Matching issue in quasi parton distribution approach

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Outline

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- Matching of quasi distributions between continuum and lattice
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Collinear factorization and PDFs

Collinear factorization - a key concept in PQCD

$$\sigma^{\rm DIS}(x, Q^2, \sqrt{s}) = \sum_{\alpha=q,\bar{q},g} C_\alpha\left(x, \frac{Q^2}{\mu^2}, \sqrt{s}\right) \otimes f_\alpha(x, \mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)$$

x : Bjorken-x, Q : momentum transfer, $\sqrt{s}$: collision energy

μ : factorization scale

Parton Distribution Functions (PDFs)

- Probability density for finding a particle with a certain longitudinal momentum fraction x of proton.
- Absorb all perturbative collinear divergences.
- Non-perturbative.
- Universal. → Predictive power of QCD !

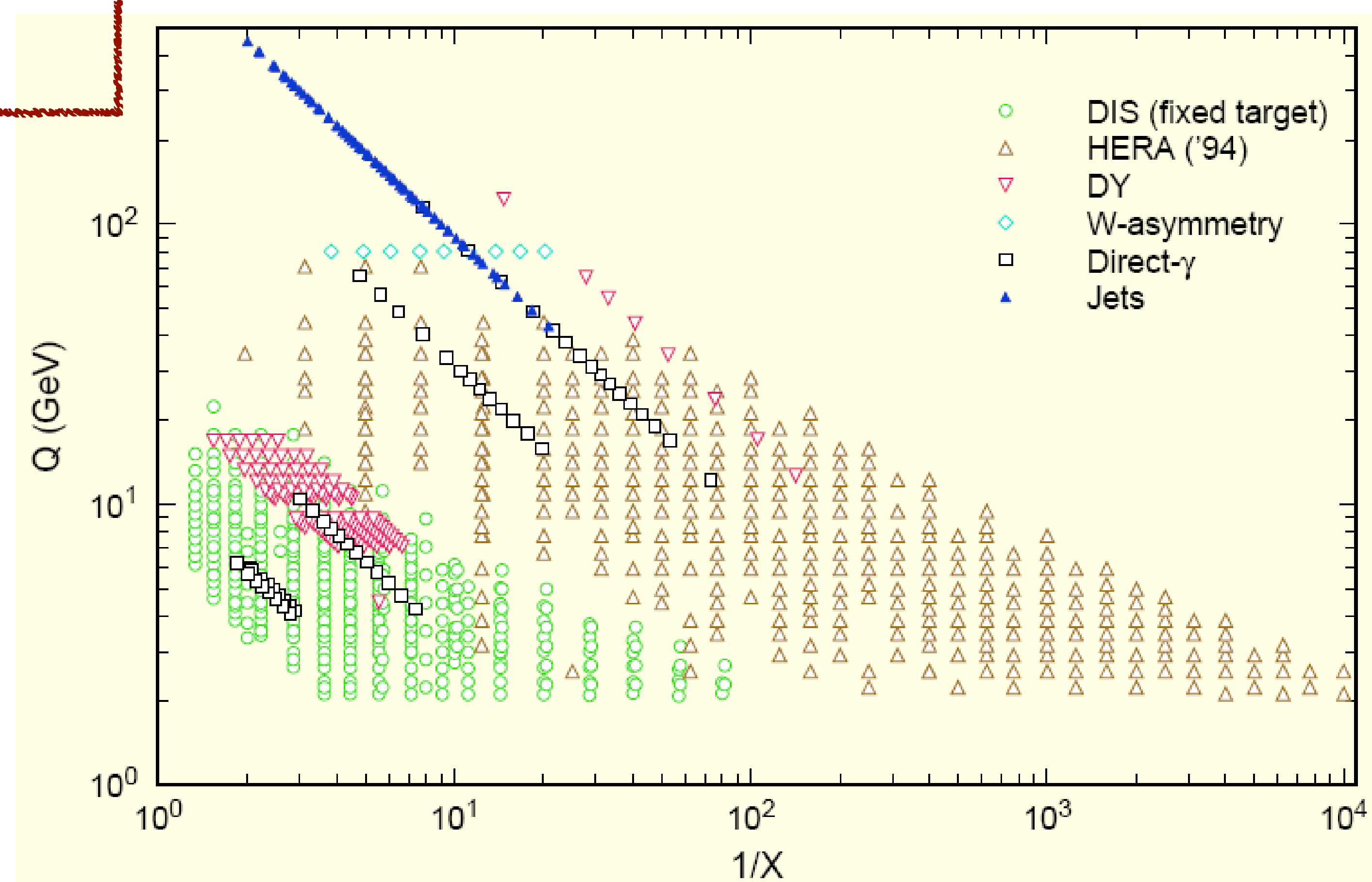
Global QCD analysis

- Extract PDFs from experiment data

PDFs at Q_0 with initial fit params $f_\alpha(x, Q_0)$

change fit params to minimize χ^2

χ^2



DGLAP evolution

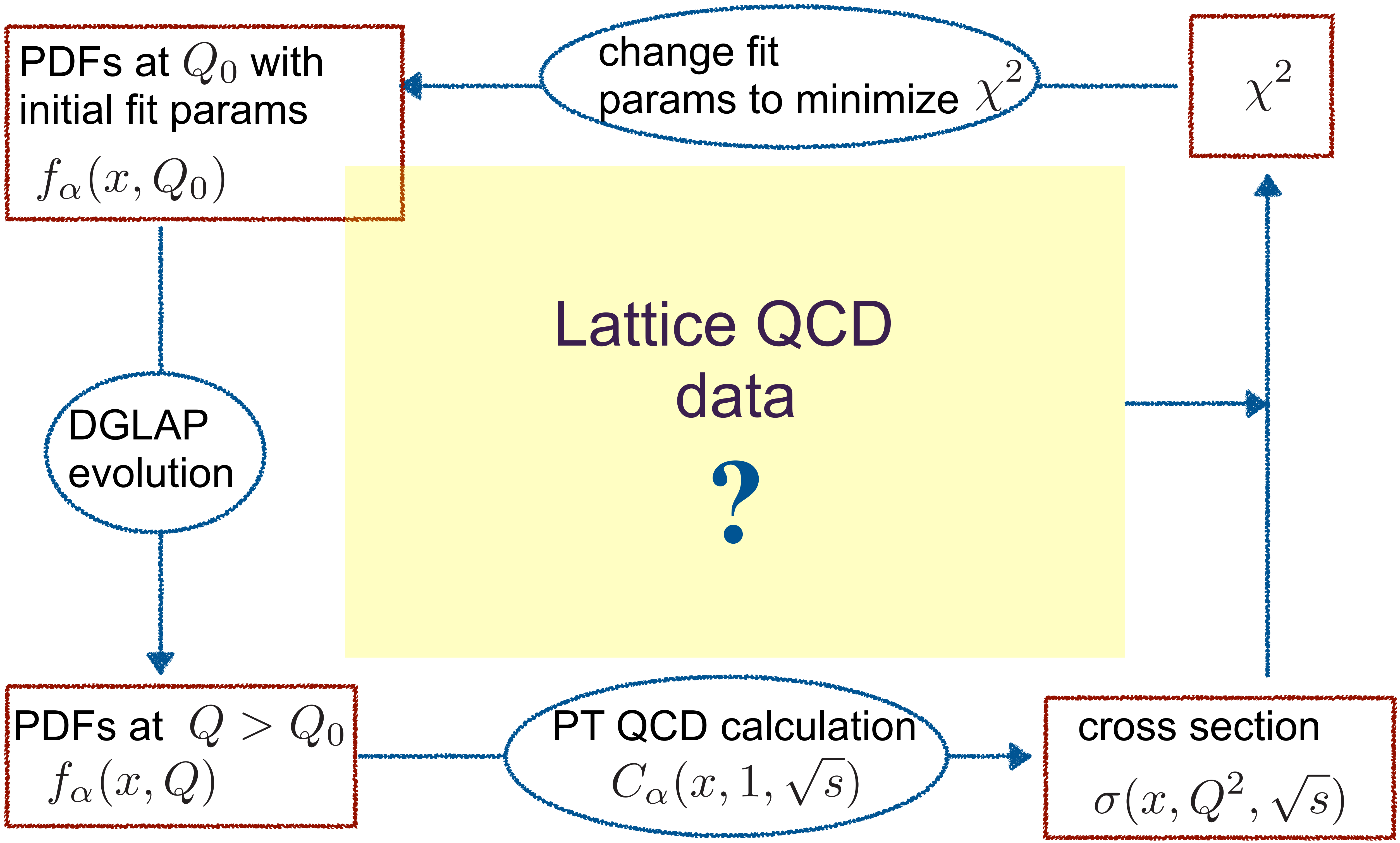
PDFs at $Q > Q_0$ $f_\alpha(x, Q)$

PT QCD calculation $C_\alpha(x, 1, \sqrt{s})$

cross section $\sigma(x, Q^2, \sqrt{s})$

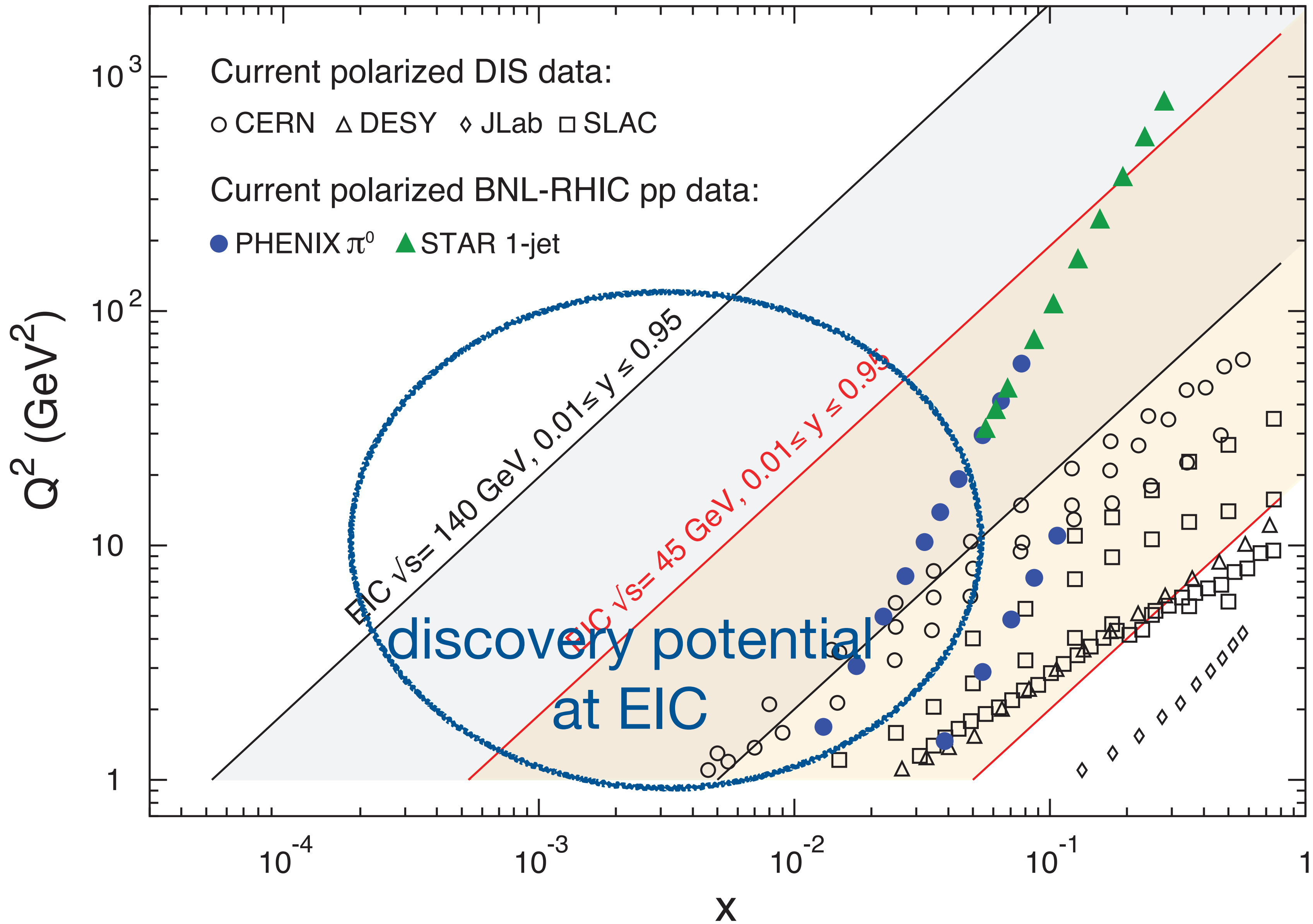
Global QCD analysis with lattice QCD

- Extract PDFs from lattice



Lattice can help?

- Electron Ion Collider (EIC) kinematic coverage



(e+P)

There would be uncovered region of x in the future experiment.

