion of the composite Higgs, arXiv: 1506.00623
  - Ma and Cacciapaglia, Fundamental Composite 2HDM: SU(N) with 4 flavours, arXiv: 1508.07014

Baryons and Large-N

- Dashen, Jenkins, and Manohar derived formulae for strange baryons in the large-N limit
  – Depends only on the spin-flavor structure of the baryons, in the QCD case of SU(2) x U(1)
- Gives a more general / less restrictive prediction for the spectrum than the quark model.

$$M = a_0 N_c + a_1 N_s + a_{21} \frac{J^2}{N_c} + a_{22} \frac{I^2}{N_c} + a_{23} \frac{N_s^2}{N_c} + \mathcal{O}\left(\frac{1}{N_c^3}\right)$$

- Do these results remain valid with fermions in mixed representations?