$|Vcb|$ from $\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}$ zero-recoil form factor using 2+1+1 flavour HISQ and NRQCD

Outline of Talk

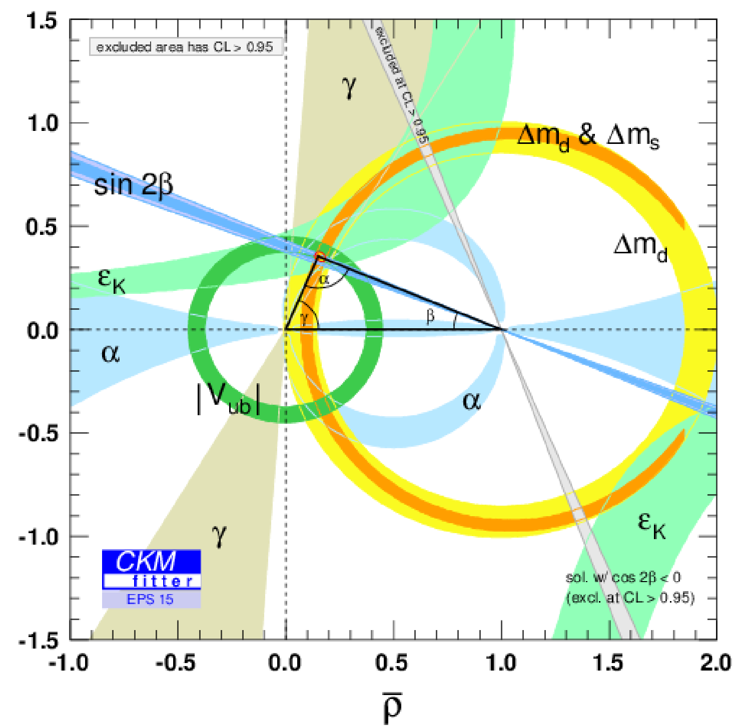
- Phenomenological Motivation
- Overview of lattice methodology
- Results
- Future work

The CKM Matrix

After EW symmetry breaking the standard model in the mass basis contains the flavour changing current

$$\mathcal{J}^{\mu,+} = \overline{u}^i_{\mathrm{L}} \gamma^\mu V^{ij}_{\mathrm{CKM}} d^j_{\mathrm{L}}$$

$$\begin{aligned} V_{cb} =& (42.46 \pm 0.88) \times 10^{-3} (\text{inclusive}) \\ & (39.45 \pm 1.42_{\text{exp}} \pm 0.88_{\text{th}}) \times 10^{-3} (B \to D) \\ & (38.94 \pm 0.49_{\text{exp}} \pm 0.58_{\text{th}}) \times 10^{-3} (B \to D^*) \end{aligned}$$



Exclusive results use lattice QCD. Here we focus on the decay to D*.

$$\bar{B}^0 \to D^{*+} l^- \bar{\nu}$$

The CKM Matrix

The determination using exclusive decays uses fits to experimental data for the differential decay rate

$$\frac{d\Gamma}{d\omega}(\bar{B} \to D^* \ell \bar{\nu}_\ell) = \frac{G_F^2 m_B^5}{48\pi^3} |V_{cb}|^2 (\omega^2 - 1)^{1/2} P(\omega) |\eta_{EW} \mathcal{F}(\omega)|^2$$

One output of such fits is the factor

$$\eta_{EW} \mathcal{F}(1) |V_{cb}|$$

The electroweak factor, accounting for box diagrams, can be calculated perturbatively with good precision. The remaining parameter is the zero recoil form factor.

The CKM Matrix

The axial and vector matrix elements we are interested in can be parametrised in terms of the four form factors

$$\langle V(p',\epsilon)|\bar{q}\gamma^\mu\gamma^5 Q|P(p)\rangle =$$
$$A_0(q^2)2M_V\epsilon^*\cdot q/q^2q^\mu$$
$$+A_1(q^2)(M_P+M_V)\left[\epsilon^{*\mu}-\epsilon^*\cdot q/q^2q^\mu\right]$$
$$-A_2(q^2)\frac{\epsilon^*\cdot q}{M_B+M_V}\left[p^\mu+p'^\mu-\frac{M_B^2-M_V^2}{q^2}q^\mu\right]$$

$$\langle V(p',\epsilon)|\bar{q}\gamma^\mu Q|P(p)\rangle =$$
$$V_0(q^2)\varepsilon^{\mu\eta\rho\kappa}\epsilon_\eta p'_\rho p_\kappa/(M_PM_V)$$

In the zero recoil limit only the A_1 term remains. It is only non-zero for spatial currents.

