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# The Roberge-Weiss endpoint in $N_f = 2 + 1$ QCD at the physical point

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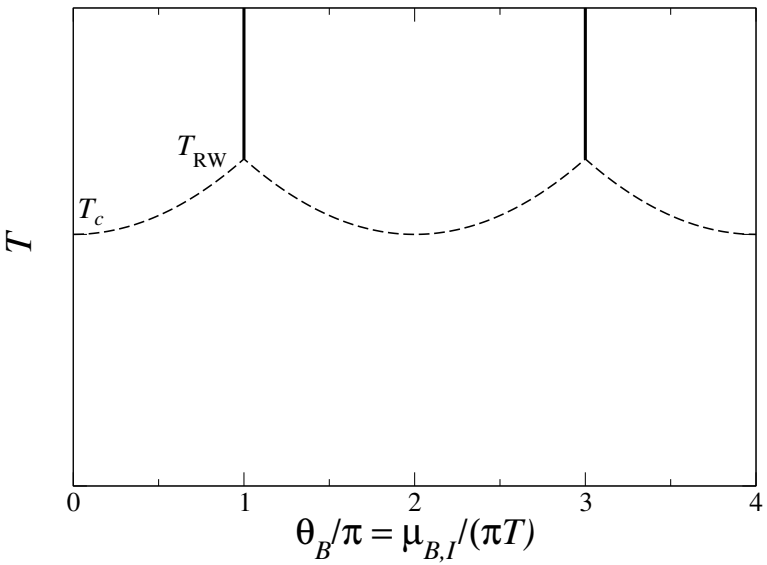
# Outline

- QCD at imaginary chemical potential and the RW transition
- Numerical setup
- Numerical results
  - Determination of the **order of the transition** [FSS of susceptibilities if $Im\, L$, $Im\, N_q$ ]
  - Continuum limit extrapolation of the **critical temperature**
  - Relation with the pseudo-critical critical line [comparison with 1507.03571] , effect on chiral observables
- Conclusions & Outlines

# QCD at imaginary chemical potential and the RW transition

Phase diagram of QCD for imaginary $\mu_B = 3\mu$




- Partition function of pure gauge theory is $Z_3$ symmetric.
- Fermions break this symmetry, but the partition function is $2\pi/3$-periodic in $\mu_I/T$,
- For $\mu_I/T = (2k+1)\pi/3$ a residual $Z(2)$ symmetry remains, which is spontaneously broken at high $T$ [R&W,86]

# Our Method

1. The determination of the order of the transition:
   - Locate the peak of the susceptibility of $Im\, L$ and $Im\, n_q$ (quark number density)

$$\begin{aligned} \chi_L &= N_t N_s^3 \left( \langle \mathrm{Im}[L]^2 \rangle - \langle |\mathrm{Im}\ [L]| \rangle^2 \right) \\ \chi_q &= N_t N_s^3 \left( \langle \mathrm{Im}[n_q]^2 \rangle - \langle |\mathrm{Im}\ [n_q]| \rangle^2 \right) \\ \chi(\beta) &\sim \frac{a}{1 + (\beta - \beta_{pc})^2 / c^2} \end{aligned}$$

   - Test 2nd (1st) order hypothesis: finite size scaling analysis with Ising 3D (1st order) critical exponents
   - Exact tricritical scaling is unlikely, and of difficult detection
2. Continuum extrapolation of $T_{RW}$:
   - evaluation of the systematic uncertainties in the location of $\beta_{pc}$

# Numerical setup

- Tree level Symanzik improved gauge action with $N_f = 2 + 1$ flavors of 2-stouted staggered fermions.
- At the physical point (line of constant physics, parameters taken from [Aoki *et al.*, 09, Borsanyi *et al.*, 10, Borsanyi *et al.* 14] )

| $N_t$ | $N_s$ | $a(\beta_c)$ (fm) |
|---|---|---|
| 4 | $16, 24, 32$ | 0.2424(6) |
| 6 | $24, 32, 40$ | 0.1714(3) |
| 8 | $32, 40$ | 0.1233(3) |
| 10 | 40 | 0.0968(2) |

