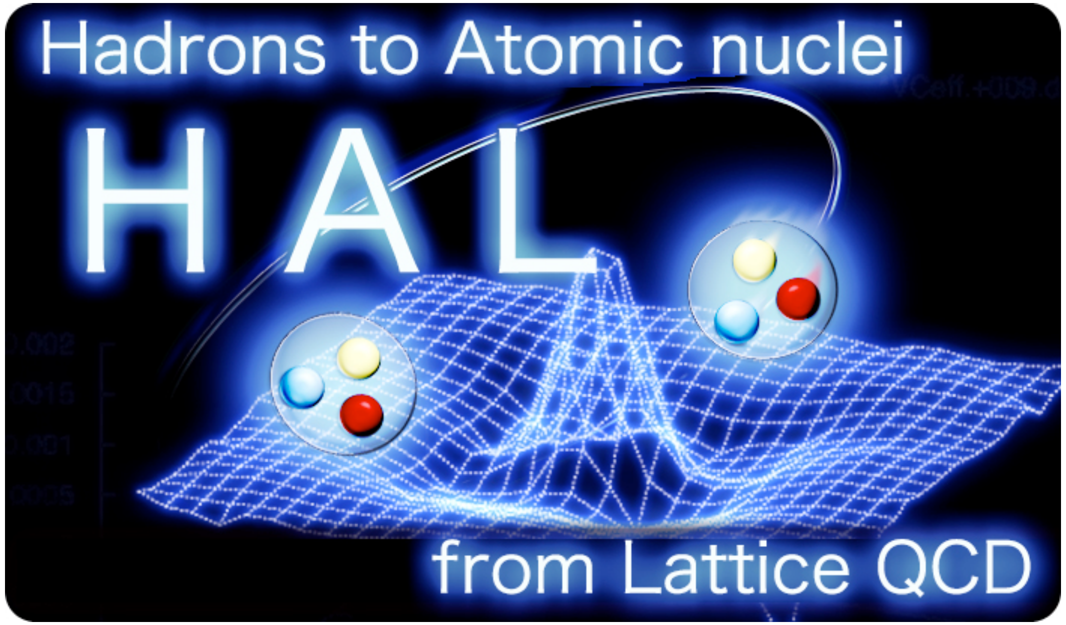
An application of stochastic LapH method to Hadron interaction in lattice QCD



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For HAL QCD Collaboration

LATTICE 2016

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- Negative parity channel
- Heavy quark hadron (T. Miyamoto's talk)
- Potential at the physical point (T.Doi, N.Ishii and, K.Sasaki and H.Nemura's talk : tomorrow 14:00~ here)
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We incorporate **stochastic LapH method** and use all-to-all propagator in order to overcome this difficulty .

Stochastic LapH method

[C. Mozrningstar, J. Bulava, J. Foley, K.J. Juge, D. Lenkner, M. Peardon, and C.H. Wong]　PRD 83 114505, 2011

Strong points.

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Weak point

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Weak point

- Relatively large cost

Laplacian Heaviside smearing (LapH smearing)

[C. Morningstar, J. Bulava, J. Foley, K.J. Juge, D. Lenkner, M. Peardon, and C.H. Wong] PRD 83 114505, 2011

In this smearing, quark field is smeared by the operator $\mathcal{S}$.

Smeared quark field $\widetilde{\psi}^{a\alpha}(x) = S^{ab}(x,y)\psi^{b\alpha}(y)$ (a, b : color index, α : spin index)

$$\mathcal{S} = \Theta\left(\sigma_s^2 + \widetilde{\Delta}\right)$$

($\sigma_s^2 > 0$: control parameter)

$$\widetilde{\Delta}^{ab}(x,y;U) = \sum_{k=1}^{3}\left\{\widetilde{U}_k^{ab}(x)\delta(y,x+\hat{k}) + \widetilde{U}_k^{ba}(x)^*\delta(y,x-\hat{k}) - 2\delta(x,y)\delta^{ab}\right\}$$

Remark : Eigenvalue of $\widetilde{\Delta}^{ab}(x,y;U) \sim -k^2 < 0$ ($\widetilde{U}$: stout-smeared gauge field)

Components whose $\widetilde{\Delta}$'s eigenvalue is lower than - σ_s^2 are cut off.

→ The contribution from excited level and inelastic region are expected to be much suppressed.

$$S = \begin{pmatrix} V_{t=0} & 0 & \dots & 0 \\ 0 & V_{t=1} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & V_{t=T-1} \end{pmatrix}^{\dagger} \begin{pmatrix} V_{t=0} & 0 & \dots & 0 \\ 0 & V_{t=1} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & V_{t=T-1} \end{pmatrix}$$

t-diagonal

→ We use V_t in actual calculation

(V_t : bases of LapH space)

Correlator with Stochastic LapH

In stochastic LapH scheme, hadron correlators are constructed by diluted noise source and sink field.

Source : $\rho^h_{a,\alpha,x,t} = V_t(a,x;n) P^{[h]}_{ts,\alpha\beta,nm} \eta(s,\beta,m)$

Sink : $\varphi^h_{f,a,\alpha,x,t} = (V_t V_t^\dagger)(a,x;b,y) D_f^{-1}(b,\alpha,y,t;c,\beta,z,s) \rho^h_{c,\beta,z,s}$

$\eta(s,\beta,m)$: Z_4 noise vector

$P^{[h]}_{ts,\alpha\beta,nm}$: Dilution projector

$V_t(a,x;n)$: LapH vector

The correlator of charged pion and ρ meson are

$$C_\pi(t,t_0) = \sum_{x,y} \langle \pi^+_{\rm s}(x,t) \pi^-_{\rm s}(y,t_0) \rangle = \sum_{x,y} \sum_{h_1,h_2} \langle \rho^{h_1\dagger} \gamma^5 \varphi^{h_2}(x,t) \rho^{h_2\dagger} \gamma^5 \varphi^{h_1}(y,t_0) \rangle_\eta$$

$$C_{\rho_k}(t,t_0) = \sum_{x,y} \langle \rho^+_{\rm s}(x,t) \rho^-_{\rm s}(y,t_0) \rangle = \sum_{x,y} \sum_{h_1,h_2} \langle \rho^{h_1\dagger} \gamma^k \varphi^{h_2}(x,t) \rho^{h_2\dagger} \gamma^k \varphi^{h_1}(y,t_0) \rangle_\eta$$

→ **All-to-all propagator** can be calculated within a reasonable cost.

