Nucleon electromagnetic and axial form factors with N_f=2 twisted mass fermions at the physical point

[a,b]C. Alexandrou, [b]M. Constantinou, [c]K. Hadjiyiannakou, [d]K. Jansen,
[b]C. Kallidonis, [b]G. Koutsou, [e]K. Ottnad, [f]A. Vaquero

[a]Department of Physics, University of Cyprus

[b]Computation-based Science and Technology Research Centre (CaSToRC), The Cyprus Institute

[c]Department of Physics, George Washington University, Washington DC

[d]DESY-Zeuthen

[e]Institut für Strahlen- und Kernphysik, Universität Bonn

[f]INFN Sezione Milano Bicocca

European Twisted Mass Collaboration

The Cyprus Institute

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CaSToRC

Outline

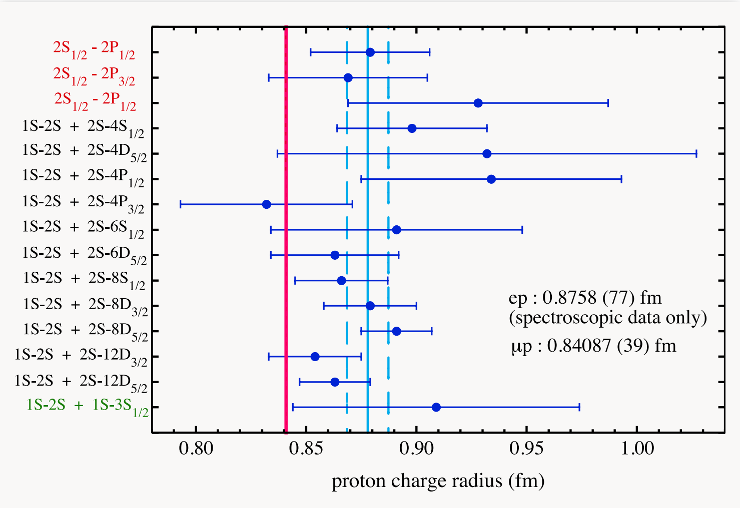
★ **Introduction and motivation**
- Proton radius
- Axial matrix elements

★ **Setup and method**
- Form factors from the lattice
- Lattice setup

★ **Results**
- Axial form factors
- Electromagnetic form factors

★ **Summary and outlook**

Introduction and motivation



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★ **Axial and Electromagnetic Form Factors**

Insight on structure of nucleon

- Slope at $Q^2 \to 0$: Electric and magnetic radii (or Dirac and Pauli)
- $G_A(Q^2=0) = g_A$ Nucleon axial charge
- $G_M(Q^2=0) = \mu_N$ Nucleon magnetic moment
- Input to determination of M_A
- $G_p(Q^2)$ test pion pole dominance expectation

★ **Proton spin puzzle**

- Discrepancy between electron scattering and muonic hydrogen Lamb shifts
- Need ~2% accuracy on $\langle r_p^2 \rangle$ to contact experiment
  - Large separations for suppressing excited state effects major challenge
  - Disconnected contributions to obtain up- and down-quark contributions — or equivalently, proton and neutron contributions

Form factor decomposition

★ Axial form factors

$$\langle N(p',s')|A^3_\mu|N(p,s)\rangle = i\sqrt{\frac{m_N^2}{E_N(\vec{p}')E_N(\vec{p})}}\bar{u}(p',s')[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q_\mu\gamma_5}{2m_N}G_p(q^2)]\frac{1}{2}u_N(p,s)$$

with:

$A^3_\mu(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\frac{\tau^3}{2}\psi(x)$, and $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$

★ Electromagnetic

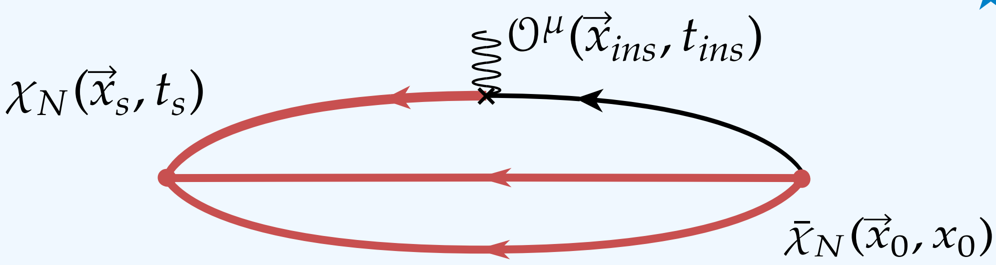
$$\langle N(p',s')|j_\mu|N(p,s)\rangle = \sqrt{\frac{m_N^2}{E_N(\vec{p}')E_N(\vec{p})}}\bar{u}(p',s')[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2)]u_N(p,s)$$

j_μ: lattice conserved current

F_1 and F_2, Dirac and Pauli form factors, alternatively, define Sachs electromagnetic form factors:

$G_E(q^2) = F_1(q^2) + \frac{q^2}{(2m_N)^2}F_2(q^2)$ $\qquad$ $G_M(q^2) = F_1(q^2) + F_2(q^2)$

Form factors from the lattice



★ **Three-point correlation function**

- Inversion through sink
- Fixed sink coordinates and momentum
- Free insertion indices (momentum, coordinate, operator)

$$G^\mu(\Gamma;\vec{q};t_s,t_{\rm ins}) = \sum_{\mathbf{x}_s \vec{x}_{\rm ins}} e^{-i\vec{p}'\vec{x}_s} e^{-i(\vec{p}'-\vec{p})\vec{x}_{\rm ins}} \Gamma^{\alpha\beta} \langle \bar{\chi}_N^\beta(\vec{x}_s;t_s) | \mathcal{O}^\mu(\vec{x}_{\rm ins};t_{\rm ins}) | \chi_N^\alpha(\vec{x}_0;t_0)\rangle$$

★ **Ratio with two-point functions:**

$$R^\mu(\Gamma;\vec{q};t_s;t_{\rm ins}) = \frac{G^\mu(\Gamma;\vec{q};t_s;t_{\rm ins})}{G(\vec{0};t_s)} \sqrt{\frac{G(\vec{0};t_s)G(\vec{q};t_s-t_{\rm ins})G(\vec{0};t_{\rm ins})}{G(\vec{q};t_s)G(\vec{0};t_s-t_{\rm ins})G(\vec{q};t_{\rm ins})}}$$

★ **Plateau method:**

$$R^\mu \xrightarrow[t_s - t_{\rm ins} \gg]{t_{\rm ins} \gg} \Pi^\mu(\Gamma;\vec{q})$$

Fit to constant when $t_s - t_{\rm ins} \gg$ $t_{\rm ins} \gg$

★ **Summation method:**

$$\sum_{t_{\rm ins}} R^\mu \xrightarrow{t_s \gg} C + \Pi^\mu(\Gamma;\vec{q}) t_s$$

Slope from linear fit

Form factor extraction

★ **Four projectors:**

- One unpolarised: $\Gamma_0 = \frac{1+\gamma_0}{4}$
- Three polarised: $\Gamma_k = i\gamma_5\gamma_k\Gamma_0$

