Determination of latent heat at the finite temperature phase transition of SU(3) gauge theory

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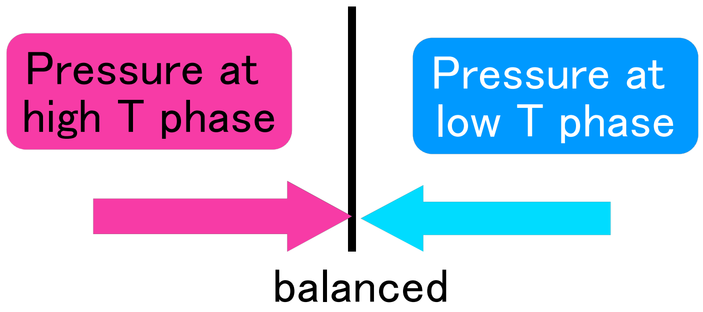
Introduction

- First order phase transitions are expected in many interesting systems of lattice field theories. e.g. high density QCD, many flavor etc..

  → important to study first order phase transitions.

- The latent heat (energy gap) the most basic quantity.
- The gap of pressure must be vanish.

  Reliability of the calculation can be confirmed.



In this talk,

- We study the equation of state at the first order phase transition of SU(3) gauge theory.
- Gaps of energy density and pressure are measured using the derivative method.
  - Volume dependence is investigated
  - Continuum extrapolation is performed.

[H. Suzuki, 2013]

- We tested the gradient flow method for the calculation of EoS.

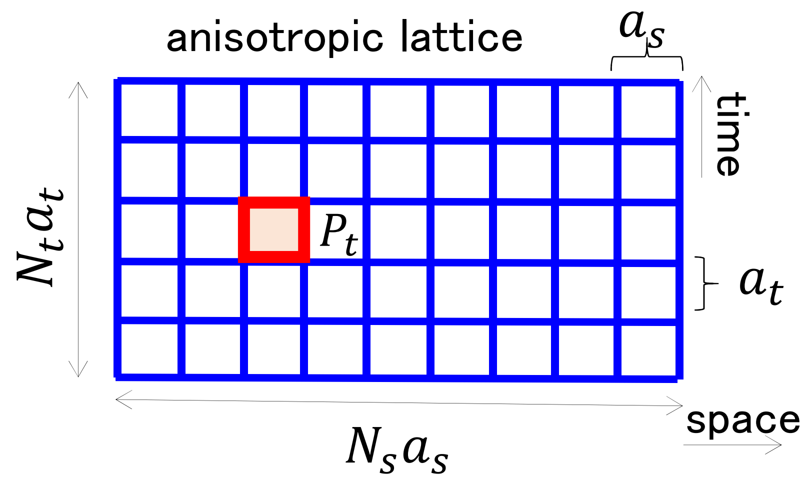
Thermodynamic quantities by the derivative method

energy density

$$\epsilon = -\frac{1}{V}\frac{\partial lnZ}{\partial T^{-1}}\Big|_V$$

pressure

$$p = T\frac{\partial lnZ}{\partial V}\Big|_T$$



temperature $\frac{1}{T} = N_t a_t$ volume $V = (N_s a_s)^3$

$$Z = \int DU\, \mathrm{e}^{-S}$$

For the SU(3) gauge theory, $S = -3N_{\mathrm{site}}(\beta_s P_s + \beta_t P_t)$

($P_{s(t)}$ space-like (time-like) plaquette)

$$\epsilon = -\frac{3N_t^4 T^4}{\xi^3}\left\{\left(a_t\frac{\partial \beta_s}{\partial a_t} - \xi\frac{\partial \beta_s}{\partial \xi}\right)(\langle P_s\rangle - \langle P\rangle_0) - \left(a_t\frac{\partial \beta_t}{\partial a_t} - \xi\frac{\partial \beta_t}{\partial \xi}\right)(\langle P_t\rangle - \langle P\rangle_0)\right\}$$

$$p = \frac{N_t^4 T^4}{\xi^3}\left\{\frac{\partial \beta_s}{\partial \xi}(\langle P_s\rangle - \langle P\rangle_0) + \frac{\partial \beta_t}{\partial \xi}(\langle P_t\rangle - \langle P\rangle_0)\right\}$$

Independent variables: a_t, $\xi = \frac{a_s}{a_t}$

$\langle P\rangle_0$: The expectation value at $T = 0$

For $\xi = 1$, the gap of the energy density

$$\frac{\Delta\epsilon}{T^4} = -3N_t^4\left\{\left(a_t\frac{\partial \beta_s}{\partial a_t} - \frac{\partial \beta_s}{\partial \xi}\right)(\langle P_s\rangle_{\mathrm{hot}} - \langle P_s\rangle_{\mathrm{cold}}) + \left(a_t\frac{\partial \beta_t}{\partial a_t} - \frac{\partial \beta_t}{\partial \xi}\right)(\langle P_t\rangle_{\mathrm{hot}} - \langle P_t\rangle_{\mathrm{cold}})\right\}$$

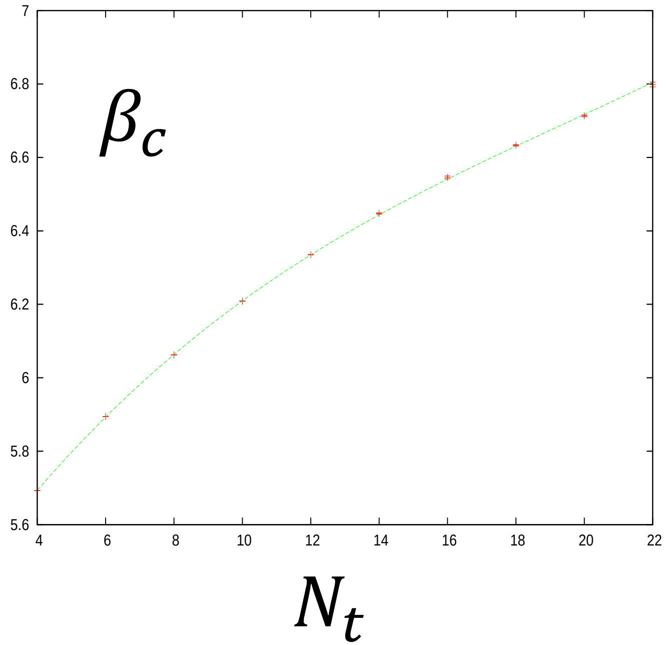
$$\boxed{a_t\frac{\partial \beta_s}{\partial a_t}, a_t\frac{\partial \beta_t}{\partial a_t}, \frac{\partial \beta_s}{\partial \xi}, \frac{\partial \beta_t}{\partial \xi}}$$

These 4 coefficients must be determined.

