Nucleon Form Factors at High Momentum Transfer

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for the LHP collaboration

LATTICE 2016

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Jefferson Lab

Outline

Nucleon form factors at high momentum / short distance

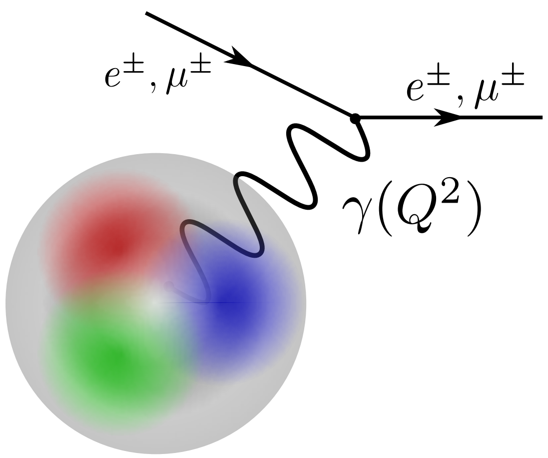
- Phenomenological motivation
  - *Perturbative Q^2-scaling limit*
  - *Experimental program at JLab*
- Challenges for high-momentum nucleon structure
  - *Signal / noise for required kinematics*
  - *Boosted (momentum) smearing*
- Details of calculation
- Preliminary results and comparison to phenomenology
- Summary and Outlook

Nucleon Vector Form Factors and GEp/GMp

$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\Big[F_1(Q^2)\,\gamma^\mu + F_2\,(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]U_P$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)$$

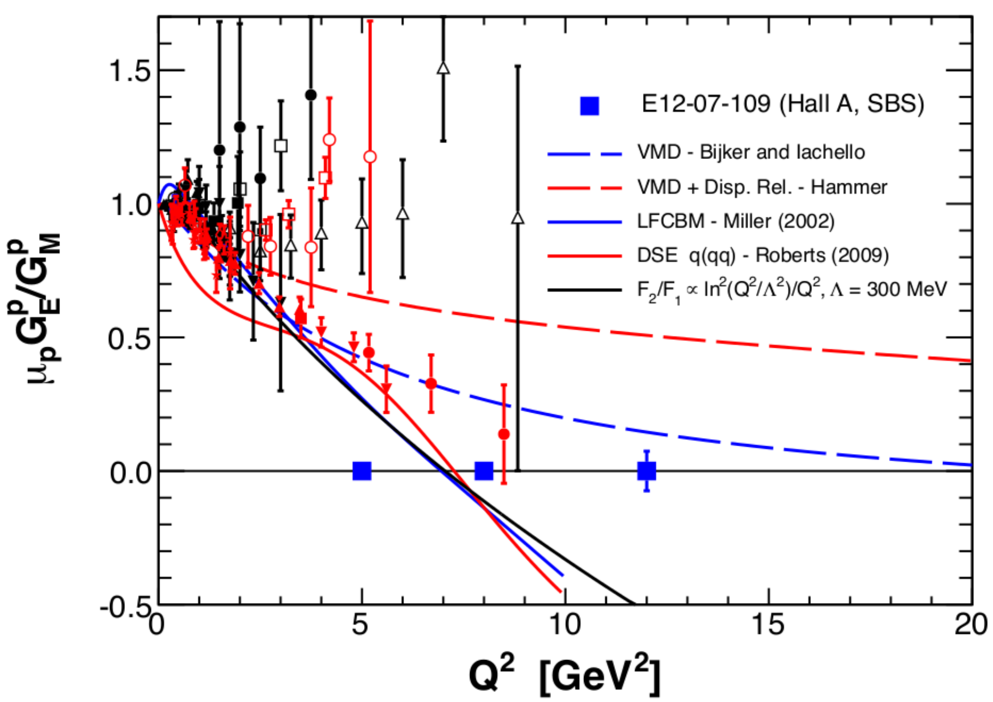
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$



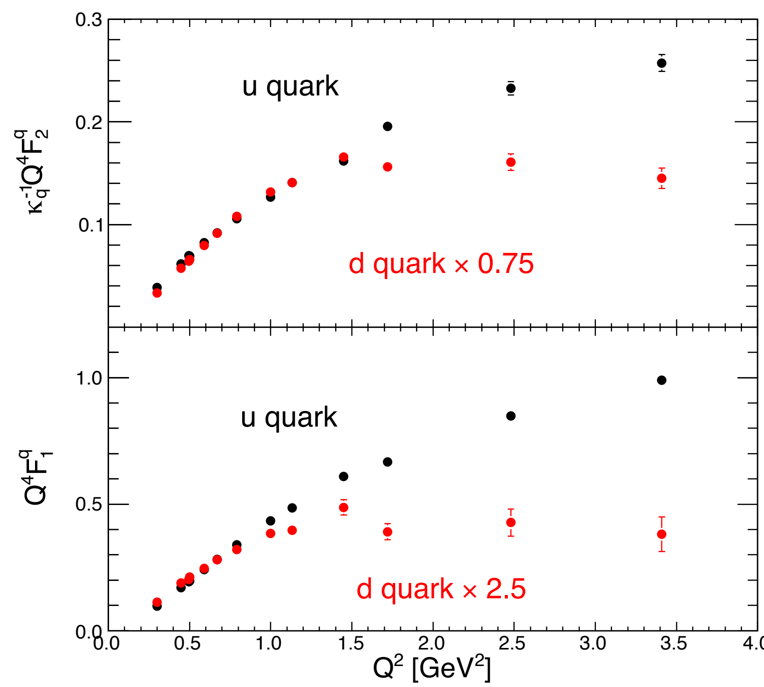
- JLab@12GeV + Super BigBite:

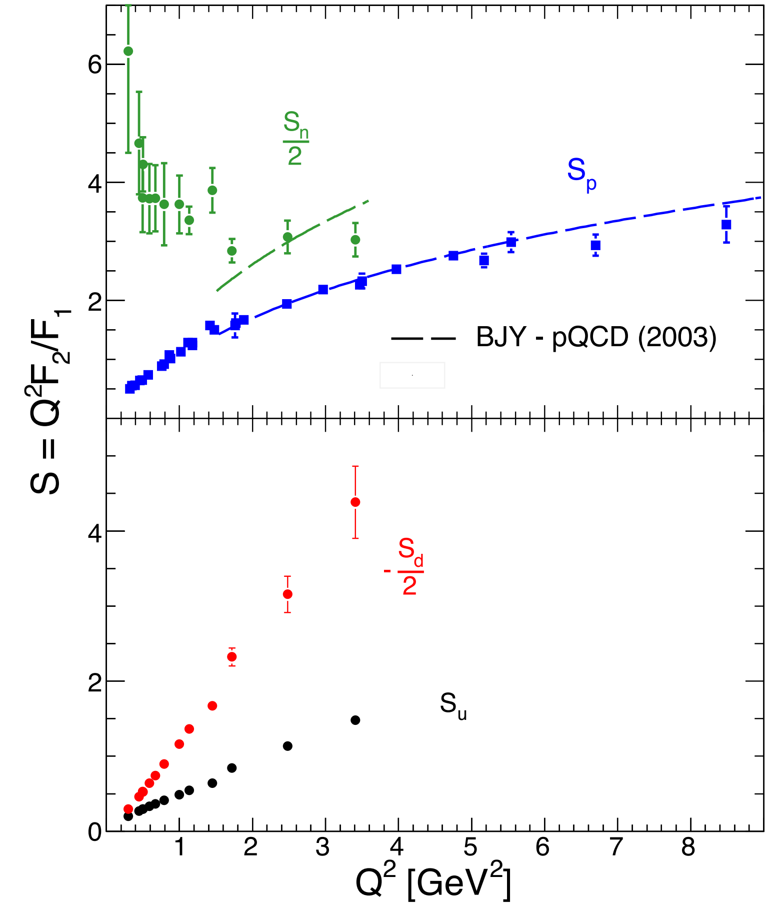
explore form factors at Q^2 up to 18 GeV^2

  - (G_E/G_M) dependence
  - *u*-, *d*-flavor contributions to form factors
  - $(F_1\,/\,F_2)$ scaling at $Q^2 \to \infty$