Lattice can help?

- Large-x: sensitive to NPT dynamics in nucleon

Testing ground for models of hadron structure

- SU(6) spin-flavor symmetry

$$d/u \longrightarrow 1/2$$

- Scalar diquark dominance

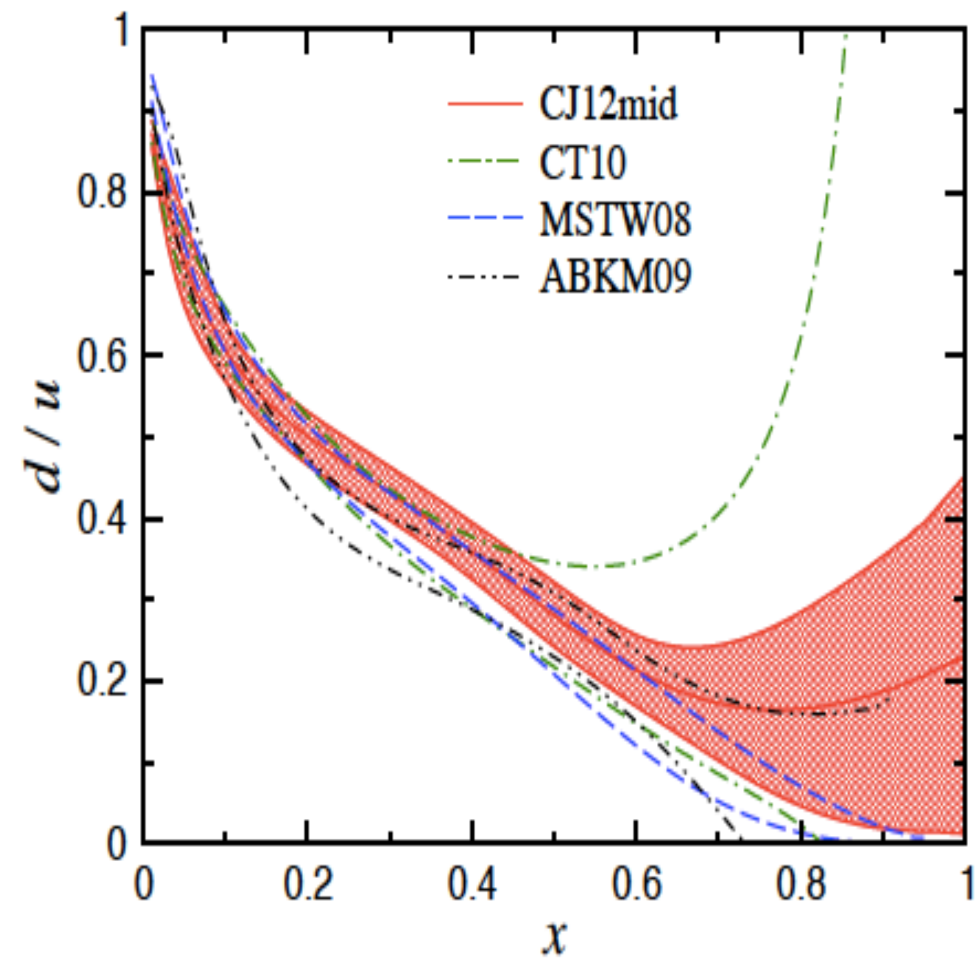
$$d/u \longrightarrow 0$$

- pQCD power counting

$$d/u \longrightarrow 1/5$$

- Local quark-hadron duality

$$d/u \longrightarrow 0.42$$



PDFs from lattice

Quark distribution by light-cone operator

$$q(x,\mu) = \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle \mathcal{N}(P)|O(\xi^-)|\mathcal{N}(P)\rangle,$$
$$O(\xi^-) = \overline{\psi}(\xi^-)\gamma^+ U_+(\xi^-,0)\psi(0)$$

- $\xi^\pm = (t \pm z)/\sqrt{2}$: light-cone coordinate
- Time-dependent. ⇨ Not calculable on the lattice directly.

