\[ |V_{cb}| \text{ from } \bar{B}^0 \rightarrow D^{*+} \ell^− \bar{\nu} \text{ zero-recoil form factor using } 2+1+1 \text{ flavour HISQ and NRQCD} \]
Outline of Talk

- Phenomenological Motivation
- Overview of lattice methodology
- Results
- Future work
The CKM Matrix

After EW symmetry breaking the standard model in the mass basis contains the flavour changing current

\[ \mathcal{J}^{\mu,+} = \bar{u}_{L}^{i} \gamma^{\mu} V_{CKM}^{ij} d_{L}^{j} \]

\[ V_{cb} = (42.46 \pm 0.88) \times 10^{-3} \text{(inclusive)} \]
\[ (39.45 \pm 1.42_{\text{exp}} \pm 0.88_{\text{th}}) \times 10^{-3} (B \rightarrow D) \]
\[ (38.94 \pm 0.49_{\text{exp}} \pm 0.58_{\text{th}}) \times 10^{-3} (B \rightarrow D^*) \]

Exclusive results use lattice QCD. Here we focus on the decay to D*.

\[ \bar{B}^{0} \rightarrow D^{\ast+} l^{-} \bar{\nu} \]
The determination using exclusive decays uses fits to experimental data for the differential decay rate

\[
\frac{d\Gamma}{d\omega} (\bar{B} \rightarrow D^* \ell \bar{\nu}_\ell) = \frac{G_F^2 m_B^5}{48\pi^3} |V_{cb}|^2 (\omega^2 - 1)^{1/2} P(\omega) |\eta_{EW} F(\omega)|^2
\]

One output of such fits is the factor

\[
\eta_{EW} F(1)|V_{cb}|
\]

The electroweak factor, accounting for box diagrams, can be calculated perturbatively with good precision. The remaining parameter is the zero recoil form factor.
The axial and vector matrix elements we are interested in can be parametrised in terms of the four form factors

\[
\langle V(p', \epsilon) | \bar{q} \gamma^\mu \gamma^5 Q | P(p) \rangle = \\
A_0(q^2)2M_V \epsilon^* \cdot q / q^2 q^\mu \\
+A_1(q^2)(M_P + M_V) \left[ \epsilon^{*\mu} - \epsilon^* \cdot q / q^2 q^\mu \right] \\
-A_2(q^2) \frac{\epsilon^* \cdot q}{M_B + M_V} \left[ p^{\mu} + p'^{\mu} - \frac{M_B^2 - M_V^2}{q^2} q^\mu \right]
\]

\[
\langle V(p', \epsilon) | \bar{q} \gamma^\mu Q | P(p) \rangle = \\
V_0(q^2) \varepsilon^{\mu\eta\rho\kappa} \epsilon_\eta p'_\rho p_\kappa / (M_P M_V)
\]

In the zero recoil limit only the $A_1$ term remains. It is only non-zero for spatial currents.
Lattice Actions

Bottom quark - NRQCD

• Non-relativistic effective theory
Lattice Actions

- Methodology

Bottom quark - NRQCD

- Non-relativistic effective theory
- Improved through $O(\alpha_s \Lambda / M_B)$
Lattice Actions

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- Simple time evolution equation – fast and simple to compute propagators
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Charm, up and down quarks - HISQ

- 2+1+1 flavours in the sea
Lattice Actions

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- 2+1+1 flavours in the sea
- Physical, relativistic charm
- Light quarks going down to physical masses
Lattice Actions

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**Charm, up and down quarks - HISQ**

- 2+1+1 flavours in the sea
- Physical, relativistic charm
- Light quarks going down to physical masses
- Improved through order $a^2$ and $(am)^4$ at leading order
Use of NRQCD means we must use several currents to match to the continuum to given order in $\Lambda/m_b$:

$$\langle c(p) | J^i | b(q) \rangle_{\text{cont.}} = \langle c(p) | C_n J^i_n | b(q) \rangle_{\text{Latt.}} + \mathcal{O}(a^2, \ldots)$$