★ **Final state at rest:** $\vec{p}' = 0, \qquad \vec{q} = -\vec{p}$

★ **Electromagnetic form factors**

$$\Pi^0(\Gamma_0;\vec{q}) = C\frac{E_N+m_N}{2m_N}G_E(Q^2)$$

$$\Pi^i(\Gamma_0;\vec{q}) = C\frac{q_i}{2m_N}G_E(Q^2)$$

$$\Pi^i(\Gamma_k;\vec{q}) = C\frac{\epsilon_{ijk}q_j}{2m_N}G_M(Q^2)$$

$$C = \sqrt{\frac{2m_N^2}{E_N(E_N+m_N)}}$$

★ **Axial form factors**

$$\Pi^i(\Gamma_k;\vec{q}) = \frac{iC}{4m_N}[\frac{q_k q_i}{2m_N}G_p(Q^2) - (E_N+m_N)\delta_{ik}G_A(Q^2)]$$

$$\sum_n^N \frac{(\sum_{m=1}^2 D_{nm}G_m - \Pi_n)^2}{w_n^2}$$

- Minimise, via the singular value decomposition of D, where G is the vector of wanted form factors

Lattice parameters

★ **Lattice ensemble**

- N_f=2 twisted mass with clover term
- $48^3 96$ lattice sites
- a = 0.093(1) fm determined from nucleon mass
- Appx. 3000 independent configurations
- m_π=0.132(1) GeV

★ **Three-point functions**

- t_s = 10a, 12a, 14a ≃ 0.9 fm, 1.1 fm, 1.3 fm
    - 580 configs. × 16 randomised source locations
    - Four polarisations: Γ_0, Γ_1, Γ_2, Γ_3

  → G_E, G_M, G_A, G_p

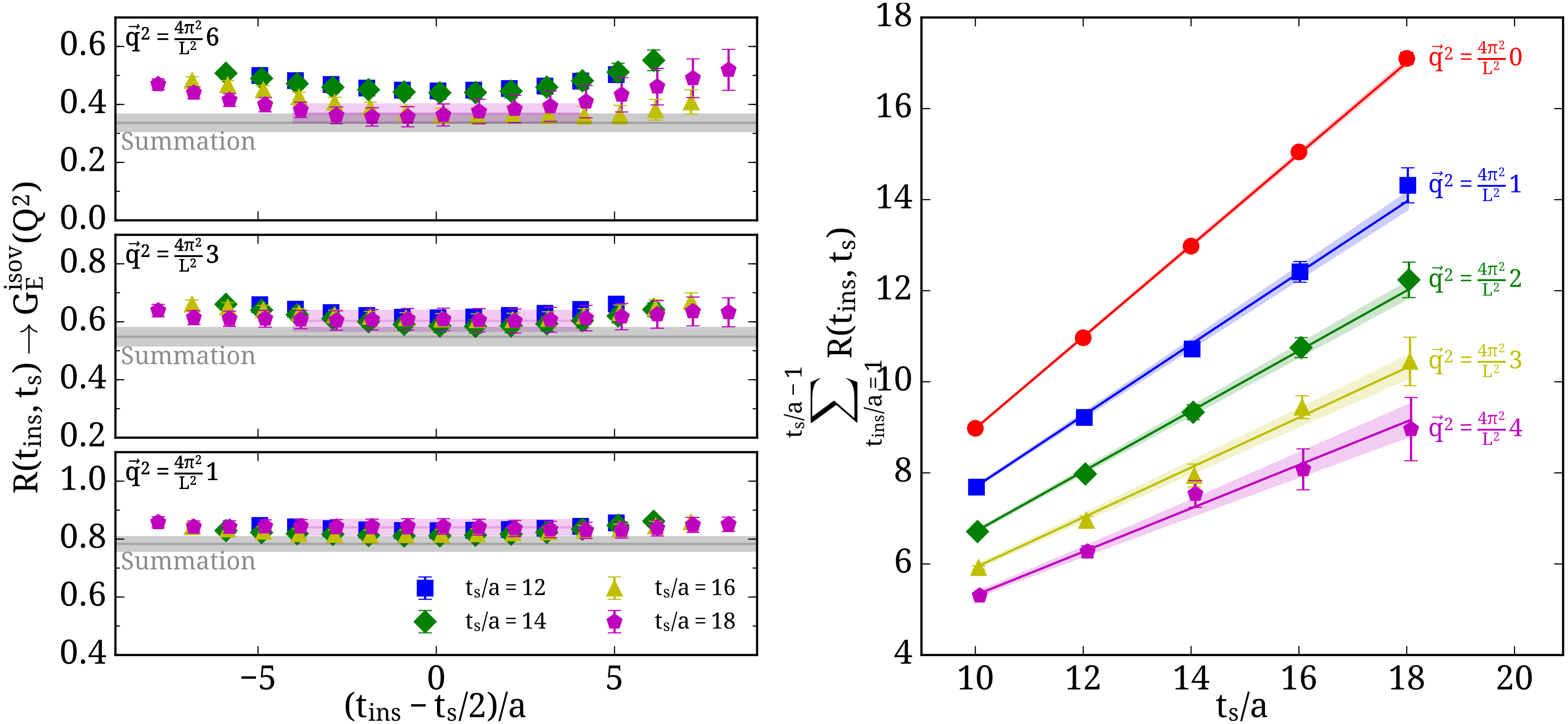
- t_s = 16a, 18a ≃ 1.5 fm, 1.7 fm
    - 530 and 725 configs. resp. × 88 randomised source locations
    - One polarisation: Γ_0

  → G_E

Plateaus and summation fits

★ Plateaus and summation method fits example

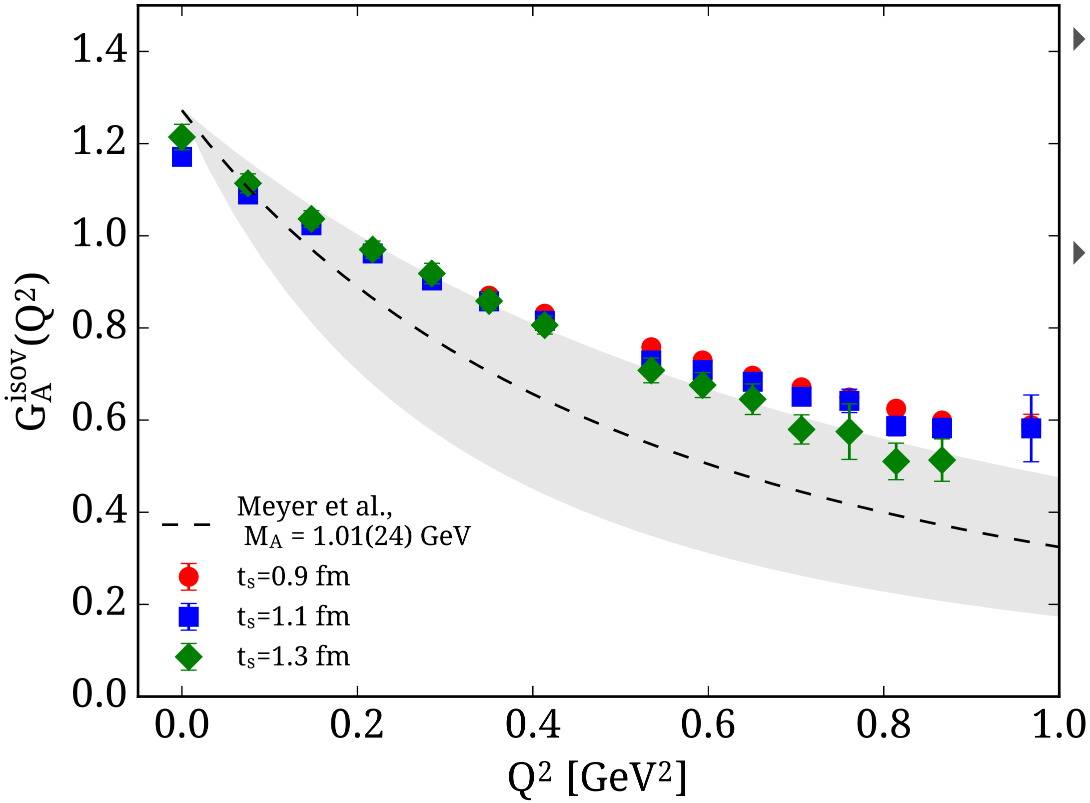
- Contact points omitted in summation
- Different statistics for different sink − source separations ⟶ fits in bootstrap throughout



