Spectrum and mass anomalous dimension of SU(2) gauge theories with fermions in the adjoint representation: from $N_f = 1/2$ to $N_f = 2$

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1. Adjoint QCD and Technicolour theories

2. Final results for minimal walking technicolour

3. New results for $N_f = 3/2$

4. Comparison with $N_f = 1$ and $N_f = 1/2$

5. Conclusions

In collaboration with I. Montvay, G. Münster, S. Piemonte, P. Giudice, A. Athenodorou, E. Bennett, B. Lucini
Conformal window for adjoint QCD

Technicolour candidates
(more “natural” EW sector):

- requirement: close to conformal (walking) behaviour, large $\gamma_m$, light scalar

$\Rightarrow$ non-perturbative problem

This work

- conformal window for adjoint representation
- conformal mass spectrum: $M \sim m^{1/(1+\gamma_m)}$
  characterised by constant mass ratios
- mass anomalous dimension $\gamma_*(N_f)$

[Dietrich, Sannino, hep-ph/0611341]
Adjoint QCD

adjoint $N_f$ flavour QCD:

$$\mathcal{L} = \text{Tr} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{i}^{N_f} \bar{\psi}_i (\slashed{D} + m) \psi_i \right]$$

$$D_{\mu} \psi = \partial_{\mu} \psi + ig [A_{\mu}, \psi]$$

- $\psi$ Dirac-Fermion in the adjoint representation
- adjoint representation allows Majorana condition $\psi = C \bar{\psi}^T$

$\Rightarrow$ half integer values of $N_f$: $2N_f$ Majorana flavours

Chiral symmetry breaking:

$$\text{SU}(2N_f) \rightarrow \text{SO}(2N_f)$$
Particle states and lattice action

Lattice action
- Wilson fermion action + stout smearing
- tree level Symanzik improved gauge action
- some results: clover improvement

Particle states
- triplet mesons $m_{PS}$, $m_S$, $m_V$, $m_{PV}$
- glueball $0^{++}$
- spin-1/2 mixed fermion-gluon state
  \[ \sum_{\mu, \nu} \sigma_{\mu \nu} \text{tr} [F^{\mu \nu} \lambda] \]
- singlet mesons $m_{a-f_0}$, $m_{a-\eta'}$
Expected behaviour of a (near) conformal theory:

- constant mass ratios
- light scalar ($0^{++}$)
- no light Goldstone ($m_{PS}$)
INTRO $N_f = 2$ $N_f = 3/2$ $N_f = 1, 1/2$ CON

Particle spectrum of Minimal Walking Technicolour: smaller lattice spacing

- remnant $\beta$ dependence
- gap between glueball and $m_{PS}$ increased
Particle spectrum of Minimal Walking Technicolour: finite size effects

- large finite size effects at small $m_{PCAC}$
- limited mass range to fit constant ratio
Particle spectrum of Minimal Walking Technicolour: singlet meson channel

- scalar singlet meson lighter or comparable to $m_{PS}$
- glueball $0^{++}$ overlap with ground state significantly better
Particle spectrum of Minimal Walking Technicolour: results for mass ratios

<table>
<thead>
<tr>
<th>State</th>
<th>( \beta = 1.5 )</th>
<th>( \beta = 1.7 )</th>
<th>[Del Debbio et al. 1512.08242]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_V )</td>
<td>1.0825(58)</td>
<td>1.051(12)</td>
<td>1.044(43)</td>
</tr>
<tr>
<td>( m_S )</td>
<td>1.285(24)</td>
<td>1.190(14)</td>
<td>1.222(52)</td>
</tr>
<tr>
<td>( m_{PV} )</td>
<td>1.329(21)</td>
<td>1.232(13)</td>
<td>1.26(35)</td>
</tr>
<tr>
<td>( m_{0^{++}} )</td>
<td>0.620(35)</td>
<td>0.398(48)</td>
<td>0.458(15)</td>
</tr>
<tr>
<td>( F_\pi )</td>
<td>0.1831(23)</td>
<td>0.15156(72)</td>
<td>0.178(5)</td>
</tr>
<tr>
<td>( m_{1/2} )</td>
<td>0.948(24)</td>
<td>0.86394(52)</td>
<td>–</td>
</tr>
<tr>
<td>( m_{PCAC} ) range</td>
<td>0.1808(22)- 0.2490(12)</td>
<td>0.2457(12)-0.26776(42)</td>
<td>0.1872(84)-0.2323(35)</td>
</tr>
<tr>
<td>( m_{PS} ) range</td>
<td>0.29986(46)- 0.58848(98)</td>
<td>0.5360(25) - 0.57247(16)</td>
<td>0.6401(11) - 1.183(1)</td>
</tr>
</tbody>
</table>