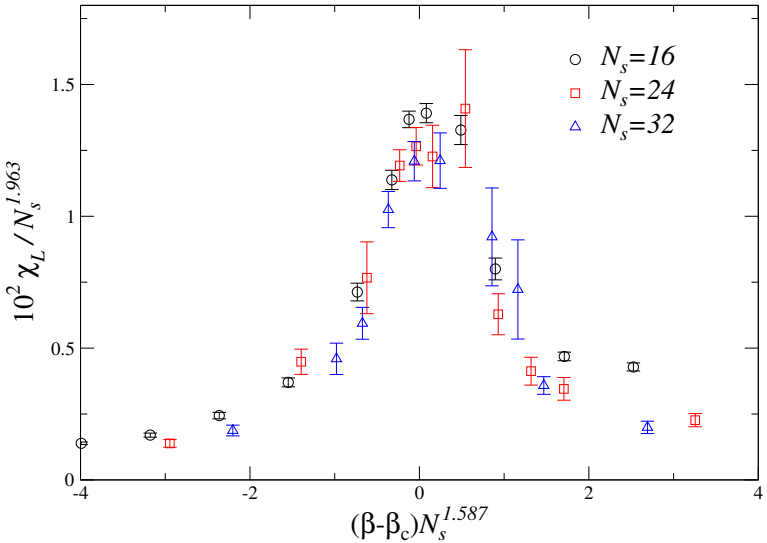
- Fermionic Observables evaluated with noisy estimators, with $\sim 10$ random vectors per quark.

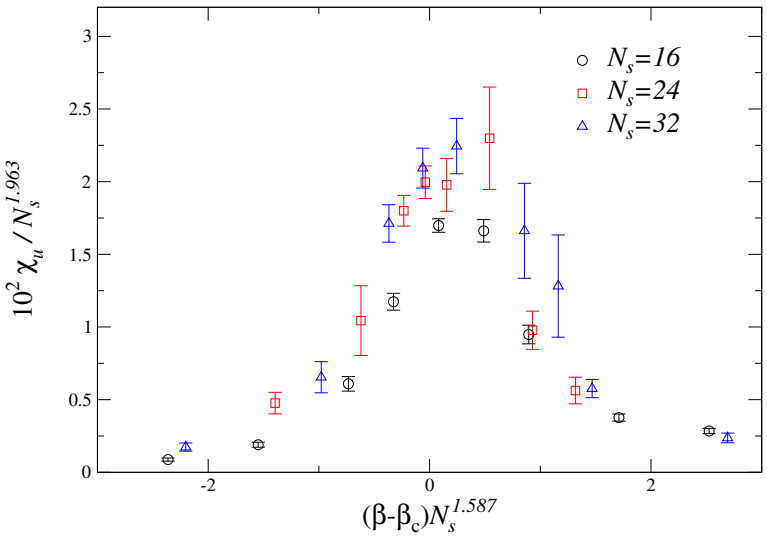
Simulations run on IBM BG-Q at CINECA and on the Zefiro cluster in Pisa.

