Nucleon spin and quark content at the physical point

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with

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Outline

Wilson twisted mass lattice QCD

Simulations by the European Twisted Mass Collaboration (ETMC)

We report on the analysis of an $N_f = 2$ ensemble of twisted mass plus a clover term simulated at a physical value of the pion mass, referred as the *Physical ensemble*, (ETMC) A. Abdel-Rehim *et al.* :1507.04936, 1507.05068, 1411.6842, 1311.4522

Parameters: lattice size $48^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 0.1312(13)$ GeV

Wilson tmQCD at maximal twist, R. Frezzotti, G. C. Rossi, JHEP 0408 (2004) 007

- Automatic $O(a)$ improvement
- No operator improvement needed, renormalization simplified $\rightarrow$ important for hadron structure

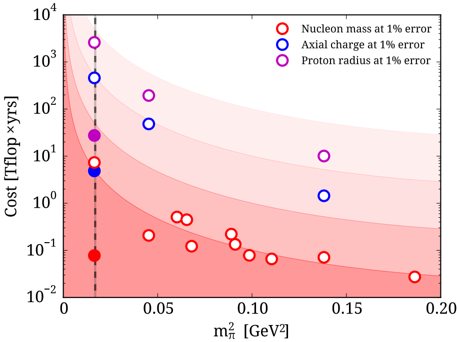
Wilson twisted mass lattice QCD

Simulations by the European Twisted Mass Collaboration (ETMC)

We report on the analysis of an $N_f = 2$ ensemble of twisted mass plus a clover term simulated at a physical value of the pion mass, referred as the *Physical ensemble*, (ETMC) A. Abdel-Rehim *et al.* :1507.04936, 1507.05068, 1411.6842, 1311.4522

Parameters: lattice size $48^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 0.1312(13)$ GeV

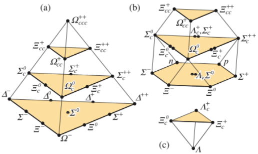
- Use exact deflation to speed-up the inversions and do multiple sources on each gauge configuration
- Use domain decomposition multi-grid (DD-αAMG) adapted for twisted mass fermions, Talk by S. Bacchio, Algorithms and Machines, Wednesday, 10:00

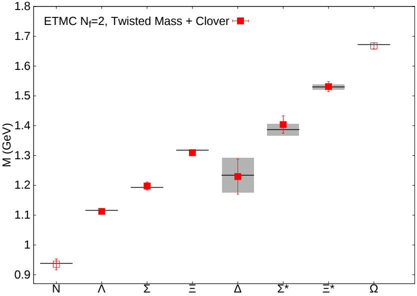


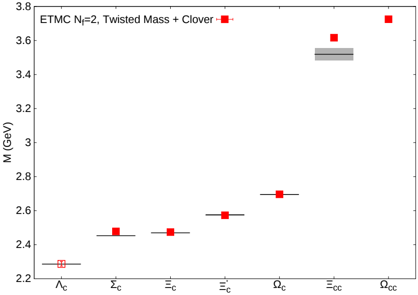
$\sim e^{(m_P - \frac{3}{2} m_\pi) t_s}$

Hadron spectrum

Results on hadron masses using the physical ensemble, 357 configurations







Recent results by other collaborations:

- Hyperons:
  - W. Bietenholz *et al.* (QCDSF-UKQCD) et al., Phys.Rev. D84, 054509 (2011), 1102.5300.
  - S. Durr *et al.*, Science 322, 1224 (2008), 0906.3599
- Charmed baryons:
  - P. Perez-Rubio, S. Collins, and G. S. Bali, arXiv:1503.08440
  - Y. Namekawa *et al.* (PACS-CS Collaboration), Phys.Rev. D87(9), 094512 (2013), 1301.4743
  - R. A. Briceno, H.-W. Lin, and D. R. Bolton, Phys.Rev. D86, 094504 (2012), 1207.3536.
  - L. Liu, H.-W. Lin, K. Orginos, and A. Walker-Loud, Phys.Rev. D81, 094505 (2010), 0909.3294