Axial form factors:
$G_A(Q^2)$, $G_p(Q^2)$

$G_A(Q^2)$

★ **Axial form factor**



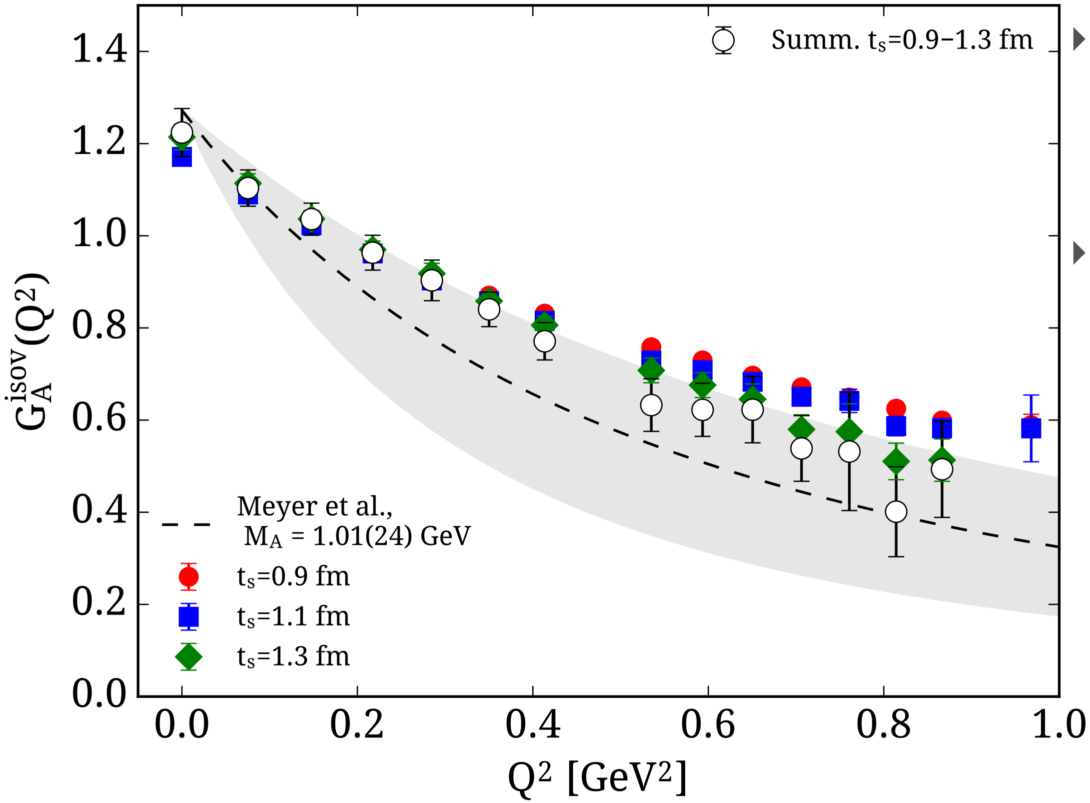
- Experimental parametrisation

$$G_A(Q^2) = \frac{g_A}{(1+\frac{Q^2}{M_A^2})^2}$$

- $M_A^{\exp}$ =1.01(24) GeV

$G_A(Q^2)$

★ **Axial form factor**



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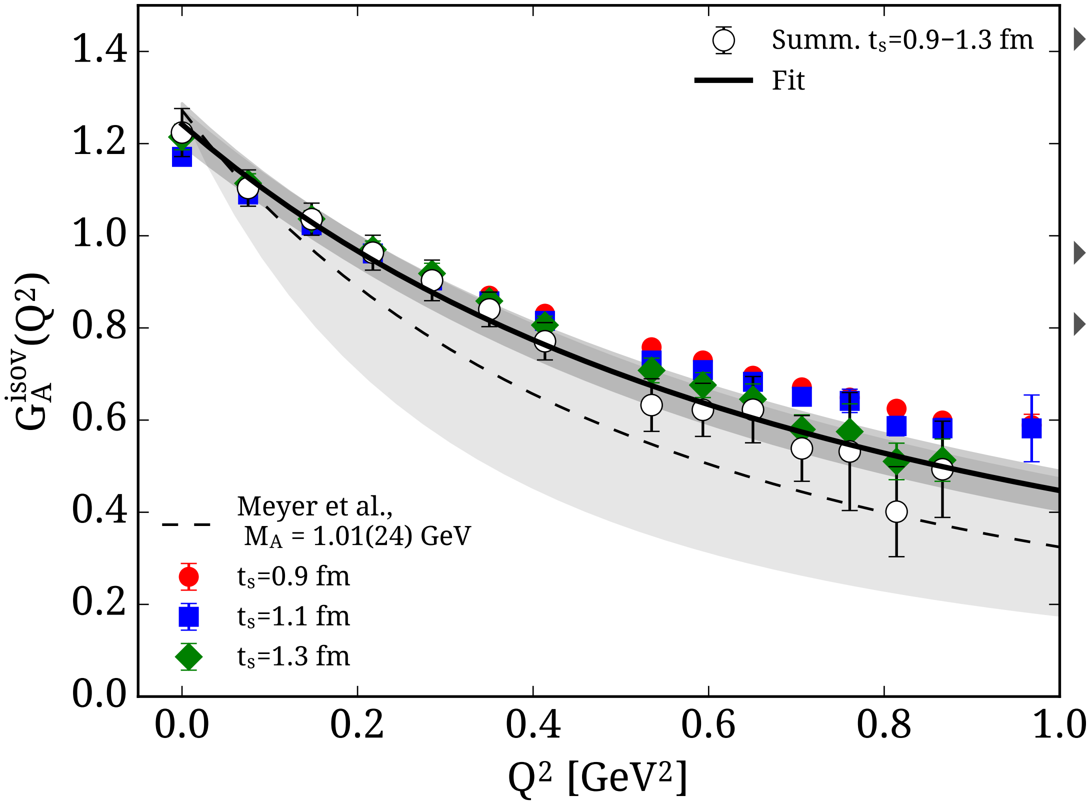