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Bottom quark - NRQCD

- Non-relativistic effective theory

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- Improved through order a^2 and $(am)^4$ at leading order

Perturbative Matching

Use of NRQCD means we must use several currents to match to the continuum to given order in Λ/m_b:

$$\langle c(p)|\mathcal{J}^i|b(q)\rangle_{\text{cont}}, = \langle c(p)|C_n J_n^i|b(q)\rangle_{\text{Latt}}, + \mathcal{O}(a^2, ...)$$

Here we use currents matched through $\mathcal{O}(\alpha_s, \Lambda_{\text{QCD}}/m_b, \alpha_s/(am_b), \alpha_s\Lambda_{\text{QCD}}/m_b)$ which requires the currents

- Christopher Monahan, Junko Shigemitsu, Ron Horgan, 1211.6966

$$J_0^i(x) = \bar{c}\gamma^i\gamma^5 Q$$
$$J_1^i(x) = -\frac{1}{2am_b}\bar{c}\gamma^i\gamma^5\gamma\cdot\Delta Q$$

However, at zero recoil Luke's theorem states that the $1/m_b$ corrections vanish. Hence we only need the leading order current and its matching coefficient – though we compute higher order currents as well.

Fits

We compute the correlation functions

$$C_{B2pt}(t)_{ij} = \langle \mathcal{O}(t)_{Bi} \mathcal{O}^\dagger(0)_{Bj} \rangle$$

$$C^{\mu\nu}_{D^*2pt}(t)_{ij} = \langle \mathcal{O}^\mu(t)_{D^*i} \mathcal{O}^{\dagger\nu}(0)_{D^*j} \rangle$$

$$C^{\mu\kappa}_{3pt}(T,t,0)_{ij} = \langle \mathcal{O}^\mu(T)_{D^*i} \mathcal{J}^\kappa(t) \mathcal{O}^\dagger(0)_{Bj} \rangle$$

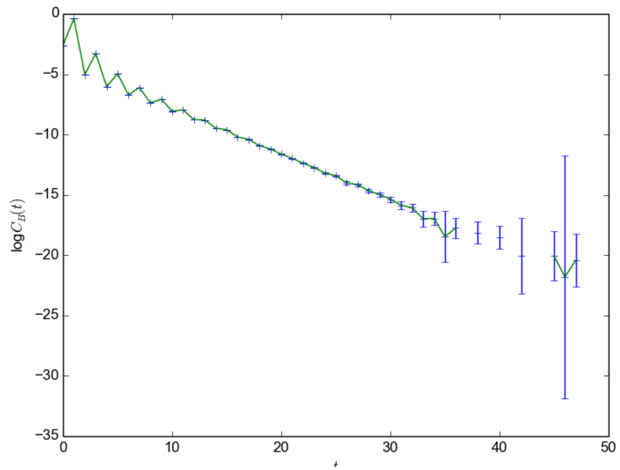
where each operator is projected onto zero spatial momentum, and i and j label different smearings.

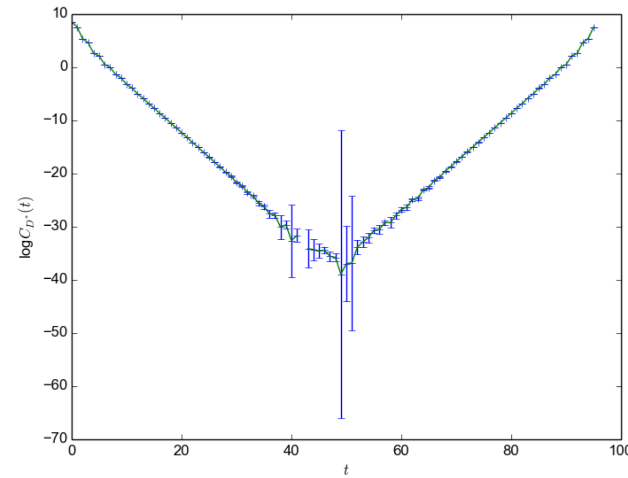
$$\mathcal{O}(x)_{Bi} = \sum_y \bar{u}(x)\Delta_i(x-y)\gamma^5 Q_b(y)$$