Nucleon charges: g_A, g_s, g_T

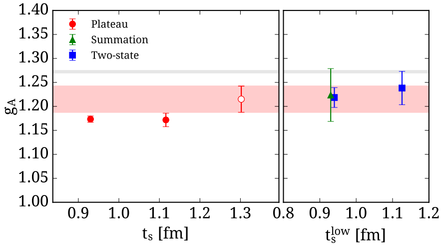
- scalar operator: $\mathcal{O}_S^a = \bar{\psi}(x)\frac{\tau^a}{2}\psi(x)$
- axial-vector operator: $\mathcal{O}_A^a = \bar{\psi}(x)\gamma^\mu\gamma_5\frac{\tau^a}{2}\psi(x)$
- tensor operator: $\mathcal{O}_T^a = \bar{\psi}(x)\sigma^{\mu\nu}\frac{\tau^a}{2}\psi(x)$

$\Longrightarrow$ extract from ratio: $\langle N(\vec{p}')\mathcal{O}_X N(\vec{p})\rangle|_{q^2=0}$ to obtain g_s, g_A, g_T

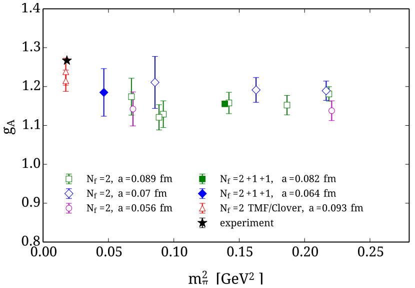
- isovector combination has no disconnected contributions;
- g_A well-known experimentally;
- Predict g_T, to be measured at JLab;
- Predict g_S

Nucleon charges: g_A

- $N_f = 2$ twisted mass plus clover, $48^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 131$ MeV
- 9264 statistics
- 3 sink-source time separations ranging from 0.9 fm to 1.5 fm



Isovector axial charge (t_s is the sink-source time separation and t_s^{low} is the lowest value of t_s used in the fits)



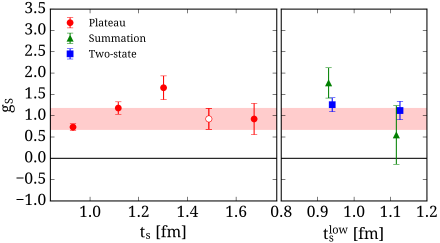
Results using twisted mass fermions

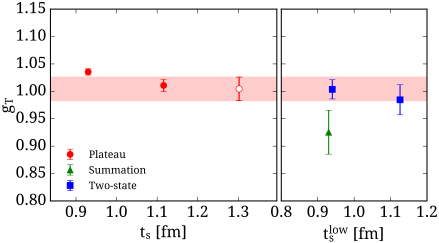
At the physical point we find from the plateau method: $g_A = 1.22(3)(2)$, where the first error is statistical and the second systematic determined by the difference between the values from the plateau and two-state fits.

A. Abdel-Rehim *et al.* (ETMC):1507.04936, 1507.05068, 1411.6842, 1311.4522

Nucleon charges: g_S, g_T

Updated results using $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm with ~ 9260 statistics for $t_s/a = 10, 12, 14$, ~ 48000 for $t_s/a = 16$ and ~ 70000 for $t_s/a = 18$.





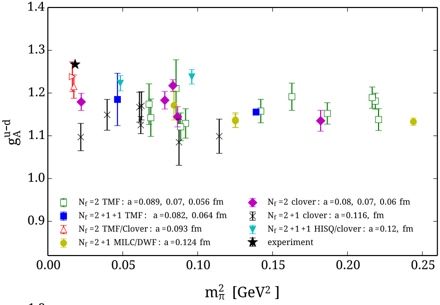
At the physical point we find from the plateau method:

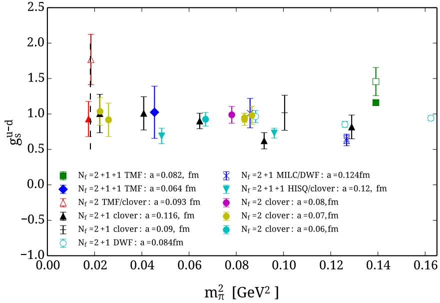
- $g_S^{\rm isov} = 0.93(25)(33)$
- $g_T^{\rm isov} = 1.00(2)(1)$

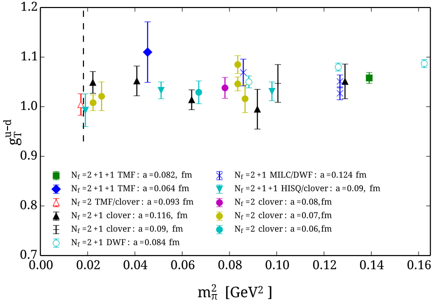
where the first error is statistical and the second systematic determined by the difference between the values from the plateau and two-state fits.

Summary of results on nucleon charges: g_A, g_S, g_T

Isovector



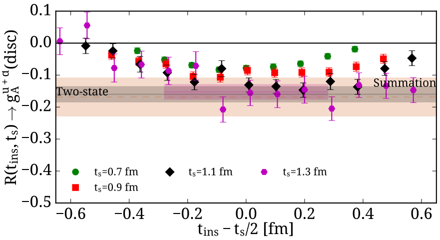




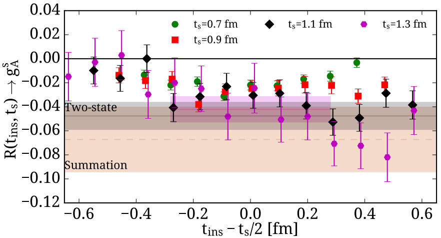
- g_A at the physical point requires further study for larger t_s. Important to keep constant error $\rightarrow$ we need large statistics
- New analysis of COMPASS and Belle data: $g_T^{u-d} = 0.81(44)$, M. R. A. Courtoy, A. Bacchettad, M. Guagnellia, arXiv: 1503.03495
- For g_S increasing the sink-source time separation to ~ 1.5 fm is crucial but more statistics are needed to settle its value.

Disconnected contributions to g_A^q

Updated results using $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm



Disconnected isoscalar axial charge



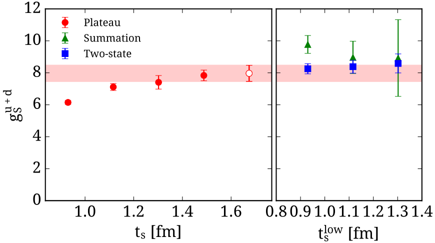
Strange axial charge, Talk by A. Vaquero, Hadron Structure, Thursday 17:30

We find from the plateau method:

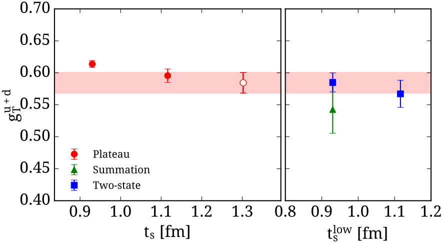
- $g_A^{u+d} = -0.15(2)$ with 854,400 statistics
- Combining with the isovector we find: $g_A^u = 0.828(21)$, $g_A^d = -0.387(21)$
- $g_A^s = -0.042(10)$ with 861,200 statistics

Scalar g_S and Tensor g_T charges

Updated results using $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm with ~ 9260 statistics for $t_s/a = 10, 12, 14$, ~ 48000 for $t_s/a = 16$ and ~ 70000 for $t_s/a = 18$.



Connected isoscalar scalar charge, $\overline{\text{MS}}$ at 2 GeV



Connected isoscalar tensor charge, $\overline{\text{MS}}$ at 2 GeV

At the physical point we find from the plateau method:

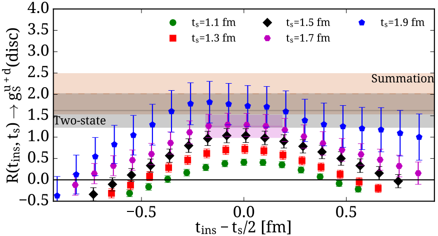
- g_S^{u+d}= 8.25(51)(13) (conn);
- g_T^{u+d}=0.584(16)(17) (conn);

where the first error is statistical and the second error on the connected is the systematic determined by the difference between the values from the plateau and two-state fits.

