Effects of magnetic fields on $q\bar{q}$ interactions

[1607.08160]

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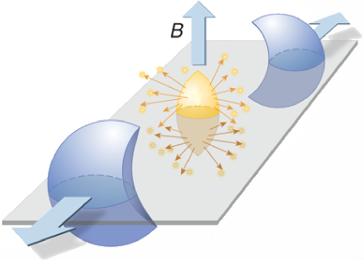
intro **physical conditions**

QCD with strong magnetic fields $eB \simeq m_\pi^2 \sim 10^{15-16}$ T

- Non-central heavy ion collisions with eB $\sim 10^{15}$T [Skokov et al. '09]
- Possible production in early universe eB $\sim 10^{16}$T [Vachaspati '91]

In heavy ion collisions

- Expected eB $\simeq$ 0.3 GeV^2 at LHC in Pb+Pb at $\sqrt{s_{NN}}$=4.5TeV and b=4fm
- Timescales depend on thermal medium properties (most pessimistic case: 0.1-0.5 fm/c)
- Spatial distribution of the field and lifetime are still debated



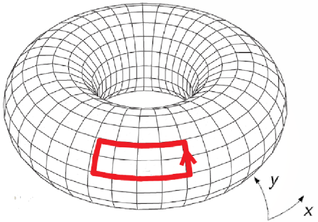
intro **turning on the B field**

An external magnetic field B on the lattice is introduced through abelian parallel transports $u_\mu(n)$

- Abelian phases enter the Lagrangian by modifying the covariant derivative

$$U_\mu(n) \to U_\mu(n) u_\mu(n)$$

- External field is fixed: non-propagating fields, no kinetic term



- Periodic boundary conditions lead to the quantization condition

$$|q_{\min}| B = \frac{2\pi b}{a^2 N_x N_y} \quad b \in \mathbb{Z}$$

intro **static potential**

In the confining phase at low temperatures, the $Q\bar{Q}$ interaction is well described by the Cornell potential

$$V_C(r) = -\frac{\alpha}{r} + \sigma r + V_0 \qquad \sigma \simeq (440\text{MeV})^2 \quad \alpha \sim 0.4$$

On the lattice:

- At T=0 it can be extracted from the Wilson loop

$$aV(a\vec{n}) = -\lim_{n_t \to \infty} \log\left(\frac{\langle W(\vec{n}, n_t+1)\rangle}{\langle W(\vec{n}, n_t)\rangle}\right)$$

- For T>0 from Polyakov loop correlators

$$F(a\vec{n}, T) \simeq -aN_t \log\langle \text{Tr}L^\dagger(\vec{r}+\vec{n})\text{Tr}L(\vec{n})\rangle$$

what about the effects of $\vec{B}$ on the potential? (a first study: [Bonati et al. '14])

T=0 **setup and continuum results at B=0**

Numerical setup

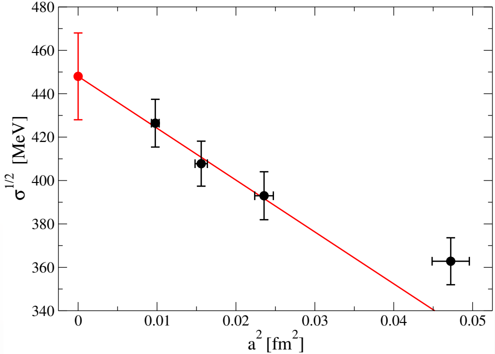
- tree-level improved gauge action
- N_f=2+1 rooted staggered fermions + stout improvement
- four lattices $48^3 \times 96$, 40^4, 32^4 and 24^4
- spacing a $\simeq$ 0.1 fm to a $\simeq$ 0.24 fm
- simulations at physical quark masses

Parameters extracted from the continuum limit at $B = 0$

$$\alpha = 0.395(22)$$

$$\sqrt{\sigma} = 448(20)\ \text{MeV}$$

$$r_0 = 0.489(20)\ \text{fm}$$



T=0 **angular dependence**

Turning on a constant uniform external field: residual rotation symmetry around $\vec{B}$ survives. Our ansatz:

$$V(r,\theta) = -\frac{\alpha(\theta,B)}{r} + \sigma(\theta,B)r + V_0(\theta,B)$$

with θ angle between quarks direction and $\vec{B}$.