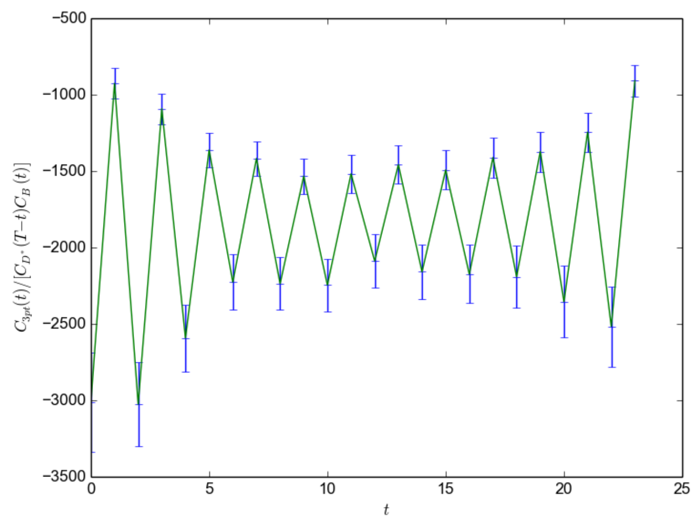
$$\mathcal{O}^\mu(x)_{D^*i} = \sum_y \bar{u}(x)\Delta_i(x-y)\gamma^\mu c(y+\mu)$$

Fits

32^3 96
T=24







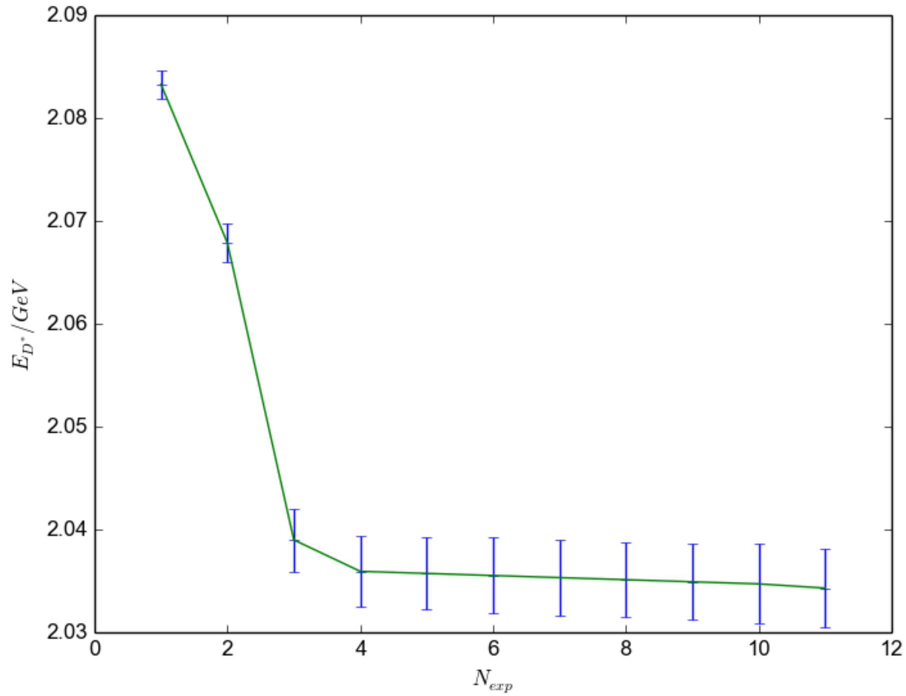
Fits

Use Bayesian fitting, minimise modified χ^2 including priors for parameters

$$\chi^2(p) = \sum_{t,t'} \Delta C(t,p)\sigma_{tt'}^{-2}\Delta C(t',p) + \sum_i (p^i - p^i_{\mathrm{prior}})^2/\sigma^2_{p^i_{\mathrm{prior}}}$$

- Constrained Curve Fitting, G.P. Lepage et al. 0110175

Allows for a very general fit function. Parameters which are not strongly determined by the data default to their prior value.