Here we use currents matched through $\mathcal{O}(\alpha_s, \Lambda_{QCD}/m_b, \alpha_s/(am_b), \alpha_s\Lambda_{QCD}/m_b)$ which requires the currents

$$J^i_0(x) = \bar{c}\gamma^i\gamma^5Q$$

$$J^i_1(x) = -\frac{1}{2am_b}\bar{c}\gamma^i\gamma^5\gamma \cdot \Delta Q$$

However, at zero recoil Luke's theorem states that the $1/m_b$ corrections vanish. Hence we only need the leading order current and its matching coefficient – though we compute higher order currents as well.
We compute the correlation functions

\[ C_{B2pt}(t)_{ij} = \langle \mathcal{O}(t)_{Bi} \mathcal{O}^\dagger(0)_{Bj} \rangle \]
\[ C_{D*2pt}^{\mu\nu}(t)_{ij} = \langle \mathcal{O}^{\mu}(t)_{D*ij} \mathcal{O}^{\dagger\nu}(0)_{D*j} \rangle \]
\[ C_{3pt}^{\mu\kappa}(T, t, 0)_{ij} = \langle \mathcal{O}^{\mu}(T)_{D*ij} \mathcal{J}^\kappa(t) \mathcal{O}^\dagger(0)_{Bj} \rangle \]

where each operator is projected onto zero spatial momentum, and i and j label different smearings.

\[ \mathcal{O}(x)_{Bi} = \sum_y \bar{u}(x) \Delta_i(x - y) \gamma^5 Q_b(y) \]
\[ \mathcal{O}^{\mu}(x)_{D*ij} = \sum_y \bar{u}(x) \Delta_i(x - y) \gamma^\mu c(y + \mu) \]
Fits

$32^3 \times 96$

$T=24$
- Methodology

Fits

Use Bayesian fitting, minimise modified $\chi^2$ including priors for parameters

$$\chi^2(p) = \sum_{t,t'} \Delta C(t, p) \sigma_{tt'}^2 \Delta C(t', p) + \sum_i (p^i - p^i_{\text{prior}})^2 / \sigma_{p^i_{\text{prior}}}^2$$

Allows for a very general fit function. Parameters which are not strongly determined by the data default to their prior value.
## Results

<table>
<thead>
<tr>
<th>$L_x \times L_t$</th>
<th>$a$ (fm)</th>
<th>$M_\pi$ (MeV)</th>
<th>$\chi^2$/dof</th>
<th>$\langle J_{\text{latt}}^0 \rangle$</th>
<th>$\mathcal{F}(1)$</th>
<th>$m_{D^*}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 $\times$ 48</td>
<td>0.1474</td>
<td>302.4</td>
<td>0.93</td>
<td>0.926(14)</td>
<td>0.833(12)</td>
<td>2059.5(4.7)</td>
</tr>
<tr>
<td>24 $\times$ 48</td>
<td>0.1463</td>
<td>215.5</td>
<td>1</td>
<td>0.960(24)</td>
<td>0.865(22)</td>
<td>2057.7(3.3)</td>
</tr>
<tr>
<td>32 $\times$ 48</td>
<td>0.1450</td>
<td>133.0</td>
<td>1</td>
<td>0.922(17)</td>
<td>0.828(16)</td>
<td>2077.1(4.2)</td>
</tr>
<tr>
<td>32 $\times$ 64</td>
<td>0.1195</td>
<td>216.5</td>
<td>1</td>
<td>0.912(11)</td>
<td>0.854(11)</td>
<td>2022.0(3.2)</td>
</tr>
<tr>
<td>48 $\times$ 64</td>
<td>0.1189</td>
<td>132.7</td>
<td>0.98</td>
<td>0.9356(94)</td>
<td>0.8730(89)</td>
<td>2036.1(3.3)</td>
</tr>
<tr>
<td>32 $\times$ 96</td>
<td>0.0884</td>
<td>306.1</td>
<td>0.94</td>
<td>0.869(11)</td>
<td>0.842(11)</td>
<td>2035.5(3.7)</td>
</tr>
<tr>
<td>64 $\times$ 96</td>
<td>0.08787</td>
<td>128.4</td>
<td>1</td>
<td>0.908(12)</td>
<td>0.882(11)</td>
<td>2004.5(3.6)</td>
</tr>
</tbody>
</table>
Once we have determined the currents on each lattice, must extrapolate to the continuum. Simultaneously fit pion mass dependence using chiral P.T. result.

