THE GEORGE WASHINGTON UNIVERSITY

WASHINGTON, DC

Two-flavor simulations of the $\rho(770)$ and the role of the $K\bar{K}$ channel

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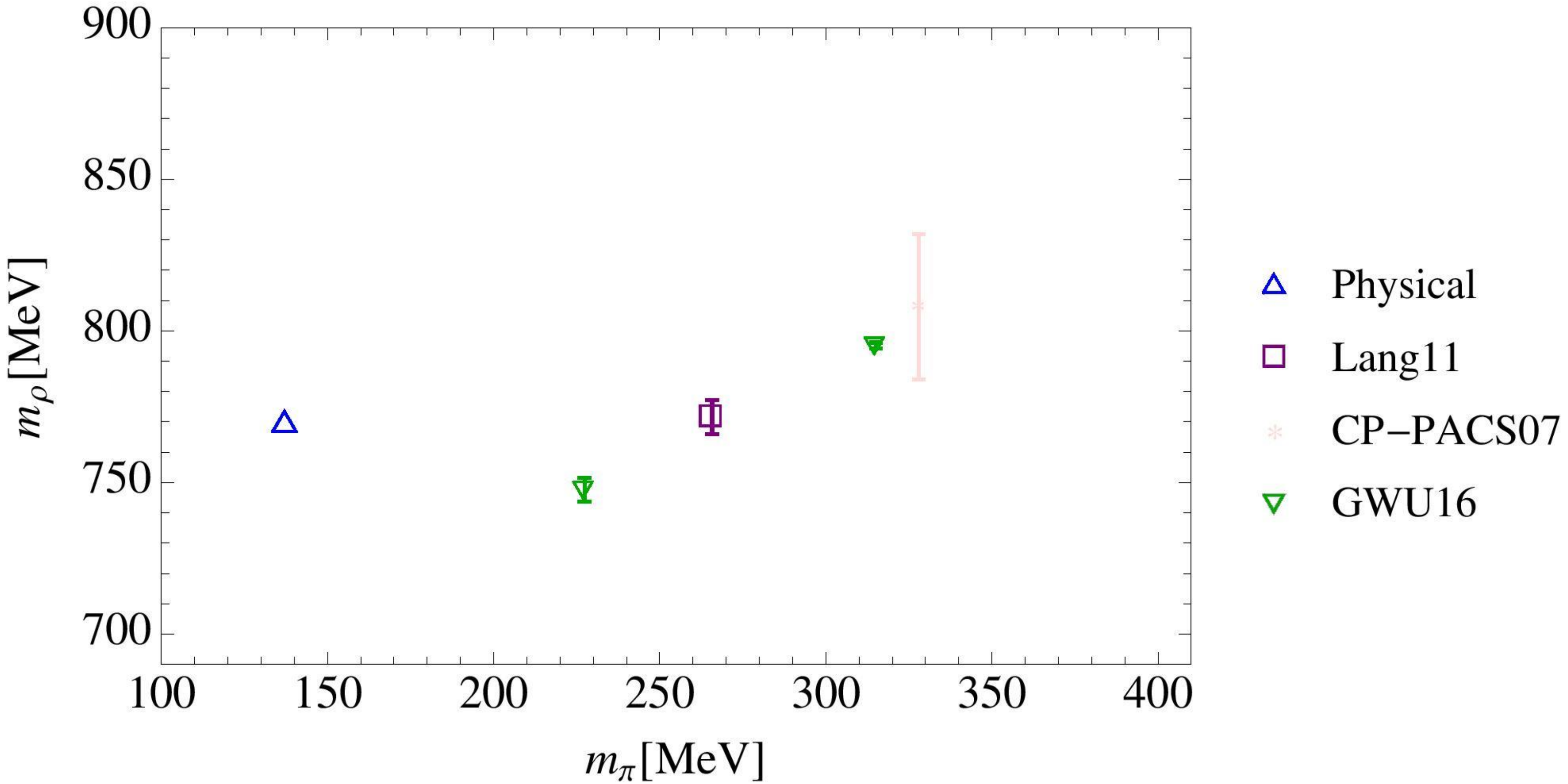
Introduction and motivation

A recent lattice QCD study (GWU16) noted that the ρ mass extracted from N_f=2 simulations is lighter than its physical value. This is also supported by an independent calculation from the RQCD Collaboration [1] very close to the physical mass.



Lang 11 [Lang, Mohler, Prelovsek, Vidmar, PRD (2011)], **CP-PACS07** [S. Aoki *et al.*, PRD (2007)]
GWU16 [Guo, Alexandru, Molina, Döring, (2016)], **RQCD16** [Bali *et al.*, PRD (2016)]

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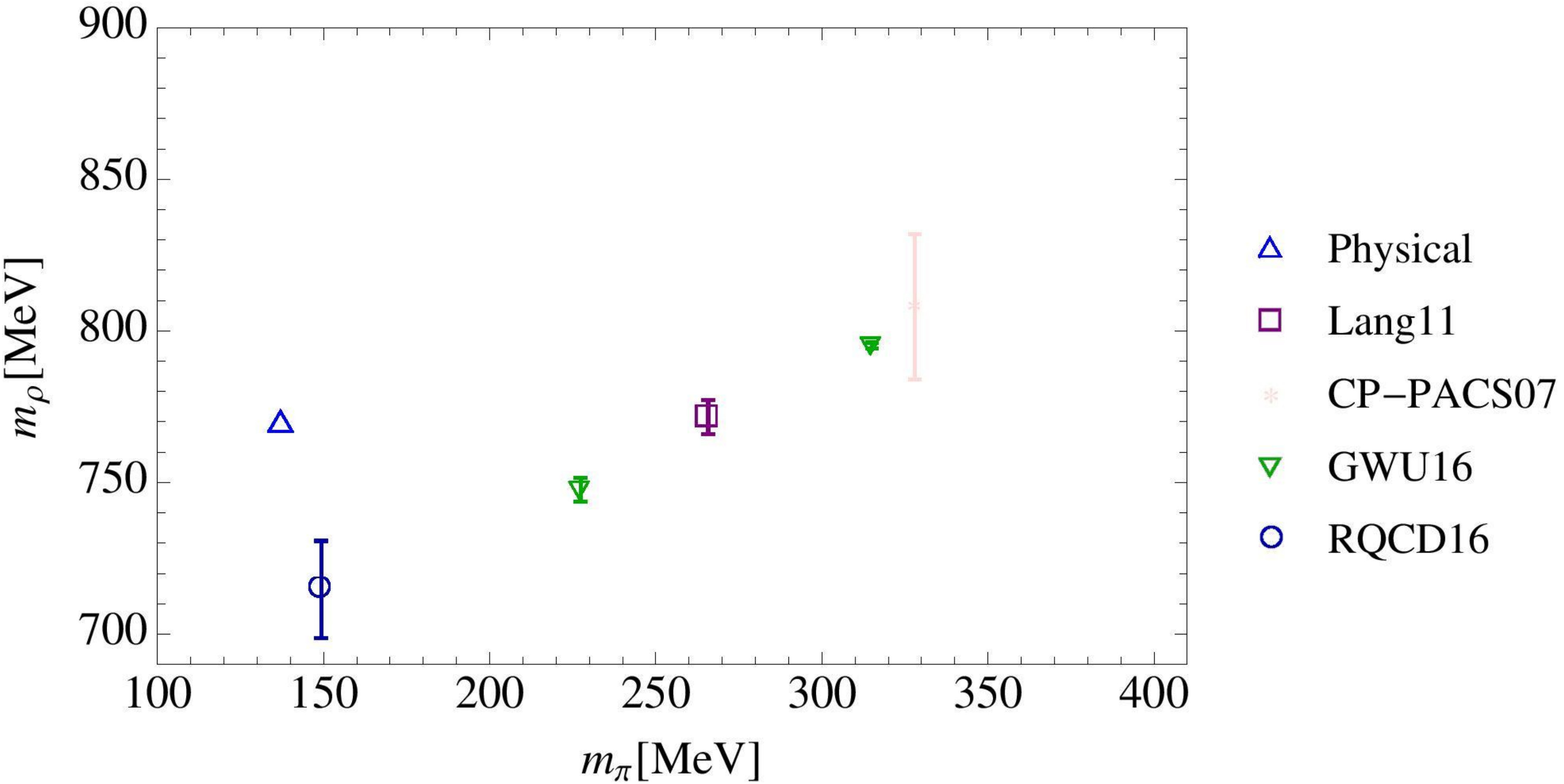

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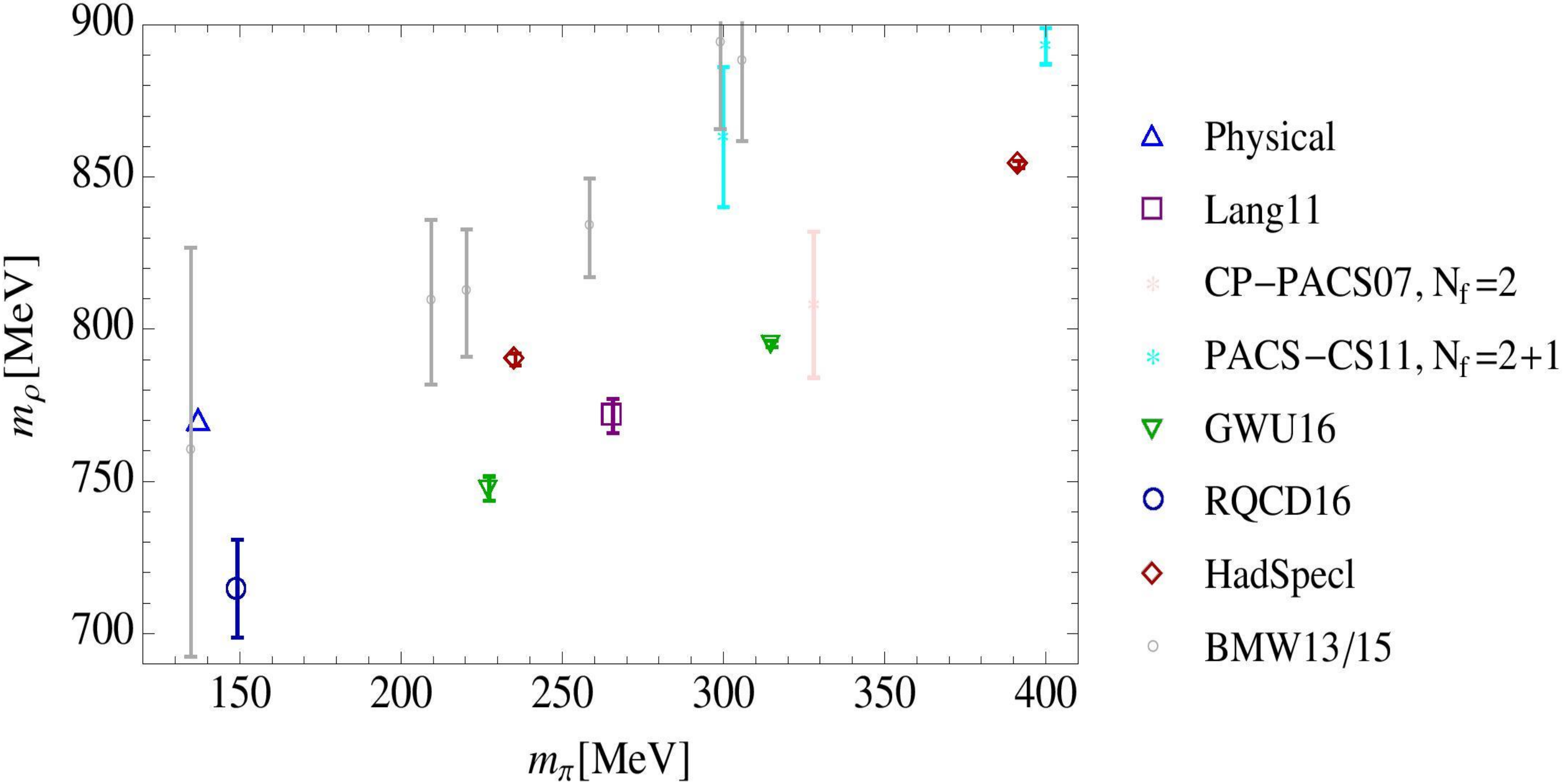