Results

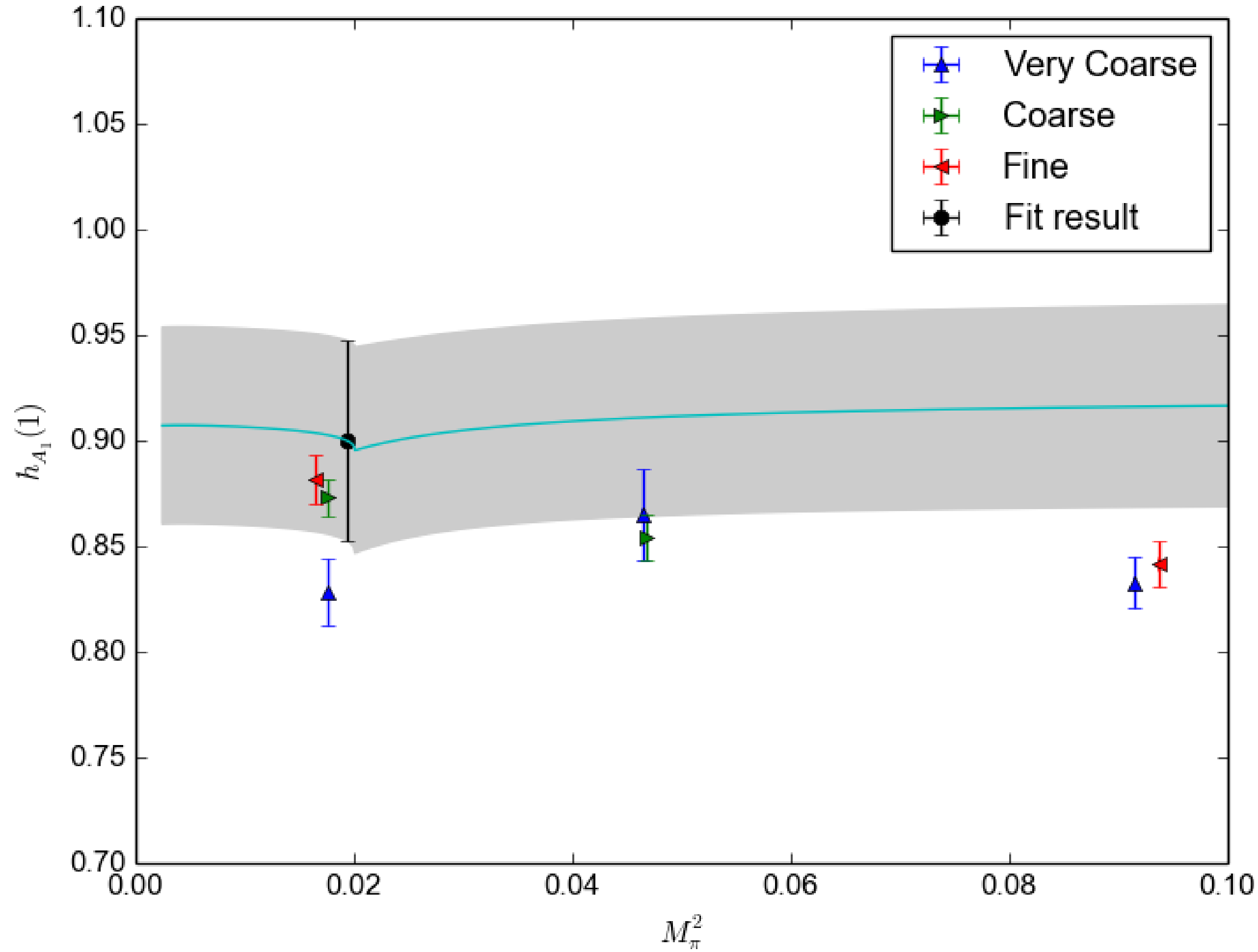
$L_x \times L_t$	$a(\mathrm{fm})$	$M_\pi(MeV)$	χ^2/dof	$\langle J^0_{\mathrm{latt}} \rangle$	$\mathcal{F}(1)$	$m_{D^*}(MeV)$
16×48	0.1474	302.4	0.93	0.926(14)	0.833(12)	2059.5(4.7)
24×48	0.1463	215.5	1	0.960(24)	0.865(22)	2057.7(3.3)
32×48	0.1450	133.0	1	0.922(17)	0.828(16)	2077.1(4.2)
32×64	0.1195	216.5	1	0.912(11)	0.854(11)	2022.0(3.2)
48×64	0.1189	132.7	0.98	0.9356(94)	0.8730(89)	2036.1(3.3)
32×96	0.0884	306.1	0.94	0.869(11)	0.842(11)	2035.5(3.7)
64×96	0.08787	128.4	1	0.908(12)	0.882(11)	2004.5(3.6)

Chiral Continuum Limit

Preliminary!

