Effects of magnetic fields on $q\bar{q}$ interactions

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intro physical conditions

QCD with strong magnetic fields $eB \sim m^2_\pi \sim 10^{15-16}$ T

- Non-central heavy ion collisions with $eB \sim 10^{15}$T [Skokov et al. ’09]
- Possible production in early universe $eB \sim 10^{16}$T [Vachaspati ’91]

In heavy ion collisions

- Expected $eB \sim 0.3$ GeV$^2$ at LHC in Pb+Pb at $\sqrt{s_{NN}}=4.5$TeV and $b=4$fm
- Timescales depend on thermal medium properties (most pessimistic case: 0.1-0.5 fm/c)
- Spatial distribution of the field and lifetime are still debated
An external magnetic field $B$ on the lattice is introduced through abelian parallel transports $u_\mu(n)$.

- Abelian phases enter the Lagrangian by modifying the covariant derivative

$$U_\mu(n) \rightarrow U_\mu(n)u_\mu(n)$$

- External field is fixed: non-propagating fields, no kinetic term

- Periodic boundary conditions lead to the quantization condition

$$|q_{\text{min}}|B = \frac{2\pi b}{a^2N_xN_y} \quad b \in \mathbb{Z}$$
**intro static potential**

In the confining phase at low temperatures, the $Q\bar{Q}$ interaction is well described by the Cornell potential

$$V_C(r) = -\frac{\alpha}{r} + \sigma r + V_0 \quad \sigma \simeq (440\text{MeV})^2 \quad \alpha \sim 0.4$$

On the lattice:

- At $T=0$ it can be extracted from the Wilson loop

$$aV(a\vec{n}) = -\lim_{n_t \to \infty} \log \left( \frac{\langle W(a\vec{n}, n_t + 1) \rangle}{\langle W(a\vec{n}, n_t) \rangle} \right)$$

- For $T>0$ from Polyakov loop correlators

$$F(a\vec{n}, T) \simeq -aN_t \log \langle \text{Tr} L^\dagger (\vec{r} + \vec{n}) \text{Tr} L(\vec{n}) \rangle$$

what about the effects of $\vec{B}$ on the potential? (a first study: [Bonati et al. ’14])
**T=0 setup and continuum results at B=0**

Numerical setup
- tree-level improved gauge action
- $N_f=2+1$ rooted staggered fermions + stout improvement
- four lattices $48^3 \times 96$, $40^4$, $32^4$ and $24^4$
- spacing $a \simeq 0.1$ fm to $a \simeq 0.24$ fm
- simulations at physical quark masses

**Parameters extracted from the continuum limit at $B = 0$**

\[ \alpha = 0.395(22) \]
\[ \sqrt{\sigma} = 448(20) \text{ MeV} \]
\[ r_0 = 0.489(20) \text{ fm} \]
$T=0$ angular dependence

Turning on a constant uniform external field: residual rotation symmetry around $\vec{B}$ survives. Our ansatz:

$$V(r, \theta) = -\frac{\alpha(\theta, B)}{r} + \sigma(\theta, B)r + V_0(\theta, B)$$

with $\theta$ angle between quarks direction and $\vec{B}$.

Angular dependence in Fourier expansion:

$$O(\theta, B) = \bar{O}(B) \left( 1 - \sum_{n=1}^{\infty} c_{2n}(B) \cos(2n\theta) \right)$$

$O = \alpha, \sigma, V_0$

General features:

- Assumption: $V(r, \theta)$ is in the Cornell form $\forall \theta$
- $c_{2n+1}$ terms vanish ($\vec{B}$ inversion $\theta \rightarrow \pi - \theta$)
$T=0$ angular dependence

Some details:
- fixed $|e|B \sim 1.0 \text{ GeV}^2$ on two lattices $aL \sim 5 \text{ fm}$ ($|\vec{b}| = 32$)
- Wilson loop averaged separately on orthogonal axes
- Access to 8 angles using three $\vec{B}$ orientations

Results:
- potential is anisotropic and $V(r, \theta)$ increases with $\theta$
- good description in terms of $c_2$'s only ($\sim 0.2 - 0.3$)
- $\bar{O}(B)$ compatible with values at $B = 0$
T=0 anisotropy in the continuum

Questions:
- Does the anisotropy survive when \( a \to 0 \)?
- Dependence to \( B \)?

