Spectrum and mass anomalous dimension of SU(2) gauge theories with fermions in the adjoint representation: from $N_f = 1/2$ to $N_f = 2$

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Southampton: July, 2016

1. Adjoint QCD and Technicolour theories
2. Final results for minimal walking technicolour
3. New results for $N_f = 3/2$
4. Comparison with $N_f = 1$ and $N_f = 1/2$
5. Conclusions

In collaboration with I. Montvay, G. Münster, S. Piemonte, P. Giudice, A. Athenodorou, E. Bennett, B. Lucini

Conformal window for adjoint QCD

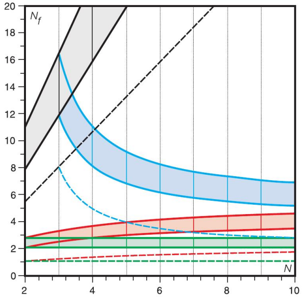
Technicolour candidates
(more "natural" EW sector):

- requirement: close to conformal (walking) behaviour, large γ_m, light scalar

$\Rightarrow$ non-perturbative probelm

This work

- conformal window for adjoint representation
- conformal mass spectrum: $M \sim m^{1/(1+\gamma_m)}$ characterised by constant mass ratios
- mass anomalous dimension $\gamma_*(N_f)$



[Dietrich, Sannino, hep-ph/0611341]

Adjoint QCD

adjoint N_f flavour QCD:

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_i^{N_f} \bar{\psi}_i(\not{D} + m)\psi_i\right]$$

$$D_\mu \psi = \partial_\mu \psi + ig[A_\mu, \psi]$$

- ψ Dirac-Fermion in the adjoint representation
- adjoint representation allows Majorana condition $\psi = C\bar{\psi}^T$

$\Rightarrow$ half integer values of N_f: $2N_f$ Majorana flavours

Chiral symmetry breaking:

$$\mathrm{SU}(2N_f) \to \mathrm{SO}(2N_f)$$

Particle states and lattice action

Lattice action

- Wilson fermion action + stout smearing
- tree level Symanzik improved gauge action
- some results: clover improvement

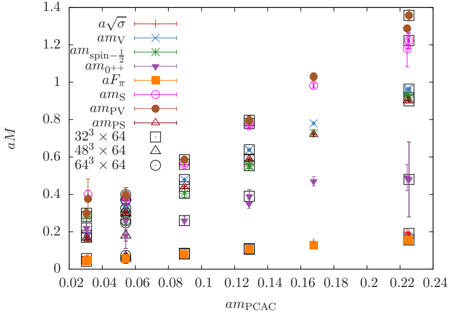
Particle states

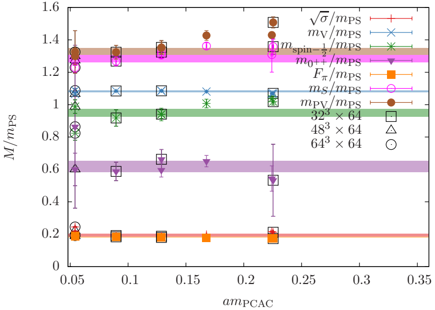
- triplet mesons m_{PS}, m_{S}, m_{V}, m_{PV}
- glueball 0^{++}
- spin-1/2 mixed fermion-gluon state

$$\sum_{\mu,\nu} \sigma_{\mu\nu} \mathrm{tr}\left[F^{\mu\nu}\lambda\right]$$

- singlet mesons $m_{\mathrm{a}-f_0}$, $m_{\mathrm{a}-\eta'}$

Particle spectrum of Minimal Walking Technicolour

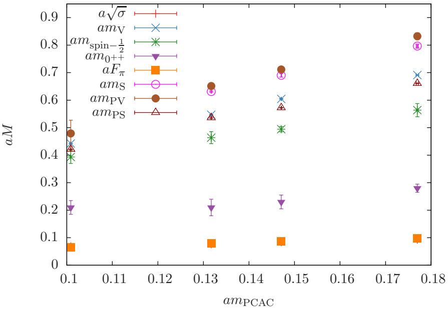


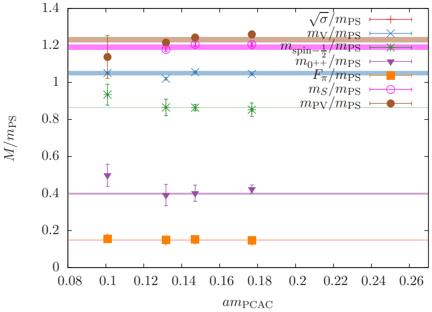


Expected behaviour of a (near) conformal theory:

- constant mass ratios
- light scalar (0^{++})
- no light Goldstone ($m_{\rm PS}$)

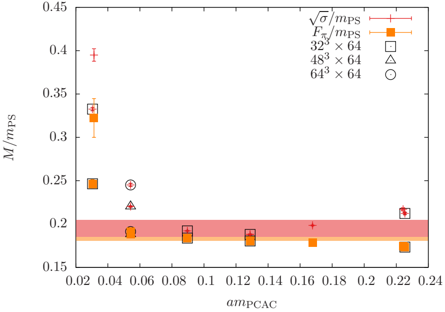
Particle spectrum of Minimal Walking Technicolour: smaller lattice spacing

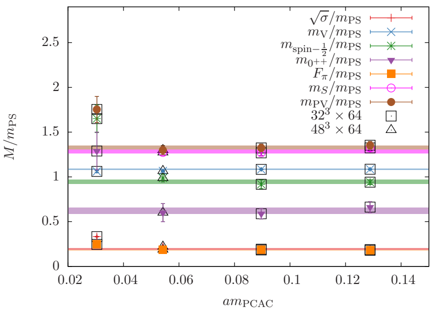




- remnant β dependence
- gap between glueball and m_{PS} increased

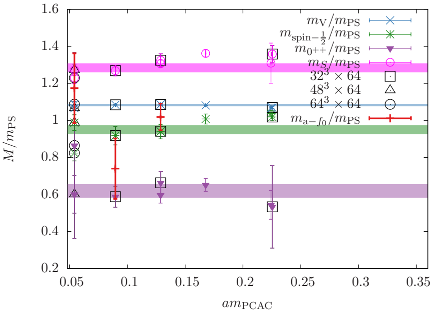
Particle spectrum of Minimal Walking Technicolour: finite size effects

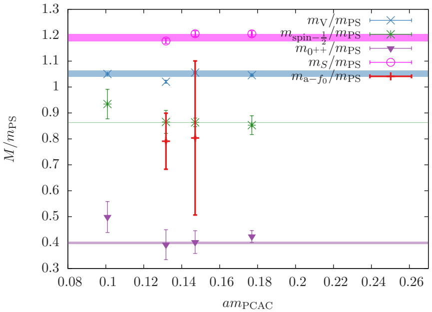




- large finite size effects at small m_{PCAC}
- limited mass range to fit constant ratio

Particle spectrum of Minimal Walking Technicolour: singlet meson channel





- scalar singlet meson lighter or comparable to m_{PS}
- glueball 0^{++} overlap with ground state significantly better

Particle spectrum of Minimal Walking Technicolour: results for mass ratios

State	$\beta = 1.5$	$\beta = 1.7$	[Del Debbio et al. 1512.08242]
m_V	1.0825(58)	1.051(12)	1.044(43)
m_S	1.285(24)	1.190(14)	1.222(52)
m_{PV}	1.329(21)	1.232(13)	1.26(35)
$m_{0^{++}}$	0.620(35)	0.398(48)	0.458(15)
F_π	0.1831(23)	0.15156(72)	0.178(5)
$m_{1/2}$	0.948(24)	0.86394(52)	–
m_{PCAC} range	0.1808(22)- 0.2490(12)	0.2457(12)-0.26776(42)	0.1872(84)-0.2323(35)
am_{PS} range	0.29986(46)- 0.58848(98)	0.5360(25) - 0.57247(16)	0.6401(11) - 1.183(1)

- significant difference between $\beta = 1.5$ and $\beta = 1.7$
- $\beta = 1.5$ results compatible with earlier investigations
- results of [Del Debbio et al., 1512.08242] between $\beta = 1.5$ and $\beta = 1.7$

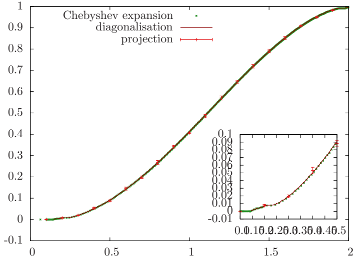
Mass anomalous dimension for Minimal Walking Technicolour: Methods

