Nucleon spin and quark content at the physical point

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with

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Outline

1 Wilson twisted mass lattice QCD

2 Hadron spectrum

3 Nucleon structure
   - Nucleon charges: $g_A, g_s, g_T$
   - First moments: $\langle x \rangle_q, \langle x \rangle_{\Delta q}, \langle x \rangle_{\delta q}$
   - Nucleon spin

4 Conclusions
Simulations by the European Twisted Mass Collaboration (ETMC)

We report on the analysis of an $N_f = 2$ ensemble of twisted mass plus a clover term simulated at a physical value of the pion mass, referred as the Physical ensemble, (ETMC) A. Abdel-Rehim et al. :1507.04936, 1507.05068, 1411.6842, 1311.4522

Parameters: lattice size $48^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 0.1312(13)$ GeV


- Automatic $O(a)$ improvement
- No operator improvement needed, renormalization simplified $\rightarrow$ important for hadron structure
Wilson twisted mass lattice QCD
Simulations by the European Twisted Mass Collaboration (ETMC)

We report on the analysis of an $N_f = 2$ ensemble of twisted mass plus a clover term simulated at a physical value of the pion mass, referred as the Physical ensemble, (ETMC) A. Abdel-Rehim et al. 
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Parameters: lattice size $48^3 \times 96$, $a = 0.093(1) \text{ fm}$, $m_\pi = 0.1312(13) \text{ GeV}$

- Use exact deflation to speed-up the inversions and do multiple sources on each gauge configuration
- Use domain decomposition multi-grid (DD-\(\alpha\)AMG) adapted for twisted mass fermions, Talk by S. Bacchio, Algorithms and Machines, Wednesday, 10:00

\[
\sim e^{(m_\pi - \frac{3}{2} m_\pi) t_s}
\]
Hadron spectrum

Results on hadron masses using the physical ensemble, 357 configurations

Recent results by other collaborations:

- **Hyperons:**
  - S. Durr *et al.*, Science 322, 1224 (2008), 0906.3599

- **Charmed baryons:**
  - Y. Namekawa *et al.* (PACS-CS Collaboration), Phys.Rev. D87(9), 094512 (2013), 1301.4743
Nucleon charges: $g_A$, $g_S$, $g_T$

- Scalar operator: $O_S^a = \bar{\psi}(x) \frac{\tau^a}{2} \psi(x)$
- Axial-vector operator: $O_A^a = \bar{\psi}(x) \gamma^\mu \gamma_5 \frac{\tau^a}{2} \psi(x)$
- Tensor operator: $O_T^a = \bar{\psi}(x) \sigma^{\mu\nu} \frac{\tau^a}{2} \psi(x)$

Extract from ratio: $\langle N(\vec{p}') O X N(\vec{p}) \rangle |_{q^2=0}$ to obtain $g_S$, $g_A$, $g_T$

- Isovector combination has no disconnected contributions;
- $g_A$ well-known experimentally;
- Predict $g_T$, to be measured at JLab;
- Predict $g_S$
Nucleon charges: \( g_A \)

- \( N_f = 2 \) twisted mass plus clover, \( 48^3 \times 96, a = 0.093(1) \text{ fm, } m_\pi = 131 \text{ MeV} \)
- 9264 statistics
- 3 sink-source time separations ranging from 0.9 fm to 1.5 fm

Isovector axial charge (\( t_s \) is the sink-source time separation and \( t_s^{\text{low}} \) is the lowest value of \( t_s \) used in the fits)

At the physical point we find from the plateau method: \( g_A = 1.22(3)(2) \), where the first error is statistical and the second systematic determined by the difference between the values from the plateau and two-state fits.

A. Abdel-Rehim et al. (ETMC):1507.04936, 1507.05068, 1411.6842, 1311.4522
Updated results using $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1) \text{ fm}$ with $\sim 9260$ statistics for $t_s/a = 10, 12, 14$, $\sim 48000$ for $t_s/a = 16$ and $\sim 70000$ for $t_s/a = 18$.