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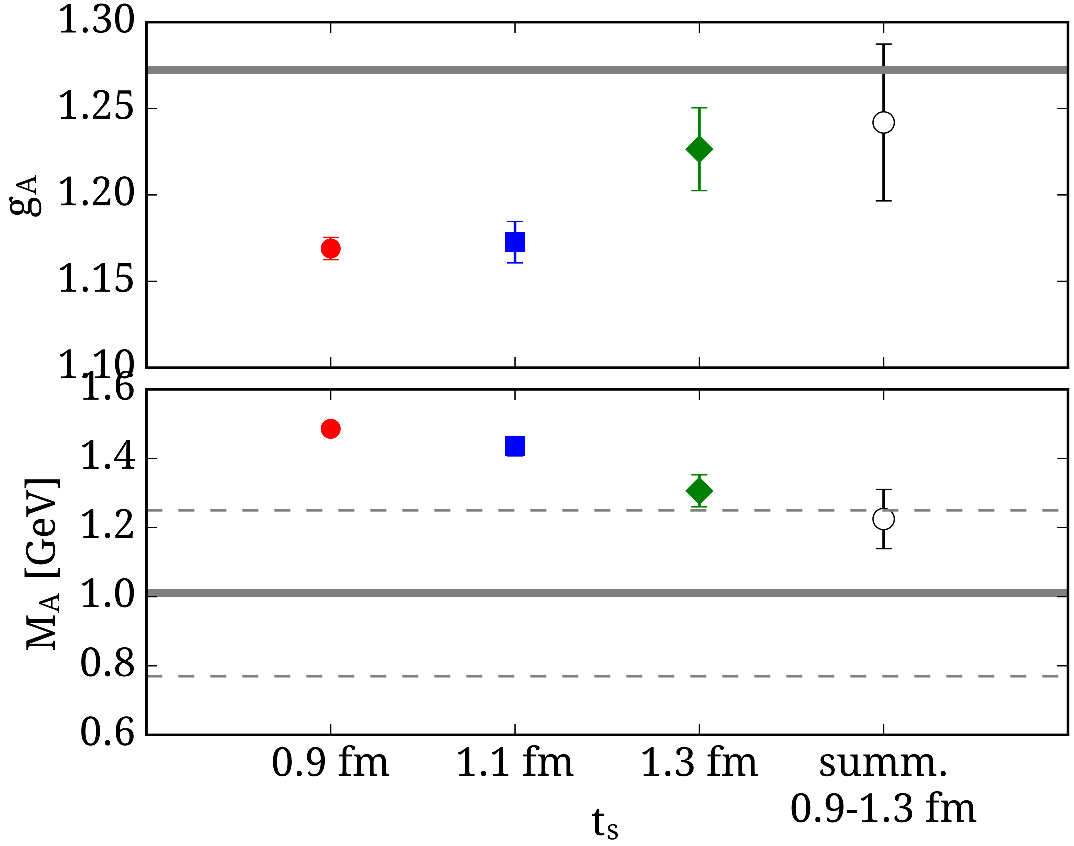
★ **Axial form factor**



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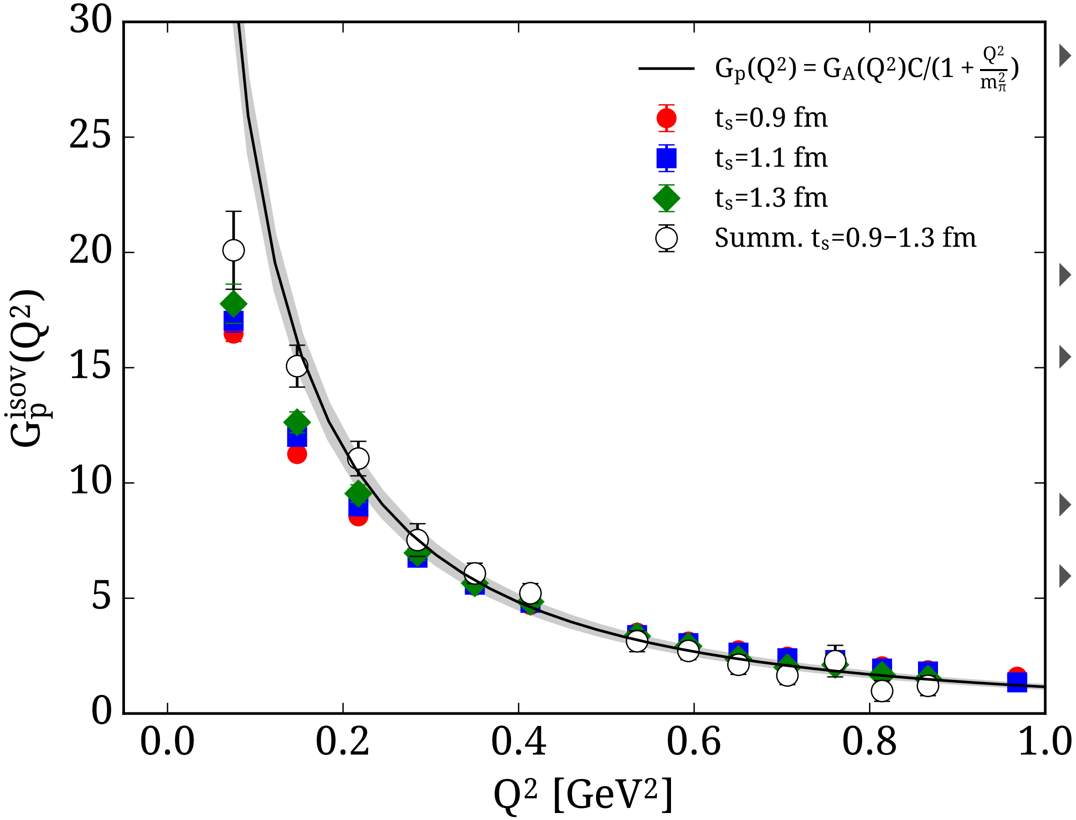
- M_A^{exp} =1.01(24) GeV
- $M_A^{\text{summ.}}$ =1.24(8) GeV

  $g_A^{\text{summ.}}$ =1.24(4)
- Suppression of excited states $\longrightarrow$ experimental values

$G_p(Q^2)$

★ Induced pseudo-scalar form factor



- Expected pole dependence:

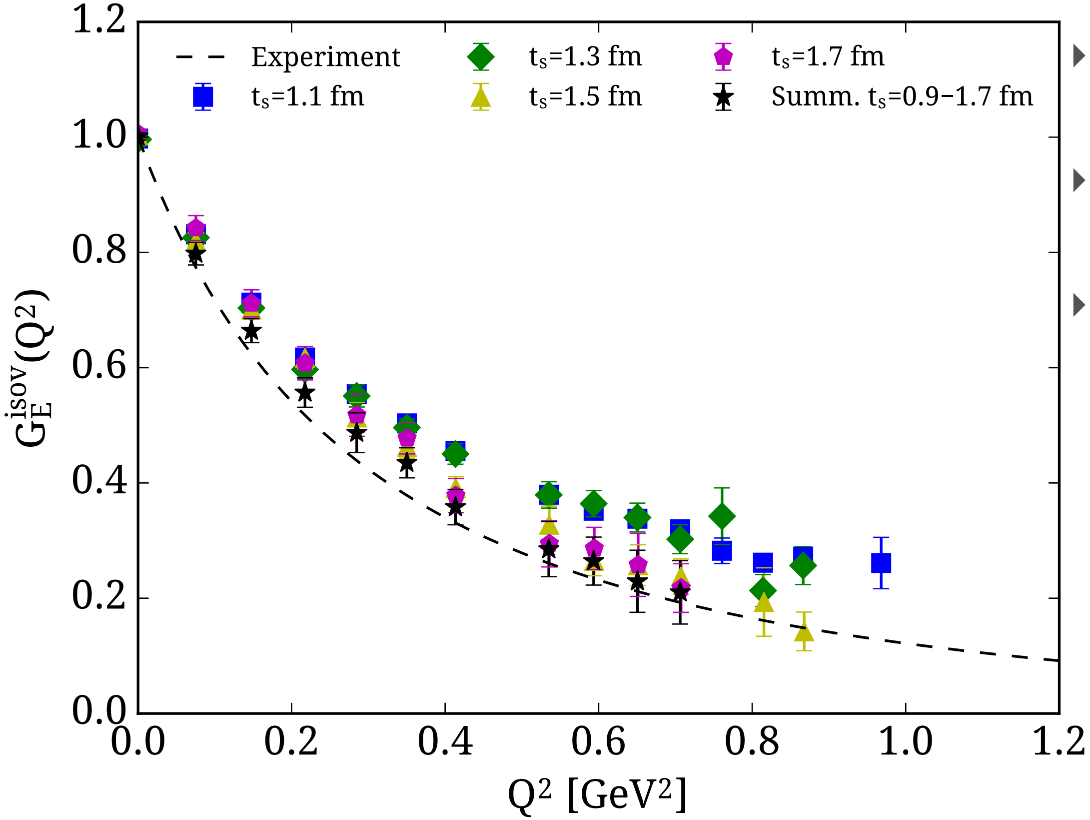
$$G_p(Q^2) = G_A(Q^2)C/(1+\frac{Q^2}{m_\pi^2})$$