A. Abdel-Rehim *et al.* (ETMC):1507.04936, 1507.05068, 1411.6842, 1311.4522

Scalar g_S and Tensor g_T charges

Updated results using $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm with ~ 9260 statistics for $t_s/a = 10, 12, 14$, ~ 48000 for $t_s/a = 16$ and ~ 70000 for $t_s/a = 18$.



Disconnected isoscalar scalar charge, $\overline{\text{MS}}$ at 2 GeV

At the physical point we find from the plateau method:

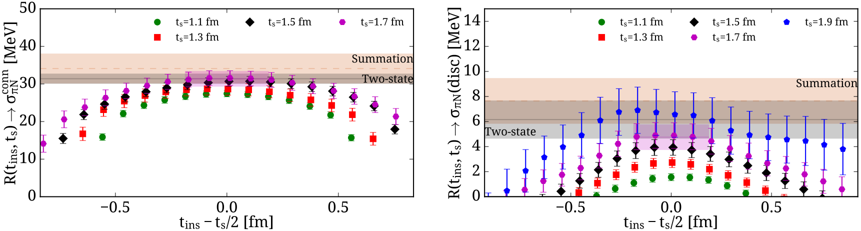
- g_S^{u+d}= 8.25(51)(13) (conn); 1.25(26) (disconn) $\rightarrow$ $g_S^u = 5.21(31)$, $g_S^d = 4.28(31)$
- g_T^{u+d}=0.584(16)(17) (conn); 0.0007(11) (disconn) $\rightarrow$ $g_T^u = 0.795(13)$, $g_T^d = -0.210(13)$

where the first error is statistical and the second error on the connected is the systematic determined by the difference between the values from the plateau and two-state fits.

A. Abdel-Rehim *et al.* (ETMC):1507.04936, 1507.05068, 1411.6842, 1311.4522

The quark content of the nucleon

- $\sigma_f \equiv m_f \langle N|\bar{q}_f q_f|N\rangle$: measures the explicit breaking of chiral symmetry
  Largest uncertainty in interpreting experiments for dark matter searches - Higgs-nucleon coupling depends on σ, J. Ellis, K. Olive, C. Savage, arXiv:0801.3656
- In lattice QCD:
  - Feynman-Hellmann theorem: $\sigma_l = m_l \frac{\partial m_N}{\partial m_l}$
    Similarly $\sigma_s = m_s \frac{\partial m_N}{\partial m_s}$
  - Direct computation of the scalar matrix element, A. Abdel-Rehim *et al.* arXiv:1601.3656, PRL116 (2016) 252001



With our increased statistics we find $\sigma_{\pi N} = 36(2)$ MeV, $\sigma_s = 37(8)$ MeV, $\sigma_c = 83(17)$ MeV

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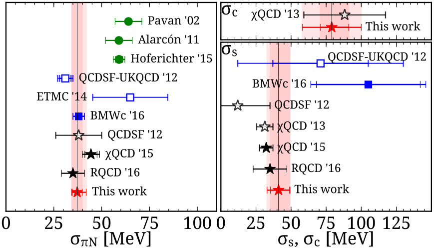

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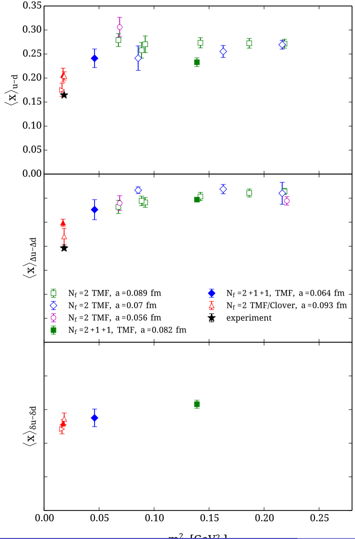
First moments of PDFs for the nucleon