Moments

$$a_n = \int_0^1 dx x^{n-1} q(x) = \frac{1}{P^{\mu_1}\dots P^{\mu_n}} \langle \mathcal{N}(P)|O^{\{\mu_1\cdots\mu_n\}}|\mathcal{N}(P)\rangle$$
$$O^{\{\mu_1\cdots\mu_n\}} = \overline{\psi}(0)\gamma^{\{\mu_1} i\overleftrightarrow{D}^{\mu_2}\dots i\overleftrightarrow{D}^{\mu_n\}}\psi(0)$$

- Written in local operators. Calculable on lattice (in principle).
- But, higher moments are difficult to be accessed.

Quasi-PDFs [Ji (2013)]

Quasi distributions

$$\widetilde{q}(\tilde{x}, \mu, P_z) = \int \frac{d\delta z}{2\pi} e^{-i\tilde{x}P_z\delta z} \langle \mathcal{N}(P_z)|\widetilde{O}(\delta z)|\mathcal{N}(P_z)\rangle,$$
$$\widetilde{O}(\delta z) = \overline{\psi}(\delta z)\gamma^z U_z(\delta z, 0)\psi(0)$$

- Separated in spatial z-direction. Calculable on lattice.
- By the limit of $P_z \to \infty$, normal distributions are recovered.

Matching (Large Momentum Effective Theory)

$$\widetilde{q}(x, \Lambda, P_z) = \int \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\Lambda}{P_z}, \frac{\mu}{P_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{P_z^2}, \frac{M^2}{P_z^2}\right)$$

- Z can be perturbatively obtained.
- Large P_z is required for small corrections.

QCD collinear factorization approach

[Ma and Qiu (2014)]

- Going back to the collinear factorization

$$\sigma^{\mathrm{DIS}}(x, Q^2, \sqrt{s}) = \sum_{\alpha=q,\bar{q},g} C_\alpha\left(x, \frac{Q^2}{\mu^2}, \sqrt{s}\right) \otimes f_\alpha(x, \mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{Q^2}\right)$$

All CO divergences are factorized into the PDFs with PT hard coefficients.

- Lattice calculable cross section

$$\widetilde{\sigma}(x, \widetilde{\mu}^2, P_z) = \sum_{\alpha=q,\bar{q},g} \widetilde{C}_\alpha\left(x, \frac{\widetilde{\mu}^2}{\mu^2}, P_z\right) \otimes f_\alpha(x, \mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{\widetilde{\mu}^2}\right)$$

All CO divergences are factorized into the PDFs with PT hard coefficients.

$\mu \longleftrightarrow \mu$ (factorization scale)

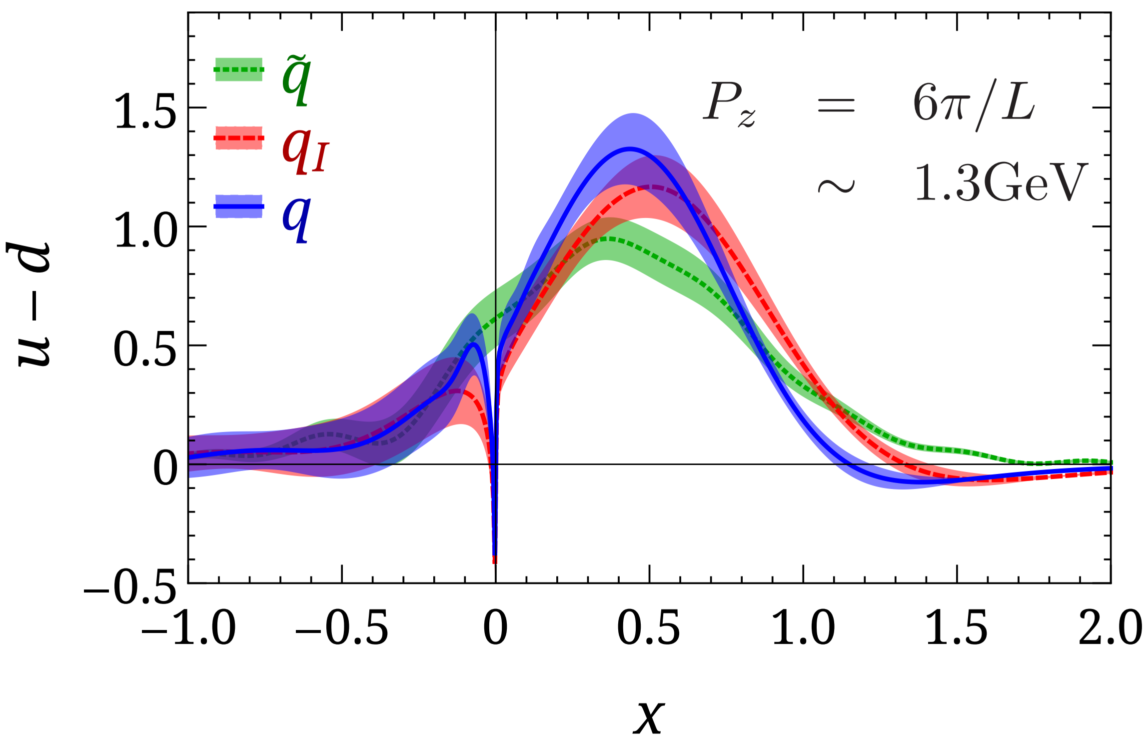
$Q \longleftrightarrow \widetilde{\mu}$ (resolution)

$\sqrt{s} \longleftrightarrow P_z$ (parameter)