# Determination of the order of the transition

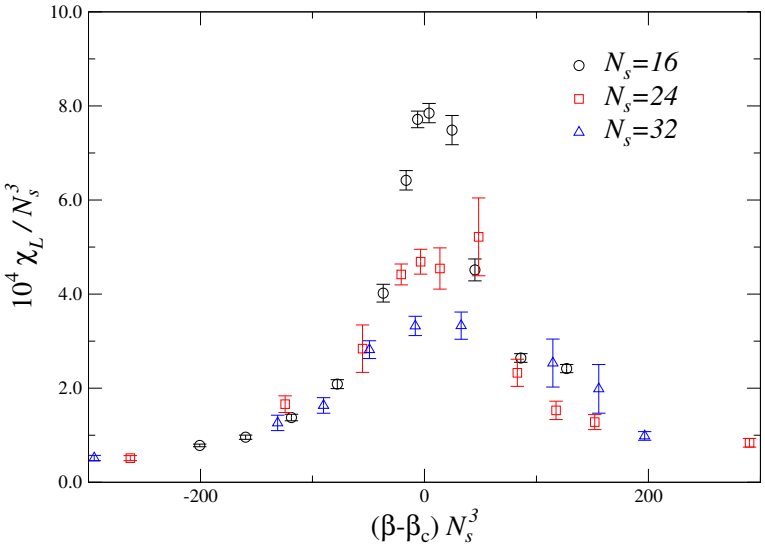
$N_t = 4$

## Ising 3D scaling



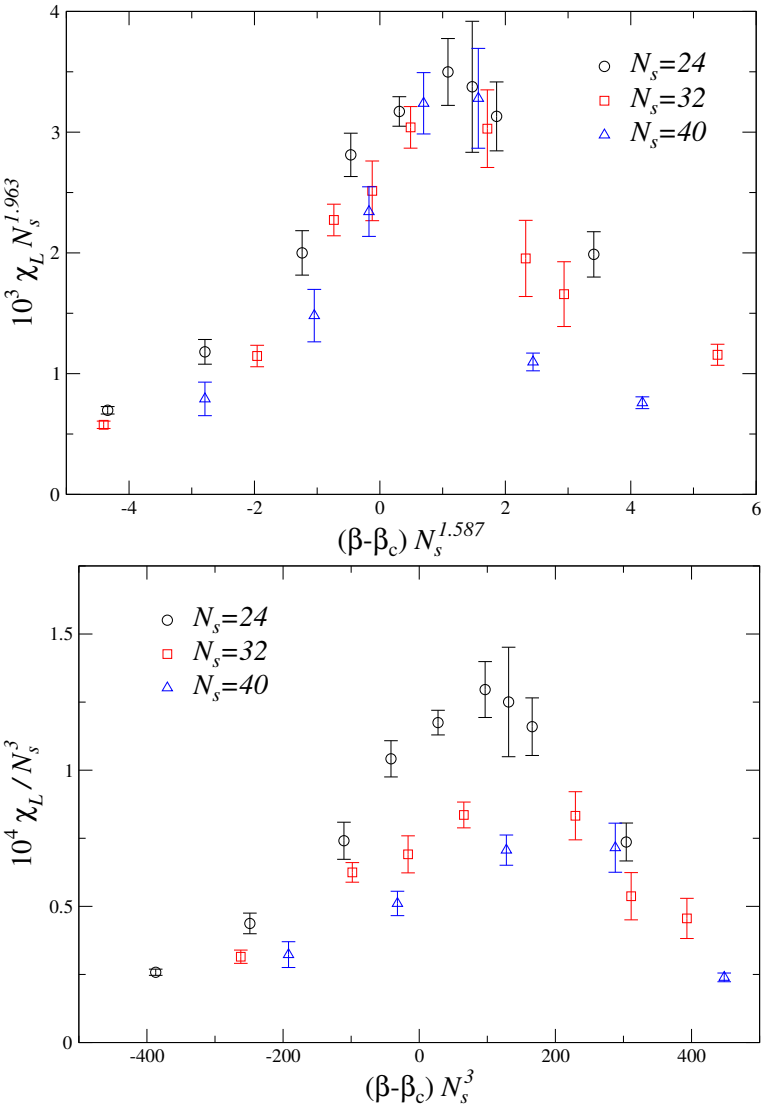

## First order scaling




**Top Left**: Polyakov loop susceptibility, 2nd order scaling
**Top Right**: Polyakov loop susceptibility, 1st order scaling
**Bottom Left**: $n_u$ susceptibility, 2nd order scaling