PACS-CS11 [Ishizuka PRD (2011)], **HadSpec** [J. J. Dudek *et al.* PRD (2013); Wilson, Briceño, Dudek, Edwards, Thomas, PRD (2015)], **BMW13/15** [Dürr *et al.* PRD (2013)]

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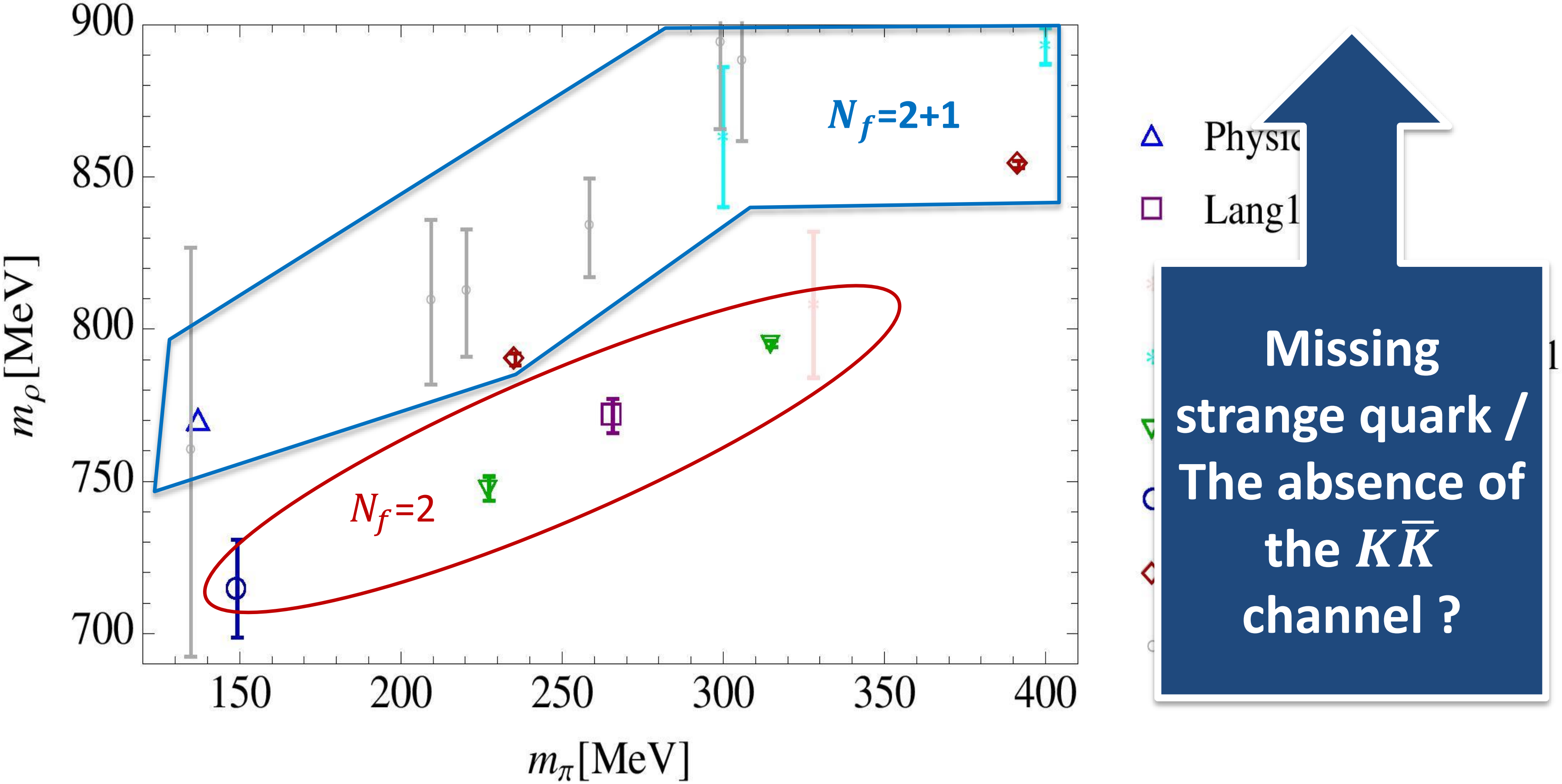

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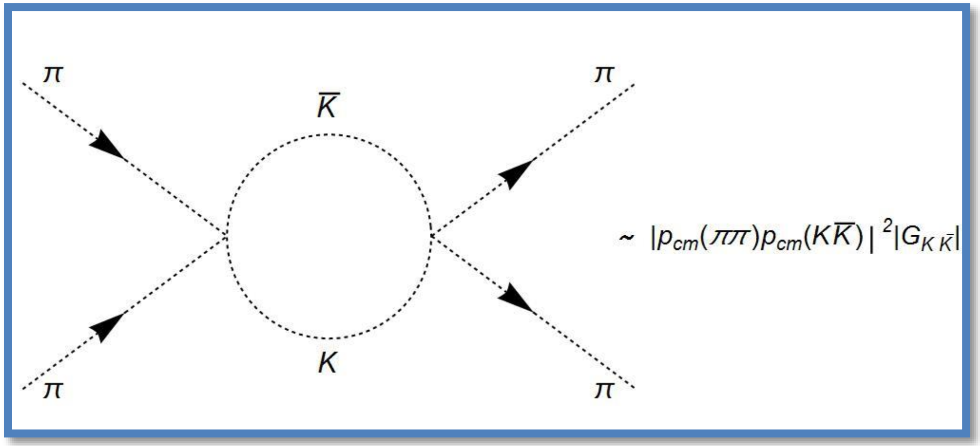
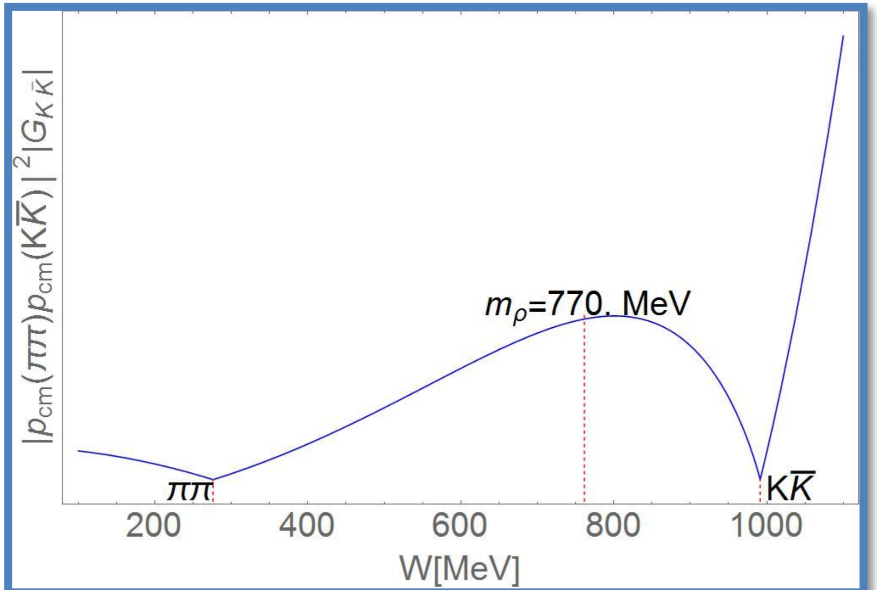
The Contribution of virtual Kaons (simple analogy)

Insertion of a $K\overline{K}$ intermediate state in scattering



Centrifugal barriers times $K\overline{K}$ propagator $G_{K\overline{K}}$



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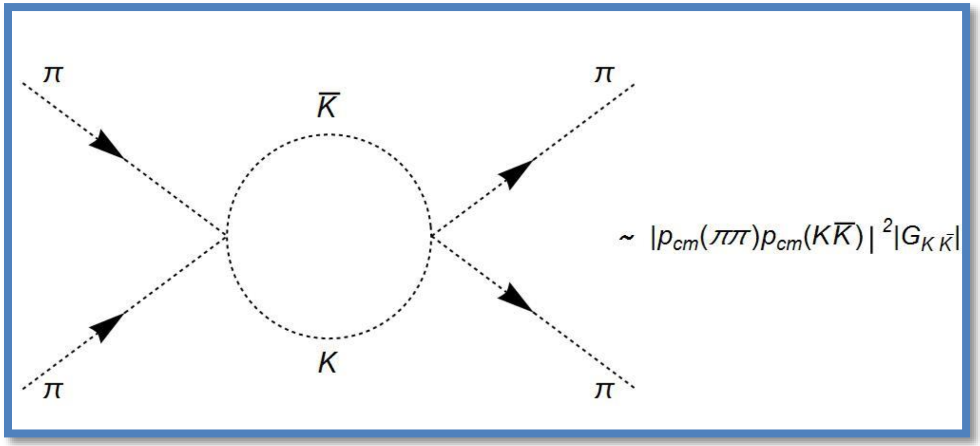

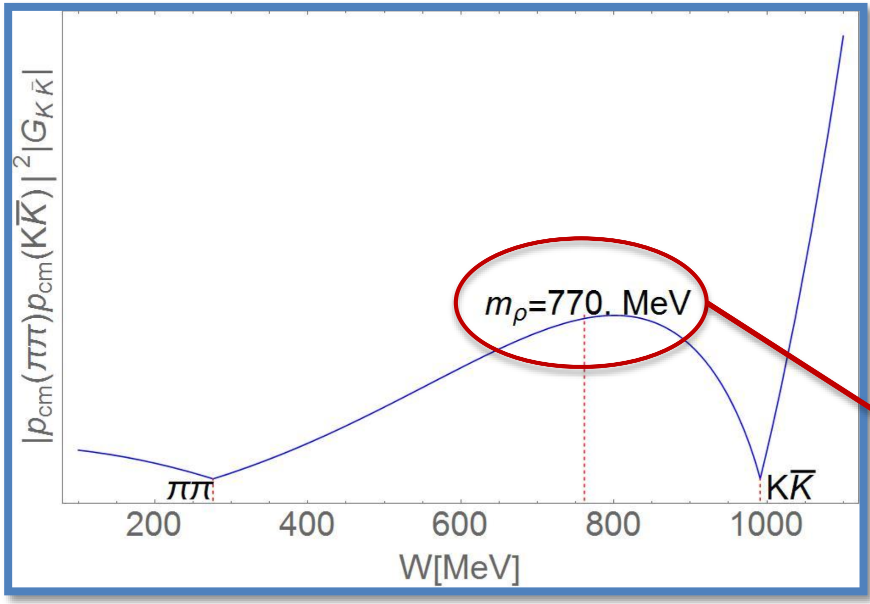

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A maximum contribution may occur close to the ρ mass, effectively shifting it.