Simplify the task:

Angular dependence is fully described by the lowest coefficients \( c_2 \s

\[ \delta \mathcal{O}(B) = \frac{\mathcal{O}_{XY}(B) - \mathcal{O}_Z(B)}{\mathcal{O}_{XY}(B) + \mathcal{O}_Z(B)} \]

For each \( \mathcal{O} = \sigma, \alpha, V_0 \) we can study its anisotropy (with \( \vec{B} \parallel \hat{z} \))

then

\[ \delta \mathcal{O} \simeq c_2^\mathcal{O} \]
$T=0$ anisotropy in the continuum

Continuum extrapolation using

\[ c_2^O = A^O \left( 1 + C^O a^2 \right) |e|B^D \left( 1 + E^O a^2 \right) \quad O = \sigma, \alpha, V_0 \]

Results:

- anisotropy $c_2^\sigma$ of the string tension survives $a \to 0$
- $c_2^\alpha$ and $c_2^V_0$ compatible with zero
- $\tilde{O}(B)$ all compatible with values at $B = 0$
**T > 0 effects on the free energy**

what about (not so) high temperatures?

**Setup:**
- Fixed $a=0.0989$ fm on lattices $48^3 \times N_t$ with $N_t=14,16,20$ ($T \lesssim T_c$)
- Several magnetic quanta $b=0$ to $b=64$ with $B//z$

**Results:**
- Anisotropy still visible but disappears at large $r$
- String tension $\sigma$ decreases
- Cornell form fits only at small $B$

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$T>0$ effects on the free energy

From our results:
- Decrease of the free energy as $B$ grows
- The effect is enhanced as $T$ reaches $T_c$

This is compatible with a decrease of $T_c$ due to $B$ [Bali et al.’12]
- Suppression of confining properties is evident before the appearance of inverse chiral magnetic catalysis
- Hence it seems to be the dominant phenomenon

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conclusions and summary

Investigation of the effect of $B$ on the $Q\bar{Q}$ interaction [arXiv:1607.08160]

- The static potential becomes anisotropic $V(r) \rightarrow V(r, \theta, B)$
- Genuine effects in the continuum limit
- Modifications mostly due to the string tension

\[ \sigma \rightarrow \sigma(B, \theta) \simeq \sigma \left( 1 - c_2^\sigma(B) \cos 2\theta \right) \]

- Anisotropy still visible at $T > 0$
- Observations agree picture with deconfinement catalysis

Possible implications:

- In meson production in heavy ion collisions [Guo et al. ’15]
- Heavy meson spectrum $c\bar{c}$ and $b\bar{b}$ [Alford and Strickland ’13, Bonati et al ’15]
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THANK YOU
backup magnetic field on the lattice

With $\vec{B} \parallel \hat{z}$, a possible choice of the abelian links is

$$u_{i; y}^f = e^{ia^2q_fB_iz_i} \quad u_{i; x}^f|_{i_x=L_x} = e^{-ia^2q_fL_xB_iz_i}$$

and all the other equal to 1.

A general $\vec{B} = (B_x, B_y, B_z)$:

- The quantization condition

$$|q_{\text{min}}|B = \frac{2\pi b}{a^2N_xN_y} \quad b \in \mathbb{Z}$$

applies separately along each coordinate axis.

- If $N_x = N_y = N_z$ the condition is the same and hence

$$\vec{B} \propto \vec{b} = (b_x, b_y, b_z)$$

- Phase in the fermion matrix is the product
backup anisotropy at T=0

The $\mathcal{O}(B)$ values are accessible computing the quantities

$$R^\mathcal{O}(|e|B) = \frac{\mathcal{O}_{XY}(|e|B) + \mathcal{O}_Z(|e|B)}{2\mathcal{O}(|e|B = 0)}$$

$$= \frac{\tilde{\mathcal{O}}(|e|B)}{\mathcal{O}(|e|B = 0)} \left(1 - \sum_{n \text{ even}} \frac{c^{\mathcal{O}}_{2n}}{c^{\mathcal{O}}_{2n}}\right) \approx \frac{\tilde{\mathcal{O}}(|e|B)}{\mathcal{O}(|e|B = 0)}$$

and are compatible with those at $B = 0$
Extension to large fields (at $a = 0.0989$ fm on $48^3 \times 96$)

- longitudinal string tension seems to vanish for $|e|B \sim 4 \text{ GeV}^2$
- problem: cut-off effects at $|e|B \sim 1/a^2 \sim 4 \text{ GeV}^2$