Angular dependence in Fourier expansion:

$$\mathcal{O}(\theta,B) = \bar{\mathcal{O}}(B)\left(1 - \sum_{n=1} c_{2n}^{\mathcal{O}}(B)\cos(2n\theta)\right) \qquad \mathcal{O} = \alpha,\sigma,V_0$$

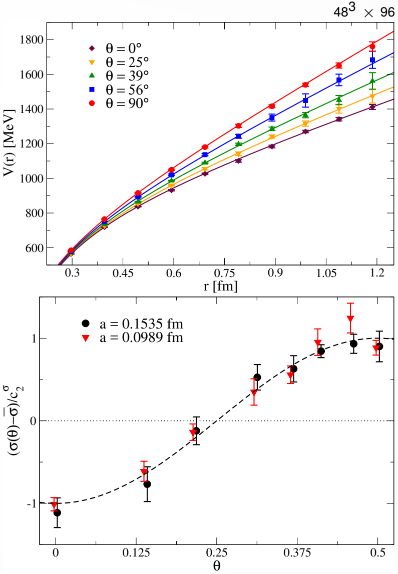
General features:

- Assumption: $V(r,\theta)$ is in the Cornell form $\forall\theta$
- c_{2n+1} terms vanish ($\vec{B}$ inversion $\theta \to \pi - \theta$)

T=0 **angular dependence**

Some details:

- fixed $|e|B \sim 1.0$ GeV^2 on two lattices $aL \sim 5$ fm ($|\vec{b}| = 32$)
- Wilson loop averaged separately on orthogonal axes
- Access to 8 angles using three $\vec{B}$ orientations



Results:

- potential is anisotropic and $V(r, \theta)$ increases with θ
- good description in terms of c_2's only ($\sim 0.2 - 0.3$)
- $\bar{\mathcal{O}}(B)$ compatible with values at $B = 0$

T=0 **anisotropy in the continuum**

Questions:

- Does the anisotropy survive when $a \to 0$?
- Dependence to B?

Simplify the task:

Angular dependence is fully described by the lowest coefficients c_2s	$\Longrightarrow$	All the informations accessed by studying the potential along two directions only

For each $\mathcal{O} = \sigma, \alpha, V_0$ we can study its anisotropy (with $\vec{B} \parallel \hat{z}$)

$$\delta^{\mathcal{O}}(B) = \frac{\mathcal{O}_{XY}(B) - \mathcal{O}_Z(B)}{\mathcal{O}_{XY}(B) + \mathcal{O}_Z(B)}$$

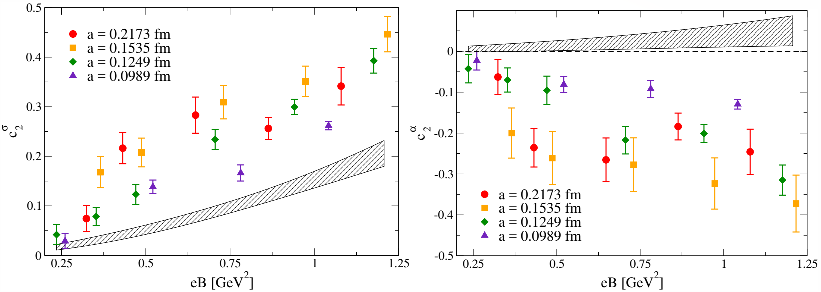
then

$$\delta^{\mathcal{O}} \simeq c_2^{\mathcal{O}}$$

T=0 **anisotropy in the continuum**

Continuum extrapolation using

$$c_2^{\mathcal{O}} = A^{\mathcal{O}}(1 + C^{\mathcal{O}} a^2)(|e|B)^{D^{\mathcal{O}}(1+E^{\mathcal{O}} a^2)} \qquad \mathcal{O} = \sigma, \alpha, V_0$$



Results:

- anisotropy c_2^σ of the string tension survives $a \to 0$
- c_2^α and $c_2^{V_0}$ compatible with zero
- $\bar{\mathcal{O}}(B)$ all compatible with values at $B = 0$

T>0 effects on the free energy

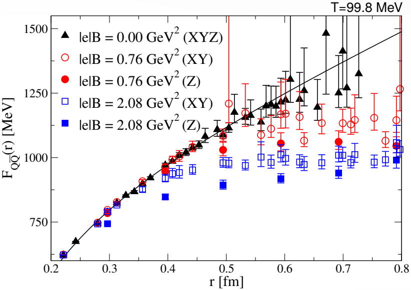
what about (not so) high temperatures?