- Single parameter fit for C
- C= 138(9) (excl. lowest Q^2)
  $C = 4[\frac{m_N^0}{m_\pi}]^2 \Rightarrow$
- $m_N^0 = 0.77(2)$ GeV
- Compare to ~0.88 GeV from HBχPT

Electromagnetic form factors

$G_E(Q^2)$

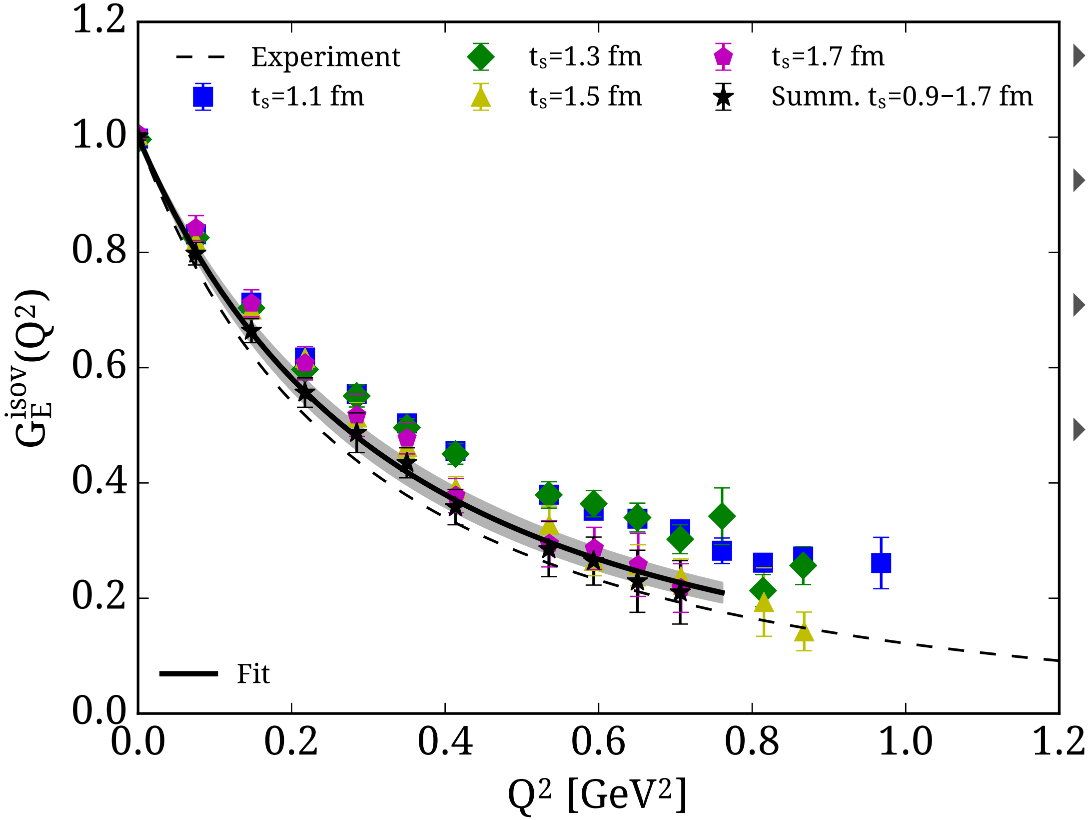
★ **Isovector electric Sachs form factor**



- Five sink-source separations, between 0.9 – 1.7 fm
- ~60000 statistics at largest separation
- Consistent approach to experiment

$G_E(Q^2)$

★ Isovector electric Sachs form factor



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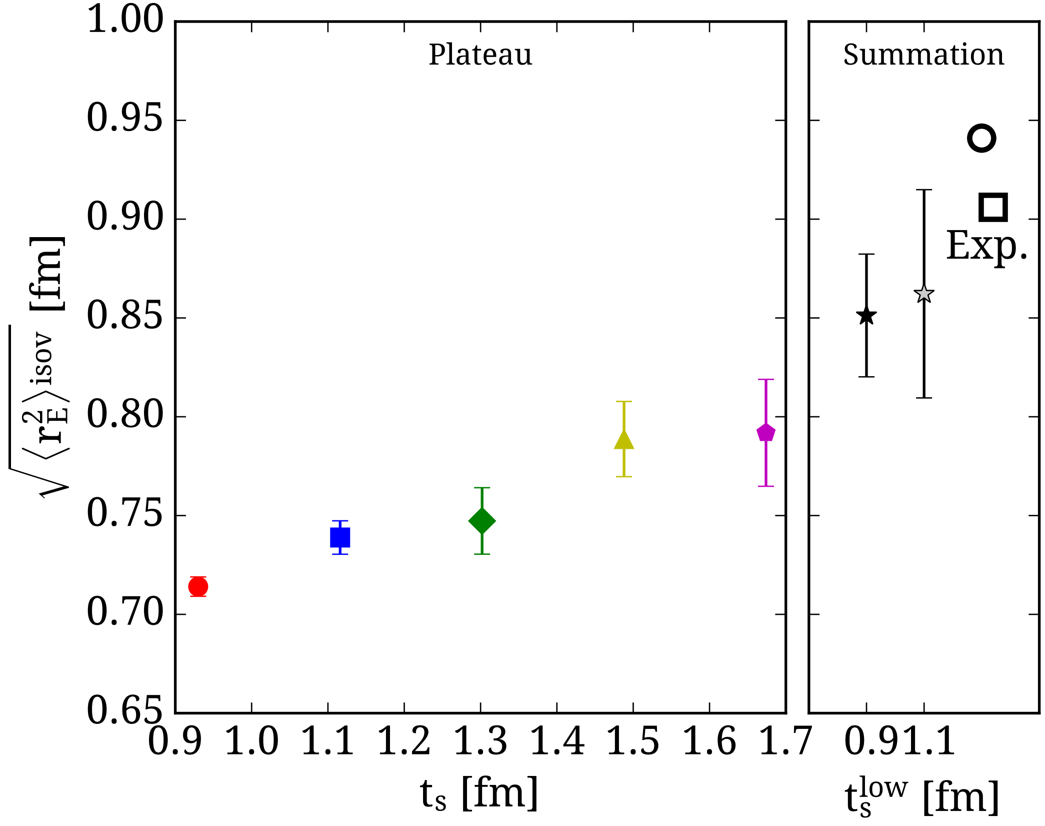
- Dipole fit:

$$G_E(Q^2) = \frac{1}{(1+\frac{Q^2}{M_E^2})}$$

$$\langle r_E^2 \rangle^{\text{isov}} = \frac{12}{M_E^2}$$

$G_E(Q^2)$

★ **Isovector electric Sachs form factor**



- Five sink-source separations, between 0.9 – 1.7 fm
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$$G_E(Q^2) = \frac{1}{(1 + \frac{Q^2}{M_E^2})}$$