Dilution technique

Noise vector $\eta(t,\alpha,n)$ in the propagator will make variance large.

→ Dilution technique conpensates this weak point

Ex.) Full dilution in temporal direction $P^{t'}_{t,s} = \delta_{t,s}\delta_{t,t'}$

$$P^{t'=1}_{t,s} = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{pmatrix}, \quad P^{t'=1}_{t,s} = \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 0 & \\ & & & \ddots \end{pmatrix}, \quad \ldots, \quad P^{t'=T-1}_{t,s} = \begin{pmatrix} 0 & & & \\ & \ddots & & \\ & & 0 & \\ & & & 1 \end{pmatrix}$$

In this case, contribution except for propagation from t_0 to t in 2-pt correlation

$$C_\pi(t,t_0) = \sum_{x,y} \langle \pi^+(x,t)\pi^-(y,t_0) \rangle = \sum_{x,y}\sum_{h_1,h_2} \langle \rho^{h_1\dagger}\gamma^5\varphi^{h_2}(x,t)\rho^{h_2\dagger}\gamma^5\varphi^{h_1}(y,t_0) \rangle_\eta$$

is **exactly** zero.

2pt correlation calculated with Stochastic LapH

[Code on the Stochastic LapH method is provided by C. Moringstar.]

The π and ρ meson 2pt correlation function draw clear exponential dump **even with 22 conf.**

Numerical setup

・2 + 1 flavor gauge configuration

by CP-PACS & JLQCD

[T. Ishikawa et.al (CP-PACS collab.) PRD 78 : 011502.2008]

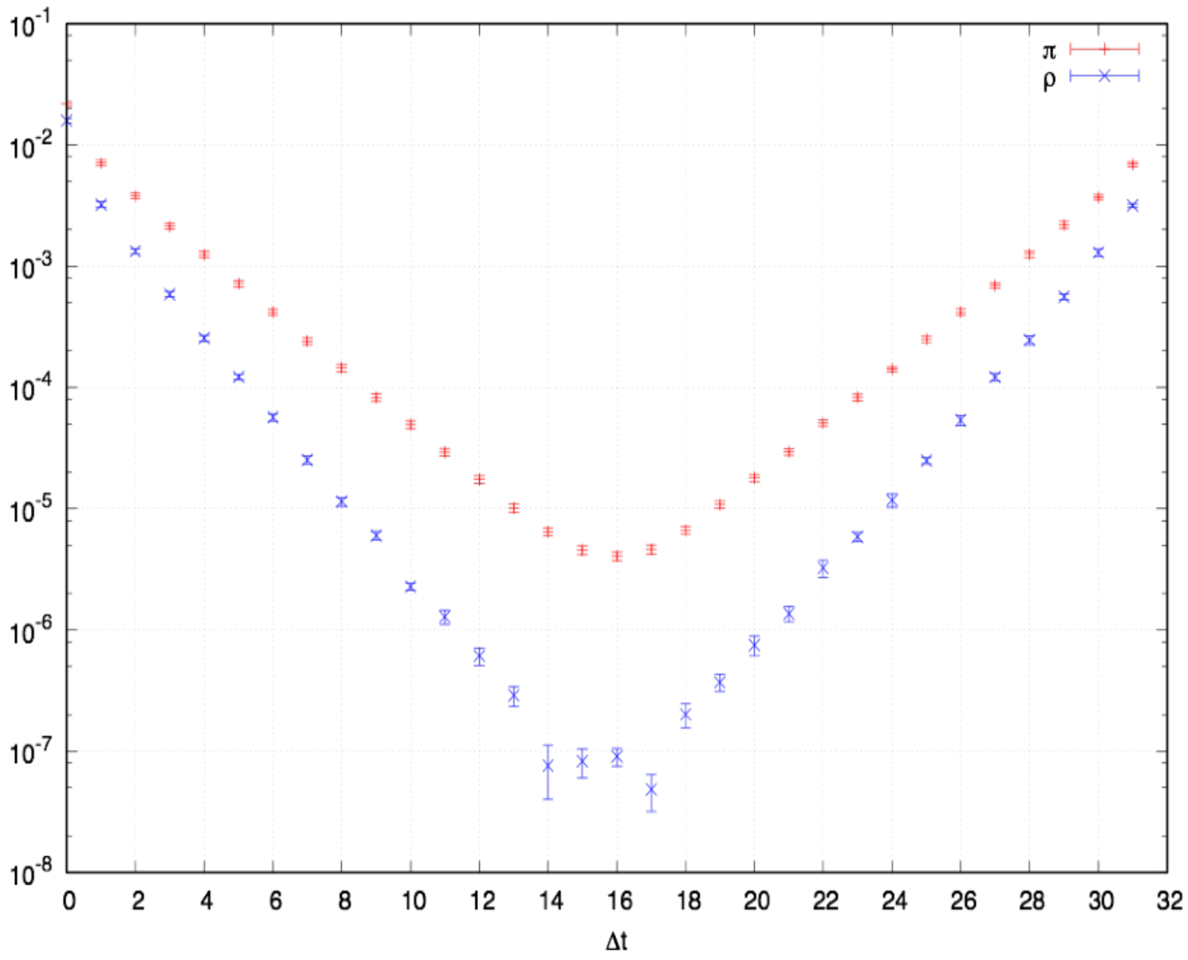
・$a = 0.1214$ fm , $16^3 \times 32$ lattice