Our conclusion: **2nd order**

# Determination of the order of the transition
## $N_t = 6$



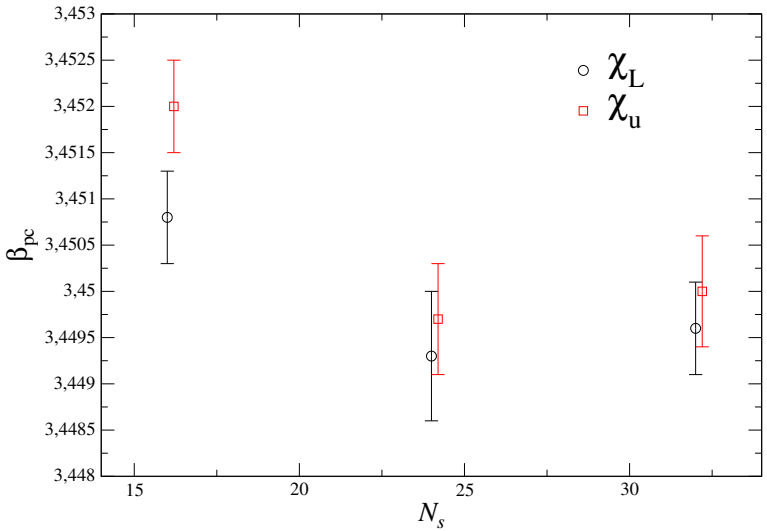

**Top**: 2nd order Ising 3D scaling
**Bottom**: 1st order scaling
(Both *Im L* susceptibility)

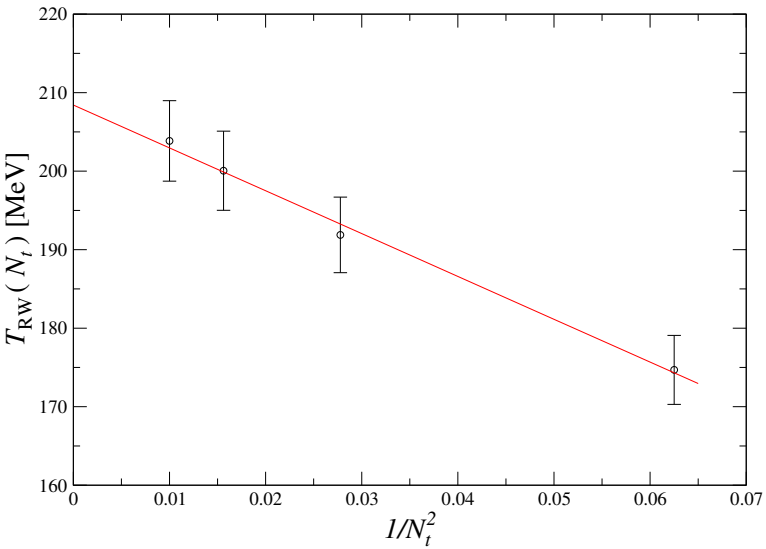
(Similar results regarding the quark number densities $n_u$, $n_d$ and $n_s$ for $N_t = 6$ are not reported)

Our conclusion: **2nd order**

# Continuum extrapolation of the critical temperature

## $N_t = 4, 6, 8$ and 10




**Top**: estimates of the pseudo-critical coupling on $N_t = 4$ lattices from different observables

**Bottom**: $T_{RW}(N_t)$ from the fits, taking into account the systematics due to the range and the choice of the observable.