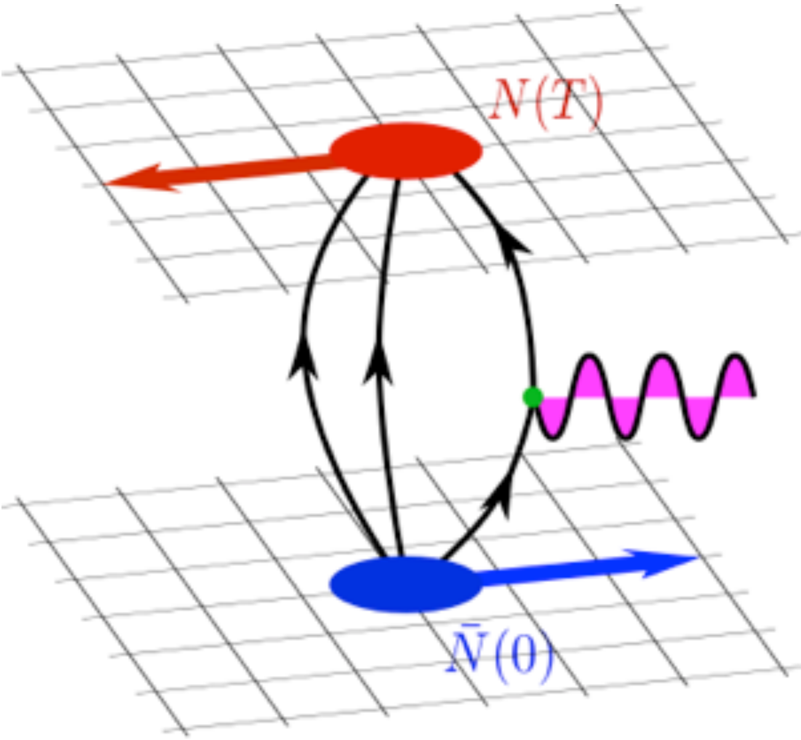
[Research Mgmt. Plan for SBS(JLab Hall A)]





[G.D.Cates, C.W.de Jager, S.Riordan, B.Wojtsekhovski, PRL106:252003, arXiv:1103.1808]

Accessing Large Q^2 : Breit Frame



Minimize $E_{in,out}$ for target Q^2:

$$Q^2 = (\vec{p}_{in} - \vec{p}_{out})^2 - (E_{in} - E_{out})^2$$

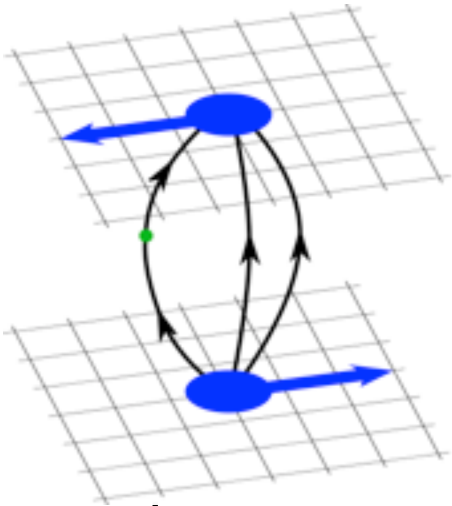
Back-to-back

$$Q^2 = 4\vec{p}^2$$

At right angle

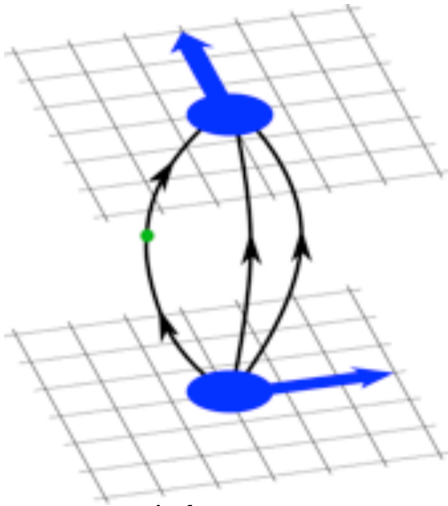
$$Q^2 = 2\vec{p}^2$$

For $Q^2 = 8\ GeV^2$



$$|\vec{p}| = \frac{1}{2}\sqrt{Q^2} \approx 1.4 \text{ GeV}$$

$E_N \approx 1.8\ GeV$



$$|\vec{p}| = \sqrt{\frac{1}{2}Q^2} \approx 2.0 \text{ GeV}$$

$E_N \approx 2.3\ GeV$

Challenges for Structure at Large Momentum

- Discretization effects

  Will need current operator improvement (in progress)

$$(V_\mu)_I = \bar{q}\gamma_\mu q + c_V\, a\partial_\nu \bar{q} i\sigma_{\mu\nu} q$$

- Stochastic noise grows faster with *T* [Lepage'89]:

$$\begin{array}{lll} \text{Signal} & \langle N(T)\bar{N}(0)\rangle & \sim e^{-E_N T} \\ \text{Noise} & \langle |N(T)\bar{N}(0)|^2\rangle - |\langle N(T)\bar{N}(0)\rangle|^2 & \sim e^{-3m_\pi T} \\ \text{Signal/Noise} & & \sim e^{-(E_N - \frac{3}{2}m_\pi)T} \end{array}$$

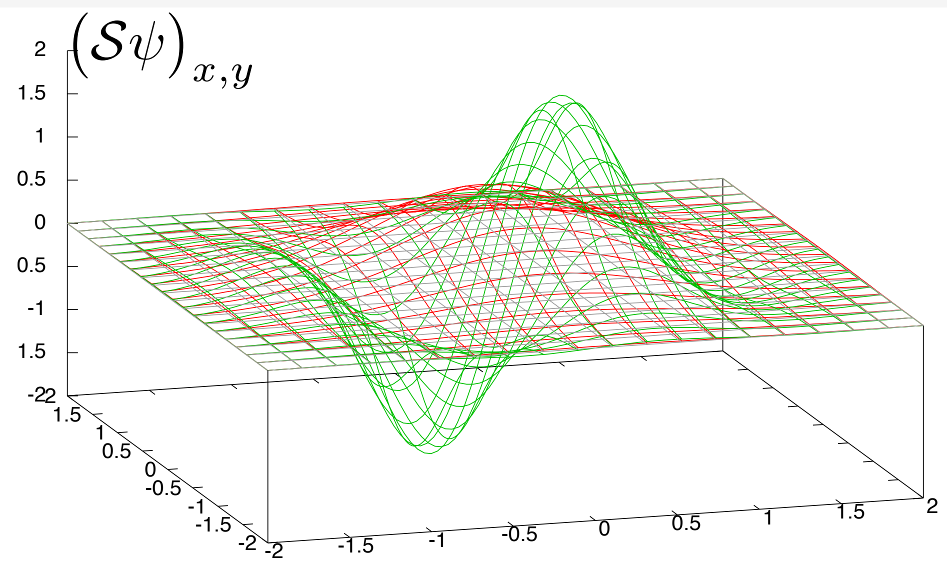
- Excited states: boosting "shrinks" the energy gap

$$E_1 - E_0 = \sqrt{M_1^2 + \vec{p}^2} - \sqrt{M_2^2 + \vec{p}^2} < M_1 - M_0$$

  - In this work : 2-exponential fits

Reduction of lattice correlator noise is crucial

High-momentum Hadron States on a Lattice



Nucleon operator is built from ≈Gaussian smeared quarks

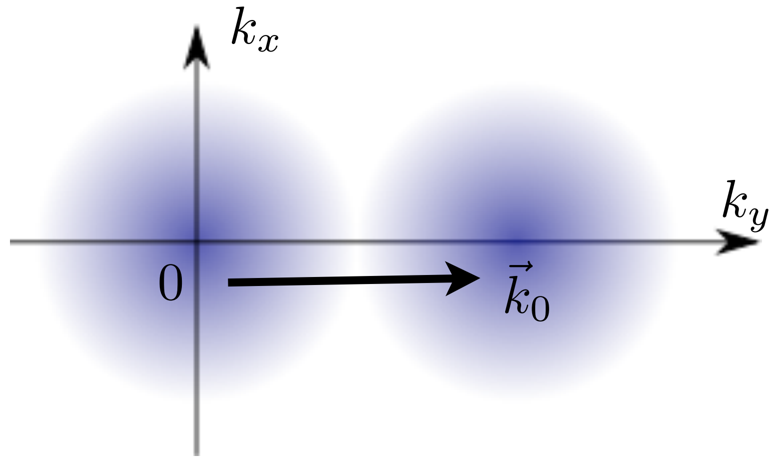
$$N_{\text{lat}}(x) = (\mathcal{S}\,u)^a_x\left[(\mathcal{S}\,d)^b_x\, C\gamma_5\,(\mathcal{S}\,u)^c_x\right]\epsilon^{abc}$$