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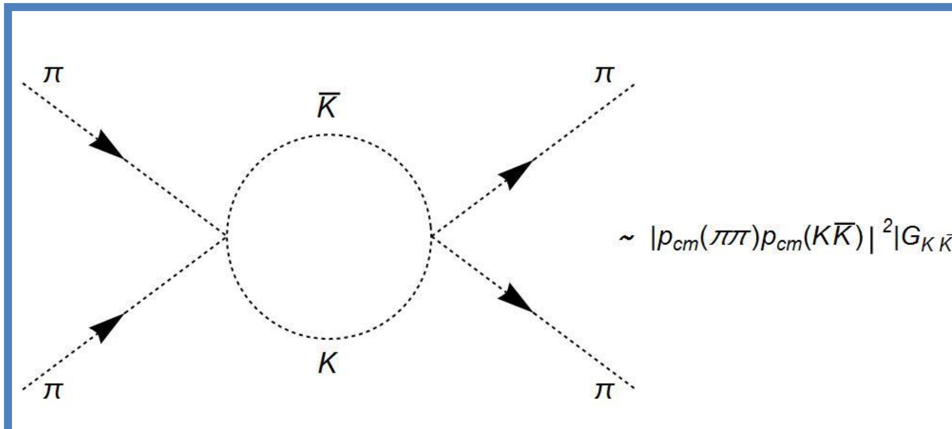

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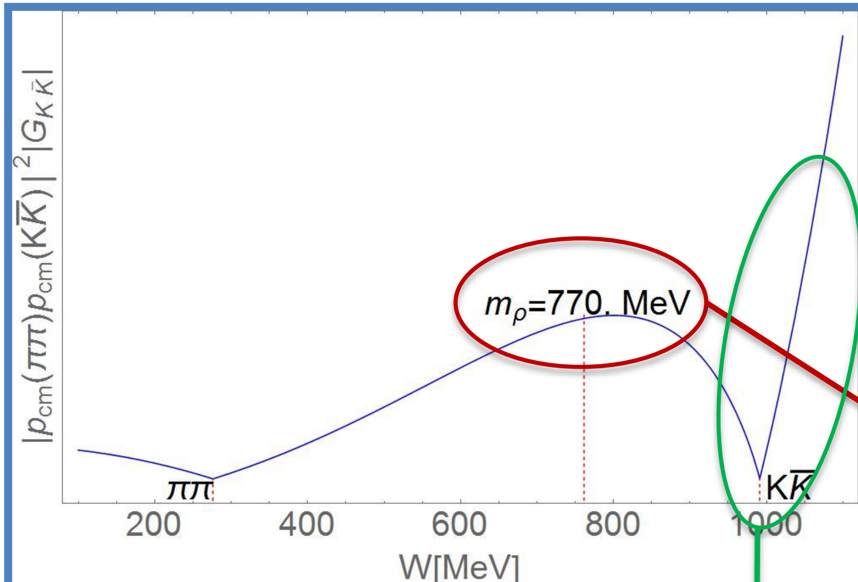
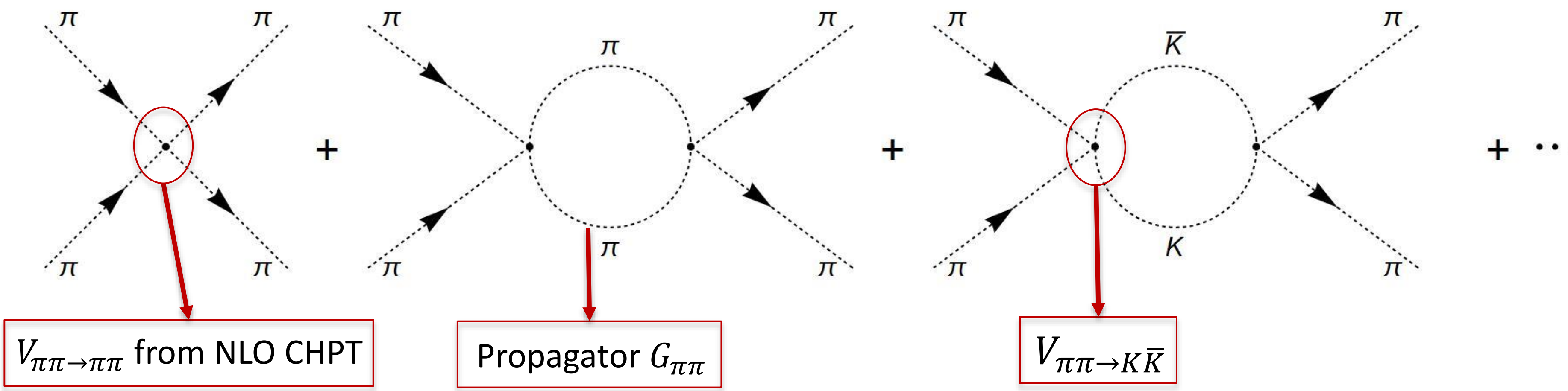


A maximum contribution may occur close to the ρ mass, effectively shifting it.

but small inelasticity ($K\overline{K}$ almost decouples above threshold) needs to be checked

Coupled channel scattering/Inverse amplitude method[1]

Unitarity in coupled channels



Organized in matrices: $\boldsymbol{V} = \begin{pmatrix} V_{\pi\pi\to\pi\pi} & V_{\pi\pi\to K\bar{K}} \\ V_{K\bar{K}\to\pi\pi} & V_{K\bar{K}\to K\bar{K}} \end{pmatrix}$, $\boldsymbol{G} = \begin{pmatrix} G_{\pi\pi} & 0 \\ 0 & G_{K\bar{K}} \end{pmatrix}$

Scattering equation: $\boldsymbol{T} = \boldsymbol{V} + \boldsymbol{TGV}$

Phenomenology: $\boldsymbol{S} = \boldsymbol{1} + i$ (factors) $\boldsymbol{T}$

$$\boldsymbol{S} = \begin{pmatrix} \eta e^{2i\delta_{\pi\pi}} & i(1-\eta^2)^{1/2} e^{i(\delta_{\pi\pi}+\delta_{K\bar{K}})} \\ i(1-\eta^2)^{1/2} e^{i(\delta_{\pi\pi}+\delta_{K\bar{K}})} & \eta e^{2i\delta_{K\bar{K}}} \end{pmatrix}$$

For SU(2) lattice data: remove $K\bar{K}$ channel and perform SU(2)/SU(3) matching of LECs.

[1]Based on Oller, Oset, Pelaez, PRD (1998).

Formulas

Workflow of chiral extrapolation

To fit the lattice phase shifts, the $K\bar{K}$ channel is removed from the coupled-channel $\pi\pi$ /$K\bar{K}$ system (including SU(3)/SU(2) matching of LECs).

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The result is extrapolated to the physical pion mass M_π = 138 MeV and then $K\bar{K}$ channel is switched on→post-diction of experimental phaseshifts.

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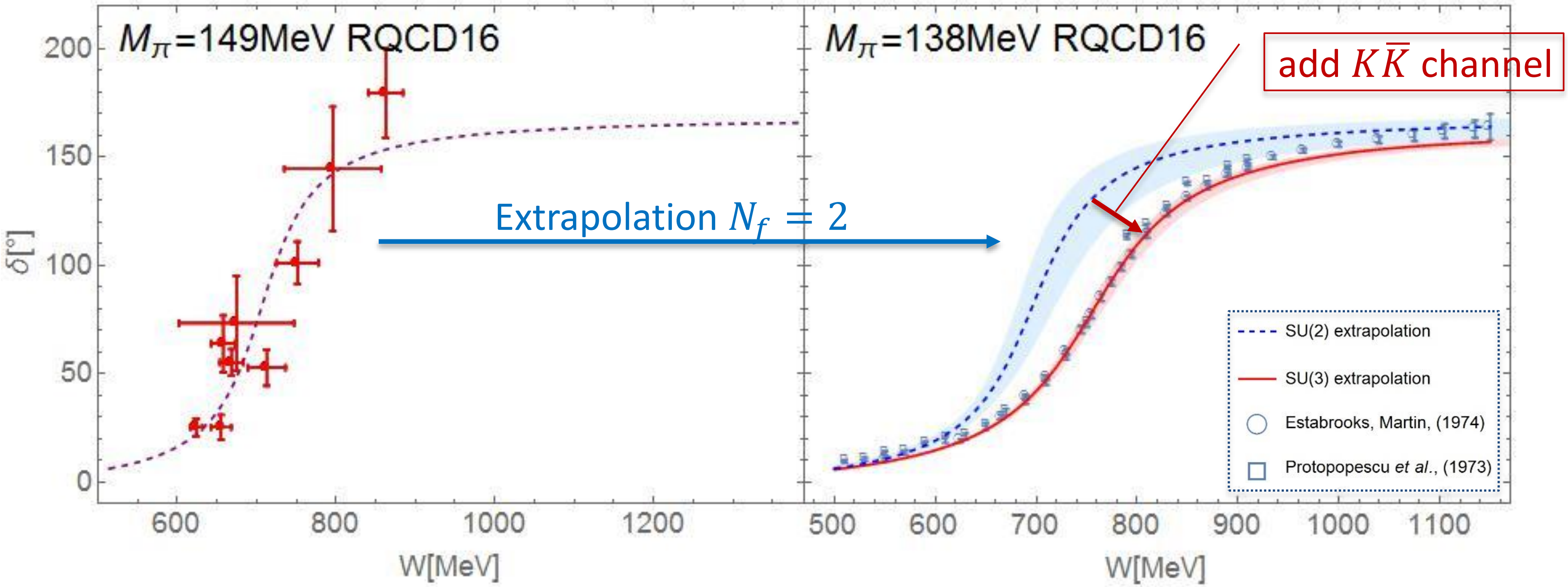
Express final results in terms of common notation, Breit-Wigner m_ρ: ρ mass, g: $\rho\pi\pi$ coupling.

Unknown LECs taken from fit to experiment

Results for the $N_f = 2$ simulations

Lattice phase shifts to be included in the fit are chosen in the maximal range around the resonance position, in which the fit passes Pearson's χ^2 test at a 90% upper confidence limit.



Experimental data: blue circles from [Estabrooks, Martin, (1974)], squares from [Protopopescu *et al.*, (1973)].