Determination of the anisotropy coefficients at $\xi = a_s/a_t = 1$

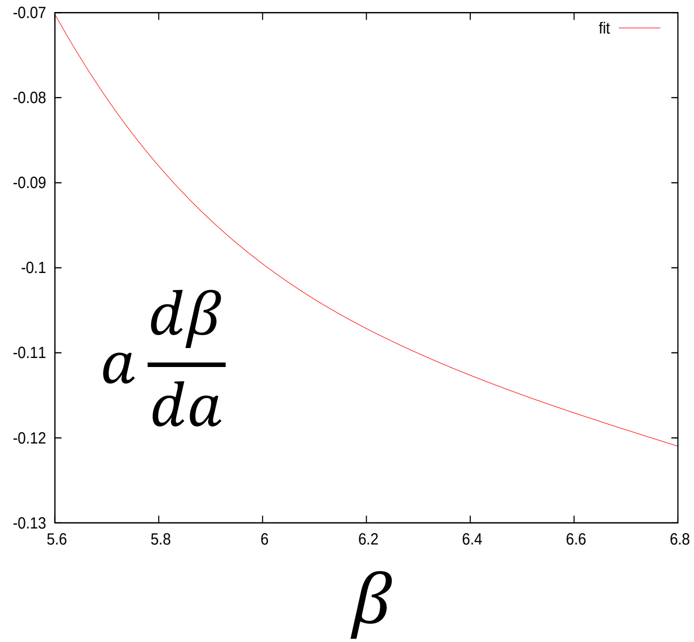
Isotropic lattice ($\beta = \beta_s = \beta_t$): $$\left(a_t \frac{\partial \beta_s}{\partial a_t}\right)_{\xi=1} = \left(a_t \frac{\partial \beta_t}{\partial a_t}\right)_{\xi=1} = a\frac{d\beta}{da}$$

$a\dfrac{d\beta}{da}$ is determined by the data of the critical β ($\beta_c(N_t)$)



$$\frac{1}{T_c} = N_t a_t$$

$$a\frac{d\beta}{da} = -N_t \frac{d\beta}{dN_t}$$



Data: Francis, kaczmarek, Laine, Neuhaus, Ohno, Phys. Rev. D 91, 096002 (2015) and our data for $N_t = 4 \sim 22$

String tension is independent of $\xi = \dfrac{a_s}{a_t}$

$$\left(\frac{\partial \beta_s}{\partial \xi} + \frac{\partial \beta_t}{\partial \xi}\right)_{a_t:\mathbf{fixed},\xi=1} = \frac{3}{2}a\frac{d\beta}{da}$$

[F. Karsch, Nucl. Phys. B205 (1982) 285]

Ratio of the anisotropy coefficients

The slope of the phase transition line in the (β_s, β_t) plane: r_t

[Ejiri, Iwasaki, Kanaya, Phys.Rev.D 58,094505 (1998)]

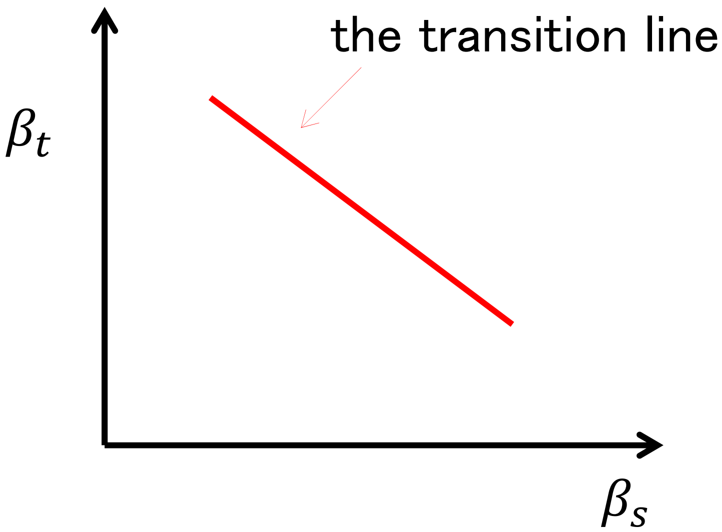
Along the phase transition line, a_t is constant

because $\frac{1}{T_c} = N_t a_t$.