・Iwasaki gauge action and Clover fermion

・**Full dilution** is used for **temporal direction.**

・**No dilution** is used for **Spin and LapH space.**

・Number of Noise for each conf. = 32



Effective mass of π and ρ

t = **2~3** is enough for inelastic region and higher component to decouple. $\because$ LapH smearing

Fitting results

$$M_\pi = 886.21\text{MeV} \pm 5.00\text{MeV}$$

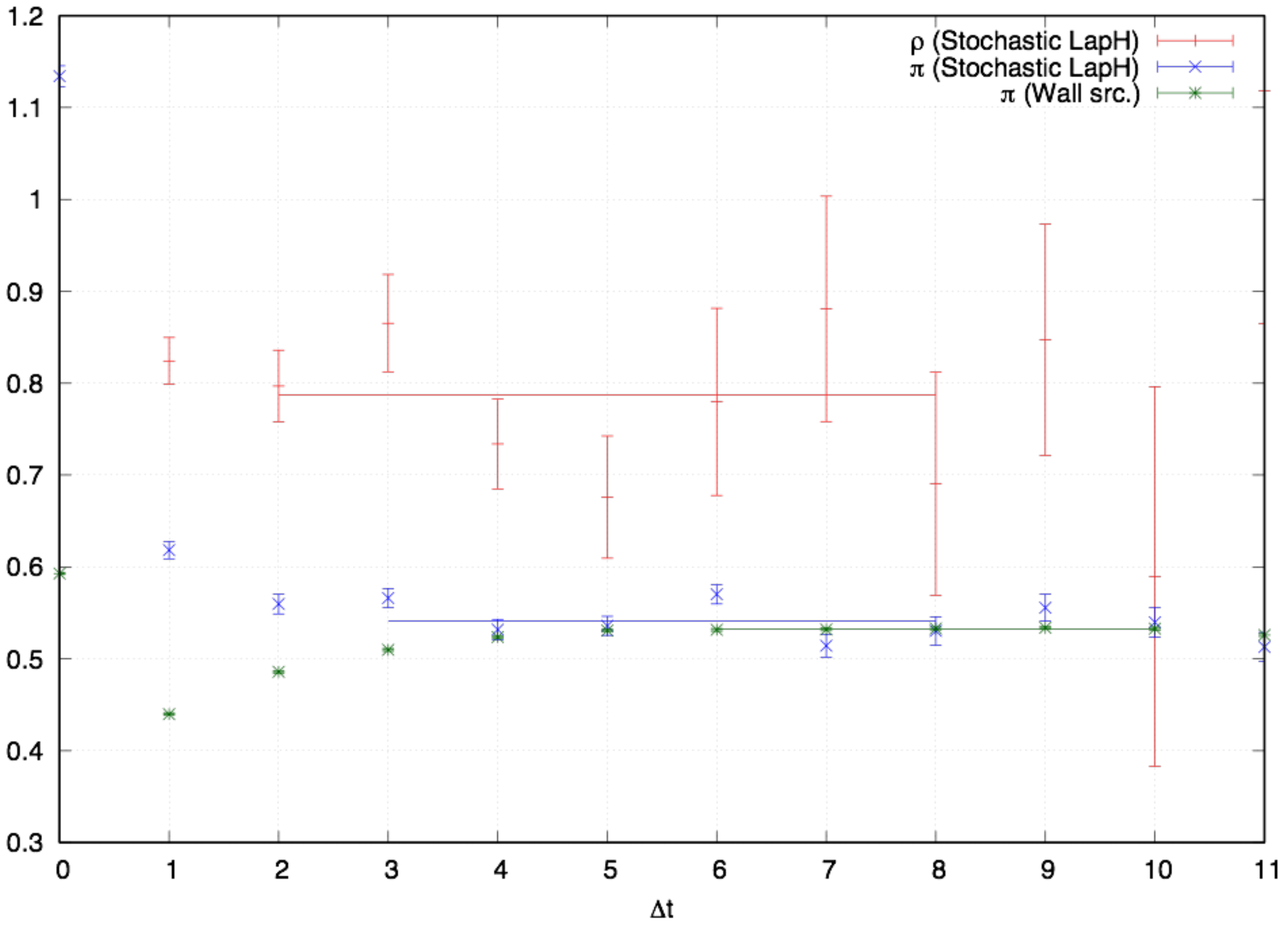
$$M_\rho = 1279.8\text{MeV} \pm 14.0\text{MeV}$$

close to Wall src. result

$$M_\pi = 871.84\text{MeV} \pm 2.07\text{MeV}$$

and published result

$$M_\pi / M_\rho = 0.7076(18)$$



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Implementation of all-to-all propagator

→ We get the access to the annihilation diagrams.