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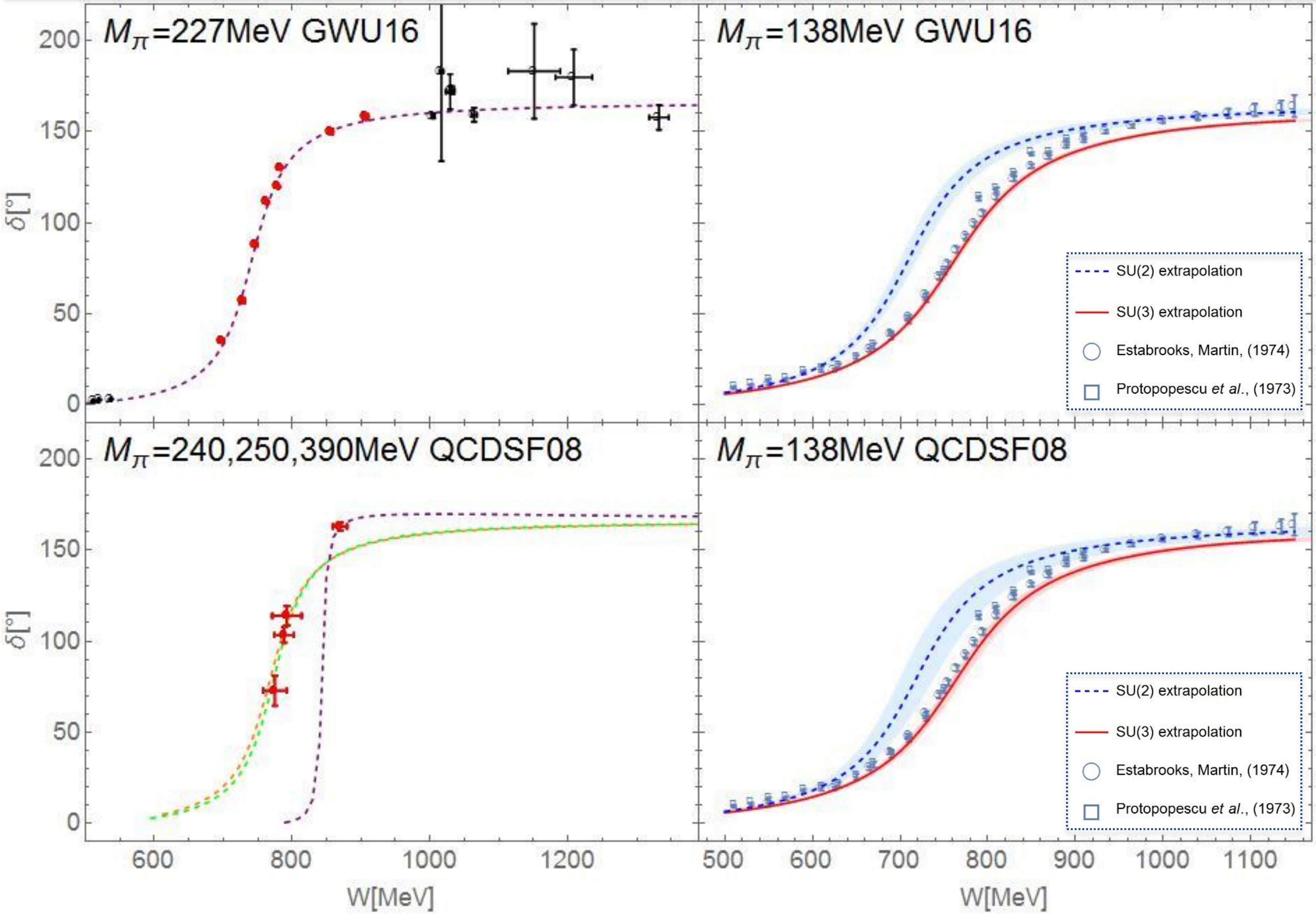

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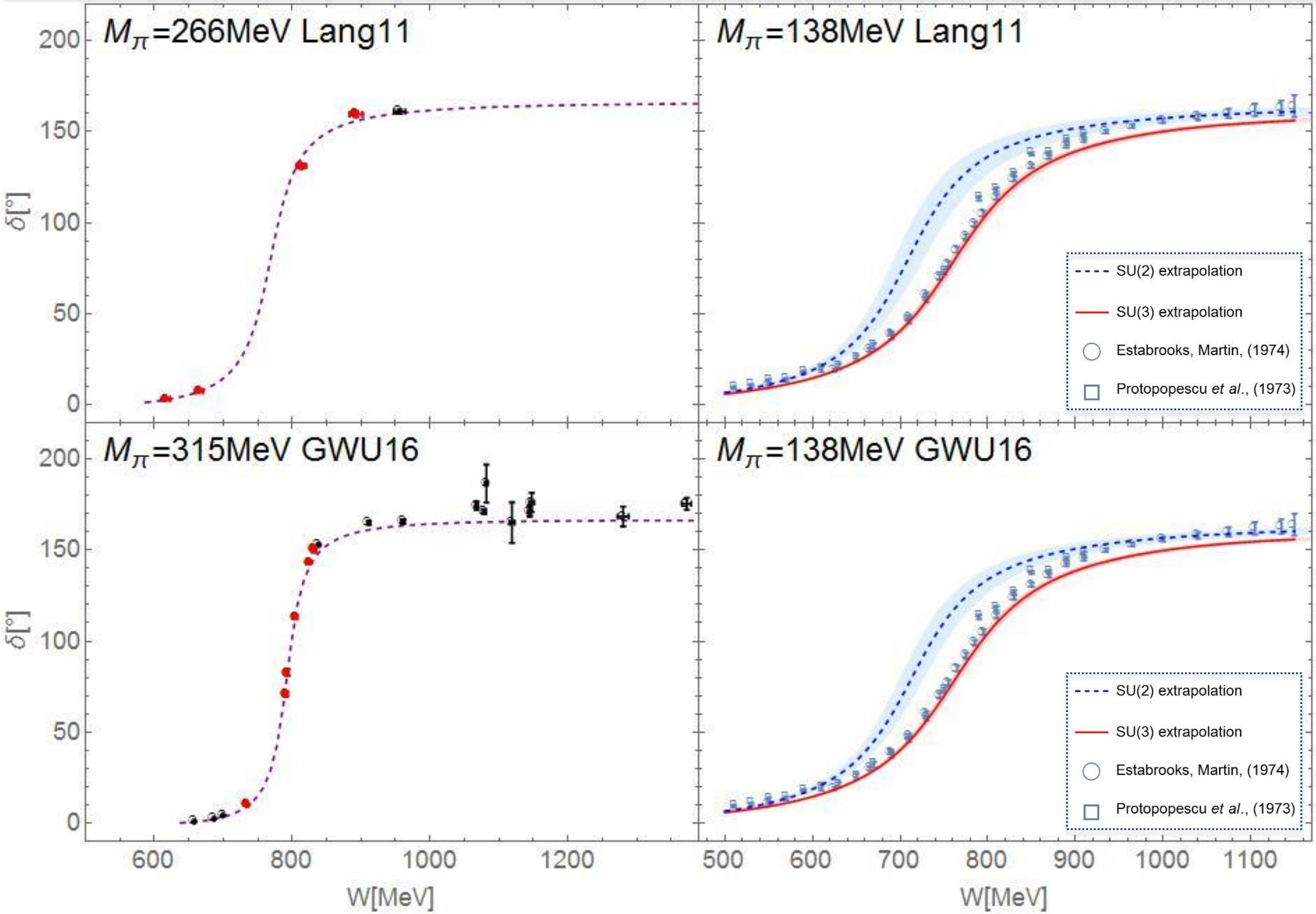

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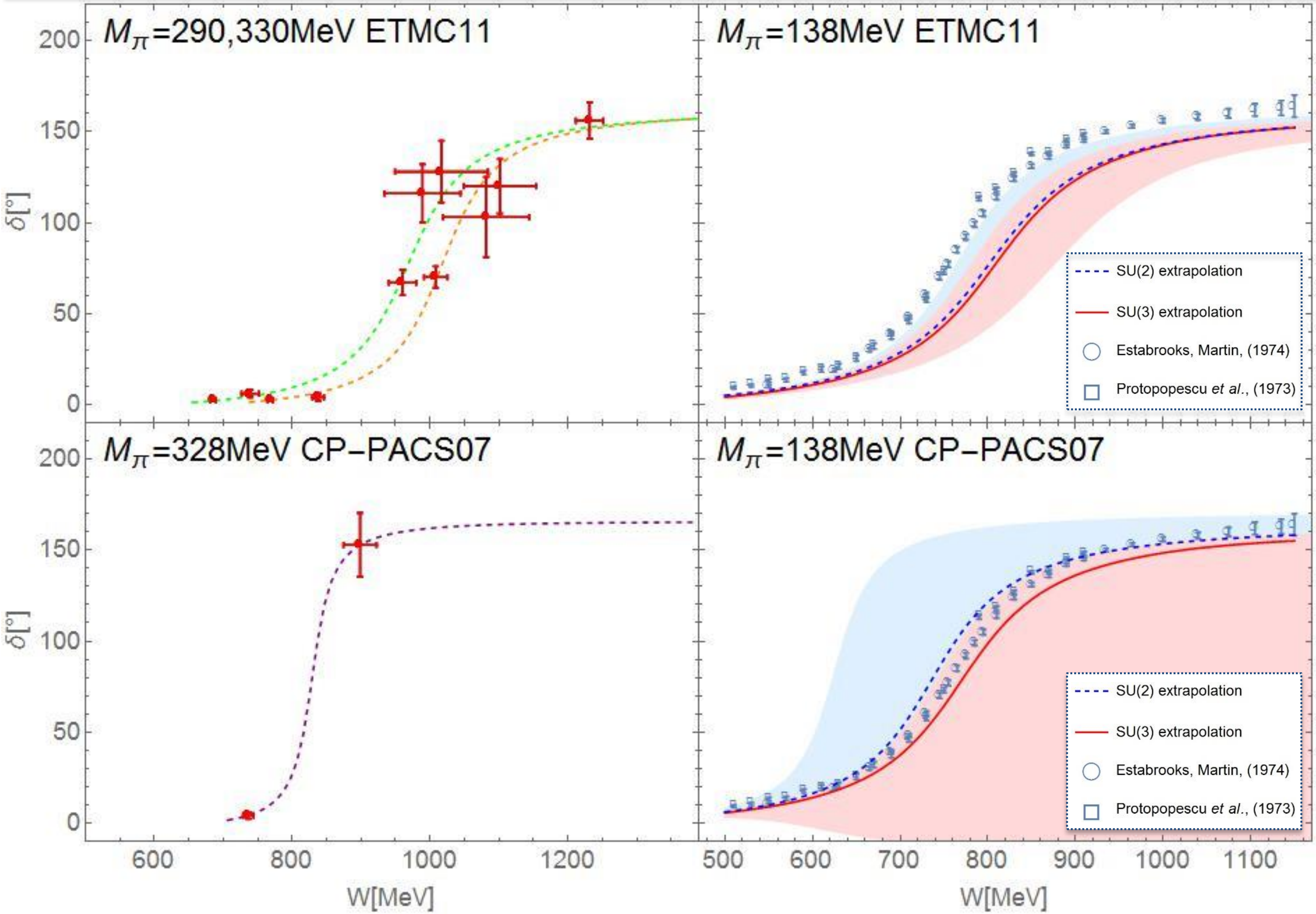
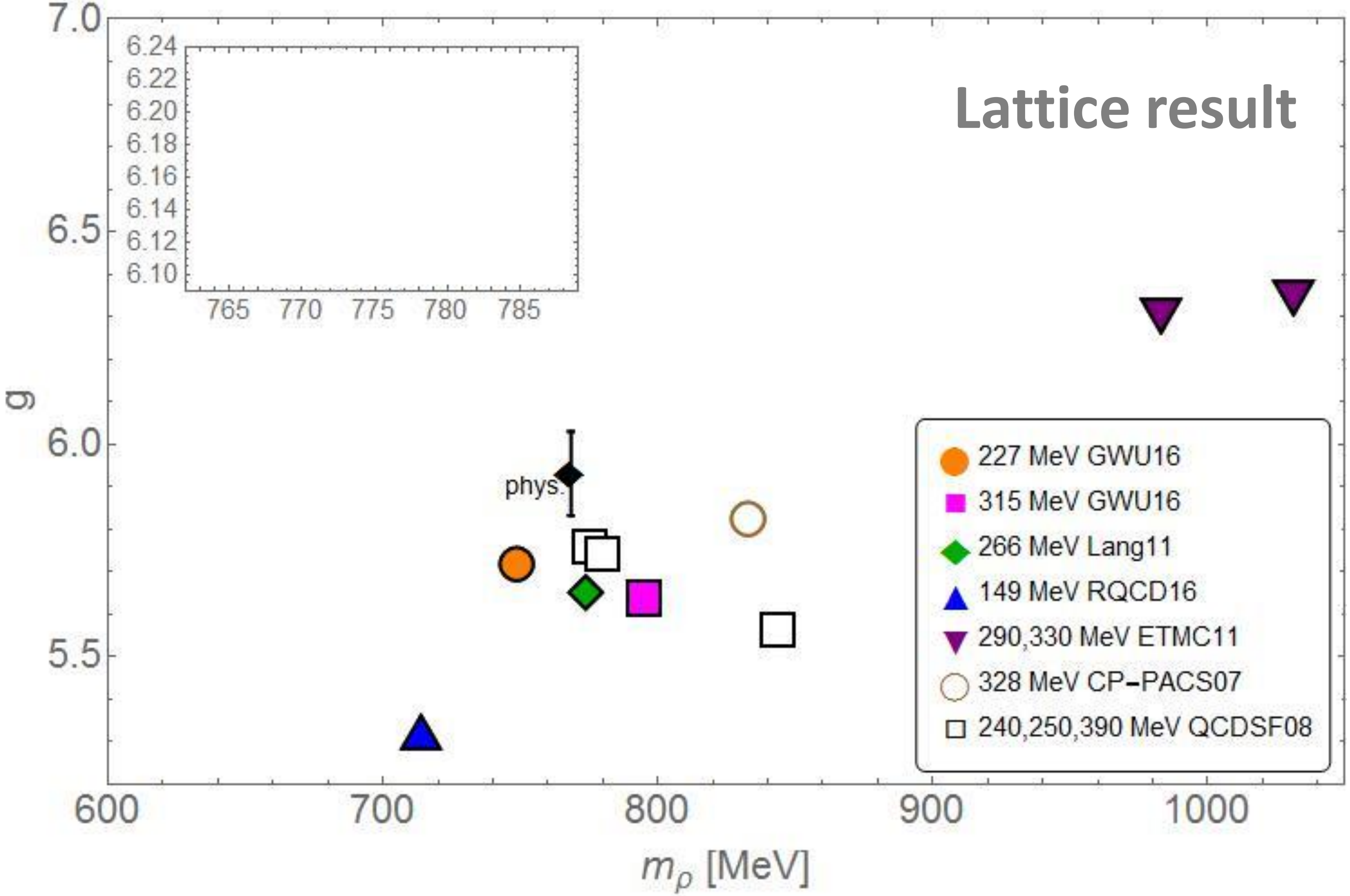