When one changes

$(\beta_s, \beta_t) \rightarrow (\beta_s + d\beta_s, \beta_t + d\beta_t)$,

$$\mathrm{d}a_t = \frac{\partial a_t}{\partial \beta_t} d\beta_s + \frac{\partial a_t}{\partial \beta_t} d\beta_t = 0$$



The slope of the transition line

$$r_t = \frac{d\beta_s}{d\beta_t} = -\frac{\left(\frac{\partial a_t}{\partial \beta_t}\right)_{\xi=1}}{\left(\frac{\partial a_t}{\partial \beta_s}\right)_{\xi=1}} = \frac{\left(\frac{\partial \beta_s}{\partial \xi}\right)_{\xi=1}}{\left(\frac{\partial \beta_t}{\partial \xi}\right)_{\xi=1}}$$

$$\begin{pmatrix} \frac{\partial \beta_s}{\partial a_t} & \frac{\partial \beta_t}{\partial a_t} \\ \frac{\partial \beta_s}{\partial \xi} & \frac{\partial \beta_t}{\partial \xi} \end{pmatrix} = \frac{1}{\left(\frac{\partial \xi}{\partial \beta_t}\right)\left(\frac{\partial a_t}{\partial \beta_s}\right) - \left(\frac{\partial \xi}{\partial \beta_s}\right)\left(\frac{\partial a_s}{\partial \beta_t}\right)} \begin{pmatrix} \frac{\partial \xi}{\partial \beta_t} & -\frac{\partial \xi}{\partial \beta_s} \\ -\frac{\partial a_t}{\partial \beta_t} & \frac{\partial a_t}{\partial \beta_s} \end{pmatrix}$$

Using the reweighting method,

(β_s, β_t)-dependence of

the Polyakov loop susceptibility is measured.

Anisotropy coefficients

From

$$\left(\frac{\partial\beta_s}{\partial\xi}+\frac{\partial\beta_t}{\partial\xi}\right)_{a_t:\mathbf{fixed},\xi=1}=\frac{3}{2}a\frac{d\beta}{da}$$

$$r_t=\frac{\left(\frac{\partial\beta_s}{\partial\xi}\right)_{\xi=1}}{\left(\frac{\partial\beta_t}{\partial\xi}\right)_{\xi=1}}$$

$$\Rightarrow\quad \underline{\left(\frac{\partial\beta_s}{\partial\xi}\right)_{\xi=1}=\frac{3r_t}{2(1+r_t)}a\frac{d\beta}{da}}\qquad \underline{\left(\frac{\partial\beta_t}{\partial\xi}\right)_{\xi=1}=\frac{3}{2(1+r_t)}a\frac{d\beta}{da}}$$

Conventional combinations of the energy density and pressure

$$\frac{\Delta(\epsilon+p)}{T^4}=3N_t^4a\frac{d\beta}{da}\frac{r_t-1}{r_t+1}\{(\langle P_s\rangle_{\mathrm{hot}}-\langle P_s\rangle_{\mathrm{cold}})-(\langle P_t\rangle_{\mathrm{hot}}-\langle P_t\rangle_{\mathrm{cold}})\}$$

$$\frac{\Delta(\epsilon-3p)}{T^4}=3N_t^4a\frac{d\beta}{da}\{(\langle P_s\rangle_{\mathrm{hot}}-\langle P_s\rangle_{\mathrm{cold}})-(\langle P_t\rangle_{\mathrm{hot}}-\langle P_t\rangle_{\mathrm{cold}})\}$$

Simulation details

Pure SU(3) gauge theory

Standard plaquette action is used.

Pseudo-heat bath algorithm + over-relaxation.

Lattice size	# β	No. of Conf.
$48^3 \times 6$	1	201200
$64^3 \times 6$	4	442000
$48^3 \times 8$	6	1220000
$64^3 \times 8$	5	4585000
$64^3 \times 12$	3	624000
$96^3 \times 12$	5	1558000

Parts of the results in Ejiri, Iwasaki, Kanaya, Phys.Rev.D 58,094505 (1998) are used in this analysis.

Simulations are performed at 1–6 β points near transition point.

High statistics data: $\sim O(10^6)$

The multi-point reweighting method is used for the measurements.

lattice size			
N_s	N_t	β	#Conf.
48	6	5.89379	201200
64	6	5.893	30000
64	6	5.89379	150000
64	6	5.894	215000
64	6	5.895	47000
48	8	6.056	200000
48	8	6.058	200000
48	8	6.06	200000
48	8	6.062	200000
48	8	6.065	220000
48	8	6.067	200000
64	8	6.0585	95000
64	8	6.061	2060000
64	8	6.063	300000
64	8	6.065	510000
64	8	6.068	1620000
64	12	63335	324000
64	12	6.355	290000
64	12	63375	10000
96	12	6.332	45000
96	12	6.334	474000
96	12	6.335	534000
96	12	6.336	336000
96	12	6.338	169000

Measurement of the slope of the transition line r_t

$$r_t = \frac{d\beta_s}{d\beta_t} = \frac{\left(\frac{\partial \beta_s}{\partial \xi}\right)_{\xi=1}}{\left(\frac{\partial \beta_t}{\partial \xi}\right)_{\xi=1}}$$

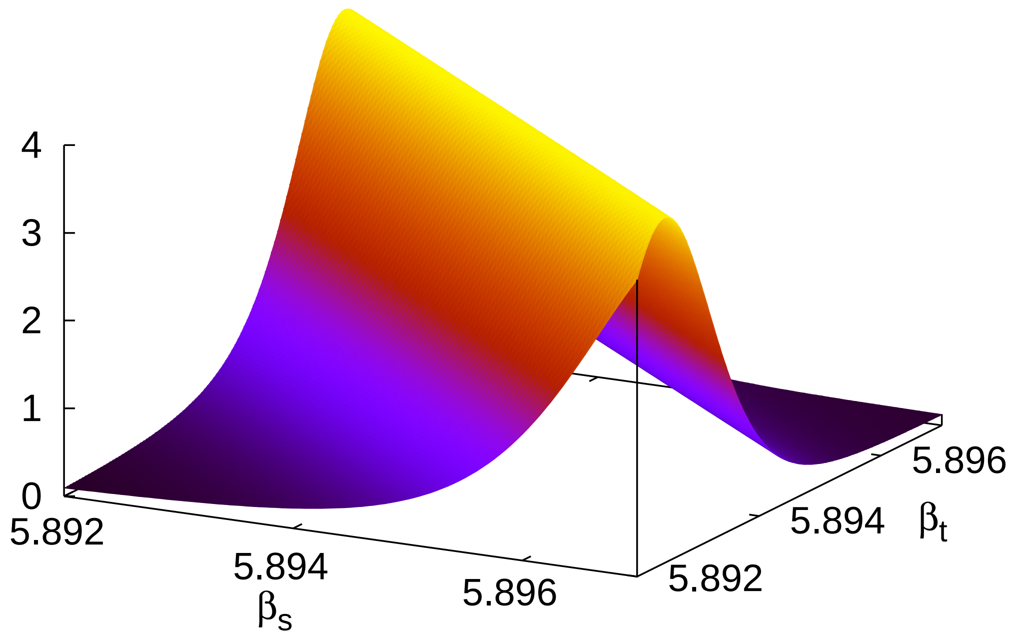
We used the reweighting method.
The slope r_t can be determined with sufficient accuracy.