Gaussian shape in momentum space :
reduced overlap with quark WFs in a boosted nucleon

$$\mathcal{S}_{\text{at-rest}} = \exp\left[-\frac{w^2}{4}(i\vec{\nabla})^2\right] \sim exp\left(-\frac{w^2\vec{k}^2_{\text{lat}}}{4}\right)$$

[orig. B.Musch; first explored in G.Bali et al, 1602.05525;
see poster by B.Lang]:
SOLUTION : improve the overlap by shifting the spatial smearing operator in momentum space (*"momentum smearing"*)

$$\mathcal{S}_{\vec{k}_0} = \exp\left[-\frac{w^2}{4}(-i\vec{\nabla} - \vec{k}_0)^2\right] \sim \exp\left(-\frac{w^2(\vec{k}_{\text{lat}} - \vec{k}_0)^2}{4}\right)$$



Modified smearing operator

$$\left[\mathcal{S}_{\vec{k}_0}(\psi)\right]_x = e^{+\vec{k}_0\vec{x}}\mathcal{S}(e^{-\vec{k}_0\vec{y}}\psi_y) \sim e^{+\vec{k}_0\vec{x}}\cdot \text{smooth fcn.}(x)$$

Modified covariant smearing operator in lattice*color space

$$\left[\mathcal{S}_{\vec{k}_0}\right]_{x,y} = e^{+i\vec{k}_0\vec{x}}\left[\mathcal{S}\right]_{x,y}e^{-i\vec{k}_0\vec{y}}$$

$\Longleftrightarrow$

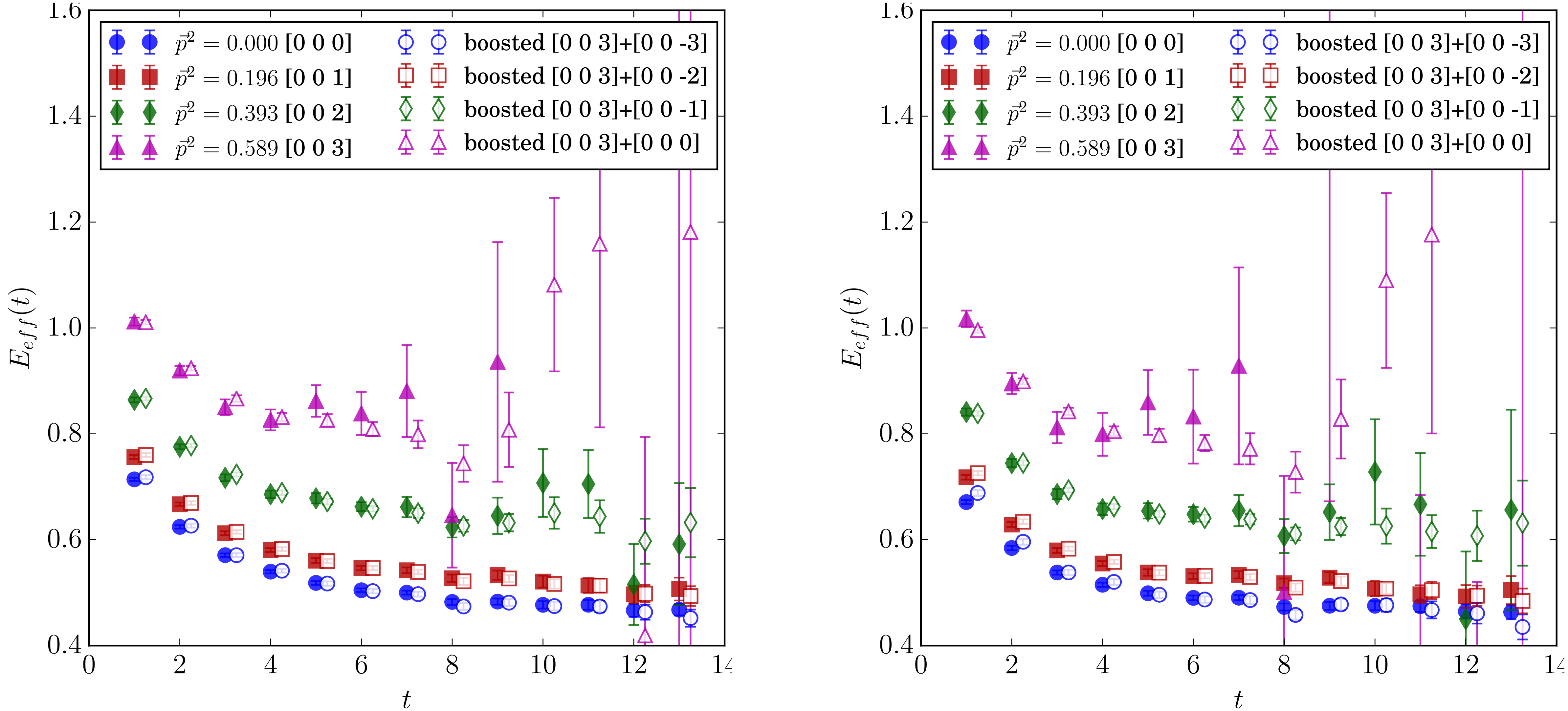
Smearing with twisted gauge links

$$\Delta_{x,y} \longrightarrow e^{+i\vec{k}_0\vec{x}}\Delta_{x,y}e^{-i\vec{k}_0\vec{y}}$$

$$U_{x,\mu} \longrightarrow e^{-ik_\mu}U_{x,\mu}$$

Signal Gain : Traditional vs. Boosted Smearing

Nucleon Effective Energy: m_π = 320 MeV, a=0.081 fm, 32^3x64



w=4.96 (N=40) ⟵ Gaussian smearing ⟶ *w=6.56 (N=70)*

- each quark is boosted with the same k=[0 0 1]
- *w=5.55 (N=45)* chosen for preliminary structure study

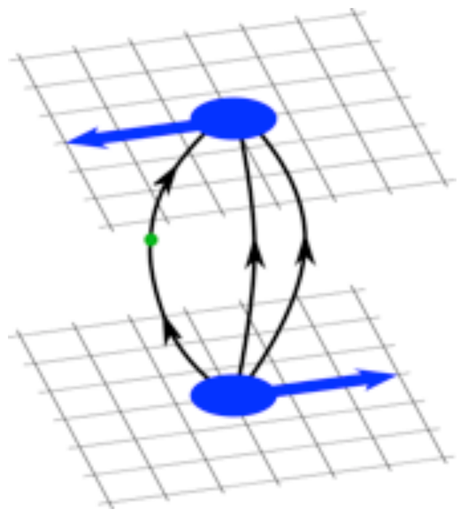
Preliminary Study : 2 Gauge Ensembles

Exploratory study with clover-improved Wilson action (WM/JLab) at m_π = 320 MeV with a=0.114 fm, a=0.081 fm

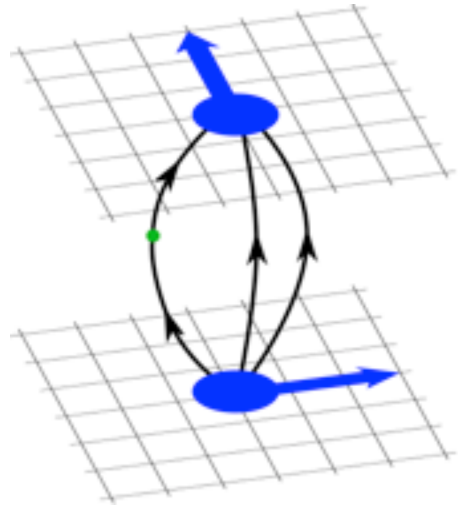
- 32^3x64
- a=0.081 fm
- *p_{min}=0.48 GeV*
- *tsep = (8 .. 12)a = 0.65 .. 0.97 fm*
- *boost-smear with [1,0,0]*
- *240*64=15,360 samples*

- 32^3x96
- a=0.114 fm
- *p_{min}=0.34 GeV*
- *tsep = (6 .. 10)a = 0.68 .. 1.14 fm*
- *boost-smear with [1,1,0]*
- *210*96=20,160 samples*