Effects of extrapolation in (m_ρ, g) plane



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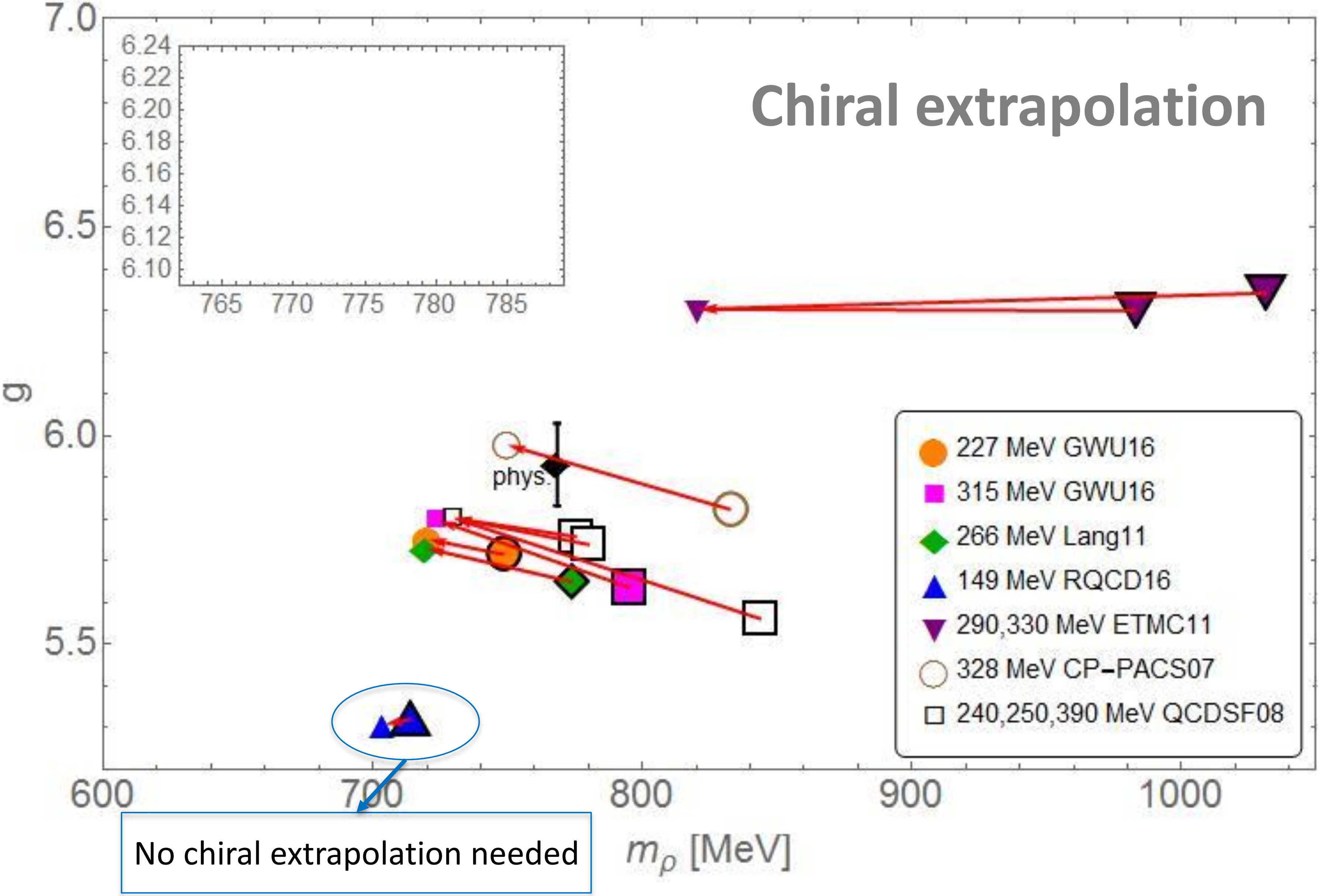

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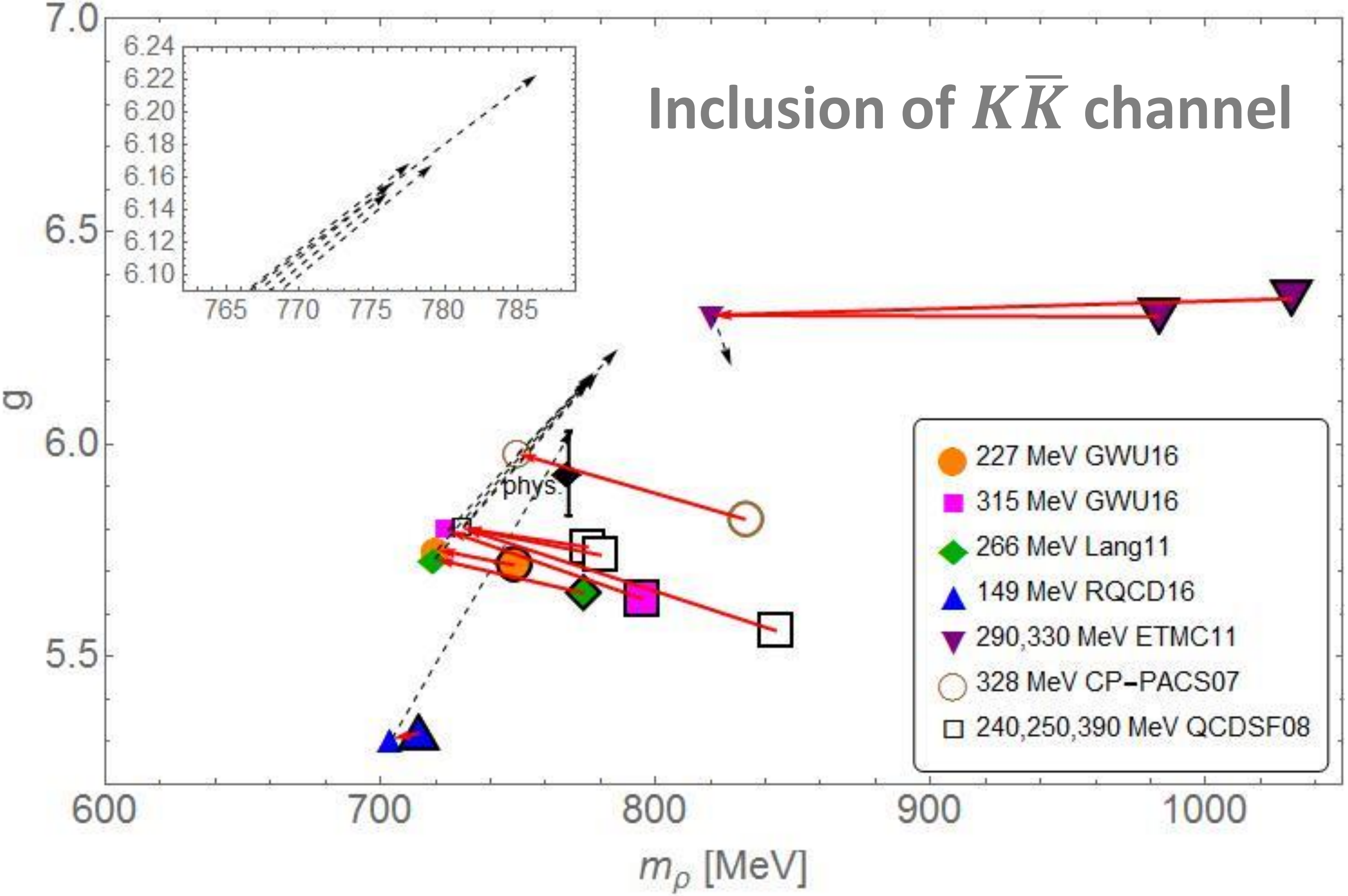