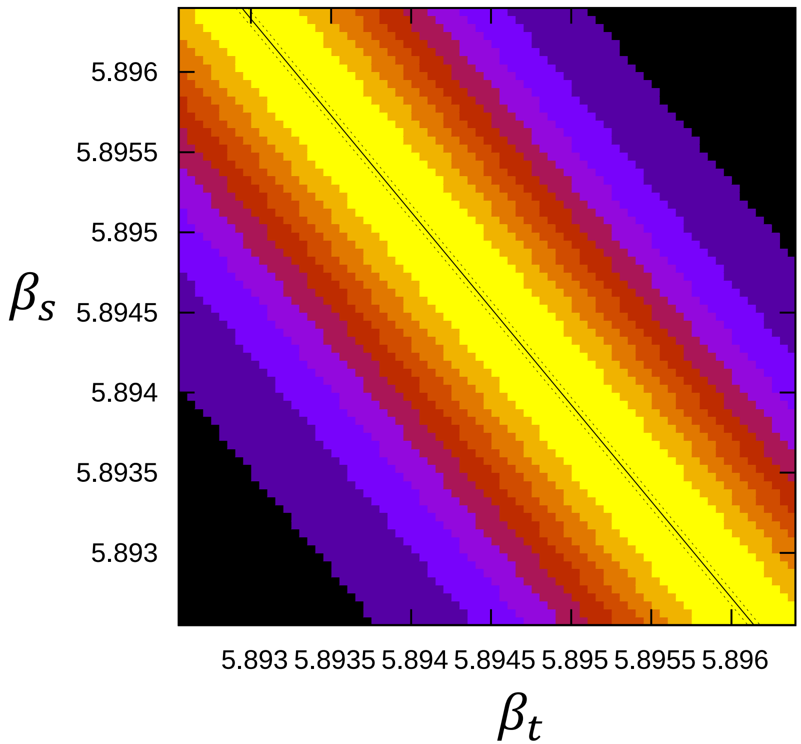
Lattice size	r_t
$48^3 \times 6$	-1.2020(39)
$64^3 \times 6$	-1.2022(52)
$48^3 \times 8$	-1.209(33)
$64^3 \times 8$	-1.255(37)
$64^3 \times 12$	-1.16(61)
$96^3 \times 12$	-1.204(53)

Order parameter: Polyakov loop $\Omega(x,t)$

Transition point:
Peak position of Polyakov loop susceptibility

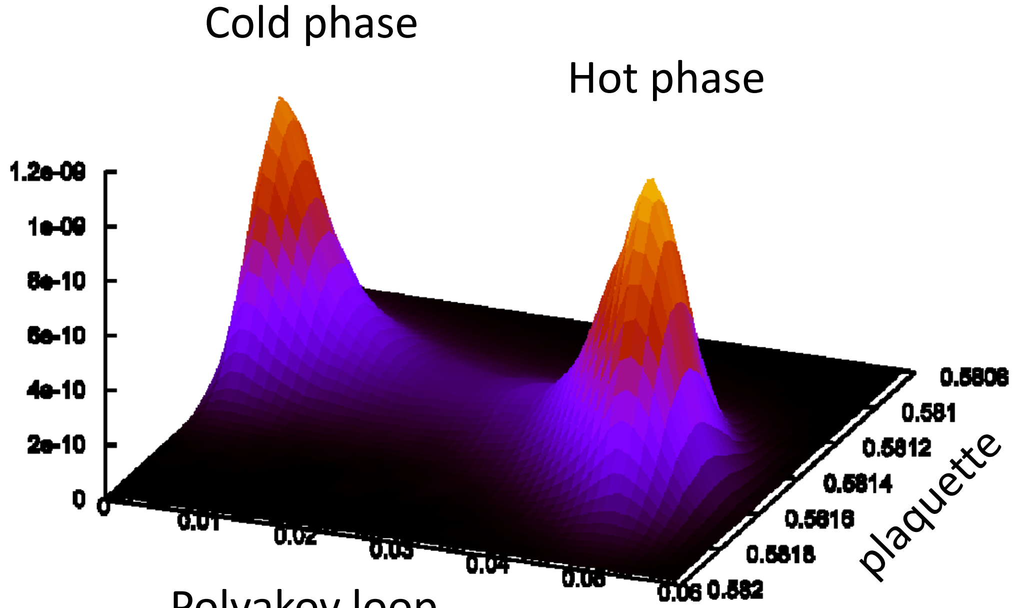
$$\chi_\Omega(\beta_s,\beta_t) = N_s^3\left(\langle\Omega^2\rangle_{(\beta_s,\beta_t)} - \langle\Omega\rangle^2_{(\beta_s,\beta_t)}\right)$$



