The QCD deconfinement critical point as a function of N_τ with $N_f = 2$ flavours of unimproved Wilson fermions

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Lattice 2016, 25th July 2016
Southampton

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Outline

T-m_q QCD phase diagram and the phase transition shift

Lattice setup and simulation details

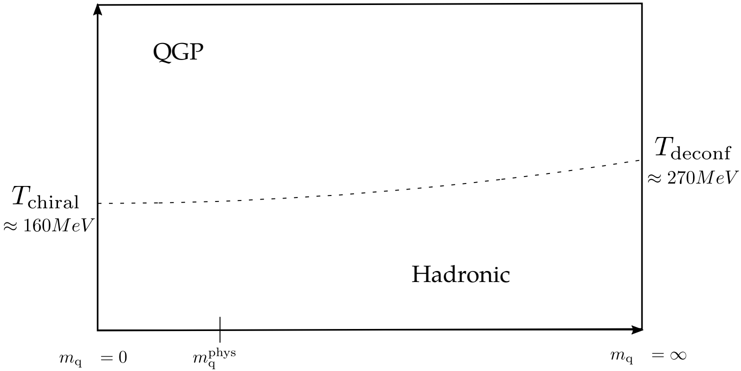
Approach to extracting the order of a phase transition

Preliminary results

Summary and perspectives

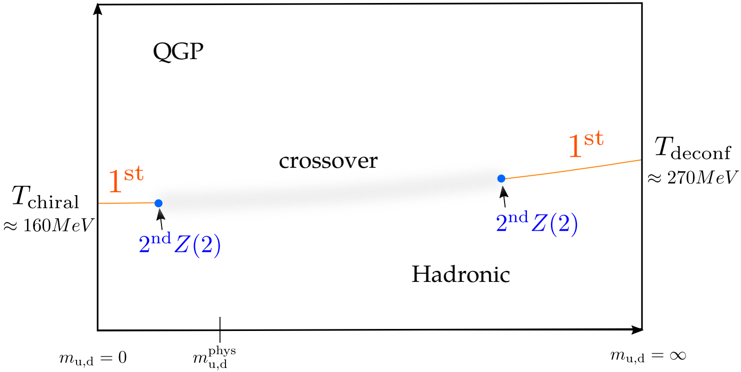
T-m_q QCD phase diagram

$\mu = 0$



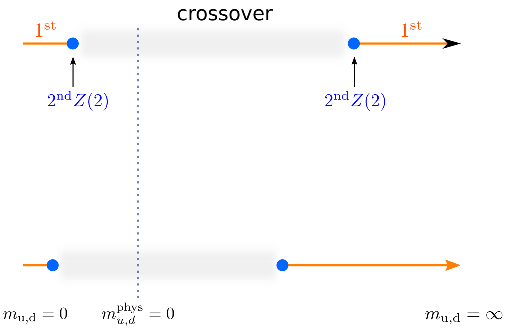
T-m_q QCD phase diagram

$\mu = 0$, $N_f = 2$ flavour studies on coarse lattices have shown:



- How does it look like in the continuum?

The 1st order region as function of the lattice spacing



$N_\tau \qquad a$

temporal size & lattice spacing

$N'_\tau > N_\tau \qquad a' < a$

O. Philipsen, C. Pinke, Phys. Rev. D 93 114507 (2016)
H. Saito, S. Ejiri, S. Aoki, T. Hatsuda, K. Kanaya, Y. Maezawa, H. Ohno and T. Umeda, Phys. Rev. D 84 054502 (2011)

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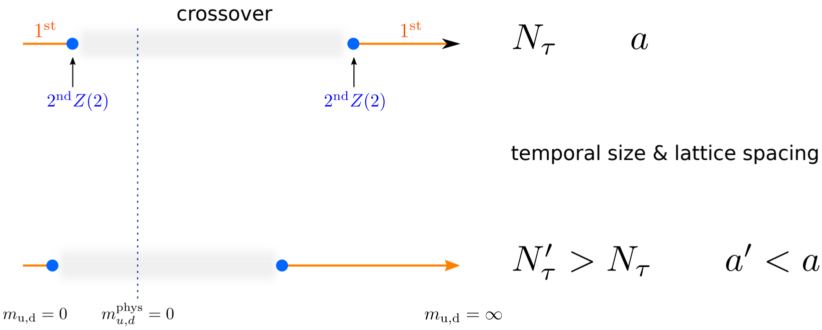

- Topic of this presentation: heavy quark mass region.
- Next talk (Alessandro Sciarra): light quark mass region.

O. Philipsen, C. Pinke, Phys. Rev. D 93 114507 (2016)
H. Saito, S. Ejiri, S. Aoki, T. Hatsuda, K. Kanaya, Y. Maezawa, H. Ohno and T. Umeda, Phys. Rev. D 84 054502 (2011)

Lattice setup / previous and ongoing studies

Former studies (heavy quark mass region):

- Wilson, $N_f = 2,\ 2+1,\ 3$
- Temporal lattice extent $N_\tau = 4$

  H. Saito, S. Ejiri, S. Aoki, T. Hatsuda, K. Kanaya, Y. Maezawa, H. Ohno and T. Umeda, Phys. Rev. D 84 054502 (2011)

Ongoing study:

- OpenCL based Hybrid Monte Carlo code CL²QCD

  A. Sciarra, M. Bach, O. Philipsen, C. Pinke (PoS Lattice2014)
  https://github.com/CL2QCD/cl2qcd.git
- $N_f = 2$ flavours of unimproved Wilson fermions
- Temporal lattice extent $N_\tau = 8$
- 6 κ values $\in [0.1100, 0.1350]$
- Up to 3 spatial lattice extents $N_\sigma \in [32, 40, 48]$ per κ.
- 3-4 β values per N_σ with 4 Markov chains per β
- $160k - 800k$ trajectories per β