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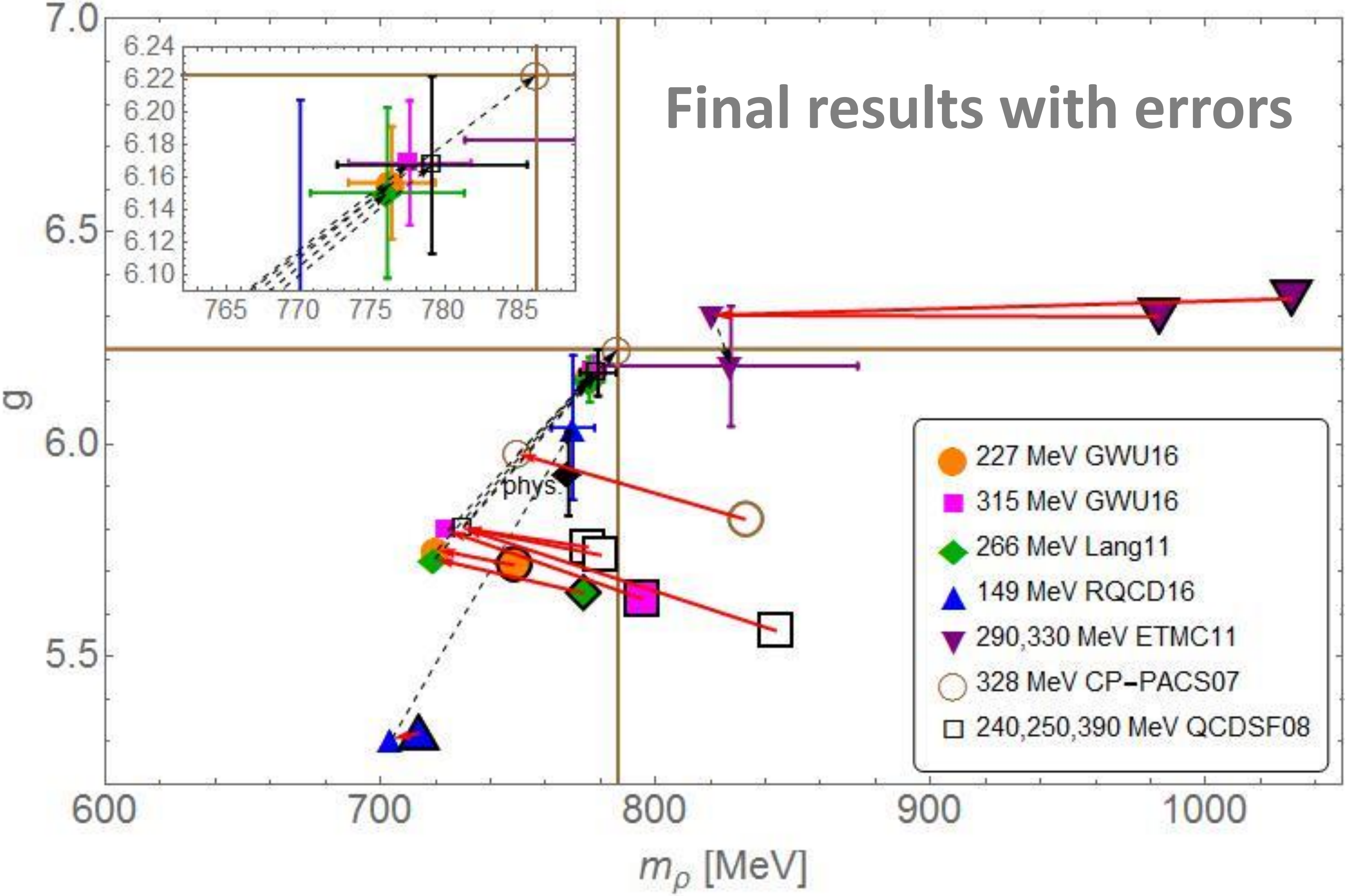
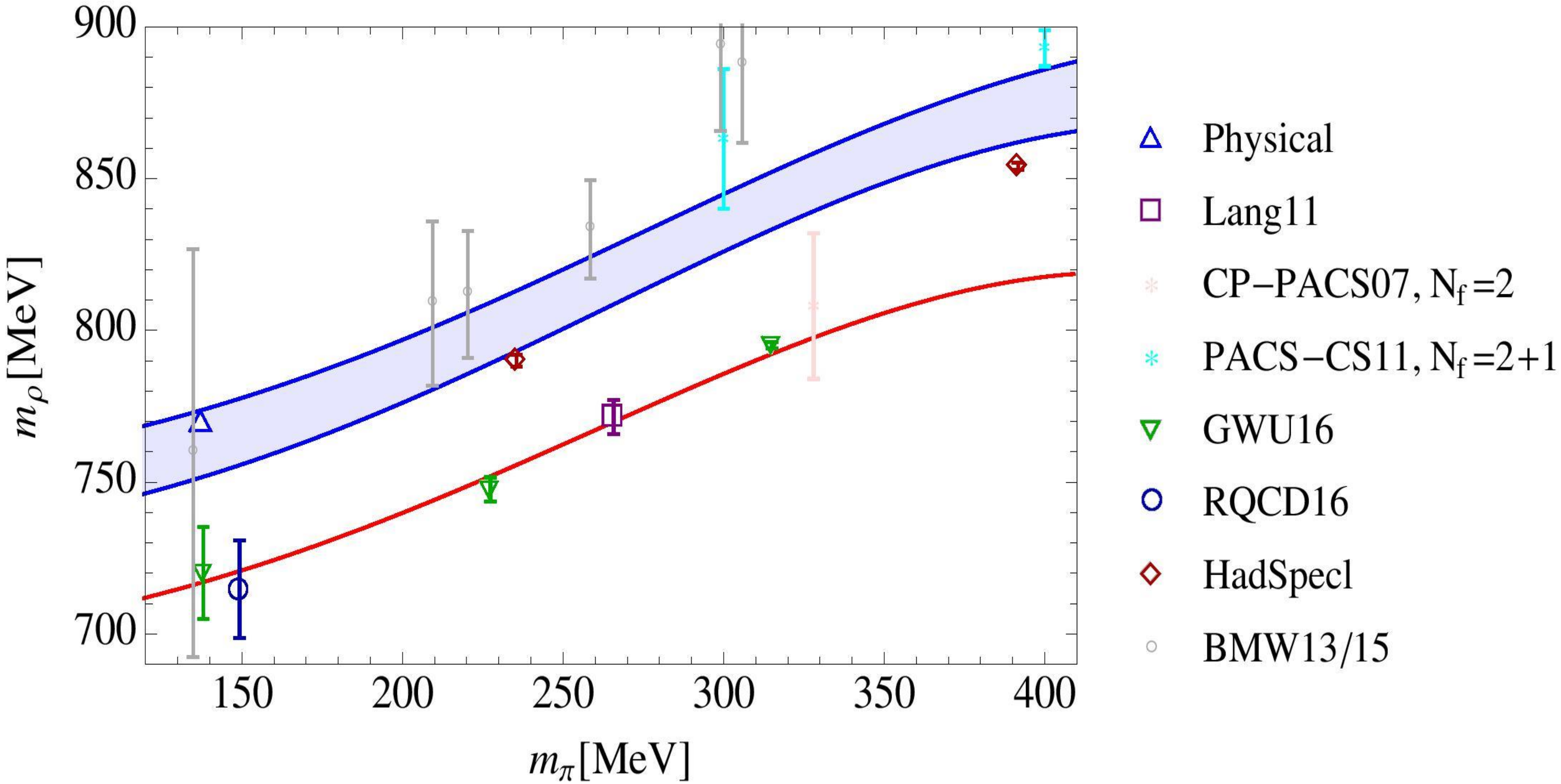
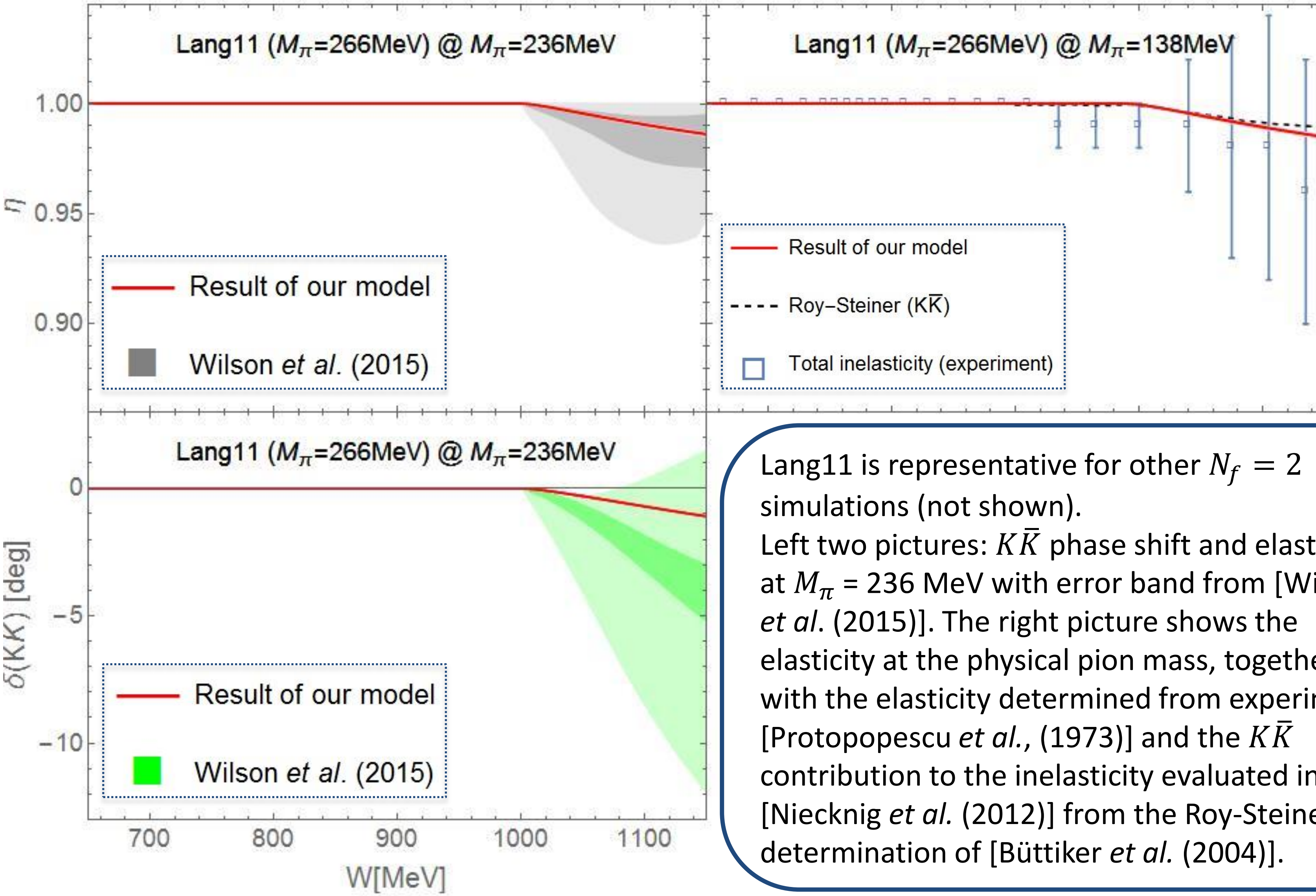


Resonance mass extrapolation

Red curve: an N_f=2 extrapolation based on the fit to (GWU16) data. Blue band: inclusion of the $K\bar{K}$ channel (systematic error only).



Inelasticity from $K\bar{K}$ channel



Lang11 is representative for other $N_f = 2$ simulations (not shown).
Left two pictures: $K\bar{K}$ phase shift and elasticity at M_π = 236 MeV with error band from [Wilson *et al*. (2015)]. The right picture shows the elasticity at the physical pion mass, together with the elasticity determined from experiment [Protopopescu *et al.*, (1973)] and the $K\bar{K}$ contribution to the inelasticity evaluated in [Niecknig *et al.* (2012)] from the Roy-Steiner determination of [Büttiker *et al.* (2004)].

Experimental inelasticity [Protopopescu *et al.*, PRD (1973)], **Roy-Steiner** [Büttiker *et al.* EPJ C (2004)].

Conclusion

The $K\bar{K}$ channel improves the extrapolations of the mass significantly except when the lattice data have large uncertainties.

Explains the systematically small lattice masses at the physical point after the chiral SU(2) extrapolation (or, directly, Bali result).

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The $K\bar{K}$ channel improves the extrapolations of the mass significantly except when the lattice data have large uncertainties.

→

Explains the systematically small lattice masses at the physical point after the chiral SU(2) extrapolation (or, directly, Bali result).

↓

Slight over-extrapolation

→

Confirmation with full 1-loop UCHPT needed

GWU (Guo/Alexandru) test: Including valence strange quark does not change ρ mass.

- If $K\bar{K}$ is really strong, the effect comes from sea quarks .
- $N_f = 2$, $N_f = 2 + 1$ simulations within same Lattice QCD simulation needed for confirmation.

Thanks

NLO CHPT Lagrangian[1]

$$\mathcal{L}_2 = \frac{f^2}{4}\langle \partial_\mu U^\dagger \partial^\mu + M(U + U^\dagger)\rangle$$

$$U(\phi) = \exp(i\sqrt{2}\Phi/f)$$

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}_\mu$$

$$\begin{aligned} \mathcal{L}_4 = \quad & L_1\langle \partial_\mu U^\dagger \partial^\mu U\rangle^2 + L_2\langle \partial_\mu U^\dagger \partial_\nu U\rangle\langle \partial^\mu U^\dagger \partial^\nu U\rangle \\ & + L_3\langle \partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\rangle + L_4\langle \partial U^\dagger \partial^\mu U\rangle\langle U^\dagger M + M^\dagger U\rangle \\ & + L_5\langle \partial_\mu U^\dagger \partial^\mu U(U^\dagger M + M^\dagger U)\rangle + L_6\langle U^\dagger M + M^\dagger U\rangle^2 \\ & + L_7\langle U^\dagger M - M^\dagger U\rangle^2 + L_8\langle M^\dagger U M^\dagger U + U^\dagger M U^\dagger M\rangle \end{aligned}$$

$$M = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix}$$

[1][Oller, Oset, Pelaez, PRD (1999)].