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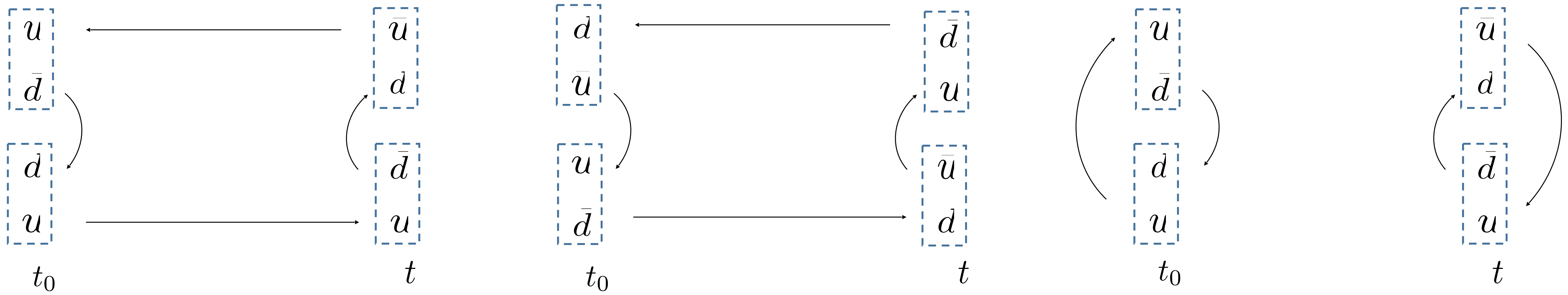
HALQCD + Stochastic LapH → Hadron interaction with annihilation



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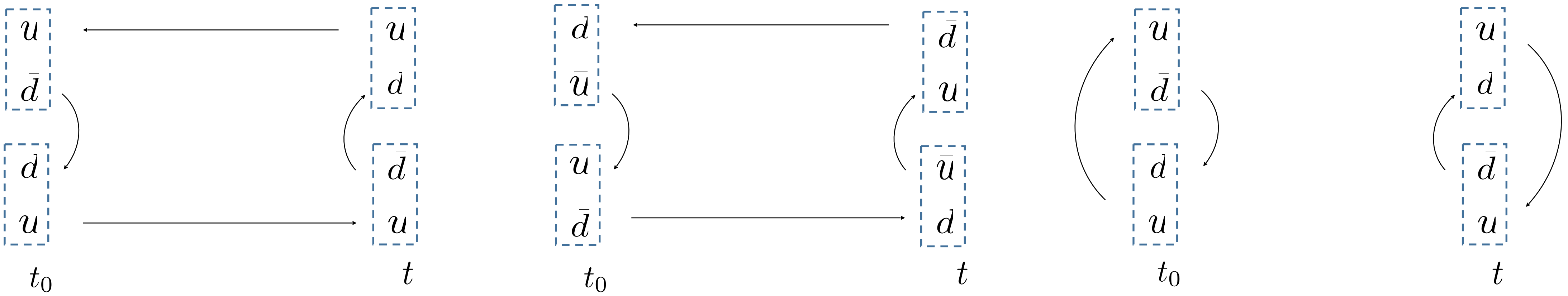


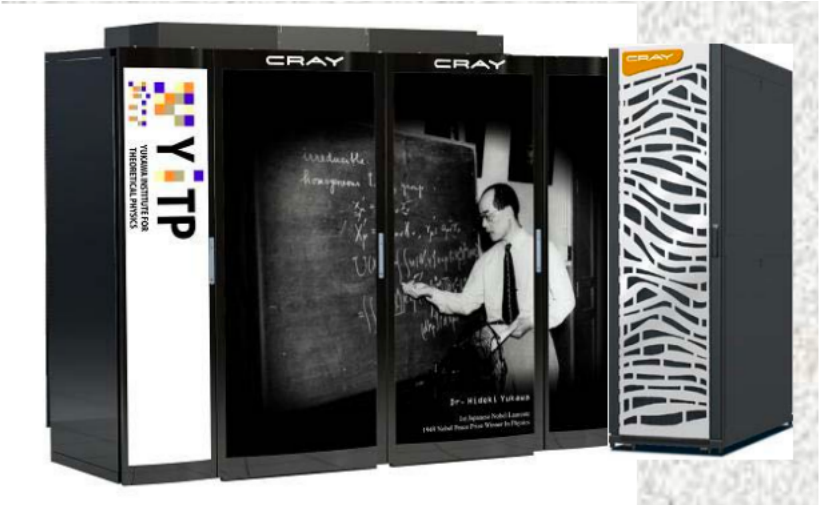
Application to

- ρ resonance
- $f_0(500)/\sigma$ meson resonance etc.

$\pi\ \pi$ (I = 2) chancel (Fisrt test, without annihilation)

Numerical setup

- 2 + 1 flavor gauge configuration by CP-PACS & JLQCD
- a = 0.1214 fm , $16^3 \times 32$ lattice
- 52conf × 32 time slice × 2 src
- Periodic Boundary Condition is used
- Cray XC40 in YITP is used.

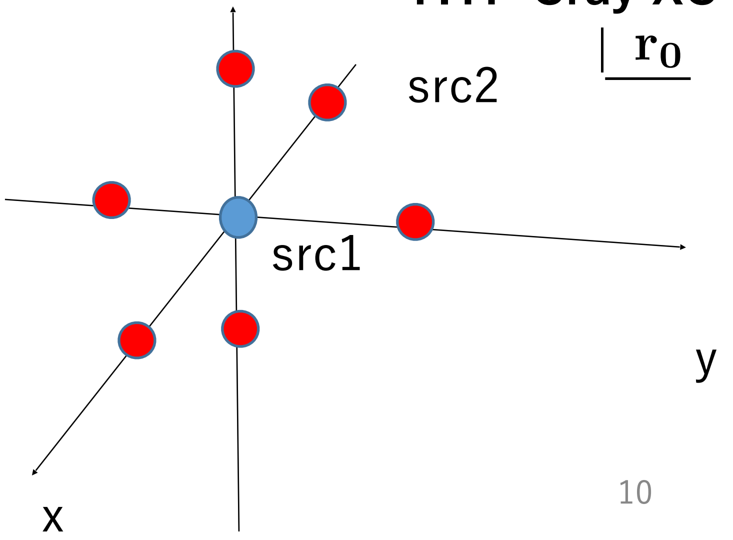


YITP Cray XC40

We computed $\pi\ \pi$ scattering with the following source setup.