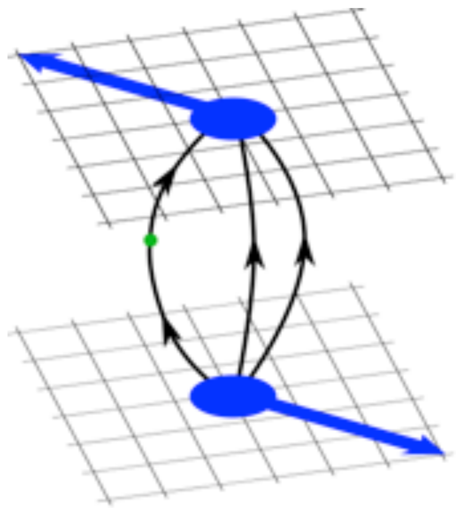
$Q^2 \lesssim 8.2\ \mathrm{GeV}^2$



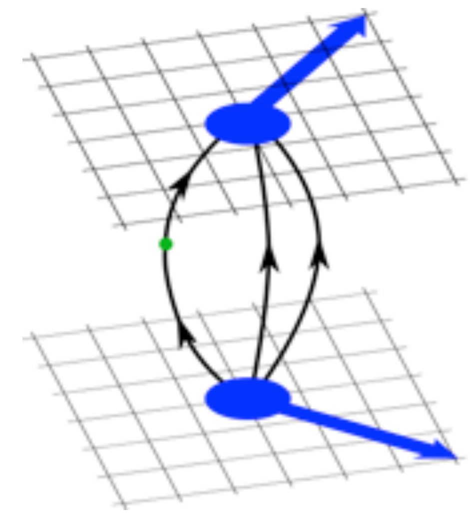
$Q^2 \lesssim 4.1\ \mathrm{GeV}^2$



$Q^2 \lesssim 8.3\ \mathrm{GeV}^2$



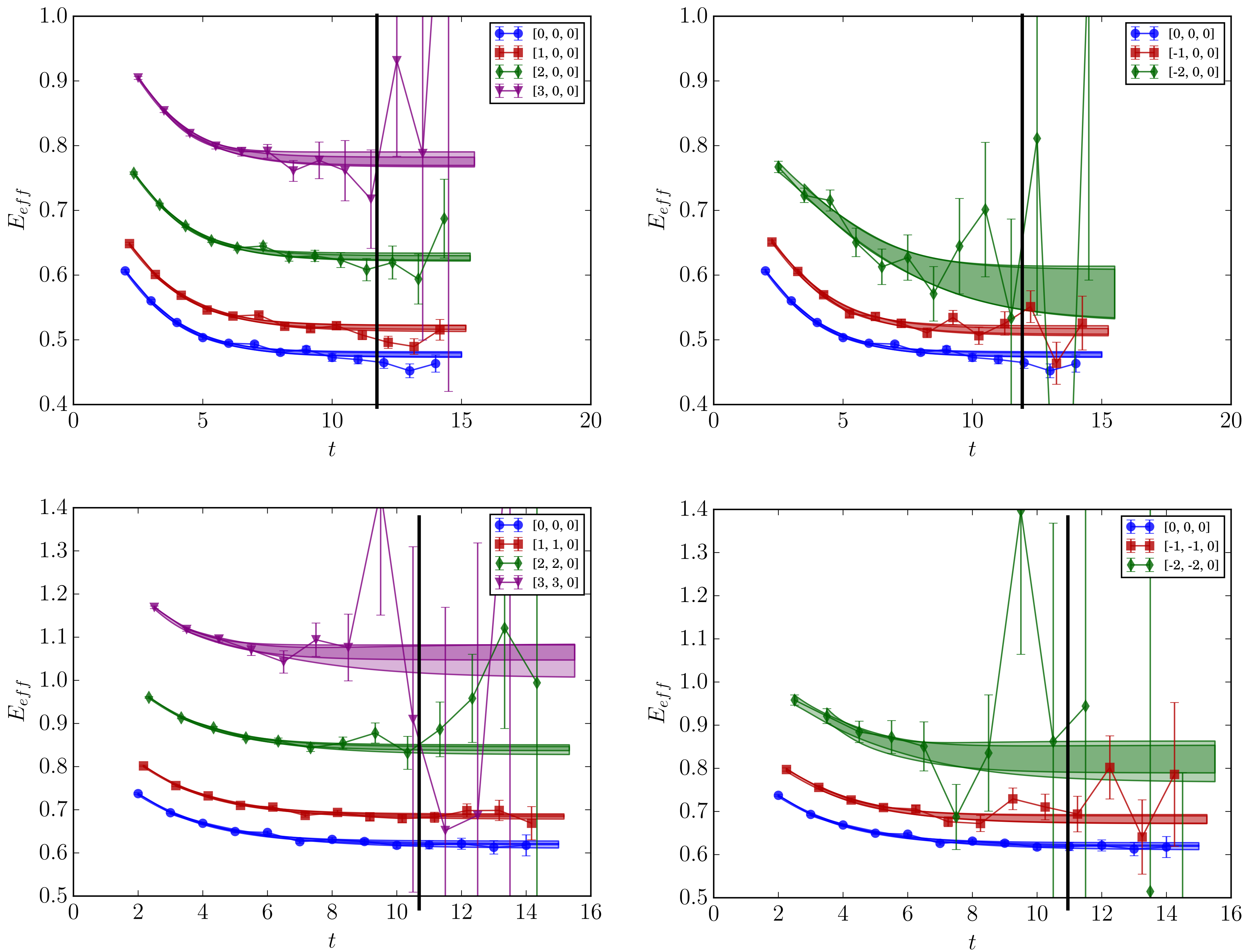
$Q^2 \lesssim 4.2\ \mathrm{GeV}^2$



- each quark is smeared with the same "boost"

Effective Energy from Boost-Smeared C2pt

$p_N \upuparrows$ boost | $p_N \updownarrows$ boost



a=0.081 fm

kboost=[1,0,0]

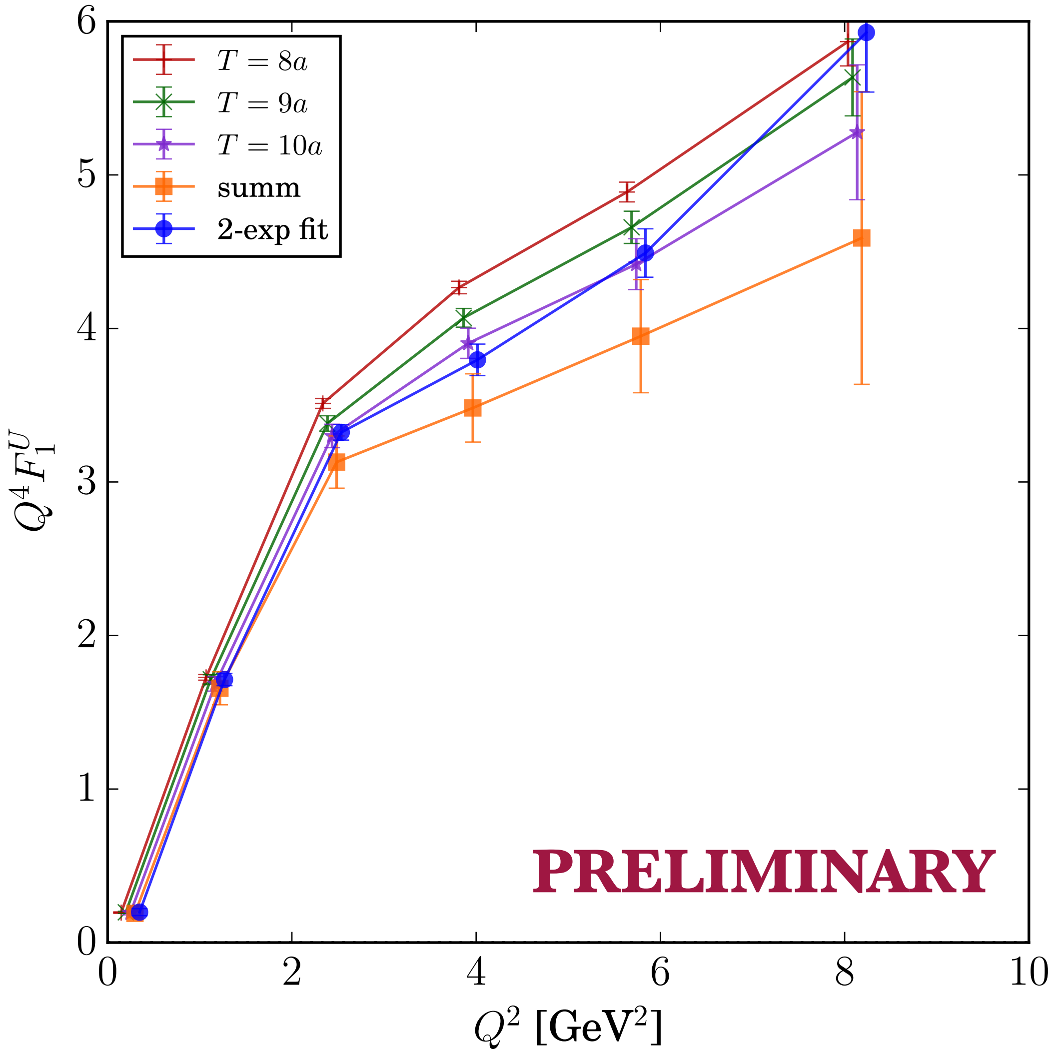
Select source-sink sep
(8 ..12)a =0.65 ..0.97 fm