Lattice quasi-PDFs, so far

- Two calculations in LMET approach

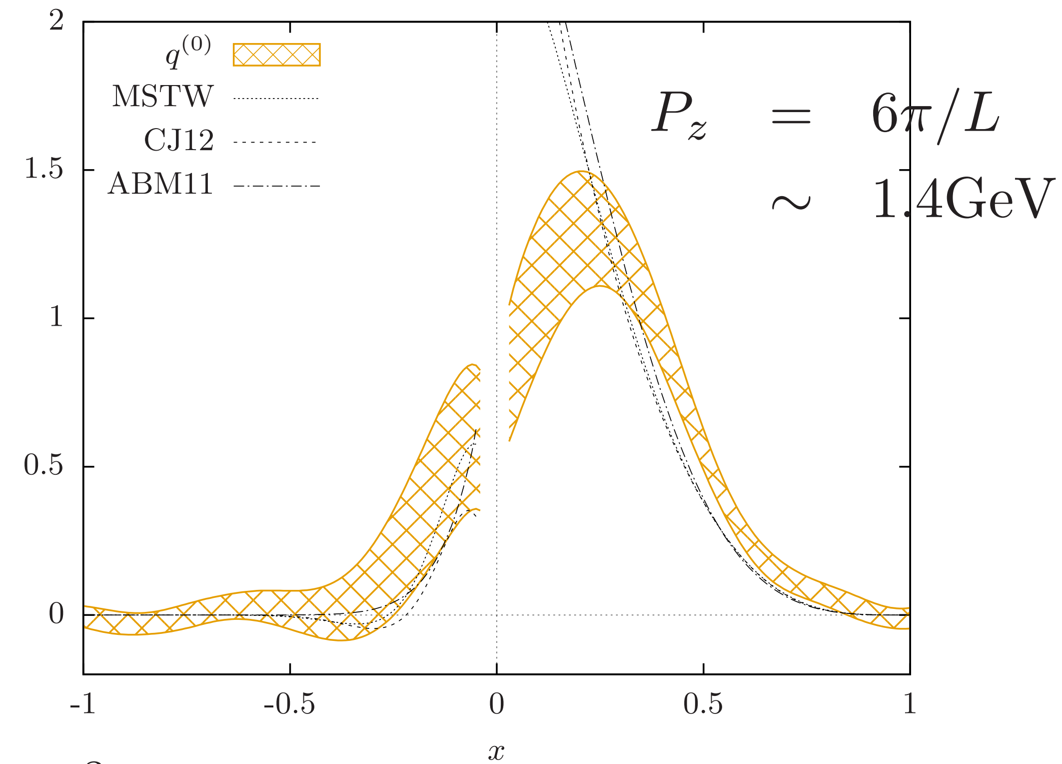
[Chen et al., arXiv:1603.06664]



$24^3 \times 64, N_f = 2+1+1$ HISQ

$a \sim 0.12\text{fm}\ (1.6\text{GeV}), m_{\text{PS}} \sim 310\text{MeV}$

[Alexandrou et al., PRD92(2015)014502]



$32^3 \times 64, N_f = 2+1+1$ Twisted Mass

$a \sim 0.082\text{fm}\ (2.4\text{GeV}), m_{\text{PS}} \sim 370\text{MeV}$

- Exploratory study.
- Two calculations look consistent with each other.

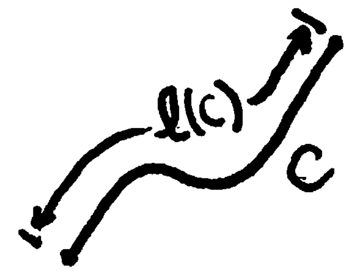
Subtracting power divergences

Power divergence

- Power divergence makes the theory ill-defined.
  (e.g. no continuum limit on lattice.)
- The power divergence must be subtracted nonperturbatively.

Renormalization of Wilson line

$$W_{\mathcal{C}} = e^{\delta m \ell(\mathcal{C})} W_{\mathcal{C}}^{\text{ren}}$$



- Well-known. [Dotsenko, Vergeles, Arefeva, Craigie, Dorn, … ('80)]
- δm : mass renormalization of a test particle moving along $\mathcal{C}$

All the power divergence is contained.

- Subtraction of the power divergence can be done by:

$$\widetilde{O}^{\text{subt}}(\delta z) = e^{-\delta m |\delta z|} \widetilde{O}(\delta z)$$

Subtracting power divergences

- Choice of δm [Musch et al. (2011)]

- One way is to use static $Q\bar{Q}$ potential $V(R)$.
- $V(R)$ is obtained from Wilson loop:

$$W_{R\times T} \propto e^{-V(R)T} \quad (T \to \text{large})$$

- Renormalization of $V(R)$:

$$V^{\text{ren}}(R) = V(R) + 2\delta m$$

- Renormalization condition we take:

$$V^{\text{ren}}(R_0) = V_0 \longrightarrow \delta m = \frac{1}{2}(V_0 - V(R_0))$$

- Power divergence free quasi distributions

$$\tilde{q}^{\text{subt}}(\tilde{x}, \mu, P_z) = \int \frac{d\delta z}{2\pi} e^{-i\tilde{x}P_z\delta z} e^{-\delta m|\delta z|} \langle \mathcal{N}(P_z)|\widetilde{O}(\delta z)|\mathcal{N}(P_z)\rangle$$

Matching between continuum and lattice

Matching for being precise

$$O^{\text{cont}} = Z O^{\text{latt}}$$

- necessary to absorb difference in renormalization.
- It can be calculable using perturbation.