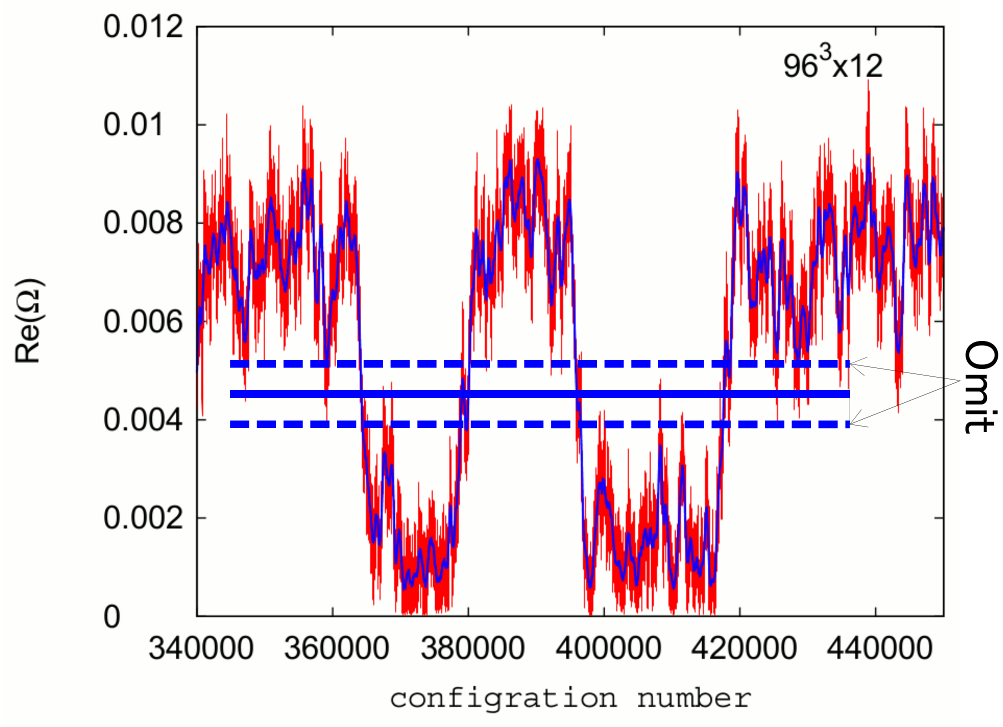


Separation of the hot and cold phases

Histogram



Time history of the Polyakov loop



- We identify the phase by the Polyakov loop.
- Two peaks in the histogram.
- Flip-flops between two phases.
- Mixed configurations are rare. (We omit mixed configurations.)

Vanishing pressure gap $\Delta p = 0$

$$\frac{\Delta p}{T^4} = N_t^4 \left\{ \frac{\partial \beta_s}{\partial \xi} (\langle P_s \rangle_{\text{hot}} - \langle P_s \rangle_{\text{cold}}) + \frac{\partial \beta_t}{\partial \xi} (\langle P_t \rangle_{\text{hot}} - \langle P_t \rangle_{\text{cold}}) \right\} = 0$$