$$MM(\mathbf{r}, t_f, \mathbf{r_0}, t_i) = \sum_{\mathbf{x},\mathbf{y}} \langle \pi^{+}_{s}(\mathbf{x}, t_f) \pi^{+}_{s}(\mathbf{x}+\mathbf{r}, t_f) \pi^{-}_{s}(\mathbf{y}, t_0) \pi^{-}_{s}(\mathbf{y}+\mathbf{r_0}, t_0) \rangle$$

$$\mathbf{r_0} = \{\pm 3\mathbf{e}_x, \pm 3\mathbf{e}_y, \pm 3\mathbf{e}_z\}$$



Time dependent HAL method [Ishii et al.,PLB712(2012)437]

We define R-correlator

$$R(\mathbf{r}, t-t_0) = e^{2m(t-t_0)} \sum_{\mathbf{x}} \langle 0|T\{B(\mathbf{x},t)B(\mathbf{x}+\mathbf{r},t)\}\bar{\mathcal{J}}(t_0)|0\rangle$$

$$= \sum_n A_n \psi_{k_n}(\mathbf{r}) e^{-(E_n-2m)(t-t_0)}$$

It satisfies time-dependent Schrödinger-like equation

$$\left[\frac{1}{4m}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right] R(\mathbf{r},t) = \int dr'^3 U(\mathbf{r},\mathbf{r}') R(\mathbf{r}',t)$$

From velocity expansion, the leading-order potential is given by

$$V(\mathbf{r}) = \frac{1}{4m}\frac{(\partial/\partial t)^2 R(\mathbf{r},t)}{R(\mathbf{r},t)} - \frac{(\partial/\partial t)R(\mathbf{r},t)}{R(\mathbf{r},t)} - \frac{H_0 R(\mathbf{r},t)}{R(\mathbf{r},t)}$$

HALQCD result on the potential of $\pi\ \pi$

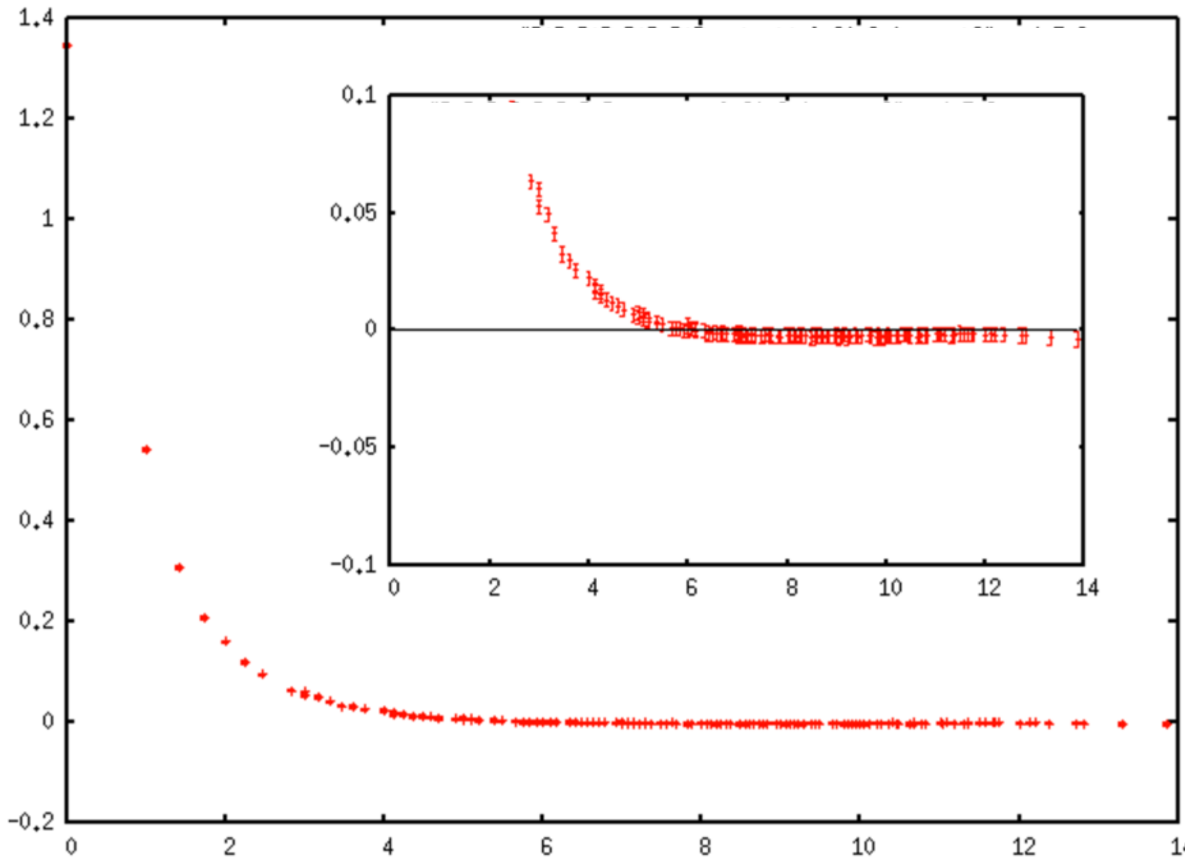
The potential in this channel is calculated with wall source.

Numerical setup

- 2 + 1 flavor gauge configuration by CP-PACS & JLQCD
- $a = 0.1214$ fm , $16^3 \times 32$ lattice
- 700 conf.

This result claims

$\pi\ \pi$ (I=2) ${}^1S_0$ channel is repulsive.



R-correlator with Stochastic LapH method

First Test

Strong correlation appears in short distance at early time.

But the peak disappear and correlation at long distance grows as time goes.