Momentum space v.s. Coordinate space

$$\tilde{q}^{\text{cont}}(\tilde{x}, \mu, P_z) = \int \frac{d\delta z}{2\pi} e^{-i\tilde{x}P_z\delta z} \langle \mathcal{N}(P_z)|\widetilde{O}(\delta z)|\mathcal{N}(P_z)\rangle^{\text{cont}}$$

$$\Updownarrow \; Z(\tilde{x}, P_z) \qquad\qquad \Updownarrow \; Z(\delta z)$$

$$\tilde{q}^{\text{latt}}(\tilde{x}, \mu, P_z) = \int \frac{d\delta z}{2\pi} e^{-i\tilde{x}P_z\delta z} \langle \mathcal{N}(P_z)|\widetilde{O}(\delta z)|\mathcal{N}(P_z)\rangle^{\text{latt}}$$

matching
in momentum space

matching
in coordinate space
(This work)

Matching between continuum and lattice

Matching pattern

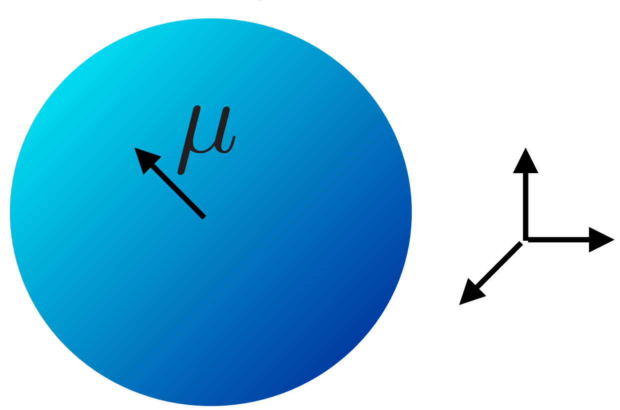
power divergence subtraction

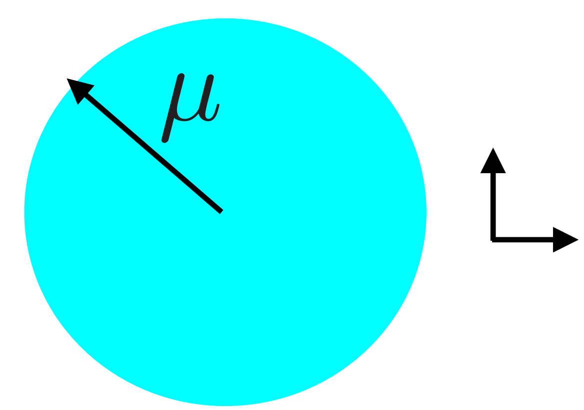
✓ No convolution-type, no mixing with different length of δz

✓ No momentum dependent factor

$$\widetilde{O}(\delta z)^{\rm cont} = Z(\delta z)\widetilde{O}(\delta z)^{\rm latt}$$

Dimensionality of UV cutoff





3d UV cutoff: $\perp = (t, x, y)$

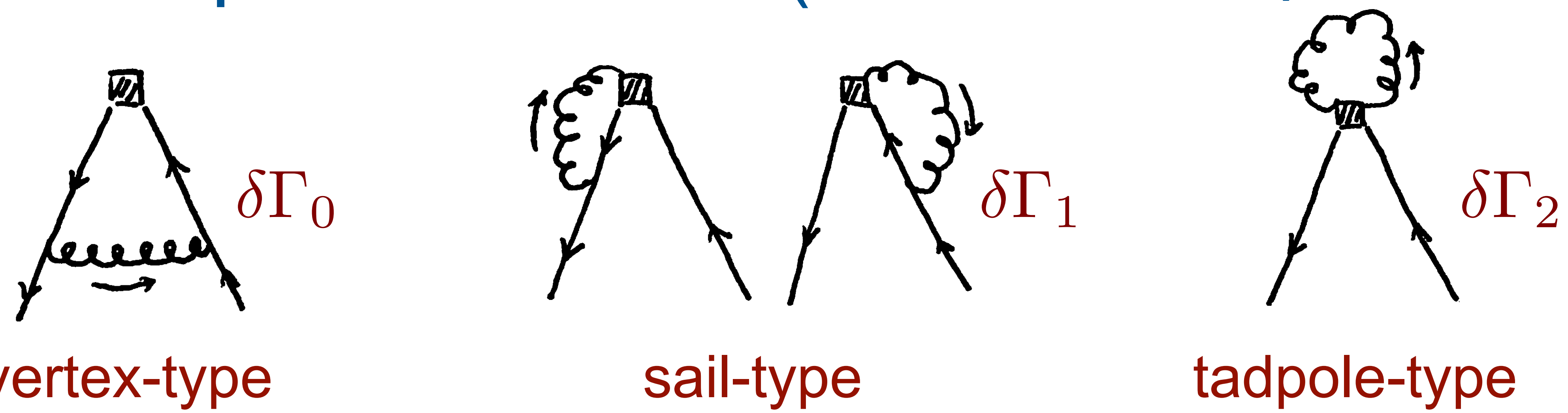
natural
in Euclidean space

2d UV cutoff: $\perp = (x, y)$

natural
in Minkowski space-time

Matching between continuum and lattice

- One-loop in continuum (3d UV cutoff)



$$\delta\Gamma_0(\delta z) = \frac{g^2 C_F}{8\pi^2}\left(\mathrm{Ei}(-k_{\perp z}) - (2+k_{\perp z})e^{-k_{\perp z}}\right)\Big|^{\mu|\delta z|}_{k_{\perp z}=\lambda|\delta z|} \xrightarrow[\delta z\to 0]{} \frac{g^2 C_F}{8\pi^2}\ln\frac{\mu}{\lambda},$$

$$\delta\Gamma_1(\delta z) = \frac{g^2 C_F}{4\pi^2}\left(\ln\frac{\mu}{\lambda} + \left(-\mathrm{Ei}(-k_{\perp z}) + e^{-k_{\perp z}}\right)\Big|^{\mu|\delta z|}_{k_{\perp z}=\lambda|\delta z|}\right) \xrightarrow[\delta z\to 0]{} 0,$$

$$\delta\Gamma_2(\delta z) = \frac{g^2 C_F}{4\pi^2}\left(\ln\frac{\mu}{\lambda} - \mathrm{Ei}(-k_{\perp z})\Big|^{\mu|\delta z|}_{k_{\perp z}=\lambda|\delta z|}\right) \xrightarrow[\delta z\to 0]{} 0.$$

$$\mathrm{Ei}(x) = -\int_{-x}^{\infty} dt\,\frac{e^{-t}}{t}$$: exponential integral

- Local case ($\delta z \to 0$) can be safely reproduced.
- Linear divergence is already subtracted.
- UV(μ) and IR(λ) regulators are introduced in $\perp = (t, x, y)$ direction.