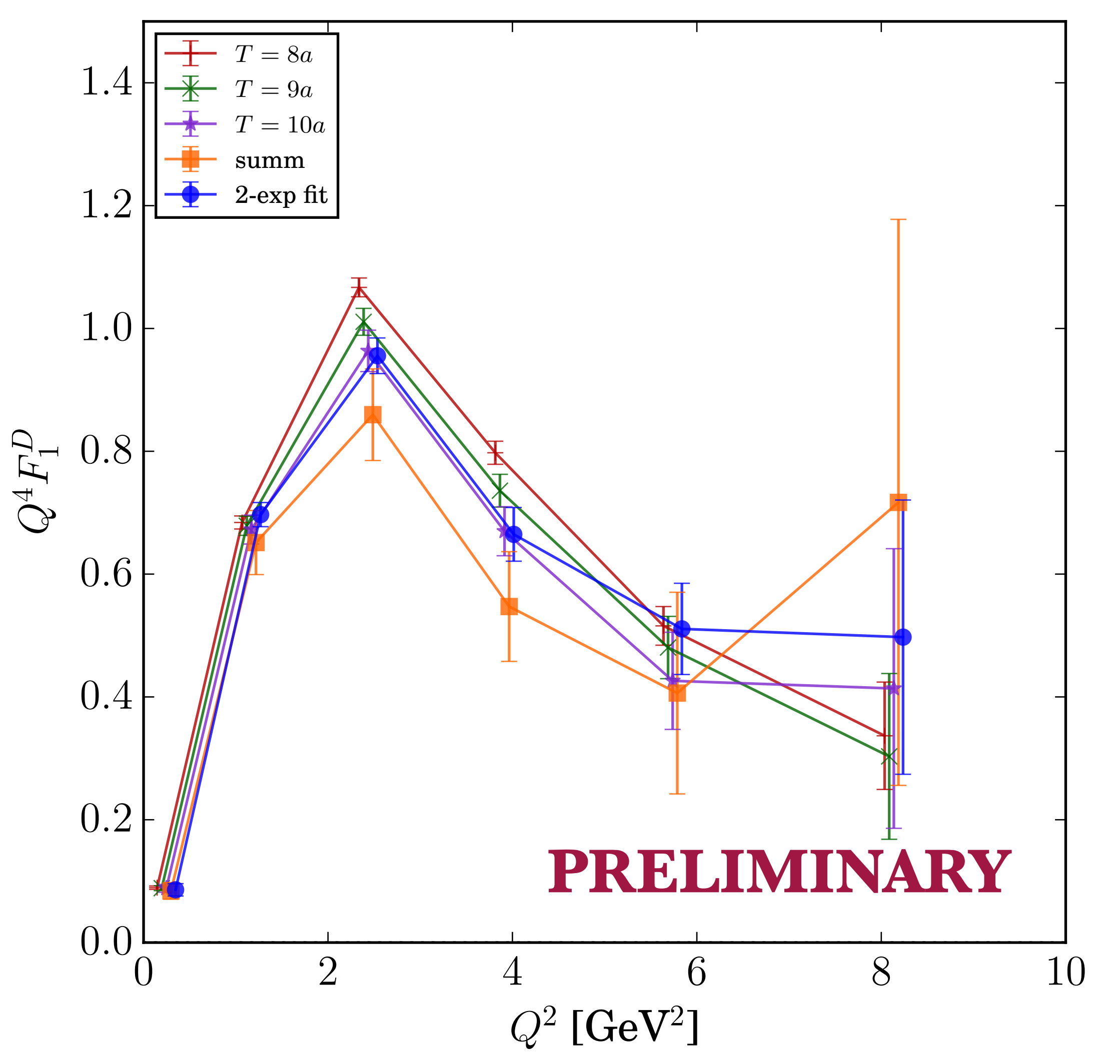
a=0.114 fm

kboost=[1,1,0]

Select source-sink sep
(6 ..10)a =0.68 ..1.1 fm

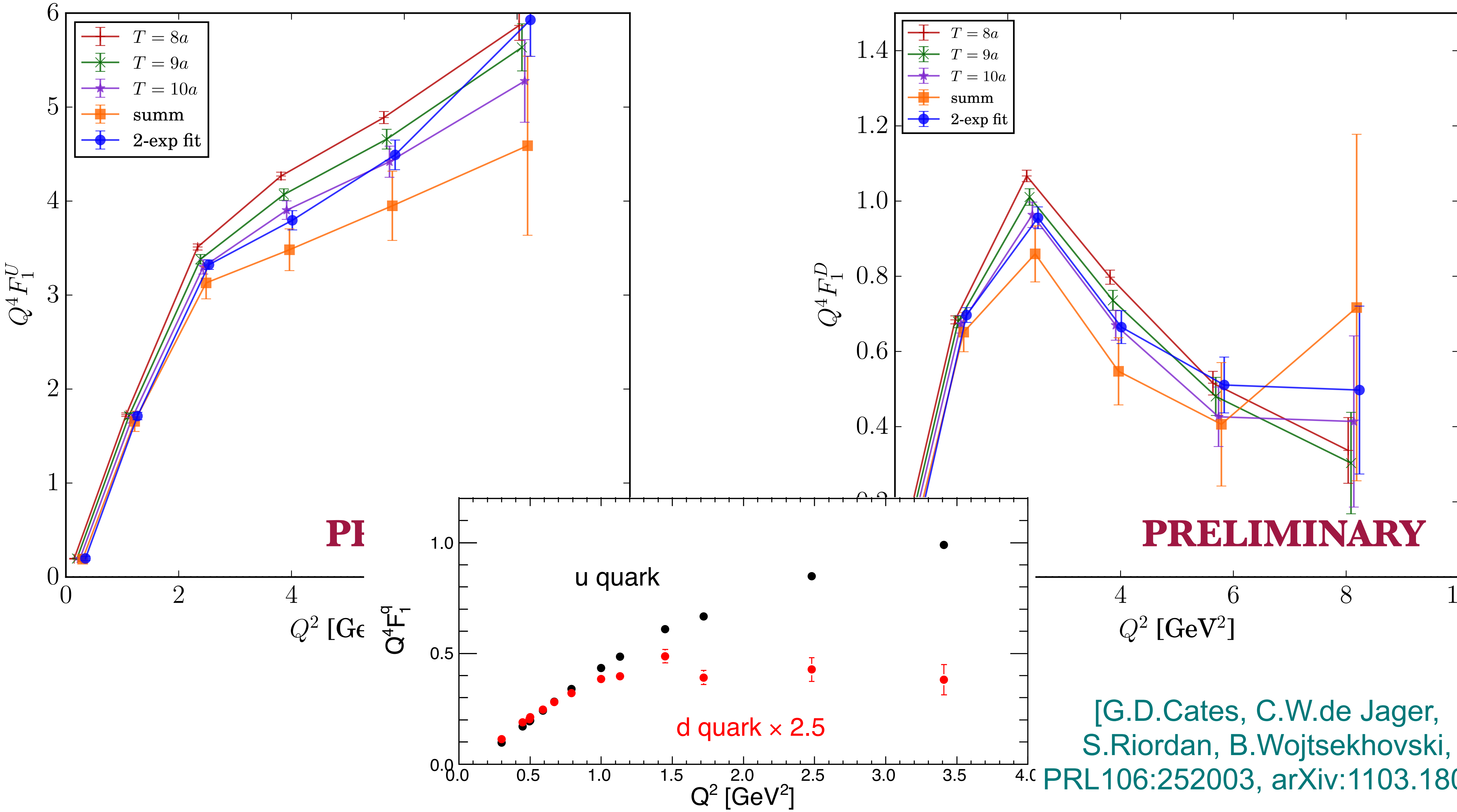
Q^2 Dependence of $F_1{}^u$ and $F_1{}^d$