- significant difference between \( \beta = 1.5 \) and \( \beta = 1.7 \)
- \( \beta = 1.5 \) results compatible with earlier investigations
- results of [Del Debbio et al., 1512.08242] between \( \beta = 1.5 \) and \( \beta = 1.7 \)
Mass anomalous dimension for Minimal Walking Technicolour: Methods

Methods for determination of $\gamma_*$:
- scaling of mass spectrum
- mode number (integrated spectral density of $D^\dagger D$ )

Methods for mode number determination:
- Chebyshev expansion of the spectral density
- consistency with [Giusti, Lüscher, 0812.3638] checked
Mass anomalous dimension for Minimal Walking Technicolour: Results

Mass spectrum:
- results cover a large range, only most precise ones considered
- larger $\beta$: tendency towards smaller $\gamma^*$

Mode number:
- $\beta = 1.5$ result consistent with [Del Debbio et al., 1512.08242] $(0.371(20))$
- $\beta = 1.7$ considerably smaller $\gamma^*$
- tendency towards clover improved results $0.20(3)$ [Rantaharju et al., 1510.03335]
INTRO

$N_f = 2$  $N_f = 3/2$  $N_f = 1, 1/2$  CON

New results for $N_f = 3/2$

- $\gamma_*$ from mass spectrum: $0.495(78)$
- $\gamma_*$ from modenumber: $\beta = 1.5$: $0.40(5)$; $\beta = 1.7$: $0.32(5)$
- light scalar, spectrum comparable to the $N_f = 2$ case
- different from MWT: spin-1/2 mass similar to $m_V$
Comparison with $N_f = 1$ and $N_f = 1/2$

<table>
<thead>
<tr>
<th>Theory</th>
<th>scalar particle</th>
<th>$\gamma_*$ small $\beta$</th>
<th>$\gamma_*$ larger $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_f = 1/2$ SYM</td>
<td>part of multiplet</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$N_f = 1$ adj QCD</td>
<td>light</td>
<td>0.92(1)</td>
<td>0.75(4)*</td>
</tr>
<tr>
<td>$N_f = 3/2$ adj QCD</td>
<td>light</td>
<td>0.40(5)*</td>
<td>0.32(5)*</td>
</tr>
<tr>
<td>$N_f = 2$ adj QCD</td>
<td>light</td>
<td>0.376(3)</td>
<td>0.274(10)</td>
</tr>
</tbody>
</table>

(*) preliminary

- SYM: SUSY provides multiplet structure of states, confining
- other theories: light scalar, light spin-1/2 state for $N_f = 2$
Conclusions

- investigation of (near) conformal theory requires careful consideration of lattice artefacts and finite size effects
- further investigations required for the complete systematics of these effects
- MWT: results point towards \( \gamma_* \) even below 0.3
- consistent behaviour: \( \gamma_* \) lower for larger \( N_f \)
- properties of interesting candidates for Technicolour extension of the standard model (MWT, UMWT)
- further consequences from relations between different theories: conformal behaviour for the adjoint representation starts at \( N_f = 1 \), indication for conformality of NMWT \( (N_f = 2 \text{ sextet}) \)

[Bergner, Ryttov, Sannino, 1510.01763]