Inverse amplitude method[1]

$$S = \begin{pmatrix} \eta e^{2i\delta_1} & i(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} \\ i(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix}$$

$$(T)_{11} = -\frac{8\pi W}{2ip_1}[(S)_{11} - 1] \ , \qquad (T)_{22} = -\frac{8\pi W}{2ip_2}[(S)_{22} - 1]$$

$$(T)_{12} = (T)_{21} = -\frac{8\pi W}{2i\sqrt{p_1 p_2}}(S)_{12}$$

$$T = [I - V^{\mathrm{IAM}} G]^{-1} V^{\mathrm{IAM}}$$

$$\boldsymbol{V} = \begin{pmatrix} V_{\pi\pi\to\pi\pi} & V_{\pi\pi\to K\bar{K}} \\ V_{K\bar{K}\to\pi\pi} & V_{K\bar{K}\to K\bar{K}} \end{pmatrix}$$

$$V^{\mathrm{IAM}} = V_2[V_2 - V_4]^{-1} V_2$$

[1][Oller, Oset, Pelaez, PRD (1999)].

G-function and V-matrices[1]

$$
\begin{aligned}
G_{ii}^{DR}(W) &= i\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2-m_1^2+i\epsilon}\frac{1}{(P-q)^2-m_2^2+i\epsilon}\\
&= \frac{1}{16\pi^2}\Big\{a(\mu)+\ln\frac{m_1^2}{\mu^2}+\frac{m_2^2-m_1^2+W^2}{2W^2}\ln\frac{m_2^2}{m_1^2}\\
&\quad +\frac{p_i}{W}\Big[\ln(\;W^2-(m_1^2-m_2^2)+2p_iW)+\ln(\;W^2+(m_1^2-m_2^2)+2p_iW)\\
&\quad -\ln(-W^2+(m_1^2-m_2^2)+2p_iW)-\ln(-W^2-(m_1^2-m_2^2)+2p_iW)\Big]\Big\}
\end{aligned}
$$

$$\bar{p}_i = \frac{\sqrt{(W^2-(m_1+m_2)^2)(W^2-(m_1-m_2)^2)}}{2W}$$

$$V_2(E) = -\begin{pmatrix} \frac{2p_\pi^2}{3f_\pi^2} & \frac{\sqrt{2}p_K p_\pi}{3f_K f_\pi} \\ \frac{\sqrt{2}p_K p_\pi}{3f_K f_\pi} & \frac{p_K^2}{3f_K^2} \end{pmatrix}$$

$$V_4(E) = -1\times \begin{pmatrix} \frac{8p_\pi^2(2\hat{l}_1 m_\pi^2-\hat{l}_2E^2)}{3f_\pi^4} & \frac{8p_\pi p_K(L_5(m_K^2+m_\pi^2)-L_3E^2)}{3\sqrt{2}f_\pi^2f_K^2} \\ \frac{8p_\pi p_K(L_5(m_K^2+m_\pi^2)-L_3E^2)}{3\sqrt{2}f_\pi^2f_K^2} & \frac{4p_K^2(10\hat{l}_1m_K^2+3(L_3-2\hat{l}_2)E^2)}{9f_K^4} \end{pmatrix}$$

$$\hat{l}_1 \equiv 2\,L_4+L_5$$
$$\hat{l}_2 \equiv 2\,L_1-L_2+L_3$$
$$l_1^r = 4L_1^r+2L_3-\tfrac{1}{24}\nu_K$$
$$l_2^r = 4L_2^r-\tfrac{1}{12}\nu_K$$
$$l_3^r = -8L_4^r-4L_5^r+16L_6^r+8L_8^r-\tfrac{1}{18}\nu_\eta$$
$$l_4^r = 8L_4^r+4L_5^r-\tfrac{1}{2}\nu_K$$

$$\Rightarrow \widehat{l_1} = l_4^r/4 + 1/8\,\upsilon_K$$
$$\widehat{l_2} = l_1^r/2 - l_2^r/4$$

[1][Guo, Alexandru, Molina, Döring, (2016)], l_i^r: [Gasser, Leutwyler, Nucl. Phys. B (1985)]

Meson mass and decay constants[1]

$$M_\pi^2 = M_{0\pi}^2\left[1+\mu_\pi-\frac{\mu_\eta}{3}+\frac{16M_{0K}^2}{f_0^2}(2L_6^r-L_4^r)\right.$$
$$\left.+\frac{8M_{0\pi}^2}{f_0^2}(2L_6^r+2L_8^r-L_4^r-L_5^r)\right],$$
$$M_K^2 = M_{0K}^2\left[1+\frac{2\mu_\eta}{3}+\frac{8M_{0\pi}^2}{f_0^2}(2L_6^r-L_4^r)\right.$$
$$\left.+\frac{8M_{0K}^2}{f_0^2}(4L_6^r+2L_8^r-2L_4^r-L_5^r)\right],$$
$$M_\eta^2 = M_{0\eta}^2\left[1+2\mu_K-\frac{4}{3}\mu_\eta+\frac{8M_{0\eta}^2}{f_0^2}(2L_8^r-L_5^r)\right.$$
$$\left.+\frac{8}{f_0^2}(2M_{0K}^2+M_{0\pi}^2)(2L_6^r-L_4^r)\right]$$
$$+M_{0\pi}^2\left[-\mu_\pi+\frac{2}{3}\mu_K+\frac{1}{3}\mu_\eta\right]$$
$$+\frac{128}{9f_0^2}(M_{0K}^2-M_{0\pi}^2)^2(3L_7+L_8^r),$$

with

$$\mu_P = \frac{M_{0P}^2}{32\pi^2 f_0^2}\log\frac{M_{0P}^2}{\mu^2}, \qquad P=\pi,K,\eta\ .$$

$$f_\pi = f_0\left[1-2\mu_\pi-\mu_K+\frac{4M_{0\pi}^2}{f_0^2}(L_4^r+L_5^r)+\frac{8M_{0K}^2}{f_0^2}L_4^r\right],$$

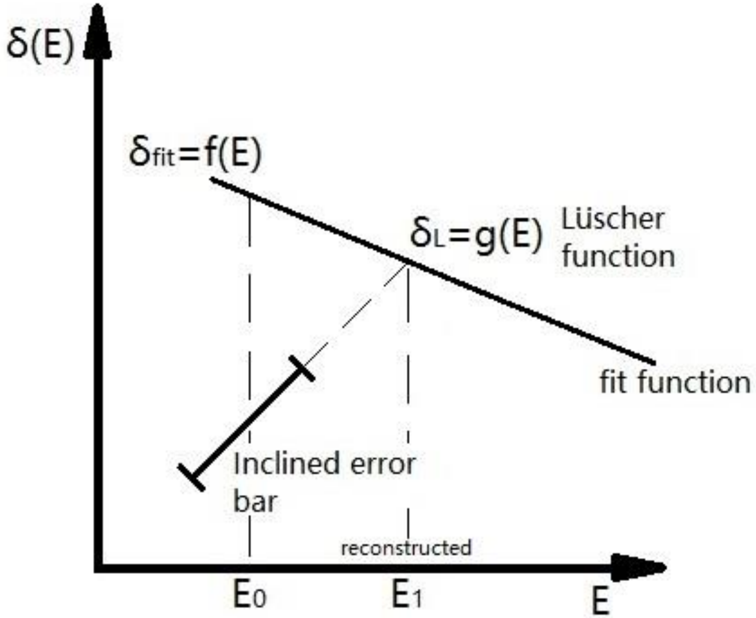
$$f_K = f_0\left[1-\frac{3\mu_\pi}{4}-\frac{3\mu_K}{2}-\frac{3\mu_\eta}{4}+\frac{4M_{0\pi}^2}{f_0^2}L_4^r\right.$$
$$\left.+\frac{4M_{0K}^2}{f_0^2}(2L_4^r+L_5^r)\right],$$

$$f_\eta = f_0\left[1-3\mu_K+\frac{4L_4^r}{f_0^2}(M_{0\pi}^2+2M_{0K}^2)+\frac{4M_{0\eta}^2}{f_0^2}L_5^r\right].$$

In the above equations, f_0 is the pion decay constant in the chiral limit, $4\pi f_0 \cong 1.2$ GeV, μ is the regularization scale, commonly fixed at $\mu = M_\rho$, and L_i^rs, with i = 1~8, are the Low Energy Constants, superscript r stand for a dependence on the regularization scale μ. L_i^rs for f_i are obtained from fit to lattice data in [Nebreda, Pelaez][1].

[1][Nebreda, Pelaez., PRD (2010)].

Inclined error bar fit



- The function $\delta_L(E) = g(E)$ returns the phase shift from an energy eigenvalue E measured on the lattice.
- Fit function $\delta_{fit}(E) = f(E)$ intersects with $g(E)$ at E_1.
- χ^2 should be measured along the inclination of the error bar.
- E_1: energy of the intersect.
- E_0: energy of the data point.
- $f'(E_0)$: the numerical derivative of fit function.
- $g'(E_0)$ can be reconstructed from error bars in E and $\delta(E)$, and determining the sign of the slope of δ_L at each measured energy.