Extracting the order of a phase transition

- Ratio of moments of fluctuations $\delta |L| = |L| - \langle |L| \rangle$:

$$B_n(\beta) = \frac{\langle (\delta |L|)^n \rangle}{\langle (\delta |L|)^2 \rangle^{\frac{n}{2}}} \qquad \lim_{V \to \infty} B_4(\beta_c) = \begin{cases} 1 & , 1^{st} \text{ order} \\ 1.604 & , 2^{nd} \text{ order Z(2)} \\ 3 & , \text{crossover} \end{cases}$$

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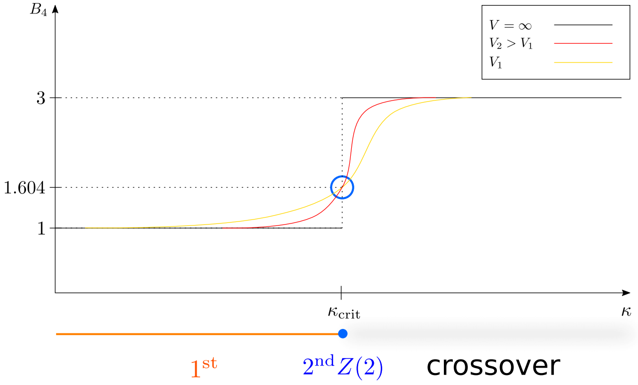
- For each κ ...
  - For each spatial extent N_σ ...
    1. Scan in temperature $T = 1/\left(a(\beta)\,N_\tau\right)$.
    2. Compute $B_3\,(\beta)$ (Skewness).
    3. Locate β_c via $B_{3,\mathrm{rew}}\,(\beta_c) = 0$.
       $\longrightarrow$ Employ Ferrenberg Swendsen reweighting to interpolate and smoothing data of $B_3\,(\beta)$.
    4. Compute $B_4\,(\beta_c)$ (Binder cumulant).

Finite Size Scaling

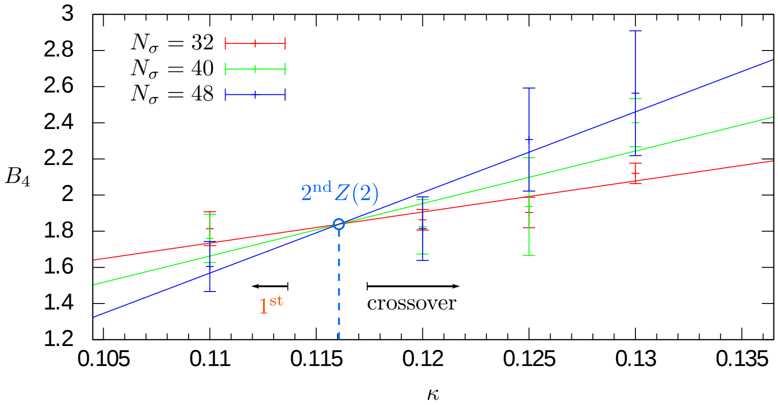
- To locate the 2nd order $Z(2)$ point plot B_4 vs κ:



$$B_4(\kappa, N_\sigma) \simeq B_4(\kappa_c, \infty) + a(\kappa - \kappa_c) N_\sigma^{1/\nu}$$

Preliminary results

$N_f = 2 \qquad N_\tau = 8 \qquad \mu = 0$



- $B_4^{\text{fit}}(\kappa_c, \infty) = 1.8387(984)$
  - 2.4σ difference to correct B_4 value.

Physical lattice size and pion mass

κ	β_c	a [fm]	am_π	m_π [MeV]	T_c [MeV]
0.1100	6.0303	0.0895(5)			275(2)
0.1300	5.9491	0.0947(6)			260(2)

- Decrease in $\kappa \longrightarrow$ increase of $\beta_c \longrightarrow$ decrease of $a(\beta_c)$.
- Physical lattice size at smallest $N_\sigma = 32$:
  - $\kappa = 0.1100 \longrightarrow V_{\kappa_{\text{min}}} = 23.5(4)\ \text{fm}^3$.
  - $\kappa = 0.1300 \longrightarrow V_{\kappa_{\text{max}}} = 27.9(5)\ \text{fm}^3$.

$$\longrightarrow V_{\kappa_{\text{max}}} \approx 1.185 \cdot V_{\kappa_{\text{min}}}$$

- Need larger N_σ for smaller κ to suppress finite size effects on B_4.

$\longrightarrow$ Problem will become more severe with larger N_τ.

Physical lattice size and pion mass

κ	β_c	a [fm]	am_π	m_π [MeV]	T_c [MeV]
0.1100	6.0303	0.0895(5)	2.1310(6)	4690(28)	275(2)
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- Need larger N_σ for smaller κ to suppress finite size effects on B_4.

  $\longrightarrow$ Problem will become more severe with larger N_τ.
- We still have $1 < am_\pi \longrightarrow$ Larger N_τ needed.

Summary and Perspectives

Summary

- We study the phase structure at $\mu = 0$ in the heavy quark mass region for $N_f = 2$ flavours of unimproved Wilson fermions ...
- ... and aim to investigate the quantitative shift of the first order regions towards smaller quark masses.
- Study currently takes place on lattices with a temporal extent of $N_\tau = 8$ for which finite size effects are more severe compared to previous studies on lattices with $N_\tau < 8$.

Perspectives

- Simulations on larger lattices will become feasible soon with ...
  - ... improvement of code (currently ongoing work on further parallelization).
  - ... advances in computational resources.
- Expand study to $N_f = 3$ flavours.