→ Consistent with the previous work that says this channel is repulsive.

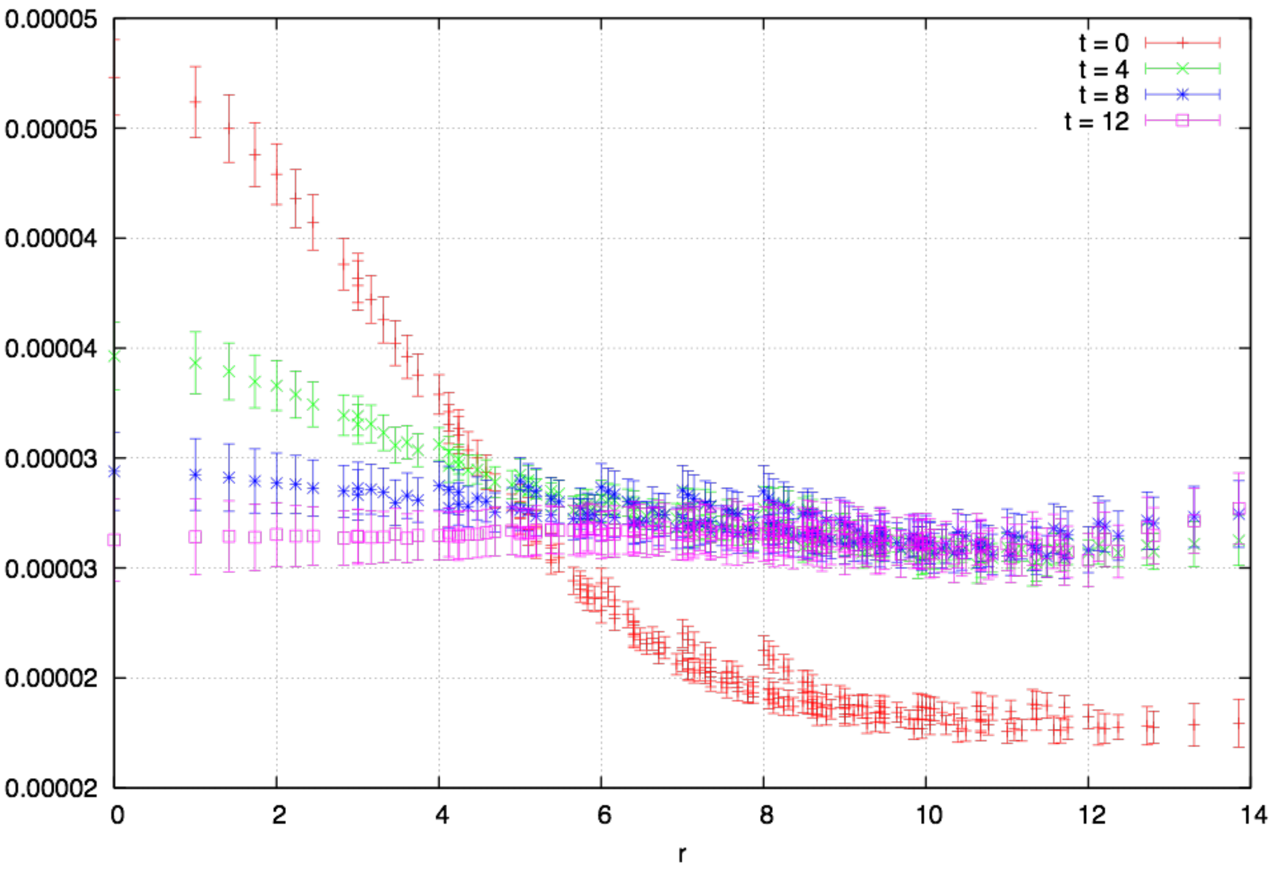
Dilution scheme (first trial)

- Time : Full dilution
- Spin : No dilution
- LapH : No dilution

In total, 32 dilution indices

(∵ 32 sitea in the temporal direction)

# of noise for each config. = 2



Spatial average of 4-pt correlation function

We took spatial sum of π 4-pt correlation.