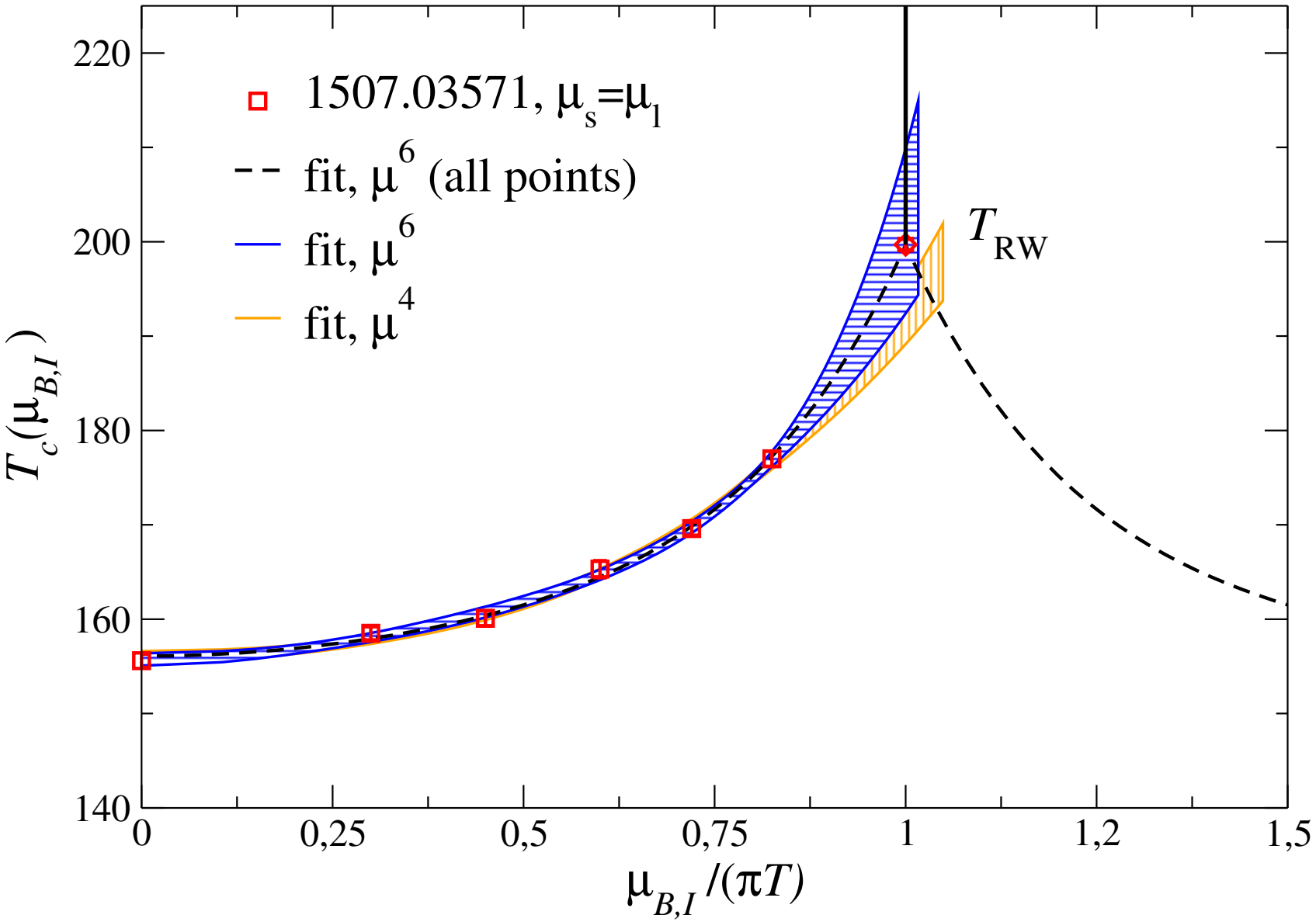
| $N_t$ | $\beta_c$ | $a(\beta_c)$ (fm) |
|---|---|---|
| 4 | 3.4498(7) | 0.2424(6) |
| 6 | 3.6310(15) | 0.1714(3) |
| 8 | 3.7540(25) | 0.1233(3) |
| 10 | 3.8600(25) | 0.0968(2) |

Note: 2%-3% error on $T_{RW}(N_t)$ due to **scale setting uncertainties**.
*This* is the *largest* source of error.