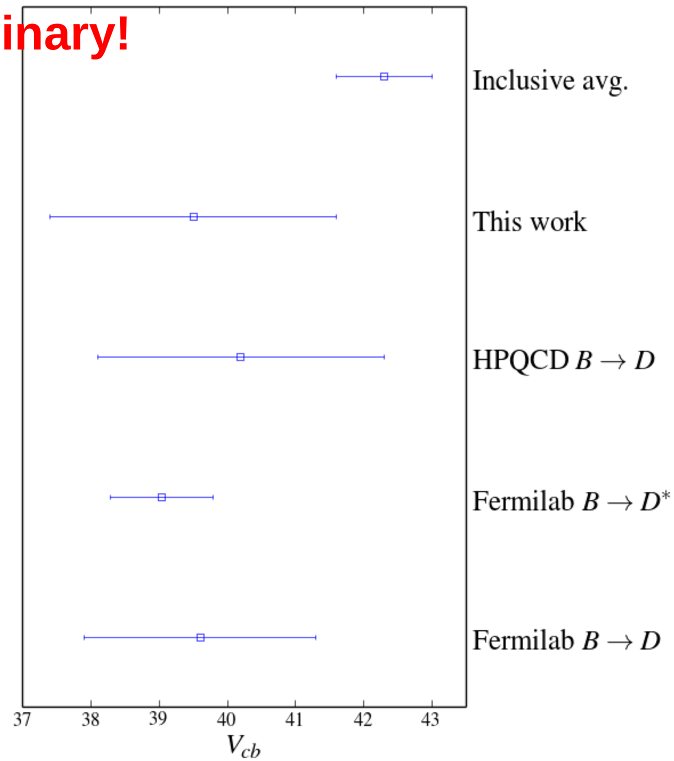
Once we have determined the currents on each lattice, must extrapolate to the continuum. Simultaneously fit pion mass dependence using chiral P.T. result.

$$\mathcal{F}(1) = 1 + \delta_a^B B + \delta_a^C C \frac{\Delta_{m_c}^2}{64\pi^2 f^2} \times \left(\ln\left[\left(\frac{M_\pi}{10 m_s^{\text{phys}}} \right)^2 \right] + F\left[\frac{-\Delta_{m_c}}{M_\pi} \right] \right) + e_1 \alpha_s^2 J_{\text{latt}}^0 + e_2 \alpha_s J_{\text{latt}}^1 + e_4 \frac{\Lambda^2}{M_b^2}$$



V_{cb} and $h_{A1}(1)$

Preliminary!



$$h_{A_1}(1) = 0.900(46)(11)$$

$$V_{cb} = 39.5(20)(5)(5) \times 10^{-3}$$

Uncertainty	Partial error (%) V_{cb}
a	0.00
$M_\pi^{\rm phys}$	0.00
M_π	0.00
$m_s^{\rm val}$	0.00
$m_c^{\rm val}$	0.00
$\alpha_s J^{(1)}$	0.07
Isospin effects	0.25
Priors	1.37
$(\Lambda_{QCD}/M_b)^2$	1.47
$\alpha_s^2 J^{(0)}$	4.71
Total systematic	5.13
Statistical	1.23
Experiment	1.27
total	5.42

Future work

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- Increase precision of V_{cb} result
- $B_s \to D_s^*$

Thank you for listening

Backup slides

Fits

Including oscillations in fits, the full form of the fit functions are

$$C_{B2pt}(t)_{ij} = \sum_{n,a=0,1} (-1)^{at} \frac{Z^{n1/2}_{B_a i} Z^{n1/2}_{B_a j}}{2M^n_{B_a}} e^{-M^n_{B_a} t}$$

$$C^{\mu\nu}_{D^*2pt}(t)_{ij} = \sum_{n,s,a=0,1} (-1)^{at} \frac{Z^{n1/2}_{D^*_a i} Z^{n1/2}_{D^*_a j}}{2M^n_{D^*_a}} \epsilon^\mu_s \epsilon^{\nu *}_s e^{-M^n_{D^*_a} t}$$

$$C^{\mu\kappa}_{3pt}(T,t,0)_{ij} = \sum_{ab=0,1} \sum_{nm,s} \frac{Z^{\frac{1}{2}}_{B_b j}}{2M^n_{B_b}} \frac{Z^{\frac{1}{2}}_{D^*_a i}}{2M^m_{D^*_a}} \epsilon^\mu_s \epsilon^{\kappa *}_s$$

$$\times (M^n_{B_b} + M^m_{D^*_a}) A^{nm}_{1ba}(q^2) e^{-M^m_{D^*_a}(T-t) - M^n_{B_b} t} (-1)^{a(T-t)+bt}$$