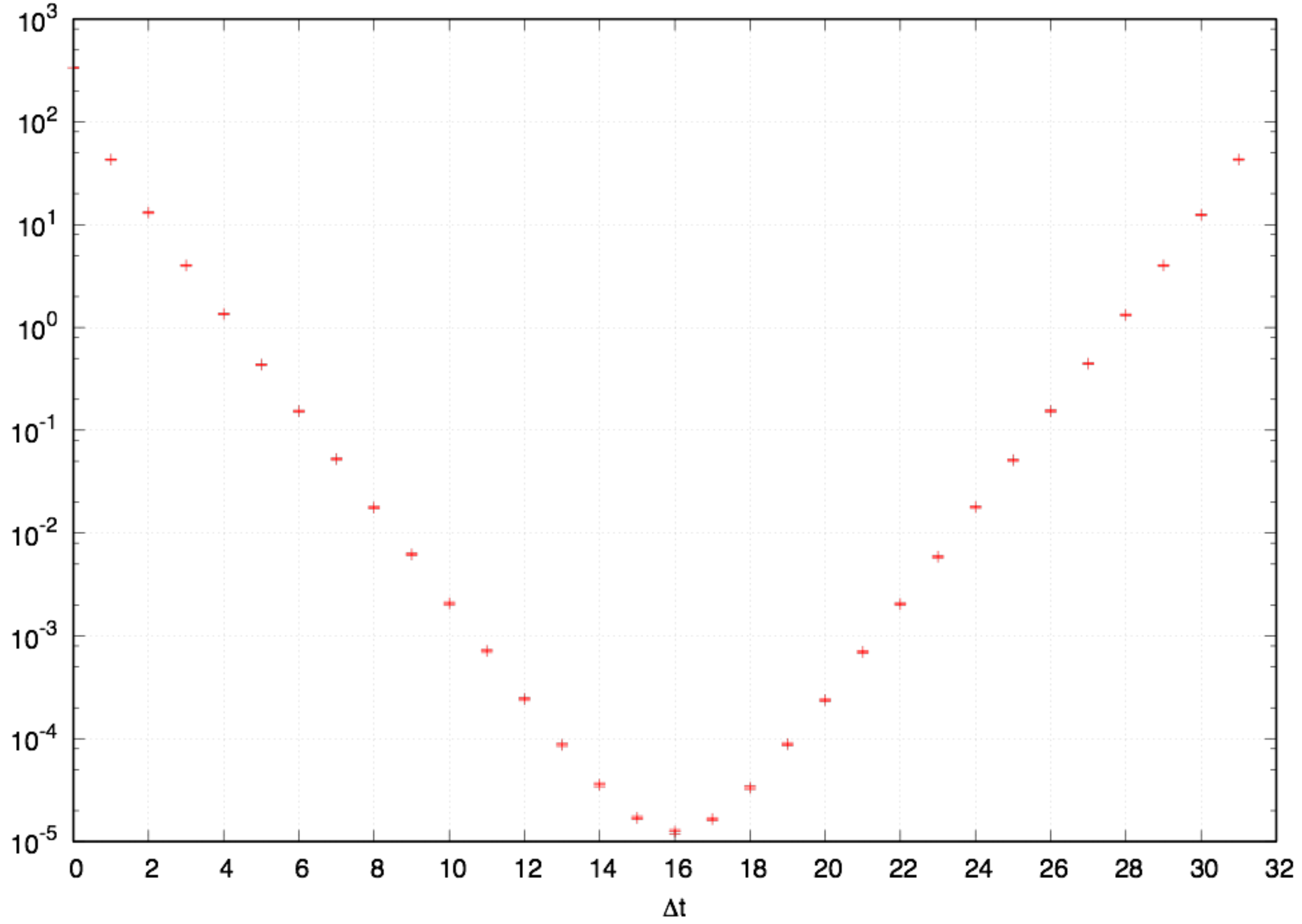
$$MM_0(\Delta t) = \sum_{\mathbf{r}} MM(\mathbf{r}, t_f, \mathbf{r}_0, t_i)$$

$$(\Delta t \equiv t_f - t_i)$$

⇩

$MM_0(\Delta t)$ shows clear exponential dump

and correct periodicity



Effective mass plot of R-correlator

Effective mass

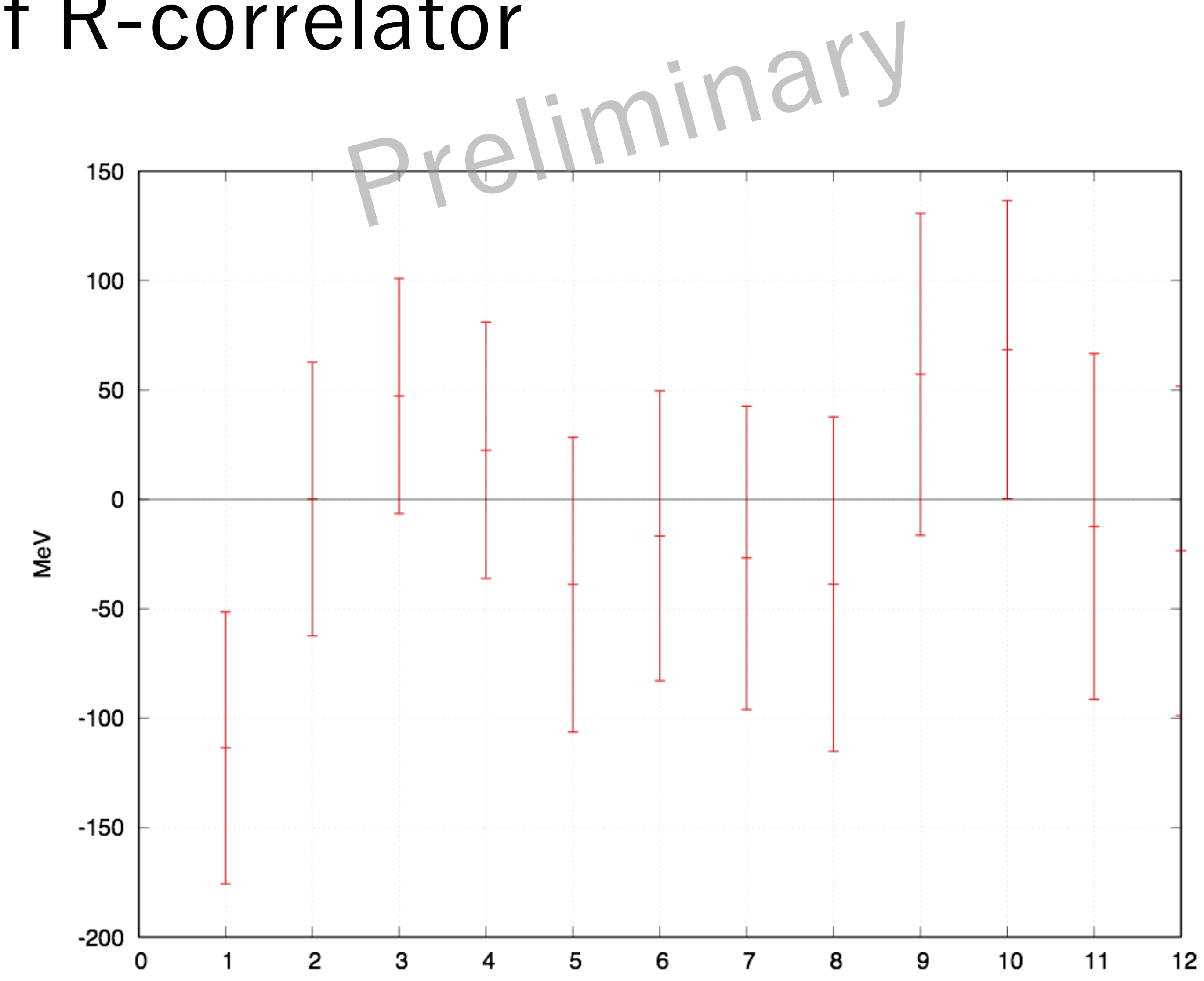
$$\Delta E(\Delta t) \equiv \log\left(\frac{\sum_{\mathbf{r}} R(\mathbf{r}, \Delta t)}{\sum_{\mathbf{r}} R(\mathbf{r}, \Delta t + 1)}\right)$$