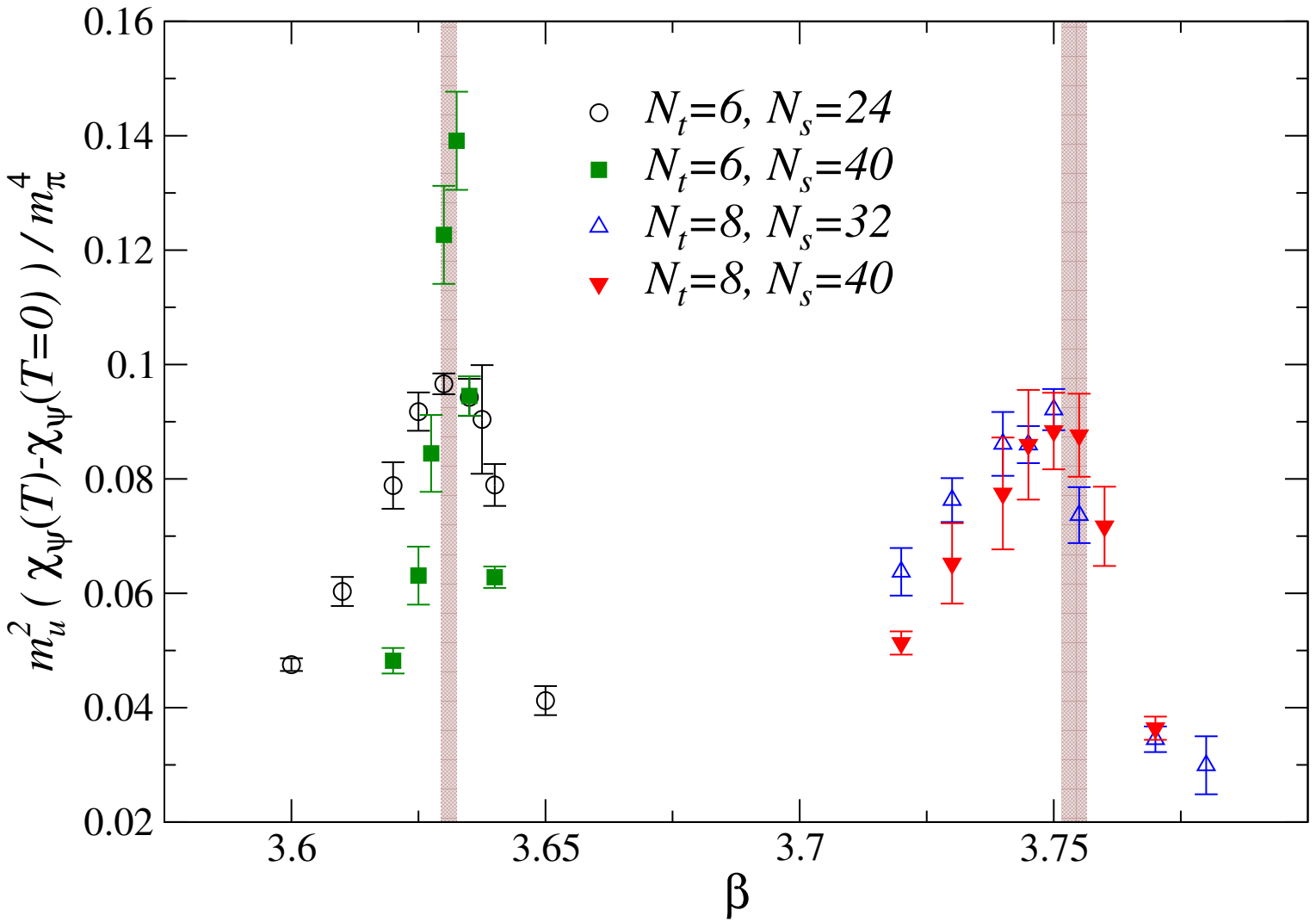
Result: **208(5) MeV**

# Comparison with the critical line at imaginary $\mu$

$N_t = 8$



# Chiral susceptibility at the RW transition




Renormalized chiral susceptibility near the RW transition.