- Unpolarized moment: $\langle x \rangle_q = \int_0^1 dx\, x\, [q(x) + \bar{q}(x)]$ $\qquad q(x) = q(x)_\downarrow + q(x)_\uparrow$
- Helicity moment: $\langle x \rangle_{\Delta q} = \int_0^1 dx\, x\, [\Delta q(x) - \Delta\bar{q}(x)]$ $\qquad \Delta q(x) = q(x)_\downarrow - q(x)_\uparrow$
- Transversity moment: $\langle x \rangle_{\delta q} = \int_0^1 dxx\, [\delta q(x) + \delta\bar{q}(x)]$ $\qquad \delta q(x) = q(x)_\perp + q(x)_\top$

Momentum fraction $\langle \mathbf{x} \rangle_{u-d}$, helicity $\langle \mathbf{x} \rangle_{´u-´d}$ and transversity $\langle \mathbf{x} \rangle_{u-d}$

Updated results using $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm with ~ 9260 statistics for $t_s/a = 10, 12, 14$, ~ 48000 for $t_s/a = 16$ and ~ 70000 for $t_s/a = 18$.



At the physical point we find in the $\overline{\text{MS}}$ at 2 GeV from the plateau method:

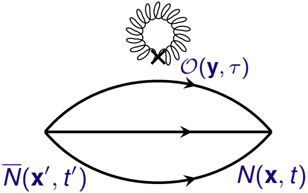
- $\langle x \rangle_{u-d} = 0.206(14)(5)$ and
  $\langle x \rangle_{u+d+s} = 0.63(12)$.
  $\langle x \rangle_{u+d+s}$ is perturbatively renormalized to one-loop due to its mixing with the gluon operator.
- $\langle x \rangle_{\Delta u - \Delta d} = 0.259(9)(10)$
- $\langle x \rangle_{\delta u - \delta d} = 0.273(17)(18)$

The first error is statistical and the second systematic determined by the difference between the values from the plateau and two-state fits.

A. Abdel-Rehim *et al.* (ETMC):1507.04936, 1507.05068, 1411.6842, 1311.4522

Gluon content of the nucleon

- Gluons carry a significant amount of momentum and spin in the nucleon
  - Compute gluon momentum fraction : $\langle x \rangle_g = A_{20}^g$
  - Compute gluon spin: $J_g = \frac{1}{2}(A_{20}^g + B_{20}^g)$
- Nucleon matrix of the gluon operator: $O_{\mu\nu} = -G_{\mu\rho}G_{\nu\rho}$
  $\rightarrow$ gluon momentum fraction extracted from
  $\langle N(0)|O_{44} - \frac{1}{3}O_{jj}|N(0)\rangle = m_N < x >_g$

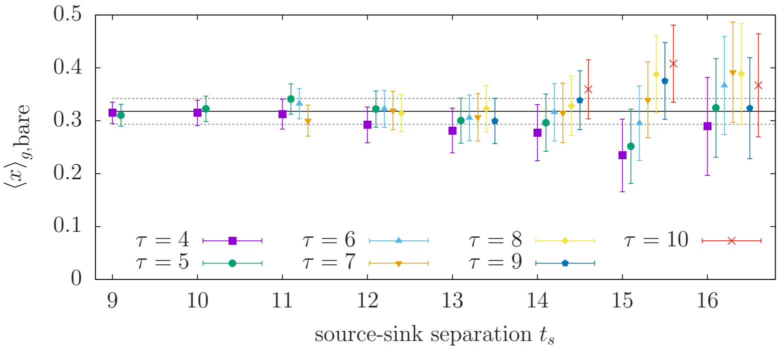


- Disconnected correlation function, known to be very noisy

$\Rightarrow$ we employ several steps of stout smearing in order to remove fluctuations in the gauge field