still has large statistical error around 0 MeV.

⇩

Lüscher's method cannot tell us this channel is repulsive or attractive.

it needs more statistics.



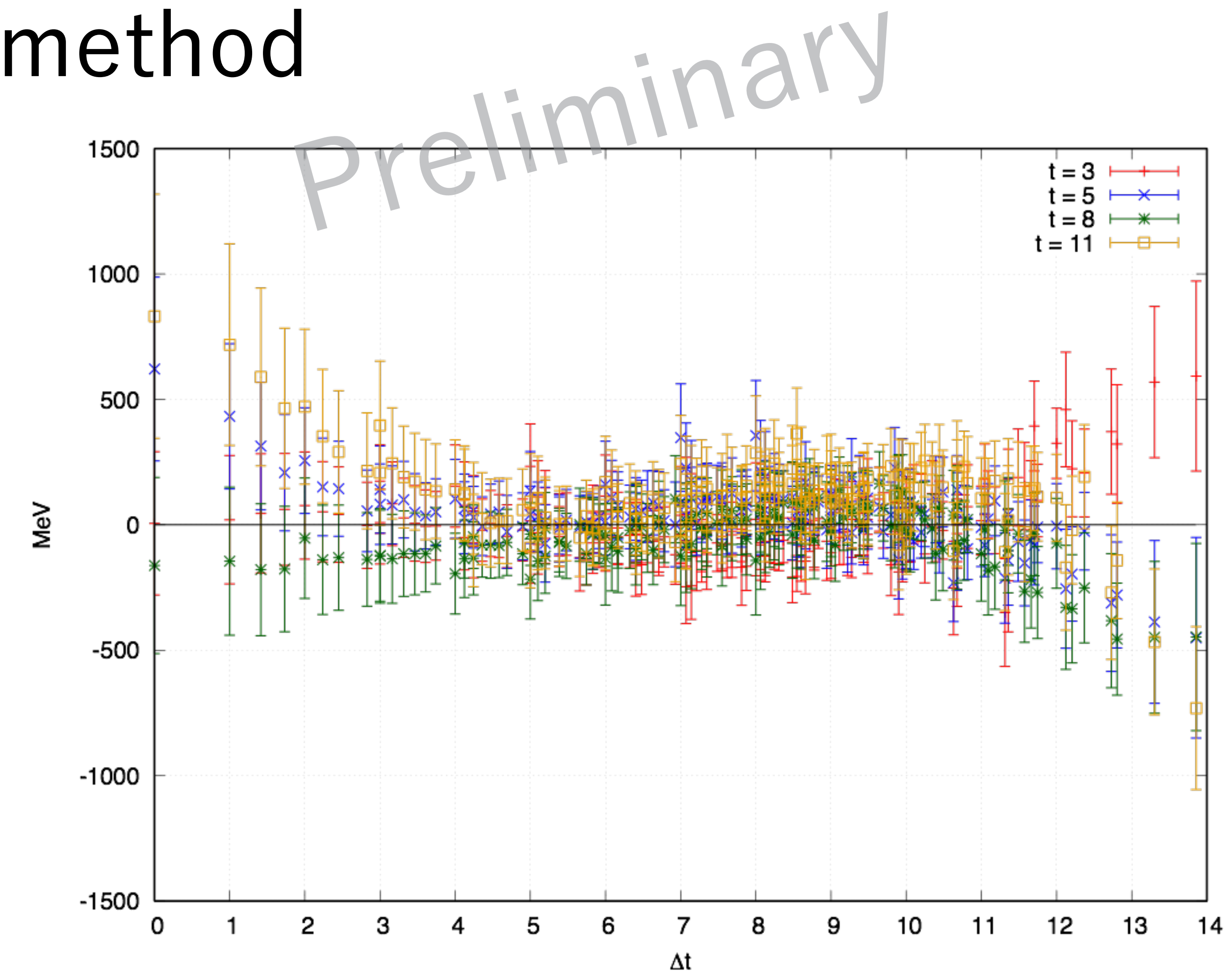
Potential with HALQCD method

We calculate potential from the R-correlator.

Short range behavior doesn't converge yet in the time evolution.

⇓

The precision of R-correlator might not be satisfactory.



The correct potential will be given as long as inelastic region's effect is negligible .

However, unfortunately potential doesn't converge in the time evolution.

⇒ We have to apply dilution in spin and LapH space to reduce statistical error in R-correlator.

Summary & Discussion

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- Stochastic LapH method achieves ground state dominance at early time.

→ Time-dependent HALQCD method will become applicable for longer temporal range.

- In the calculation of 4pt correlation, large dilution might be necessary.

→ Trade-off with calculation time

Efficient code is necessary. → We are writing computation time ~ (dilution index)^3 code

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The combination of HALQCD method and Stochastic LapH method will be a strong probe for the channels which include annihilation diagram.