The values of $\beta_c$ obtained for $N_t = 6, 8$ are also plotted.

# Conclusions & Outlooks

## Conclusions

- The transition at the physical value of $m_\pi$ seems to be second order: we verified this explicitly on $N_t = 4, 6$ lattices.

  Since $m_\pi^{\text{tric,light}}$ decreases with $n_t \to \infty$ [Philipsen,Pinke, '14, Cuteri *et al.* ,16], this picture is likely to hold in the continuum

- Our continuum limit of the RW transition temperature: $T_{RW} = 208(5)$ MeV.

  The large source of error is scale setting.

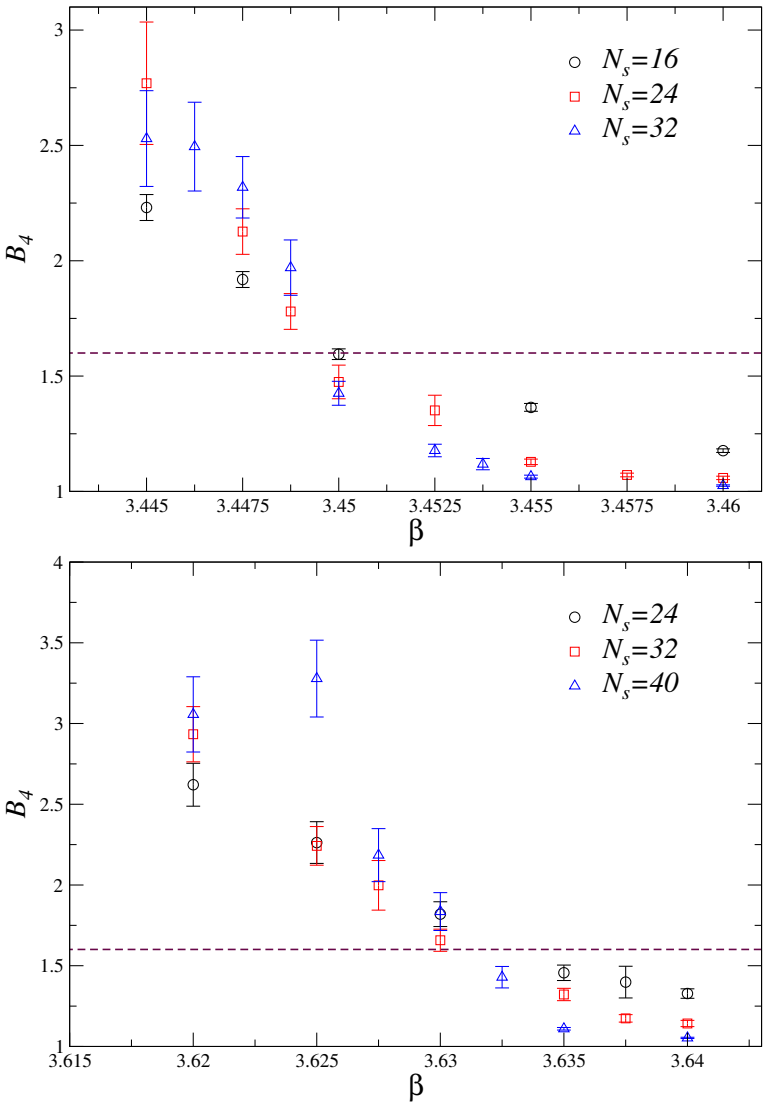
- The value of $T_{RW}$ we found is compatible with the analytic continuation of the critical line up to $\mu_s/T = \mu_l/T = \pi/3$ ($N_t = 8$ data from [1507.03571])