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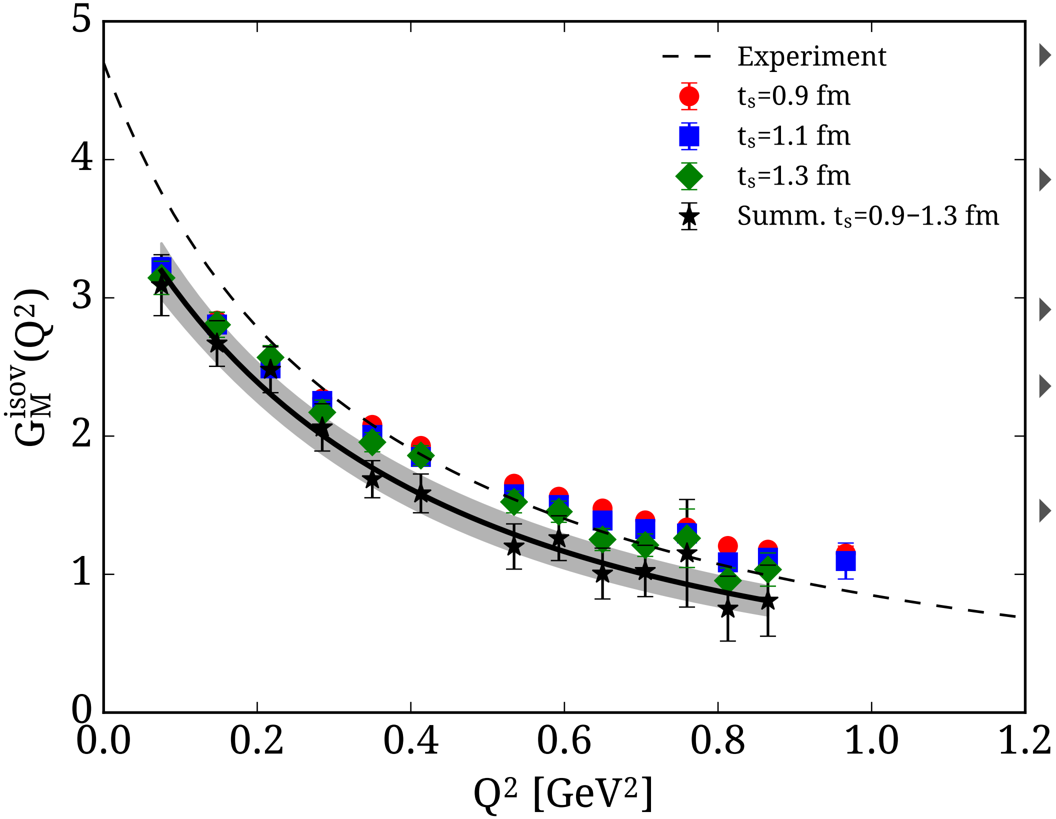
$G_M(Q^2)$

★ Isovector magnetic Sachs form factor



- Three sink-source separations, between 0.9 – 1.3 fm
- ~9300 statistics at largest separation
- Underestimated $G_M(0)$
- Similar radius behaviour as electric case
- Two-parameter fit:

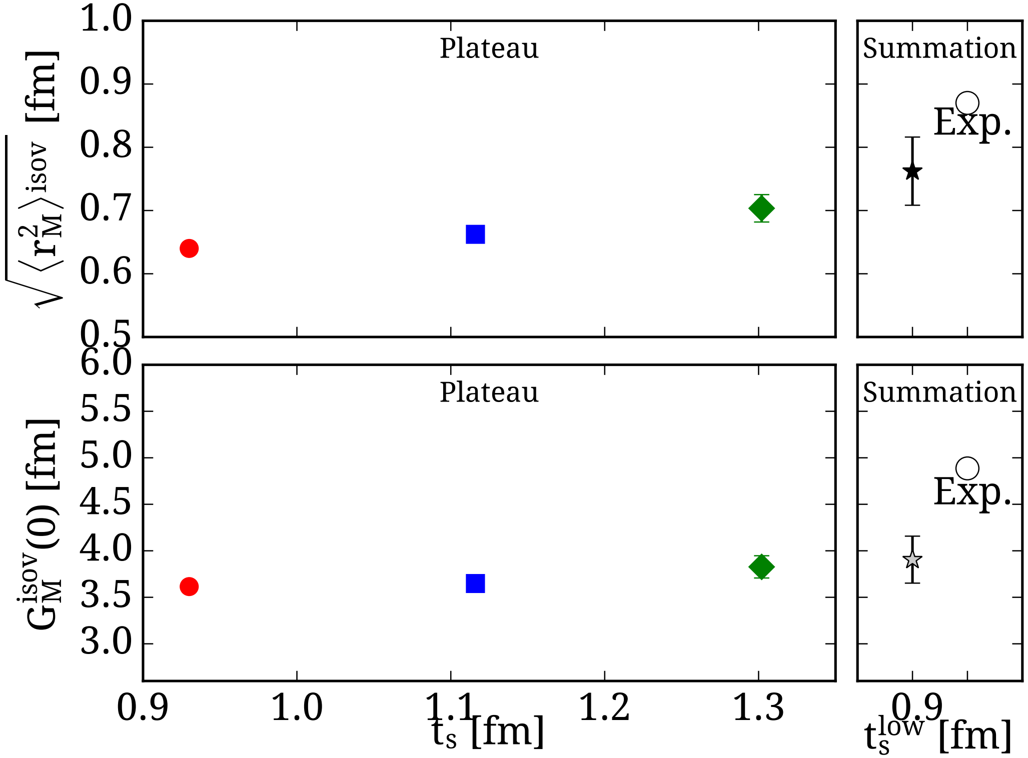
$$G_M(Q^2) = \frac{G_M(0)}{(1+\frac{Q^2}{M_M^2})^2}$$

$$\langle r_M^2 \rangle^{\text{isov}} = \frac{12}{M_M^2}$$

$$\mu_N^p - \mu_N^n = G_M(0)$$

$G_M(Q^2)$

★ **Isovector magnetic Sachs form factor**



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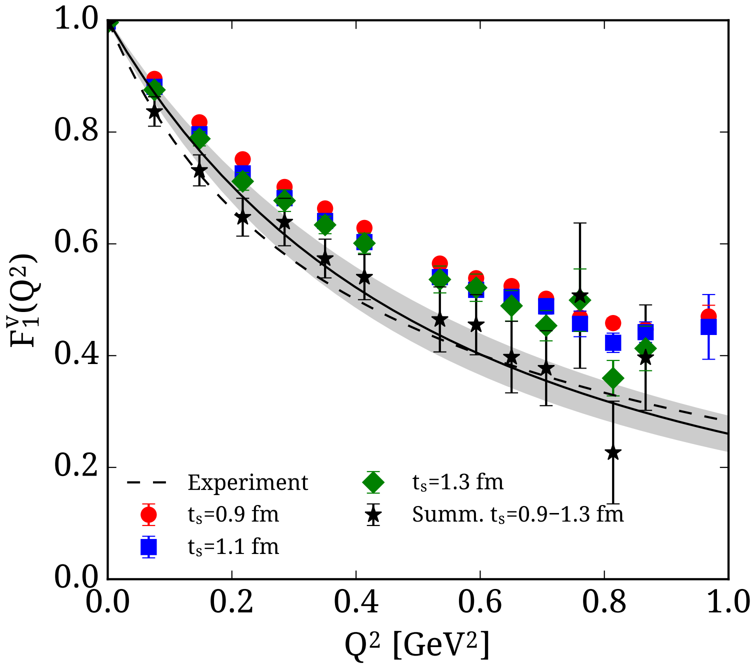
$$\langle r_M^2 \rangle^{\text{isov}} = \frac{12}{M_M^2}$$