- expect $F_1(Q^2) \sim Q^4$ scaling [Lepage, Brodsky (1979)]
- Both form factors overshoot experiment (x3-4)

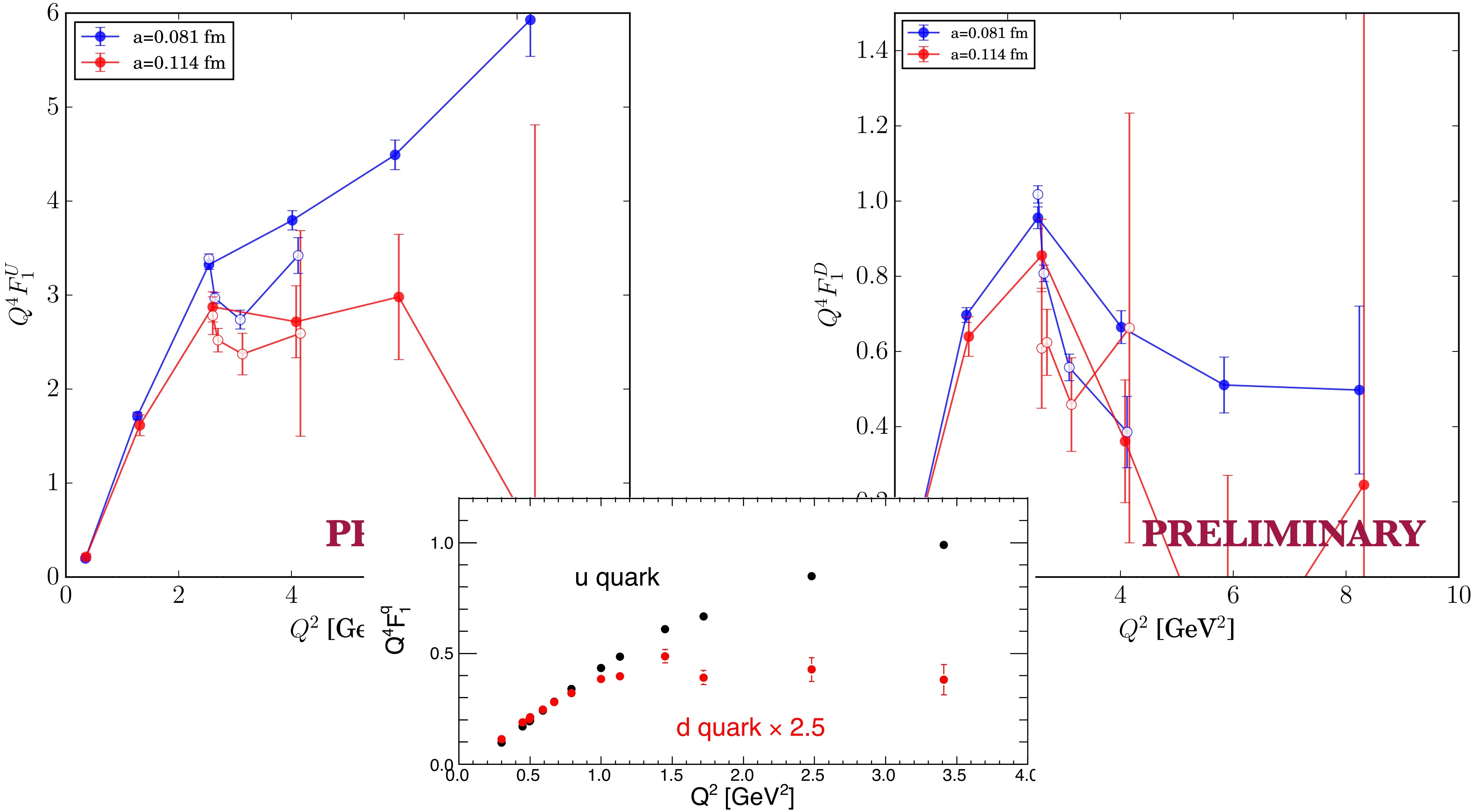
Q^2 Dependence of F_1^u and F_1^d



[G.D.Cates, C.W.de Jager, S.Riordan, B.Wojtsekhovski, PRL106:252003, arXiv:1103.1808]

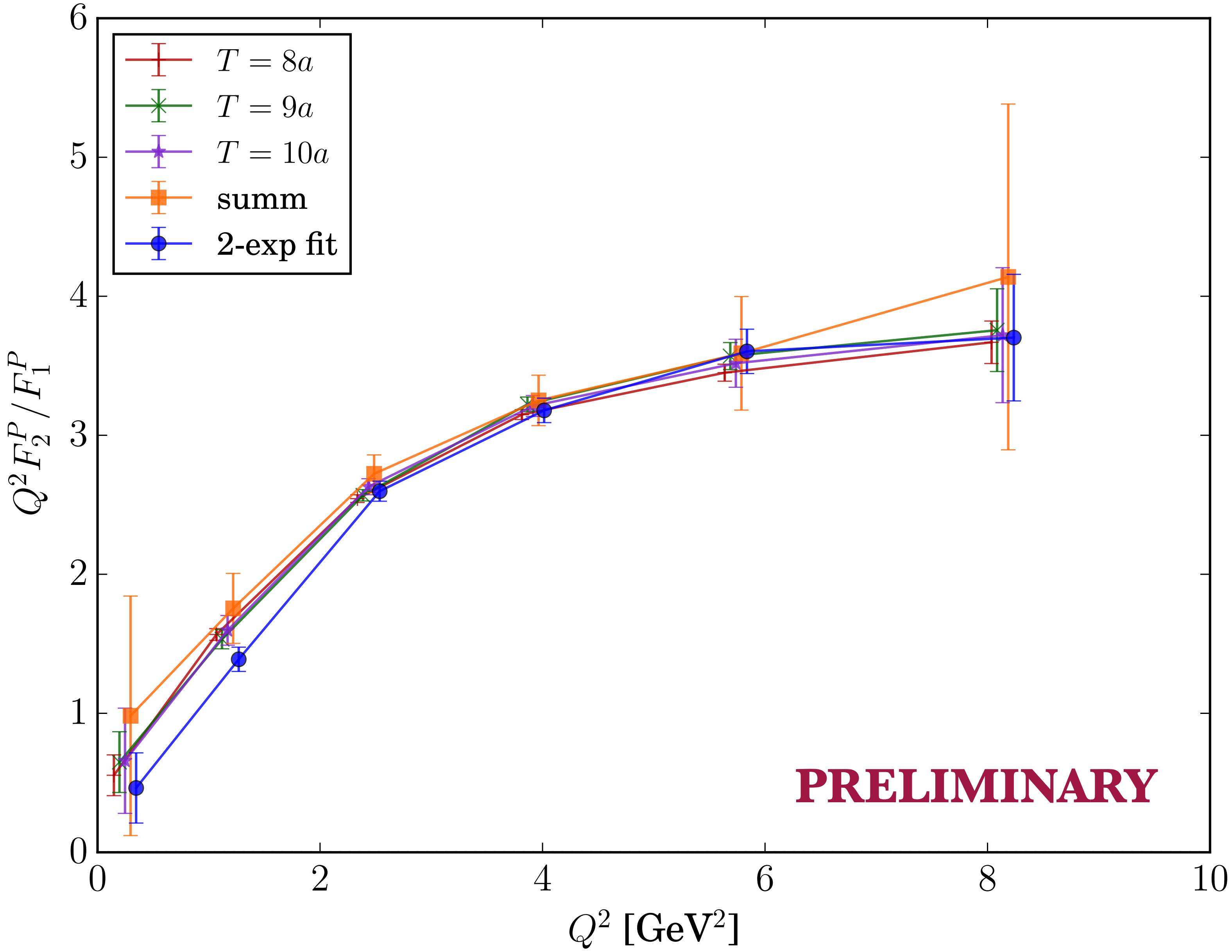
- expect $F_1(Q^2) \sim Q^4$ scaling [Lepage, Brodsky (1979)]
- Both form factors overshoot experiment (x3-4)