Matching between continuum and lattice

One-loop in continuum (2d UV cutoff)

$$\delta\Gamma_0(\delta z) = -\frac{g^2 C_F}{16\pi^2}\int_{-\infty}^{\infty} dk_0 \left(k_\perp + \frac{1}{\sqrt{k_0^2+1}}\right) e^{-\sqrt{k_0^2+1}k_\perp}\Bigg|_{k_\perp=\lambda|\delta z|}^{\mu|\delta z|} \xrightarrow[\delta z\to 0]{} \frac{g^2 C_F}{8\pi^2}\ln\frac{\mu}{\lambda},$$

$$\delta\Gamma_1(\delta z) = \frac{g^2 C_F}{4\pi^2}\left(\ln\frac{\mu}{\lambda} + \frac{1}{2}\int_{-\infty}^{\infty} dk_0 \, \frac{e^{-\sqrt{k_0^2+1}k_\perp}}{\sqrt{k_0^2+1}}\Bigg|_{k_\perp=\lambda|\delta z|}^{\mu|\delta z|}\right) \xrightarrow[\delta z\to 0]{} 0,$$

$$\delta\Gamma_2(\delta z) = \frac{g^2 C_F}{4\pi^2}\Bigg(\ln\frac{\mu}{\lambda} + \frac{1}{2}\int_{-\infty}^{\infty} dk_0 \left(\frac{e^{-\sqrt{k_0^2+1}k_\perp}}{\sqrt{k_0^2+1}} + k_\perp \mathrm{Ei}\left[-\sqrt{k_0^2+1}k_\perp\right]\right)\Bigg|_{k_\perp=\lambda|\delta z|}^{\mu|\delta z|}\Bigg) \xrightarrow[\delta z\to 0]{} 0.$$

- Local case ($\delta z \to 0$) can be safely reproduced.
- Complex expressions, but similar behavior to 3D cutoff case.
- UV(μ) and IR(λ) regulators are introduced in $\perp = (x,y)$ direction.

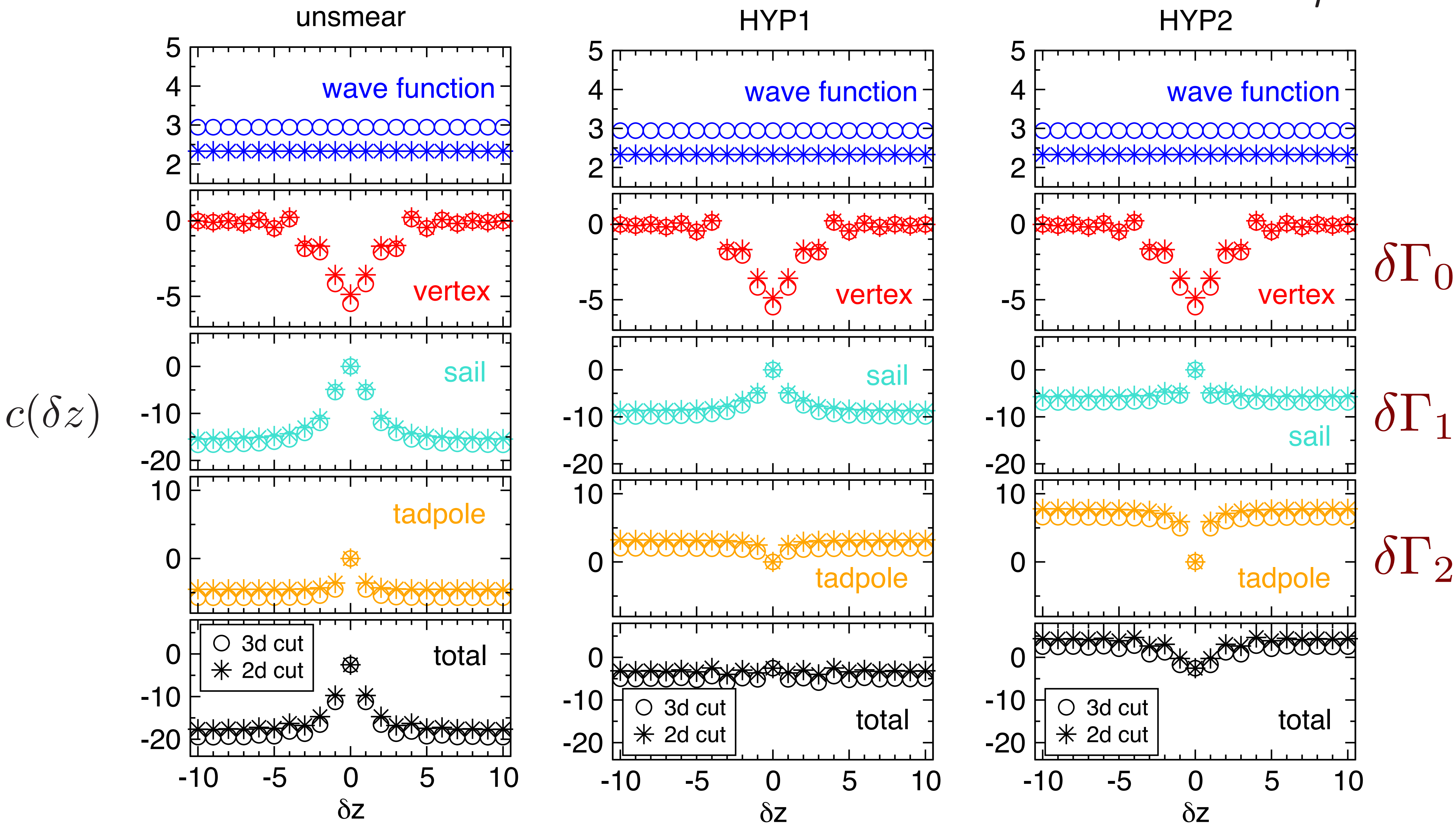
Matching between continuum and lattice

One-loop matching coefficients: an example

- Naive fermion is used.
- Link smearing (HYP1, HYP2)