\[
\mathcal{F}(1) = 1 + \delta_a^B B + \delta_a^C C \frac{\Delta^2 m_c}{64 \pi^2 f^2} \times \\
\left( \ln \left( \frac{M_\pi}{10m^\text{phys}_s} \right)^2 + F \left[ \frac{-\Delta m_c}{M_\pi} \right] \right) + C_1 \alpha_s^2 J^0_{\text{latt}} + C_2 \alpha_s J^1_{\text{latt}} + C_3 \frac{\Lambda^2}{M_b^2}
\]
V_{cb} and h_{A_1}(1)

Preliminary!

\begin{center}
\begin{tabular}{l|c|c}
\hline
\text{Inclusive avg.} & \text{This work} & \text{HPQCD } B \to D \\
\hline
37 & 38 & 39 \\
\hline
40 & 41 & 42 \\
\hline
43 & & \\
\hline
\end{tabular}
\end{center}

\[ h_{A_1}(1) = 0.900(46)(11) \]

\[ V_{cb} = 39.5(20)(5)(5) \times 10^{-3} \]

\begin{center}
\begin{tabular}{l|c}
\hline
\text{Uncertainty} & \text{Partial error (\%)} \\
\hline
\text{Isospin effects} & 0.25 \\
\text{Priors} & 1.37 \\
\text{Partial error} & 1.47 \\
\text{Total systematic} & 4.71 \\
\text{Statistical} & 5.13 \\
\text{Experiment} & 1.27 \\
\text{total} & 5.42 \\
\hline
\end{tabular}
\end{center}
Future work

- Recent confirmation of $R(D^*)$ deviation of $\sim 2\sigma$ by LHCb run 1 from standard model prediction
Future work

- Recent confirmation of $R(D^\ast)$ deviation of $\sim 2\sigma$ by LHCb run 1 from standard model prediction

- Work currently being done to constrain leading $\alpha_s^2$ error using heavy-HISQ extrapolation.
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- Increase precision of $V_{cb}$ result
Future work

- Recent confirmation of $R(D^*)$ deviation of $\sim 2\sigma$ by LHCb run 1 from standard model prediction

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- Increase precision of $V_{cb}$ result

- $B_s \rightarrow D_s^*$
Thank you for listening
Backup slides
Including oscillations in fits, the full form of the fit functions are

\[ C_{B_{2 pt}}(t)_{i,j} = \sum_{n,a=0,1} (-1)^a t \frac{Z_{B_{ai}}^{n1/2} Z_{B_{aj}}^{n1/2}}{2 M_{B_a}^n} e^{-M_{B_a}^n t} \]

\[ C_{D_{2 pt}}^{\mu\nu}(t)_{i,j} = \sum_{n,s,a=0,1} (-1)^a t \frac{Z_{D_{ai}}^{n1/2} Z_{D_{aj}}^{n1/2}}{2 M_{D_a}^n} \epsilon_s^\mu \epsilon_{s}^{\nu*} e^{-M_{D_a}^n t} \]

\[ C_{3 pt}^{\mu\kappa}(T, t, 0)_{i,j} = \sum_{a b=0,1} \sum_{n m s} \frac{Z_{B_{bj}}^{1/2} Z_{D_{ai}}^{1/2}}{2 M_{B_b}^n 2 M_{D_a}^m} \epsilon_s^\mu \epsilon_{s}^{\kappa*} \]

\[ \times (M_{B_b}^n + M_{D_a}^m) A_{1 ba}^{nm}(q^2) e^{-M_{D_a}^n (T-t) - M_{B_b}^m t} (-1)^a (T-t) + bt \]