Q^2 Dependence of F_1^u and F_1^d: a=0.081 vs 0.114 fm



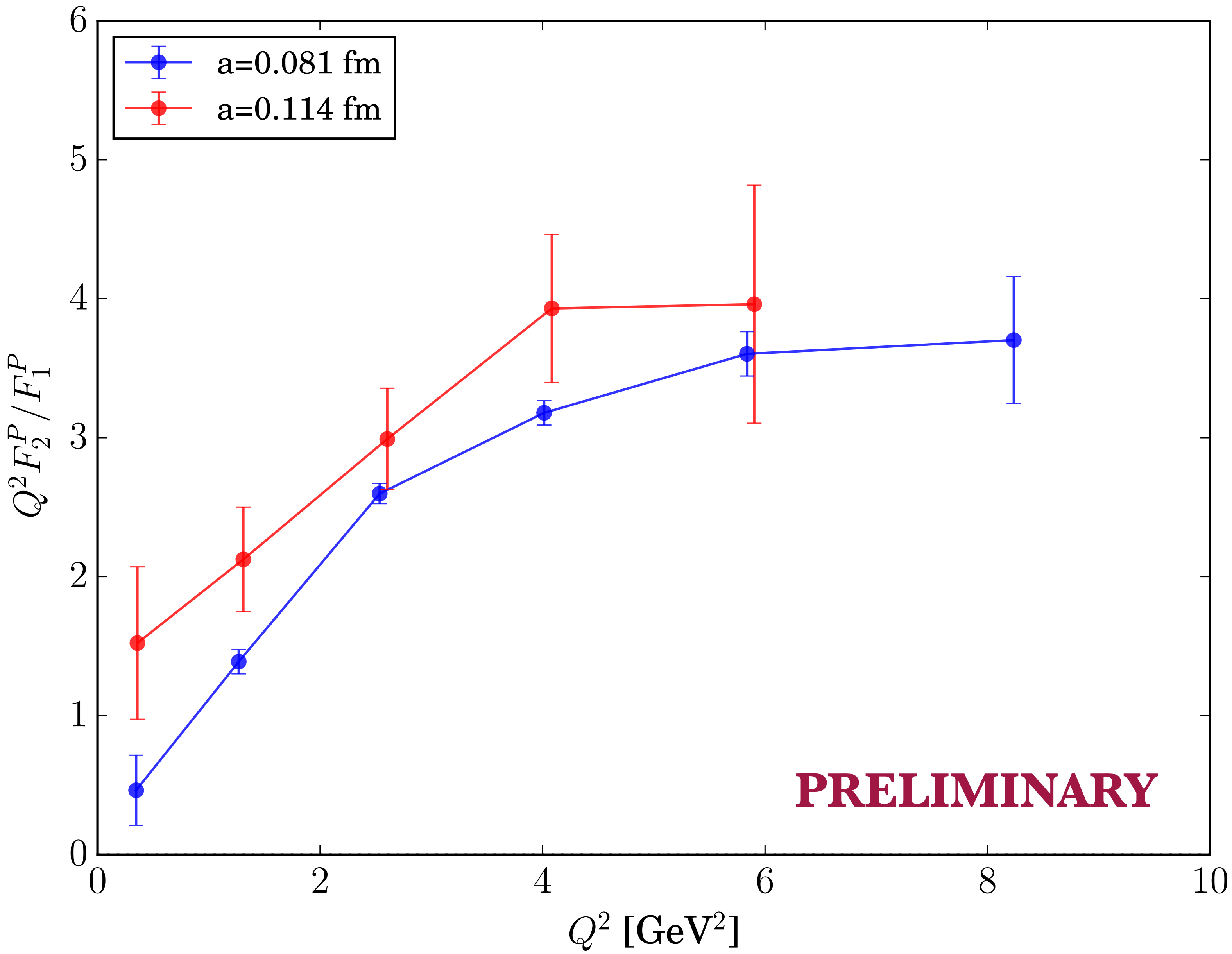
- expect $F_1(Q^2) \sim Q^4$ scaling [Lepage, Brodsky (1979)]
- Both form factors overshoot experiment (x3-4)

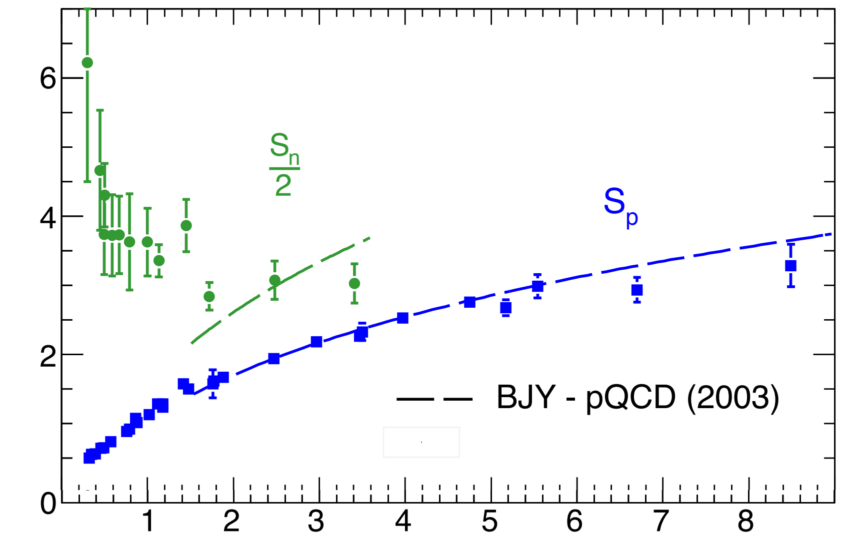
Q^2F_2/F_1 for Proton



- expect $Q^2 F_1(Q^2)/F_2(Q^2)$ ~ log[Q^2/Λ^2] scaling [Belitsky, Ji, Yuan (2003)]
- Qualitative behavior of F1u, F1d agrees with phenomenology