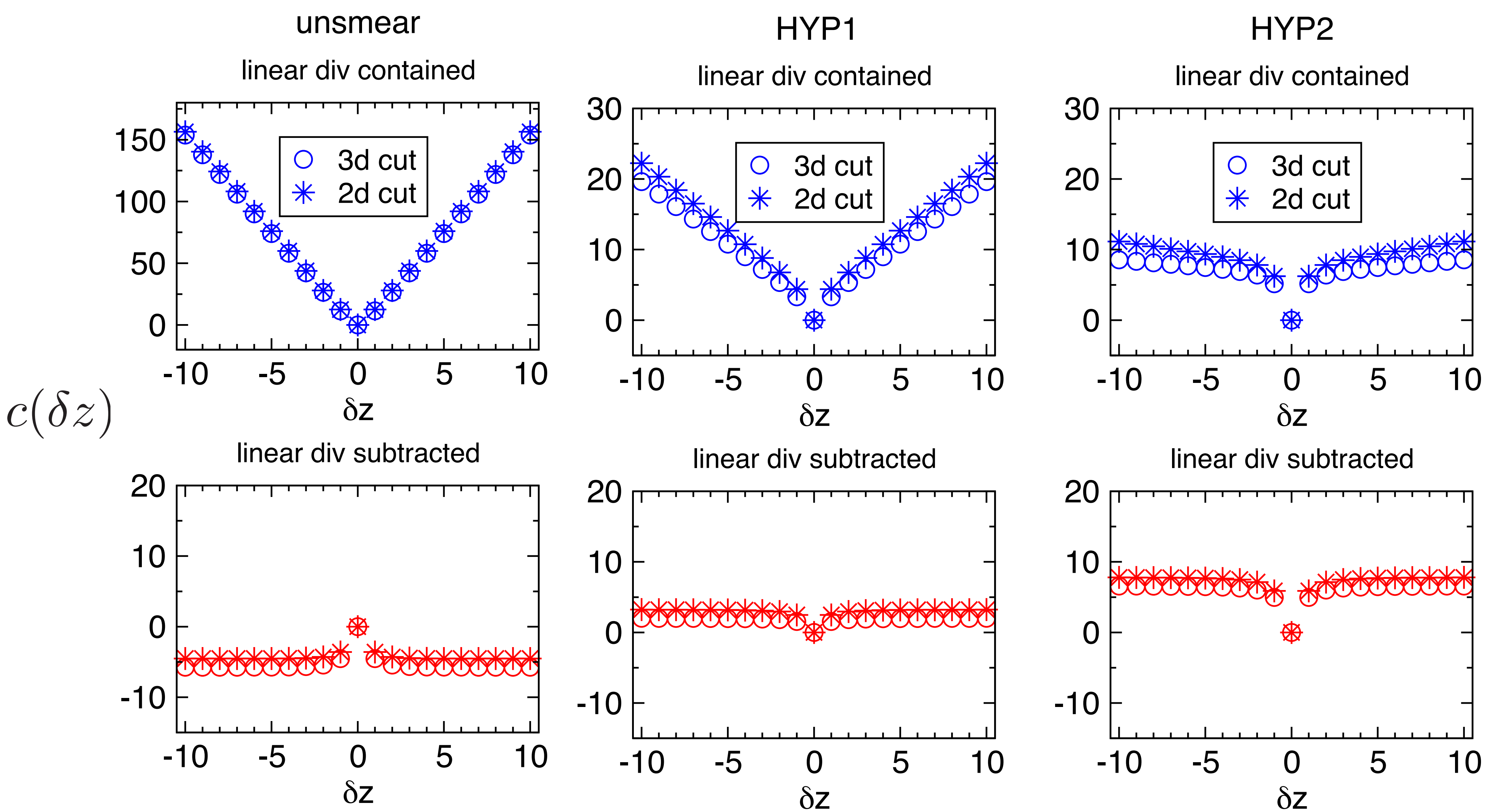
$$Z(\delta z) = 1 + \frac{g^2}{(4\pi)^2} C_F c(\delta z) + O(g^4)$$

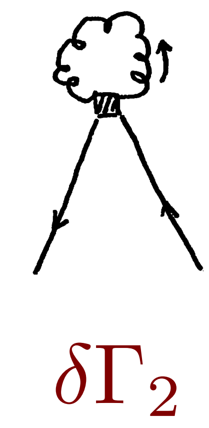
$$\mu = a^{-1}$$



Matching between continuum and lattice

▶ Effects of link smearing





- Linear divergence is observed when it is not subtracted.
- HYP2 removes the linear divergence in large part in the matching.

Summary and outlook

- New approach for lattice calculation of PDFs has been proposed:
  - quasi-PDFs with LMET approach [Ji (2013)]
  - lattice cross section with collinear factorization approach [Ma and Qiu (2014)]
- For precise calculation, there are several important steps:
  - power divergence subtraction
  - lattice-continuum matching (PT, NPT)
  - continuum limit
- Global QCD analysis with lattice QCD could support EIC.
- Transverse momentum dependent parton densities (TMDs) and Generalized parton distributions (GPDs) could be also addressed by defining lattice calculable cross section toward full scan of 3D structure of nucleons.