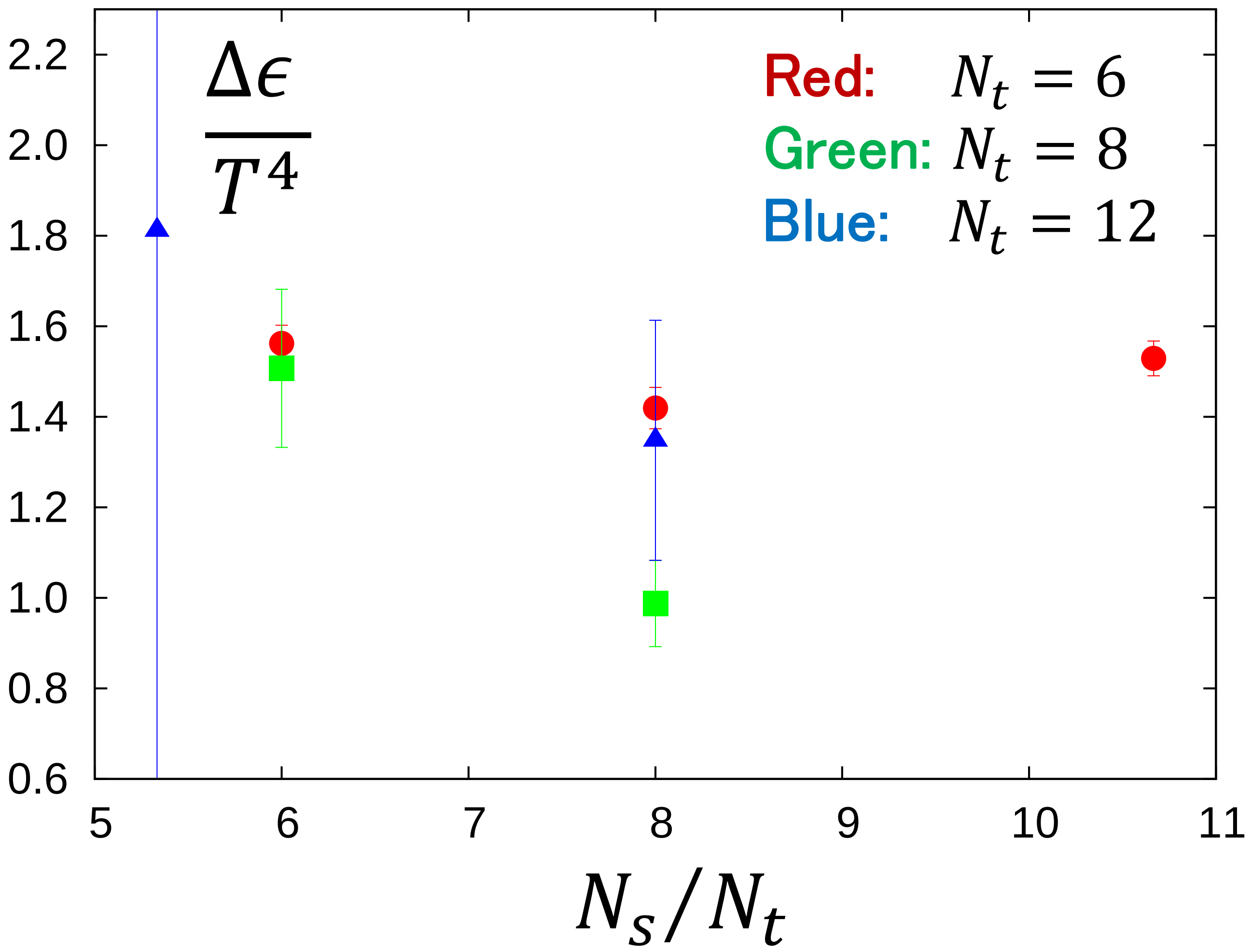
Condition for $\Delta p = 0$

$$\frac{\frac{\partial \beta_s}{\partial \xi}}{\frac{\partial \beta_t}{\partial \xi}} = r_t = -\frac{\langle P_t \rangle_{\text{hot}} - \langle P_t \rangle_{\text{cold}}}{\langle P_s \rangle_{\text{hot}} - \langle P_s \rangle_{\text{cold}}}$$

lattice	r_t	$\frac{\langle P_t \rangle_{hot} - \langle P_t \rangle_{cold}}{\langle P_s \rangle_{hot} - \langle P_s \rangle_{cold}}$
$48^3 \times 6$	-1.2020(39)	1.216(50)
$64^3 \times 6$	-1.2022(52)	1.2053(38)
$48^3 \times 8$	-1.209(33)	1.204(14)
$64^3 \times 8$	-1.255(37)	1.2344(66)
$64^3 \times 12$	-1.16(61)	1.324(84)
$96^3 \times 12$	-1.204(53)	1.283(53)

The pressure gap is zero on each finite lattice.

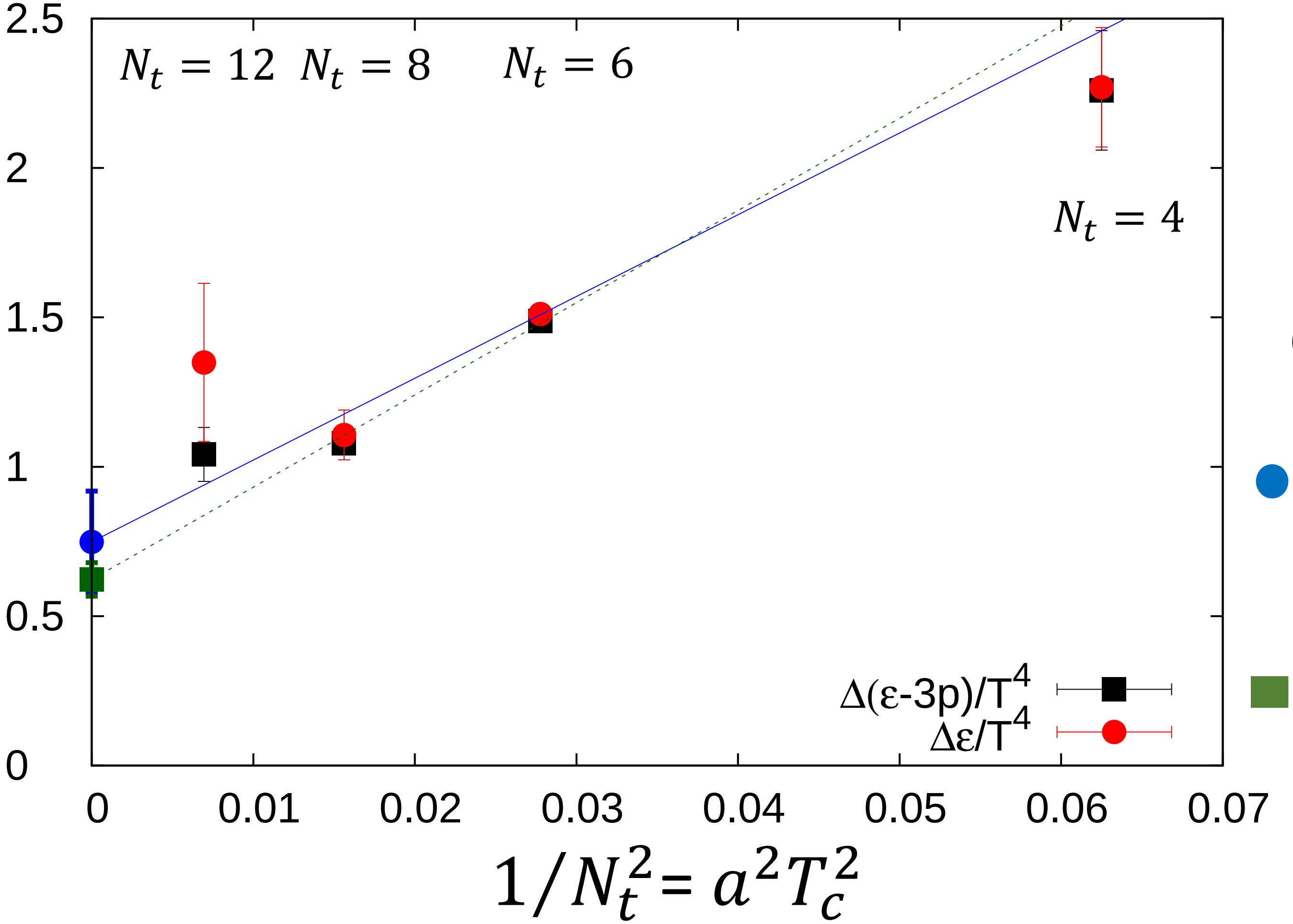
Volume-dependence of $\Delta\epsilon$



Because the correlation length is finite at the first order transition, $\Delta\epsilon$ should be constant for large volume (> correlation length).
We did a constant fit as a function of N_s/N_t for each N_t.