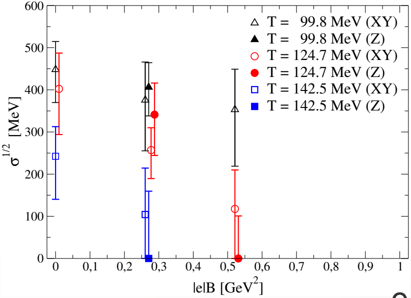
Setup:

- Fixed a=0.0989 fm on lattices $48^3 \times N_t$ with N_t=14,16,20 ($T \lesssim T_c$)
- Several magnetic quanta b=0 to b=64 with B//z

Results:

- Anisotropy still visible but disappears at large *r*
- String tension σ decreases
- Cornell form fits only at small *B*





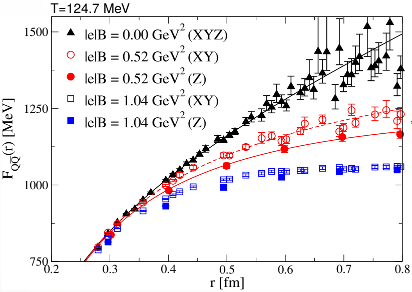
T>0 **effects on the free energy**

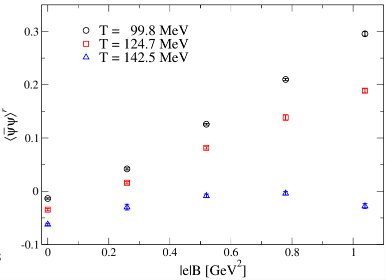
From our results:

- Decrease of the free energy as *B* grows
- The effect is enhanced as *T* reaches T_c

This is compatible with a decrease of T_c due to *B* [Bali et al.'12]

- Suppression of confining properties is evident before the appearance of *inverse chiral magnetic catalysis*
- Hence it seems to be the dominant phenomenon





conclusions **and summary**

Investigation of the effect of B on the $Q\bar{Q}$ interaction [arXiv:1607.08160]

- The static potential becomes anisotropic $V(r) \to V(r, \theta, B)$
- Genuine effects in the continuum limit
- Modifications mostly due to the string tension

$$\sigma \to \sigma(B, \theta) \simeq \sigma\Big(1 - c_2^{\sigma}(B) \cos 2\theta\Big)$$

- Anisotropy still visible at $T > 0$
- Observations agree picture with deconfinement catalysis

Possible implications:

- In meson production in heavy ion collisions [Guo et al. ’15]
- Heavy meson spectrum $c\bar{c}$ and $b\bar{b}$ [Alford and Strickland ’13, Bonati et al ’15]

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THANK YOU

backup **magnetic field on the lattice**

With $\vec{B} \parallel \hat{z}$, a possible choice of the abelian links is

$$u^f_{i;\,y} = e^{ia^2 q_f B_z i_x} \quad u^f_{i;\,x}\big|_{i_x = L_x} = e^{-ia^2 q_f L_x B_z i_y}$$

and all the other equal to 1.

A general $\vec{B} = (B_x, B_y, B_z)$:

- The quantization condition

$$|q_{\min}|B = \frac{2\pi b}{a^2 N_x N_y} \quad b \in \mathbb{Z}$$

  applies separately along each coordinate axis.
- If $N_x = N_y = N_z$ the condition is the same and hence $\vec{B} \propto \vec{b} = (b_x, b_y, b_z)$
- Phase in the fermion matrix is the product

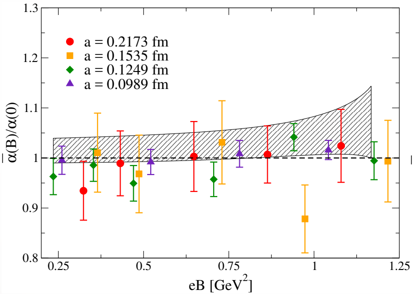
backup **anisotropy at T=0**

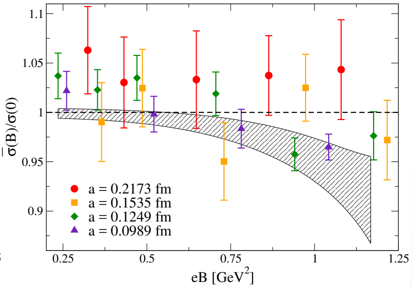
The $\mathcal{O}(B)$ values are accessible computing the quantities

$$R^{\mathcal{O}}(|e|B) = \frac{\mathcal{O}_{XY}(|e|B) + \mathcal{O}_Z(|e|B)}{2\mathcal{O}(|e|B=0)}$$

$$= \frac{\bar{\mathcal{O}}(|e|B)}{\mathcal{O}(|e|B=0)}\left(1 - \sum_{n \text{ even}} c_{2n}^{\mathcal{O}}\right) \simeq \frac{\bar{\mathcal{O}}(|e|B)}{\mathcal{O}(|e|B=0)}$$

and are compatible with those at $B=0$





backup **large B**

Extension to large fields (at $a = 0.0989$ fm on $48^3 \times 96$)

- longitudinal string tension seems to vanish for $|e|B \sim 4$ GeV^2
- problem: cut-off effects at $|e|B \sim 1/a^2 \sim 4$ GeV^2