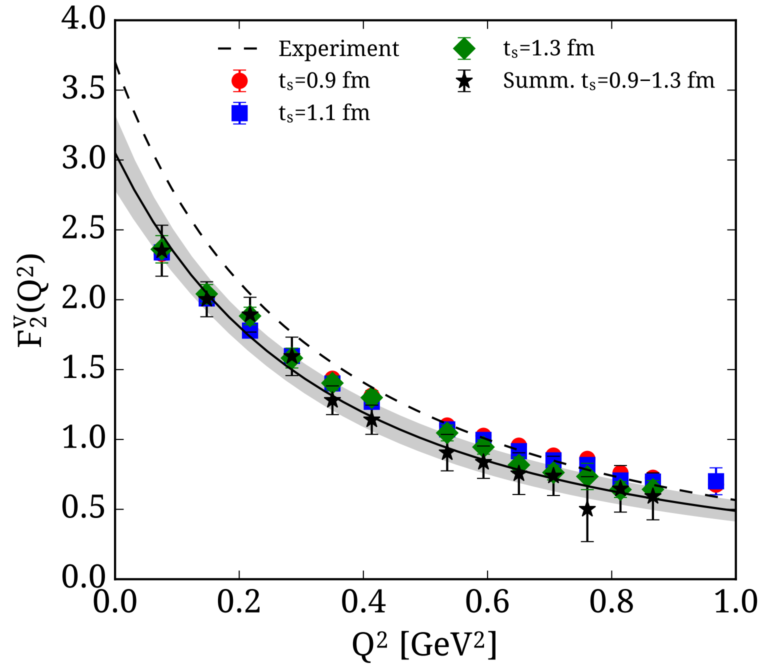
$$\mu_N^p - \mu_N^n = G_M(0)$$

Dirac and Pauli isovector form factors

Only for common sink – source separations between G_E and G_M

$$F_1(Q^2) = \frac{G_E(Q^2) + \tau G_M(Q^2)}{1+\tau} \qquad F_2(Q^2) = \frac{G_M(Q^2) - G_E(Q^2)}{1+\tau} \qquad \tau = \frac{Q^2}{(2m_N)^2}$$





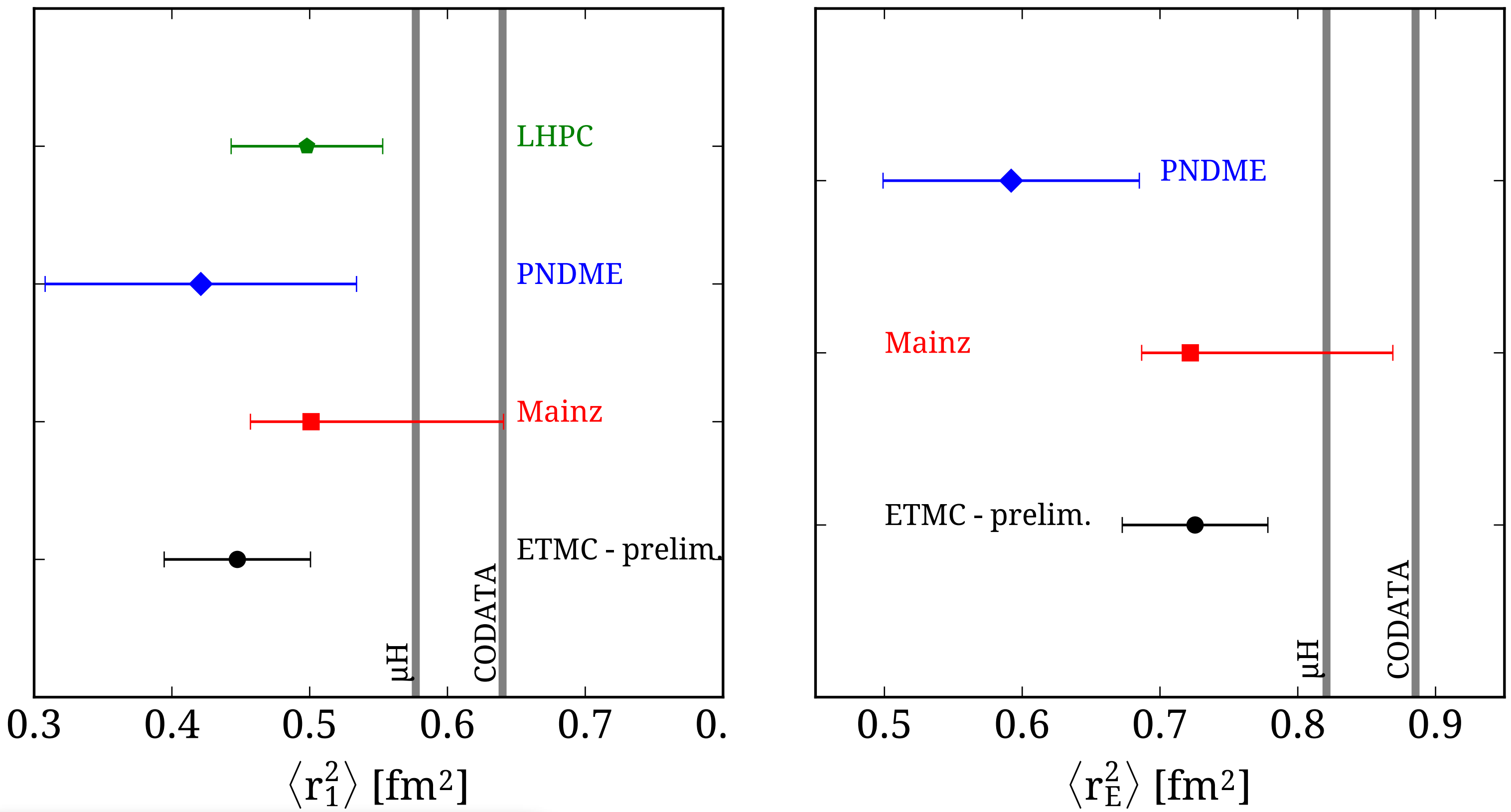
Fit to dipole form:

$$F_i(Q^2) = \frac{F_i(0)}{(1+\frac{Q^2}{M_i^2})^2}$$

$F_1(0) = 1$, $F_2(0)$ allowed to vary

Electric and Dirac radii

Comparison of recent results near or at physical pion mass



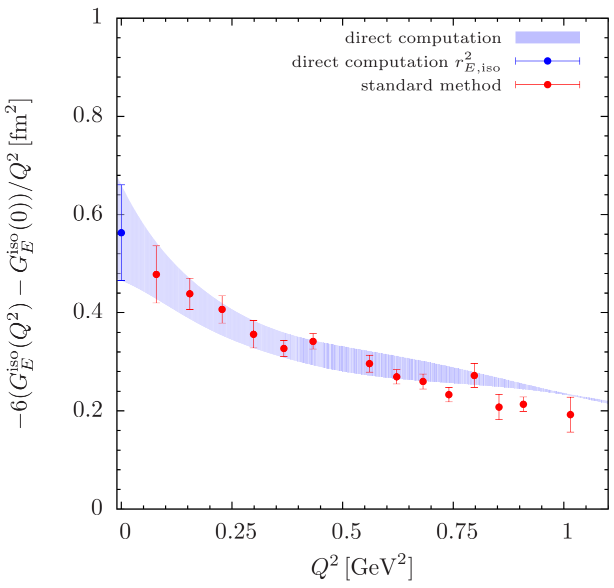
- LHPC, arXiv:1404.4029
- Mainz, arXiv:1504.04628
- PNDME, arXiv:1306.5435