Continuum extrapolation of the latent heat

Fit the data at $N_t = 6, 8, 12$ with a linear function of $1/N_t^2$ assuming $O(a^2)$ error.



continuum limit

$$\frac{\Delta\epsilon}{T^4} = 0.75 \pm 0.17$$

$$\frac{\Delta\epsilon - 3\Delta p}{T^4} = 0.623 \pm 0.056$$

EoS by the Gradient Flow (H. Suzuki, 2013)

[H. Suzuki, Prog. Theor. Exp. Phys. 2014, 083B03 (2013); FlowQCD, Phys. Rev. D90, 011501(2014)]

- Smeared field strength: $F_{\mu\nu} \xrightarrow[\text{Gradient Flow}]{} G_{\mu\nu}$

- Dim. 4 operators: $E(t,x) = \frac{1}{4}\delta_{\mu\nu}G_{\rho\sigma}(t,x)G_{\rho\sigma}(t,x)$

$$U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\rho\sigma}(t,x)G_{\rho\sigma}(t,x)$$

- Energy momentum tensor

$$T^R_{\mu\nu} = \lim_{t\to 0}\left\{\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}\left[E(t,x) - \langle E(t,x)\rangle_0\right]\right\}$$

$$\alpha_U(t) = g^2[1 + 2b_0 s_1 g^2 + \cdots] \qquad \alpha_E(t) = \frac{1}{2b_0}[1 + 2b_0 s_2 g^2 + \cdots]$$

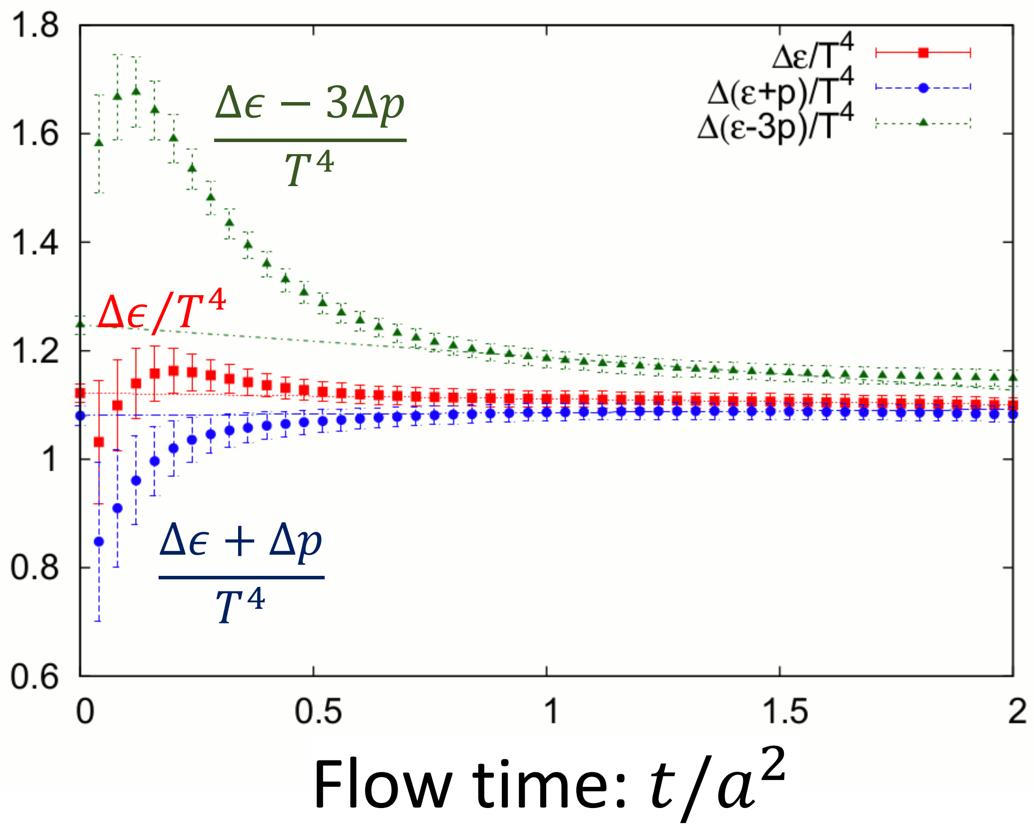
g: running coupling constant with $\overline{\mathrm{MS}}$ scheme based on the 4-loop beta function.

- Energy density and Pressure

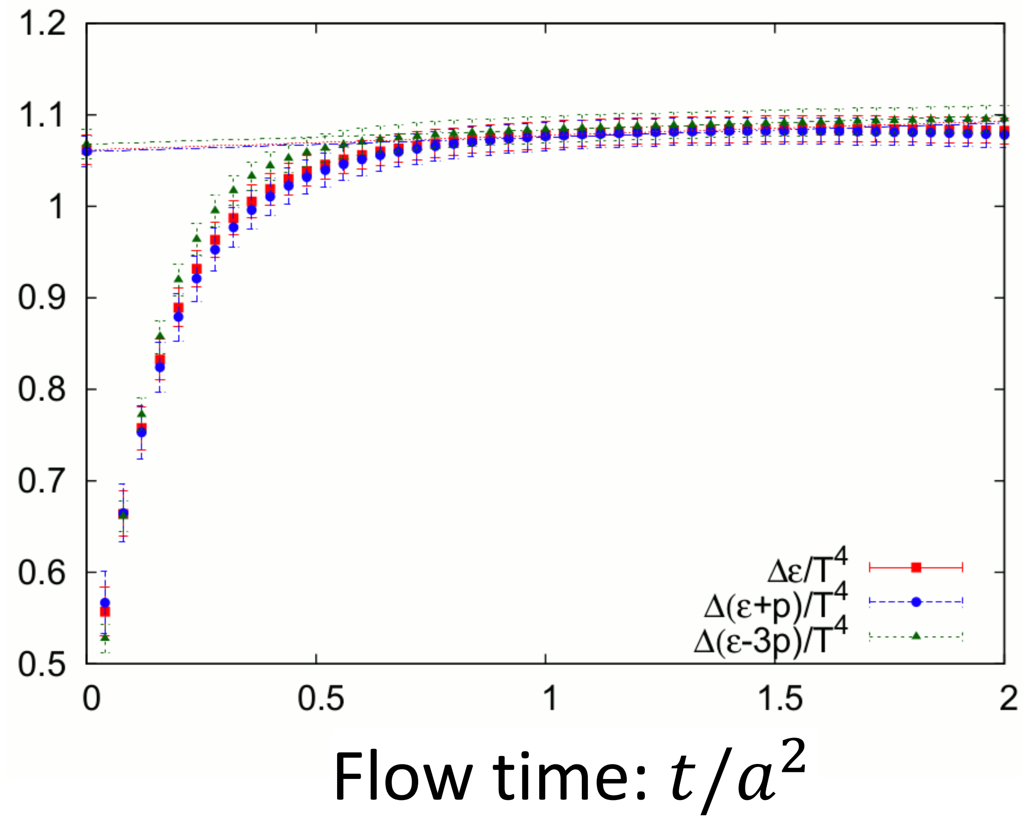
$$\epsilon = \langle T_{00}\rangle \qquad p = \frac{1}{3}\sum_i \langle T_{ii}\rangle$$