- Results are computed on the $N_f = 2$ ensemble at the physical point, $m_\pi = 131$ MeV, $a = 0.093$ fm, $V = 48^3 \times 96$, A. Abdel-Rehim *et al.* (ETMC):1507.04936
- The methodology was tested for $N_f = 2 + 1 + 1$ twisted mass at $m_\pi = 373$ MeV, C. Alexandrou, V. Drach, K. Hadjiyiannakou, K. Jansen, B. Kostrzewa, C. Wiese, PoS LATTICE2013 (2014) 289

Results for the gluon content

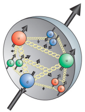


- 2094 gauge configurations with 100 different source positions each $\rightarrow$ more than 200 000 measurements
- Due to mixing with the quark singlet operator, the renormalization and mixing coefficients had to be extracted from a one-loop perturbative lattice calculation, M. Constantinou and H. Panagopoulos
- $\langle x \rangle_{g,\text{bare}} = 0.318(24) \xrightarrow{\text{Renormalization}} < x >^R_g = Z_{gg} < x >_g + Z_{gq} < x >_{u+d+s}$ $= 0.317(24)(13)$. The renormalization is perturbatively done using two-levels of stout smearing. The systematic error is the difference between using one- and two-levels of stout smearing.
- Momentum sum is satisfied: $\sum_q \langle x \rangle_q + \langle x \rangle_g = \langle x \rangle^{CI}_{u+d} + \langle x \rangle^{DI}_{u+d+s} + \langle x \rangle_g = 0.945(126)(13)$

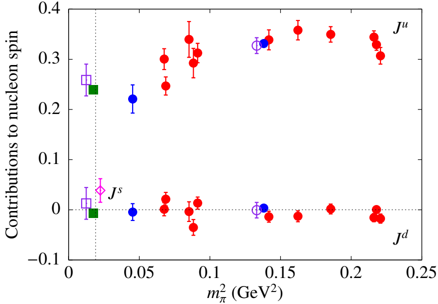
Nucleon spin?

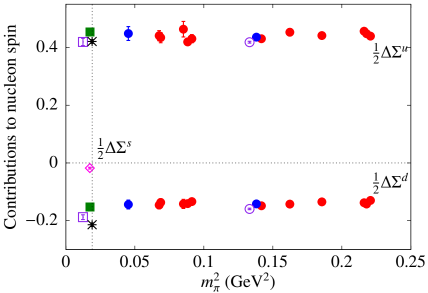
Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^G$

$J^q = A^q_{20}(0) + B^q_{20}(0)$ and $\Delta\Sigma^q = g^q_A$



Disconnected contribution using $\mathcal{O}(210500)$ statistics





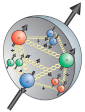
$\Longrightarrow$ Total spin for u-quarks $J^u < 0.3$ and for d-quark $J^d \sim 0$

- $\Delta\Sigma^{u,d}$ consistent with experimental values after disconnected contributions are included

Nucleon spin?

Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^G$

$J^q = A_{20}^q(0) + B_{20}^q(0)$ and $\Delta\Sigma^q = g_A^q$



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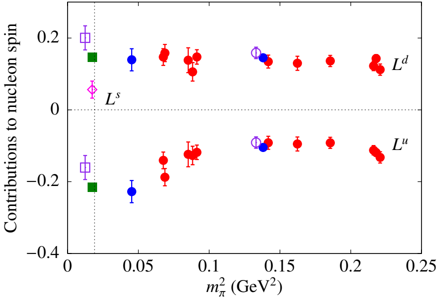

- Diconnected contributions affect the value of L^q
- Preliminary results: L^u and L^d increase if disconnected are included

Conclusions

- Results at the physical point are now directly accessible
  High statistics and careful cross-checks are needed $\rightarrow$ noise reduction techniques are crucial e.g. AMA, TSM, smearing etc
- Evaluation of quark loop diagrams has become feasible even at the physical point!
- Confirmation of experimentally known quantities such as g_A will enable reliable predictions of others $\rightarrow$ provide insight into the structure of hadrons and input that is crucial for new physics such as the nucleon σ-terms, g_s and g_T

Thank you for your attention