At the physical point we find from the plateau method:

- $g_{S}^{\text{isov}} = 0.93(25)(33)$
- $g_{T}^{\text{isov}} = 1.00(2)(1)$

where the first error is statistical and the second systematic determined by the difference between the values from the plateau and two-state fits.
Summary of results on nucleon charges: $g_A$, $g_S$, $g_T$

Isovector

- $g_A$ at the physical point requires further study for larger $t_s$. Important to keep constant error $\rightarrow$ we need large statistics

- New analysis of COMPASS and Belle data:
  $g_T^{u-d} = 0.81(44)$, M. R. A. Courtoy, A. Bacchetta, M. Guagnellia, arXiv: 1503.03495

- For $g_S$ increasing the sink-source time separation to $\sim 1.5$ fm is crucial but more statistics are needed to settle its value.
Disconnected contributions to $g_A^q$

Updated results using $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm

Disconnected isoscalar axial charge

We find from the plateau method:

- $g_A^{u+d} = -0.15(2)$ with 854,400 statistics
- Combining with the isovector we find: $g_A^u = 0.828(21)$, $g_A^d = -0.387(21)$
- $g_A^s = -0.042(10)$ with 861,200 statistics

Strange axial charge, Talk by A. Vaquero, Hadron Structure, Thursday 17:30
Scalar $g_S$ and Tensor $g_T$ charges

Updated results using $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm with $\sim 9260$ statistics for $t_s/a = 10, 12, 14$, $\sim 48000$ for $t_s/a = 16$ and $\sim 70000$ for $t_s/a = 18$.

At the physical point we find from the plateau method:

- $g_{SU}^{ud} = 8.25(51)(13)$ (conn);
- $g_{Tu}^{ud} = 0.584(16)(17)$ (conn);

where the first error is statistical and the second error on the connected is the systematic determined by the difference between the values from the plateau and two-state fits.

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**Scalar $g_S$ and Tensor $g_T$ charges**

Updated results using $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm with $\sim 9260$ statistics for $t_s/a = 10, 12, 14, \sim 48000$ for $t_s/a = 16$ and $\sim 70000$ for $t_s/a = 18$.

![Graph showing the results of the calculations](image)

Disconnected isoscalar scalar charge, $\overline{MS}$ at 2 GeV

At the physical point we find from the plateau method:

- $g_{u+d}^S = 8.25(51)(13)$ (conn); $1.25(26)$ (disconn) → $g_S^u = 5.21(31), g_S^d = 4.28(31)$
- $g_{u+d}^T = 0.584(16)(17)$ (conn); $0.0007(11)$ (disconn) → $g_T^u = 0.795(13), g_T^d = -0.210(13)$

where the first error is statistical and the second error on the connected is the systematic determined by the difference between the values from the plateau and two-state fits.

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The quark content of the nucleon

- $\sigma_f \equiv m_f \langle N|\bar{q}_f q_f|N\rangle$: measures the explicit breaking of chiral symmetry
  Largest uncertainty in interpreting experiments for dark matter searches - Higgs-nucleon coupling depends on $\sigma$, J. Ellis, K. Olive, C. Savage, arXiv:0801.3656

- In lattice QCD:
  - Feynman-Hellmann theorem: $\sigma_f = m_f \frac{\partial m_N}{\partial m_f}$
  - Similarly $\sigma_s = m_s \frac{\partial m_N}{\partial m_s}$

With our increased statistics we find $\sigma_{\pi N} = 36(2)$ MeV, $\sigma_s = 37(8)$ MeV, $\sigma_c = 83(17)$ MeV
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First moments of PDFs for the nucleon

- **Unpolarized moment:** $\langle x \rangle_q = \int_0^1 dx \ x \left[ q(x) + \bar{q}(x) \right]$ 
  \[ q(x) = q(x)_{\downarrow} + q(x)_{\uparrow} \]

- **Helicity moment:** $\langle x \rangle_{\Delta q} = \int_0^1 dx \ x \left[ \Delta q(x) - \Delta \bar{q}(x) \right]$ 
  \[ \Delta q(x) = q(x)_{\downarrow} - q(x)_{\uparrow} \]

- **Transversity moment:** $\langle x \rangle_{\delta q} = \int_0^1 dx x \left[ \delta q(x) + \delta \bar{q}(x) \right]$ 
  \[ \delta q(x) = q(x)_{\perp} + q(x)_{\top} \]
Momentum fraction \( \langle x \rangle_{u-d} \), helicity \( \langle x \rangle_{u-d} \) and transversity \( \langle x \rangle_{u-d} \)

Updated results using \( N_f = 2 \) twisted mass fermions with a clover term at a physical value of the pion mass, \( 48^3 \times 96 \) and \( a = 0.093(1) \) fm with \( \sim 9260 \) statistics for \( t_s/a = 10, 12, 14, \sim 48000 \) for \( t_s/a = 16 \) and \( \sim 70000 \) for \( t_s/a = 18 \).

At the physical point we find in the \( \overline{\text{MS}} \) at 2 GeV from the plateau method:

- \( \langle x \rangle_{u-d} = 0.206(14)(5) \) and \( \langle x \rangle_{u+d+s} = 0.63(12) \).