EoS by gradient flow method (preliminary)

$96^3 \times 12$ lattice



$G^2_{\mu\nu}$ defined by plaquette

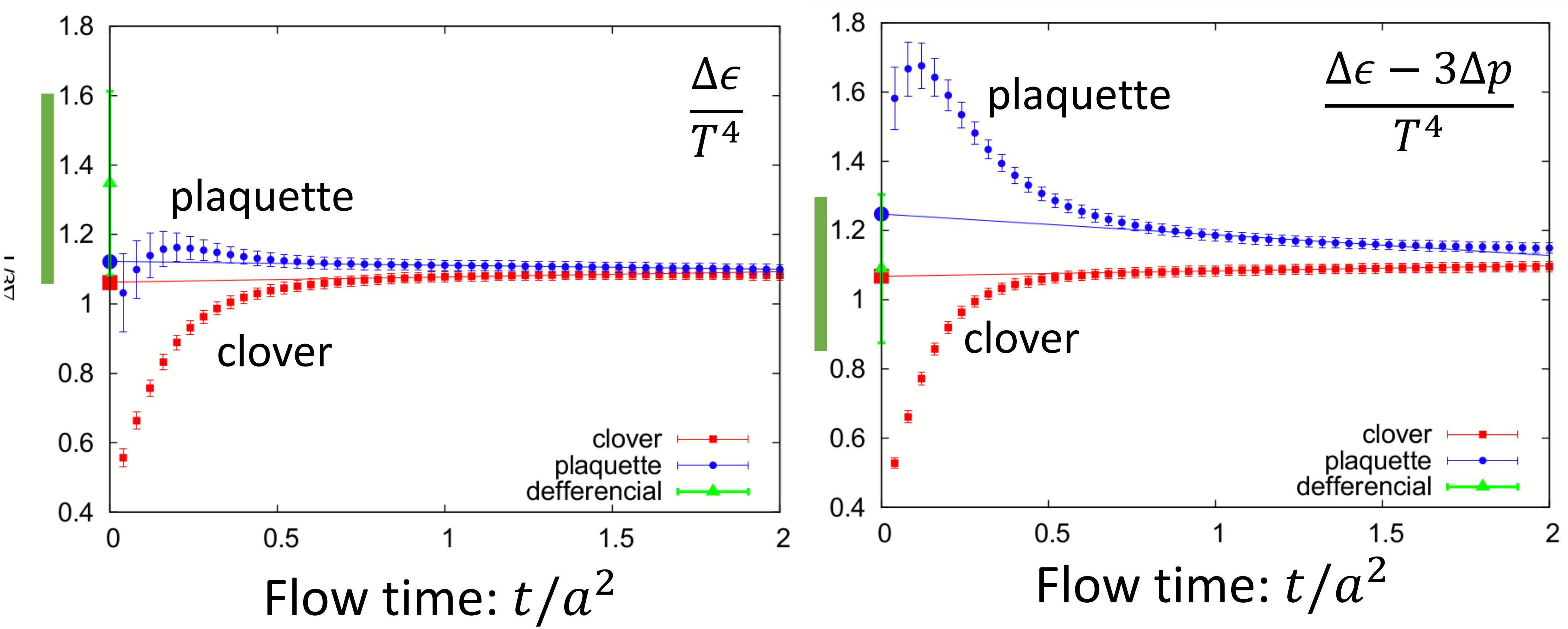


$G^2_{\mu\nu}$ defined by clover tem

- $\Delta\epsilon/T^4$ becomes constant at long flow time.
- As flow time proceeds, Δp vanishes. ($\Delta\epsilon - 3\Delta p \approx \Delta\epsilon + \Delta p$)

EoS by the gradient flow method (preliminary)

$96^3 \times 12$ lattice



- In the t=0 limit, the results by the gradient flow method and the derivative method are consistent within the error.

are the results by the derivative method on $96^3 \times 12$ lattice.

Conclusions and Outlook

- We study the equation of state at the first order phase transition of SU(3) gauge theory.
- Gaps of energy density and pressure are measured using the derivative method.
  - We confirmed that the pressure gap is zero on each finite lattice.
  - The result of the latent heat in the continuum limit is

$$\frac{\Delta\epsilon}{T^4} = 0.75 \pm 0.17 \qquad \frac{\Delta(\epsilon - 3p)}{T^4} = 0.623 \pm 0.056$$

- We tested the gradient flow method for the calculation of EoS.
  - As flow time increases, $\Delta\epsilon/T^4$ becomes constant and Δp vanishes.
  - In the t=0 limit, the latent heat is consistent with that by the derivative method on the finite lattice.