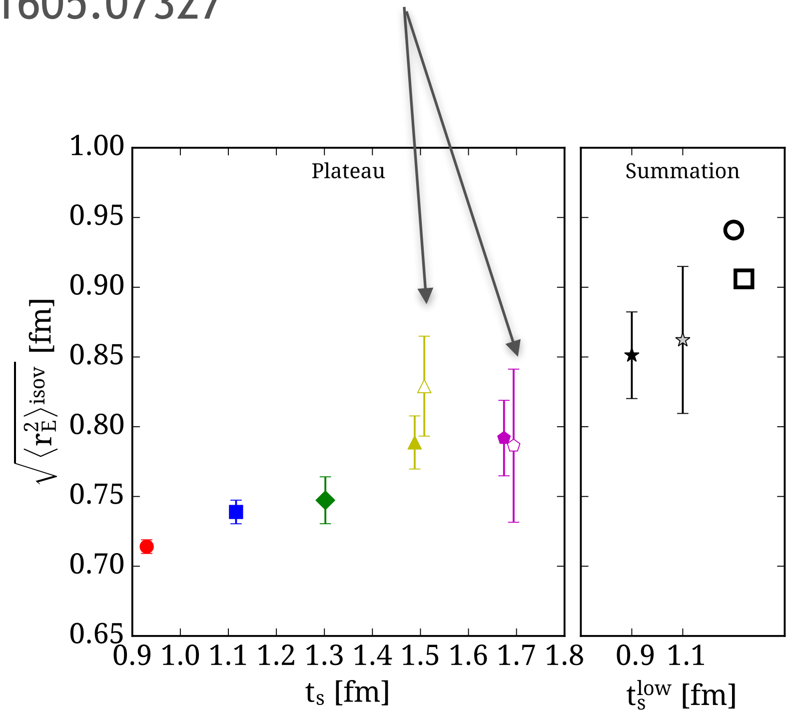
Consistent trend towards larger radii with increasing sink-source separations

Conclusions — outlook

★ **Outlook: position space methodS for radius**

- Preliminary results using few momenta
- Radius consistent within large error
- Applied to 370 MeV in arXiv:1605.07327





Conclusions — outlook

★ **Direct physical point calculation of axial and EM form factors**

- Separations up to 1.3 or 1.7 fm
- Still observe excited state effects in radii

★ **Axial form factors**

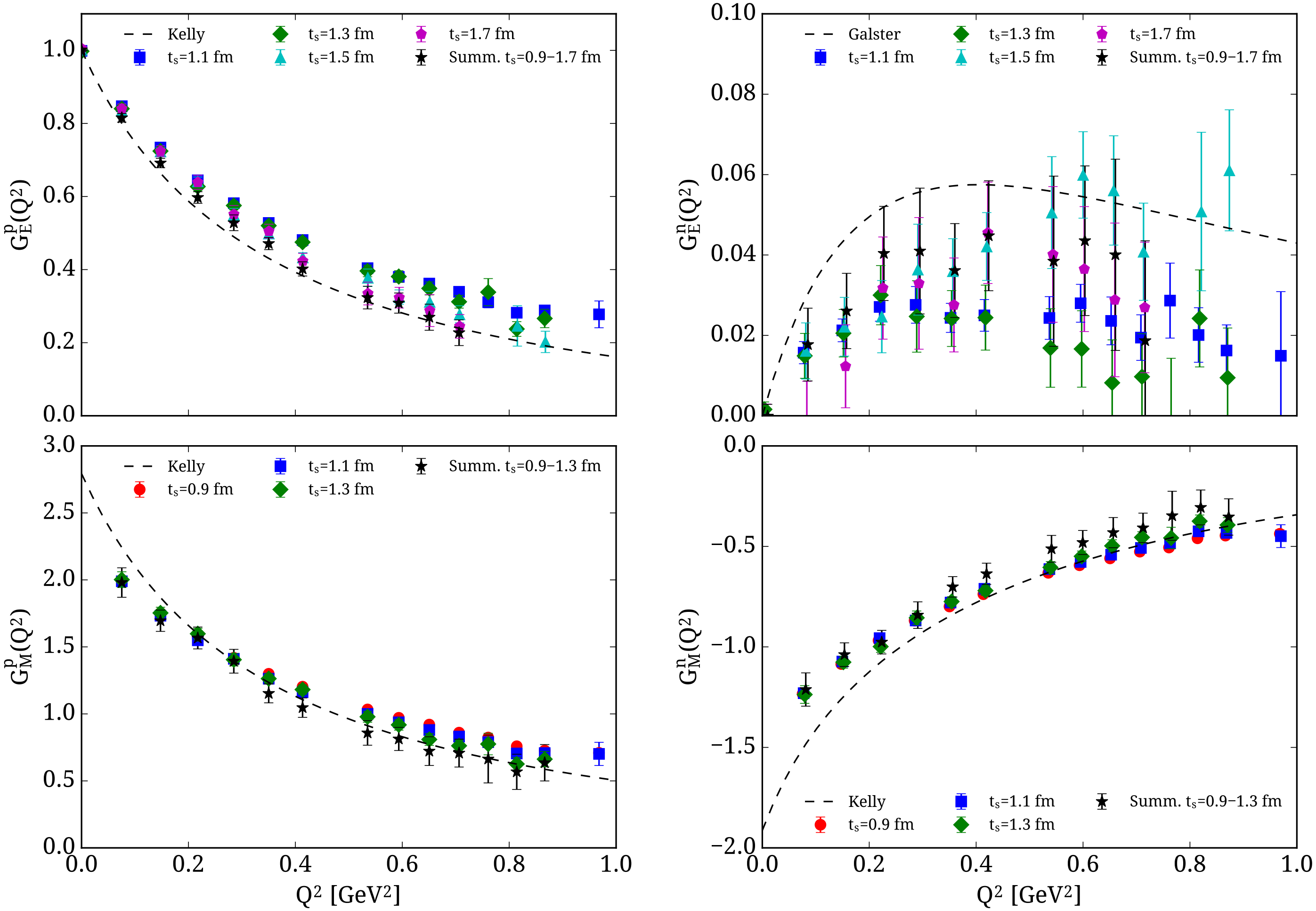
- Axial pole mass within error of latest analysis of experimental data
- Excited state continuation $\longrightarrow$ larger pole mass

★ **Electromagnetic form factors**

- Radii approaching experimental values with increasing separations
- Still more statistics and larger separations needed though
- May need ~2 fm separation and $O(10^5)$ statistics
- $G_M(Q^2 \longrightarrow 0)$ still puzzling, excited state effects seem mild

BONUS

Proton and neutron [ignoring disconnected]



Disconnected diagram contributions being assessed

[see talk by A. Vaquero, Thu. 17:30]

Error of form factors and radii

★ **Variance estimate**

- Note that different sink-source separations have different statistics
- Exponential suppression: (m_N-$3/2m_\pi$) consistent with nucleon mass at all momentum transfers