\( \langle x \rangle_{u+d+s} \) is perturbatively renormalized to one-loop due to its mixing with the gluon operator.

- \( \langle x \rangle_{\Delta u-\Delta d} = 0.259(9)(10) \)
- \( \langle x \rangle_{\delta u-\delta d} = 0.273(17)(18) \)

The first error is statistical and the second systematic determined by the difference between the values from the plateau and two-state fits.

A. Abdel-Rehim et al. (ETMC):1507.04936, 1507.05068, 1411.6842, 1311.4522
Gluon content of the nucleon

- Gluons carry a significant amount of momentum and spin in the nucleon
  - Compute gluon momentum fraction: \( \langle x \rangle_g = A_{20}^g \)
  - Compute gluon spin: \( J_g = \frac{1}{2} (A_{20}^g + B_{20}^g) \)

- Nucleon matrix of the gluon operator: \( O_{\mu \nu} = -G_{\mu \rho} G_{\nu \rho} \)
  - gluon momentum fraction extracted from
    \[ \langle N(0) | O_{44} - \frac{1}{3} O_{jj} | N(0) \rangle = m_N \langle x \rangle_g \]

- Disconnected correlation function, known to be very noisy
  \( \Rightarrow \) we employ several steps of stout smearing in order to remove fluctuations in the gauge field

- Results are computed on the \( N_f = 2 \) ensemble at the physical point, \( m_\pi = 131 \text{ MeV}, \ a = 0.093 \text{ fm}, \ V = 48^3 \times 96 \), A. Abdel-Rehim et al. (ETMC):1507.04936

- The methodology was tested for \( N_f = 2 + 1 + 1 \) twisted mass at \( m_\pi = 373 \text{ MeV} \), C. Alexandrou, V. Drach, K. Hadjiyiannakou, K. Jansen, B. Kostrzewa, C. Wiese, PoS LATTICE2013 (2014) 289
Results for the gluon content

- 2094 gauge configurations with 100 different source positions each → more than 200,000 measurements
- Due to mixing with the quark singlet operator, the renormalization and mixing coefficients had to be extracted from a one-loop perturbative lattice calculation, M. Constantinou and H. Panagopoulos
- \( \langle x \rangle_{g, \text{bare}} = 0.318(24) \) \( \xrightarrow{\text{Renormalization}} \) \( \langle x \rangle_{g} = Z_{gg} \langle x \rangle_{g} + Z_{gq} \langle x \rangle_{u+d+s} = 0.317(24)(13) \). The renormalization is perturbatively done using two-levels of stout smearing. The systematic error is the difference between using one- and two-levels of stout smearing.
- Momentum sum is satisfied: \( \sum_q \langle x \rangle_q + \langle x \rangle_g = \langle x \rangle_{u+d}^{CI} + \langle x \rangle_{u+d+s}^{DI} + \langle x \rangle_g = 0.945(126)(13) \)
Nucleon spin?

Spin sum: \( \frac{1}{2} = \sum_q \left( \frac{1}{2} \Delta \Sigma^q + L^q \right) + J^G \)

\[ J^q = A_{20}^q(0) + B_{20}^q(0) \text{ and } \Delta \Sigma^q = g_A^q \]

Disconnected contribution using \( \mathcal{O}(210500) \) statistics

\[ \Rightarrow \text{Total spin for } u\text{-quarks } J^u < 0.3 \text{ and for } d\text{-quark } J^d \sim 0 \]

\( \Delta \Sigma^{u,d} \) consistent with experimental values after disconnected contributions are included
Nucleon spin?

Spin sum: \( \frac{1}{2} = \sum_q \left( \frac{1}{2} \Delta \Sigma^q + L^q \right) + J^G \)

\[ J^q = A_{20}^q(0) + B_{20}^q(0) \text{ and } \Delta \Sigma^q = g_A^q \]

Disconnected contribution using \( \mathcal{O}(210500) \) statistics

Disconnected contributions affect the value of \( L^q \)

Preliminary results: \( L^u \) and \( L^d \) increase if disconnected are included
Conclusions

- Results at the physical point are now directly accessible
  High statistics and careful cross-checks are needed $\rightarrow$ noise reduction techniques are crucial e.g. AMA, TSM, smearing etc

- Evaluation of quark loop diagrams has become feasible even at the physical point!

- Confirmation of experimentally known quantities such as $g_A$ will enable reliable predictions of others $\rightarrow$ provide insight into the structure of hadrons and input that is crucial for new physics such as the nucleon $\sigma$-terms, $g_s$ and $g_T$
Thank you for your attention