$$f(E_1) = g(E_1)$$

$$f(E_0) + f'(E_0)(E_1 - E_0) = g(E_0) + g'(E_0)(E_1 - E_0)$$

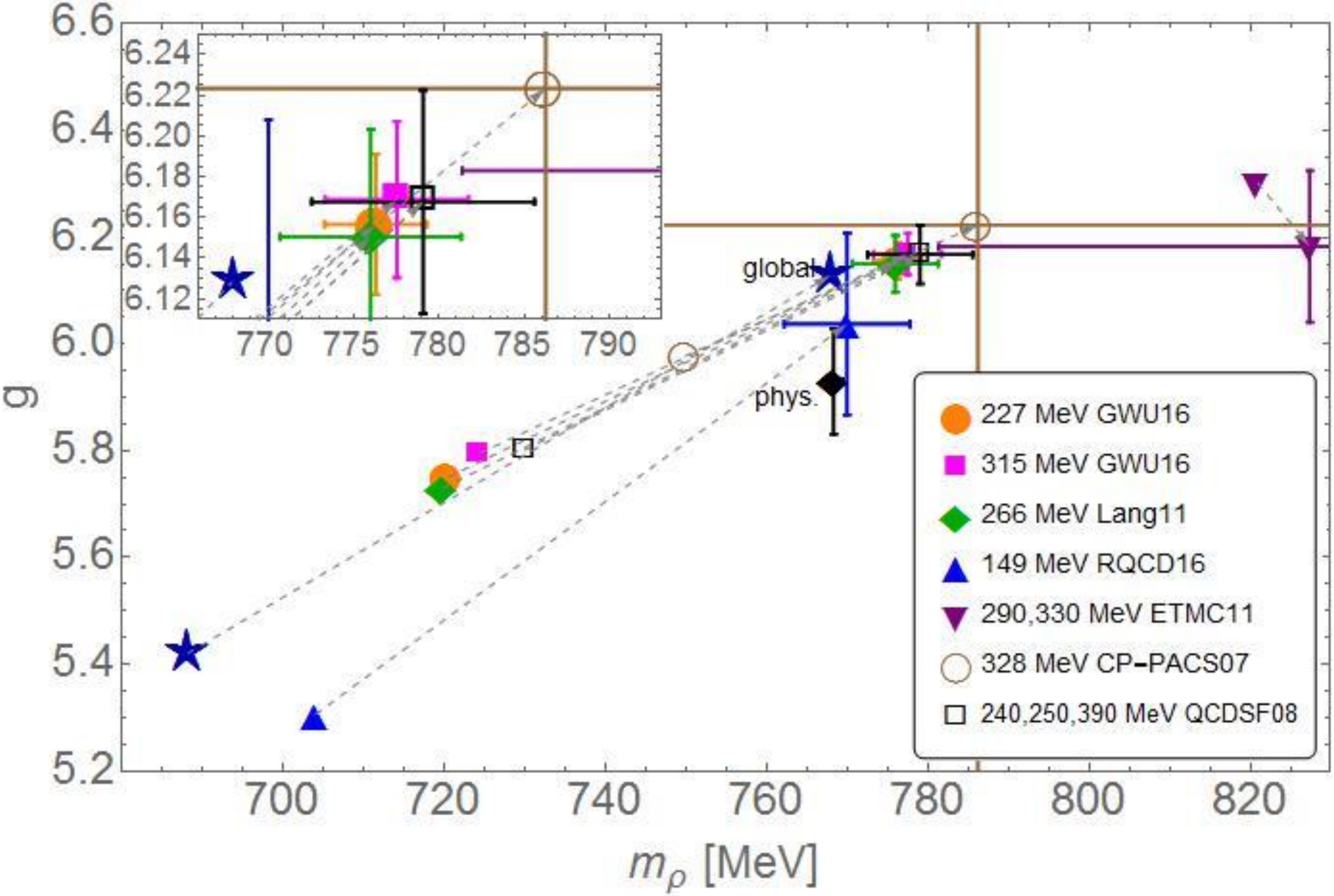
$$f'(E_0)(E_1 - E_0) - g'(E_0)(E_1 - E_0) = g(E_0) - f(E_0)$$

$$E_1[f'(E_0) - g'(E_0)] - E_0[f'(E_0) - g'(E_0)] = g(E_0) - f(E_0)$$

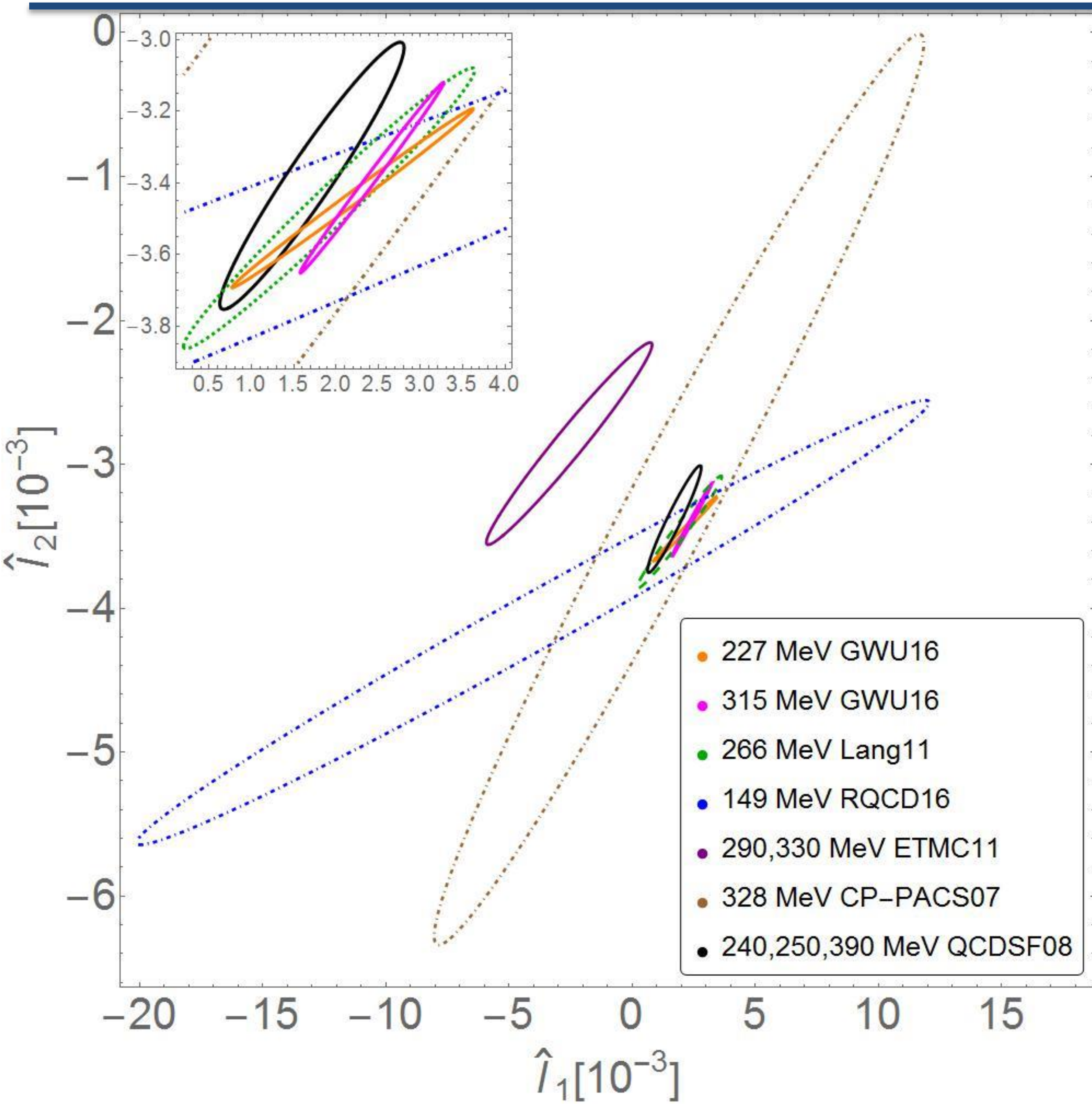
$$\Rightarrow E_1 = \frac{g(E_0) - f(E_0)}{f'(E_0) - g'(E_0)} + E_0$$

$$\Rightarrow \chi^2 = \frac{(E_1 - E_0)^2}{\Delta E^2} = \frac{(g(E_0) - f(E_0))^2}{\Delta E^2} \frac{1}{(f'(E_0) - g'(E_0))^2}$$

Effects of extrapolation in (m_ρ, g) plane



68% confidence ellipses in l_1, l_2



- **Linear combinations of LECs:**
  $\widehat{l_1} = 2L_4 + L_5$,
  $\widehat{l_2} = 2L_1 - L_2 + L_3$.
- **The error ellipses from RQCD, GWU (M_π = 227 MeV and M_π = 315 MeV), Lang et al., and CP-PACS all have a common overlap.**
- **The ellipse from QCDSF is very slightly off, while the one from ETMC is clearly incompatible.**
- **The contribution of kaon tadpole** (Gasser, Leutwyler, Nucl. Phys. B, 1985) **is very tiny.**

Breit-Wigner ρ masses of the ρ meson

	Mπ	BW	BW,converted	N_f=2 extrapolated	N_f=2+1 extrapolated
RQCD[1]	149	715(16)(21)	714	704	770(8)(3)
GWU16[2]	227	749.2(1.6)(15)	749	721	776(3)(10)
	315	795.5(0.7)(16)	795	724	778(4)(11)
QCDSF08[5]	240	770(10)	776	730	779(7)(6)
	250	784(10)	781		
	390	846(10)	844		
Lang11[3]	266	772(6)(8)	774	720	776(5)(9)
ETMC11[6]	290	980(31)	983	821	827(46)(0.4)
	330	1033(31)	1031		
CP-PACS07[7]	328	808(24)(25)	833	750	786(>100)(>100)

Second column: Pion masses of respective studies [MeV]. Third column: ρ masses as quoted or extracted from pictures in the respective publications. Fourth column: Breit-Wigner ρ masses as reconstructed from our UCHPT solutions at unphysical pion masses. Fifth column: SU(2) extrapolated fits at physical pion mass. Sixth column: Final results after including the $K\bar{K}$ channel. Uncertainties (statistical, systematic) only quoted for these cases.

Scattering lengths and effective ranges

	Exp./ CHPT	RQCD 16	GWU16		QCDSF 08	Lang11	ETMC 11	CP-PACS07
			M_π = 227	M_π = 315				
$10a_1^1$	0.38 $\pm$0.02 (exp.)	0.37 $\pm$ 0.31	0.349$\pm$ 0.003	0.350$\pm$ 0.001	0.345$\pm$ 0.003	0.348$\pm$ 0.005	0.367$\pm$ 0.003	0.34 $\pm$ 0.67
$100b_1^1$	0.48 $[\mathcal{O}(p^4)]$ 0.79 $[\mathcal{O}(p^6)]$	1.14$\pm$2 9.1	0.683$\pm$ 0.065	0.627$\pm$ 0.023	0.695$\pm$ 0.113	0.656$\pm$ 0.136	0.911 $\pm$ 0.092	0.701$\pm$ 40

Scattering lengths a_1^1 and effective ranges b_1^1. The experimental value, the $\mathcal{O}(p^4)$, and the $\mathcal{O}(p^6)$ results are taken from Ref. [12].

Ref. [12][Bijnens, Colangelo, Ecker, Gasser, Sainio, Nucl. Phys. B (1997)].

Estimate systematic uncertainties

$$V_2(E) = -\begin{pmatrix} \frac{2p_\pi^2}{3f_\pi^2} & \frac{\sqrt{2}p_K p_\pi}{3f_K f_\pi} \\ \frac{\sqrt{2}p_K p_\pi}{3f_K f_\pi} & \frac{p_K^2}{3f_K^2} \end{pmatrix}$$

$$V_4(E) = -1\times \begin{pmatrix} \frac{8p_\pi^2(2\hat{l}_1 m_\pi^2 - \hat{l}_2 E^2)}{3f_\pi^4} & \frac{8p_\pi p_K(L_5(m_K^2+m_\pi^2)-L_3 E^2)}{3\sqrt{2}f_\pi^2 f_K^2} \\ \frac{8p_\pi p_K(L_5(m_K^2+m_\pi^2)-L_3 E^2)}{3\sqrt{2}f_\pi^2 f_K^2} & \frac{4p_K^2(10\hat{l}_1 m_K^2+3(L_3-2\hat{l}_2) E^2)}{9f_K^4} \end{pmatrix}$$

$\widehat{l_1}$ and $\widehat{l_2}$ in green circles: from fit to $N_f = 2$ lattice phase shifts.

$\widehat{l_1}$ and $\widehat{l_2}$ in purple circles:

1. from fit to experimental phase shifts.
2. from fit to $N_f = 2$ lattice phase shifts.

→Do both, the difference is estimated as systematic/model error.

L_3 and L_5 in red circles: from fit to experimental phase shifts.

Further tests

Results of different subtractions

We test the dependence of the results on the value of the subtraction constant, changing it from the default value $a = -1.28$[1] to $a = -0.8$ and $a = -1.7$. The global fits to experimental phase shifts visibly deteriorate for these extreme values, e.g., for πK scattering, but barely change in the ρ channel (less than 10 MeV in m_ρ and less than 0.08 in g) as experimental phase-shift data are more precise.

Effects of the kaon tadpoles

If one goes from SU(2) to SU(3), actually the $\pi\pi \to \pi\pi$ transition changes by a tiny bit, given by so-called kaon tadpoles[2] we have checked that they indeed change nothing (resonance positions by < 1 MeV).

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