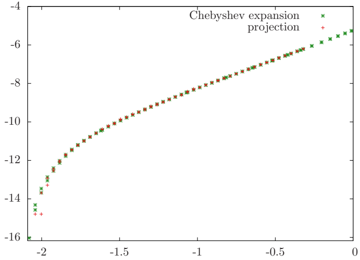
Methods for determination of γ_*:

- scaling of mass spectrum
- mode number (integrated spectral density of $D^\dagger D$)

Methods for mode number determination:

- Chebyshev expansion of the spectral density
- consistency with [Giusti, Lüscher, 0812.3638] checked





Mass anomalous dimension for Minimal Walking Technicolour: Results

Mass spectrum:

- results cover a large range, only most precise ones considered
- larger β: tendency towards smaller γ_*

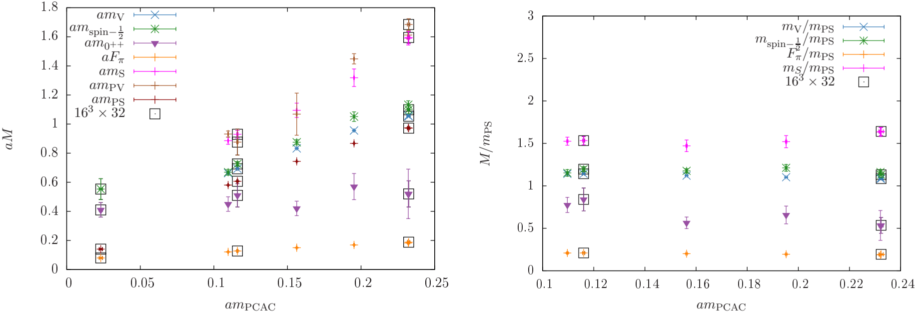
Observable	β	γ_*
m_{PS}	1.5	0.2958(45)
m_V	1.5	0.295(26)
F_π	1.5	0.391(20)
average	1.5	0.300(20)
m_{PS}	1.7	0.289(17)
m_V	1.7	0.263(28)
F_π	1.7	0.265(12)
average	1.7	0.272(11)

Mode number:

- $\beta = 1.5$ result consistent with [Del Debbio et al., 1512.08242] (0.371(20))
- $\beta = 1.7$ considerably smaller γ_*
- tendency towards clover improved results 0.20(3) [Rantaharju et al., 1510.03335]

$N_s \times N_t$	β	κ	γ
24×64	1.5	0.1325	0.39(3)
32×64	1.5	0.1335	0.38(1)
48×64	1.5	0.1344	0.380(10)
32×64	1.5	0.1350	0.375(4)
average	1.5		0.376(3)
32×64	1.7	0.1285	0.270(15)
32×64	1.7	0.1290	0.260(20)
32×64	1.7	0.1300	0.285(15)
average	1.7		0.274(10)

New results for $N_f = 3/2$



- γ_* from mass spectrum: 0.495(78)
- γ_* from modenumber: $\beta = 1.5$: 0.40(5); $\beta = 1.7$: 0.32(5)
- light scalar, spectrum comparable to the $N_f = 2$ case
- different from MWT: spin-1/2 mass similar to m_V

Comparison with $N_f = 1$ and $N_f = 1/2$

Theory	**scalar particle**	γ_* small β	γ_* larger β
$N_f = 1/2$ SYM	part of multiplet	–	–
$N_f = 1$ adj QCD	light	0.92(1)	0.75(4)*
$N_f = 3/2$ adj QCD	light	0.40(5)*	0.32(5)*
$N_f = 2$ adj QCD	light	0.376(3)	0.274(10)

(∗ preliminary)

- SYM: SUSY provides multiplet structure of states, confining
- other theories: light scalar, light spin-1/2 state for $N_f = 2$

Conclusions

- investigation of (near) conformal theory requires careful consideration of lattice artefacts and finite size effects
- further investigations required for the complete systematics of these effects
- MWT: results point towards γ_* even below 0.3
- consistent behaviour: γ_* lower for larger N_f
- properties of interesting candidates for Technicolour extension of the standard model (MWT, UMWT)
- further consequences from relations between different theories: conformal behaviour for the adjoint representation starts at $N_f = 1$, indication for conformality of NMWT ($N_f = 2$ sextet)

  [Bergner, Ryttov, Sannino, 1510.01763]