## Future developments

- Locate $m_\pi^{tric,light}$ and $m_\pi^{tric,heavy}$ (and possibly perform continuum limit)
- Investigate the relation between the chiral and the RW transitions

# Backup - Determination of the order of the transition

## FSS - Binder cumulant of *Im L*



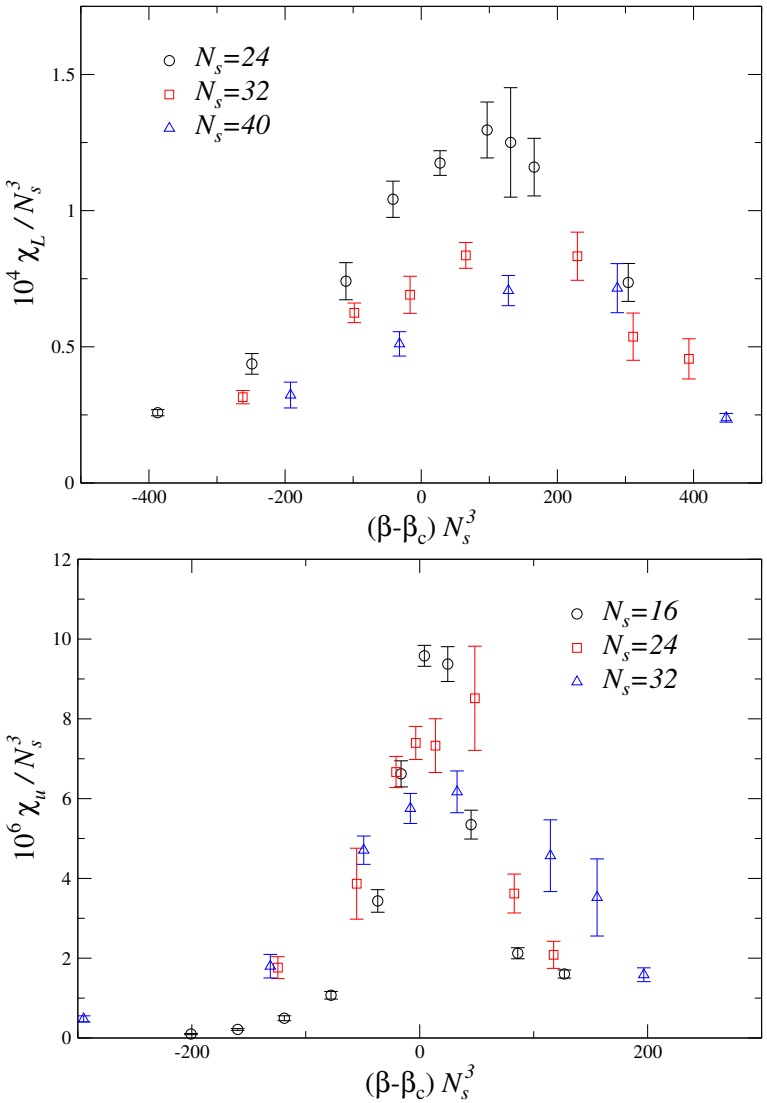

The Binder cumulant is defined as

$$B_4 = \frac{\langle (\mathrm{Im}L)^4 \rangle}{\langle (\mathrm{Im}L)^2 \rangle^2}$$

Expected value for the 3D Ising universality class: $B_4 = 1.604(1)$

Top: $N_t = 4$
Bottom: $N_t = 6$

# Backup - First order scaling




Top: $N_t = 6$, $Im\, L$ susceptibility

Bottom: $N_t = 6$, $Im\, n_u$ susceptibility

# Backup - Monte Carlo Histories of $Im\,L$

$N_t = 4$