Q2F2/F1, Comparison to pQCD scaling

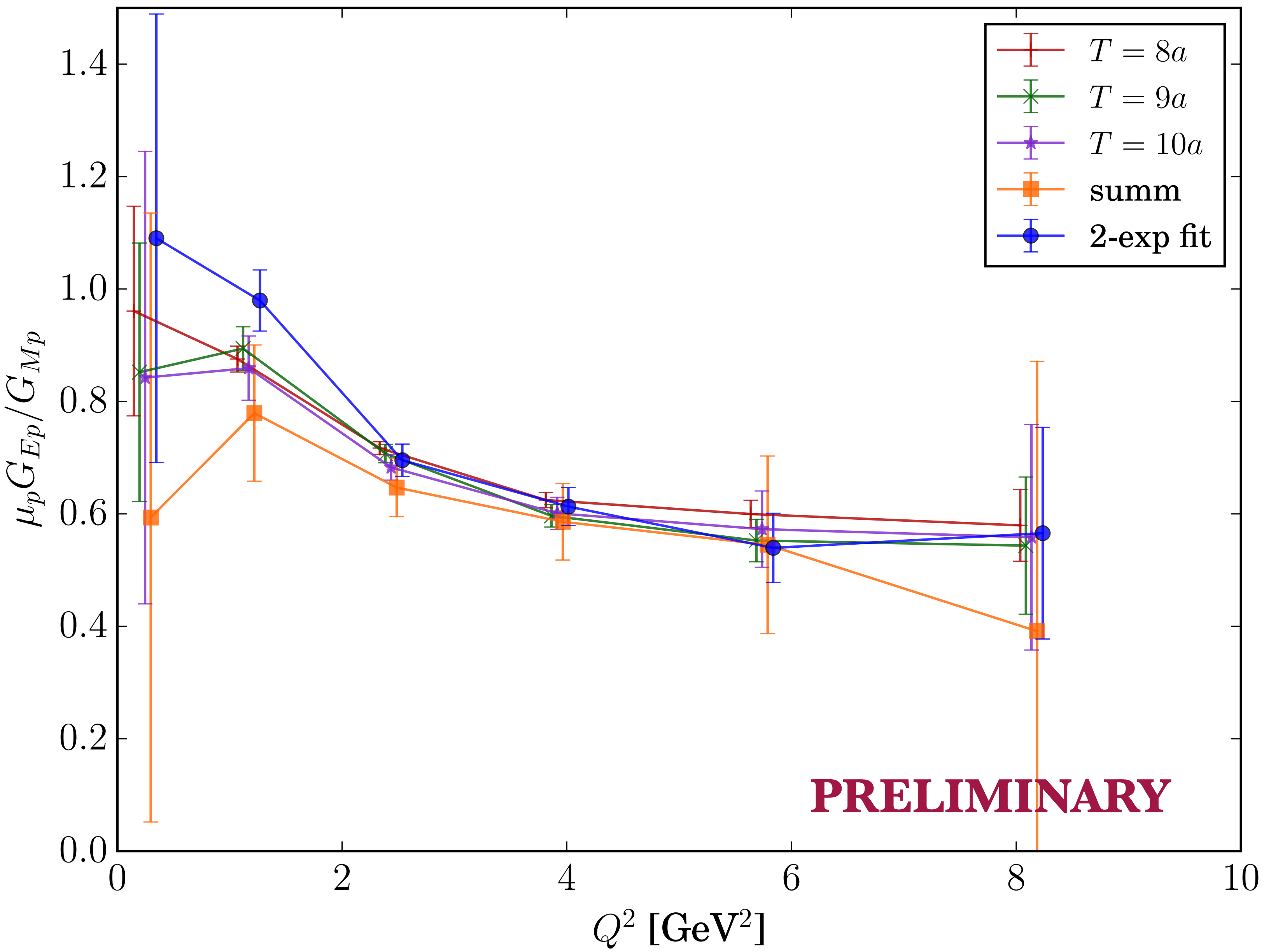




[G.D.Cates, C.W.de Jager, S.Riordan, B.Wojtsekhovski, PRL106:252003, arXiv:1103.1808]

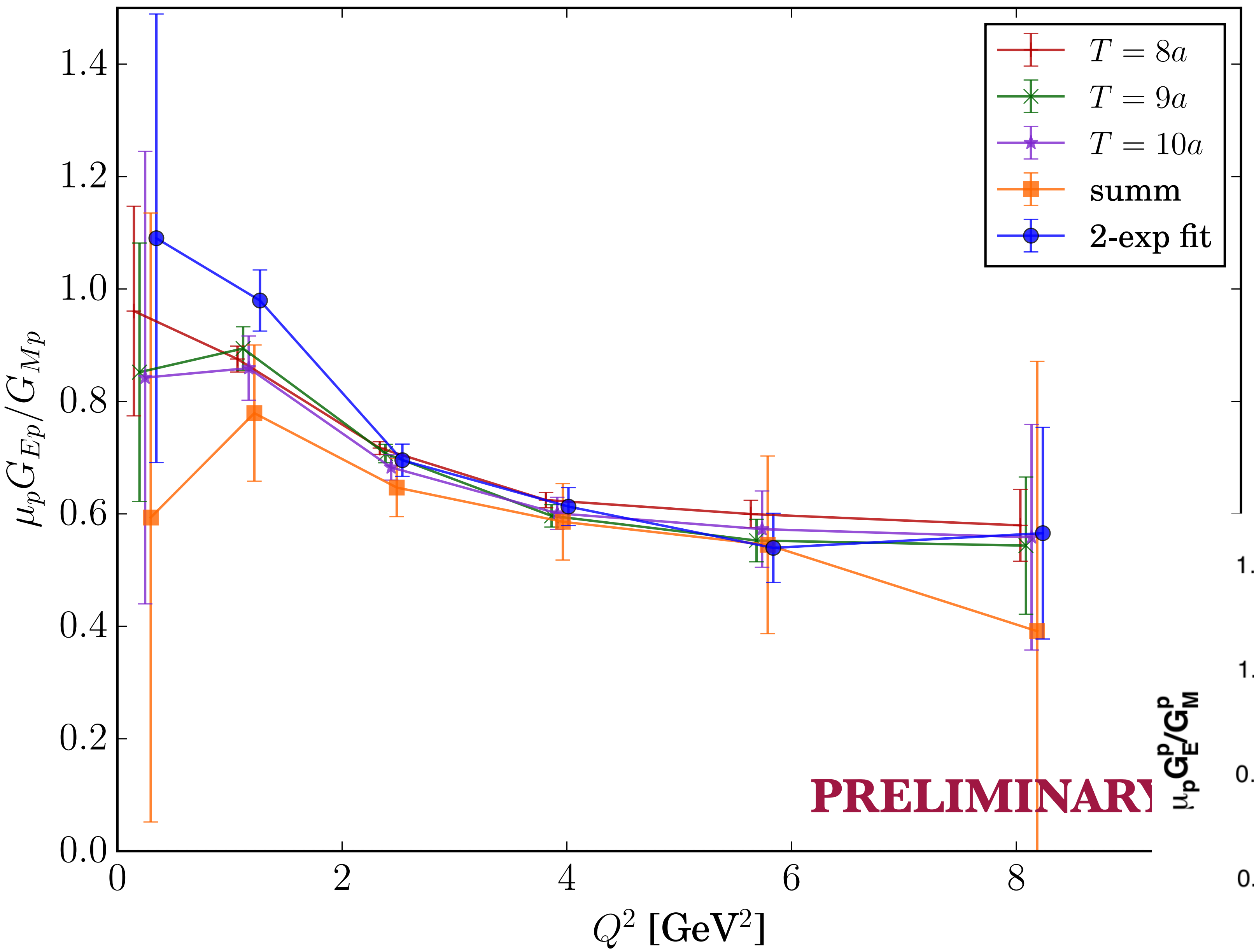
- expect $Q^2 F_1(Q^2)/F_2(Q^2) \sim \log[Q^2/\Lambda^2]$ scaling [Belitsky, Ji, Yuan (2003)]
- Qualitative behavior of F1u, F1d agrees with phenomenology

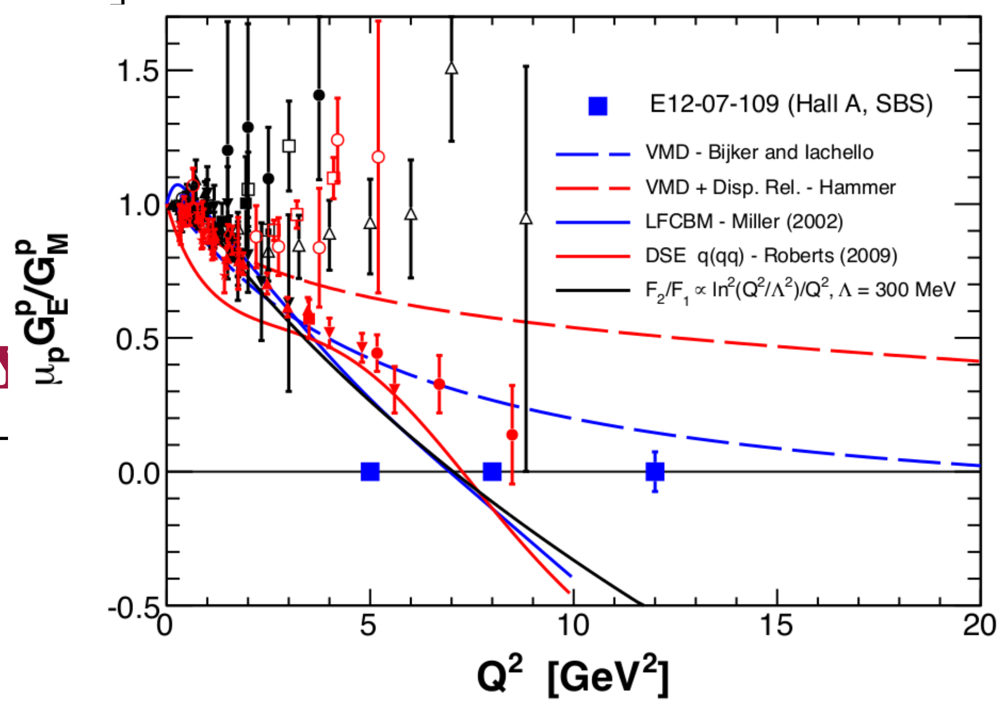
G_{Ep}/G_{Mp} for Proton



- Need to evaluate disconnected diagrams and operator improvement term

GEp/GMp





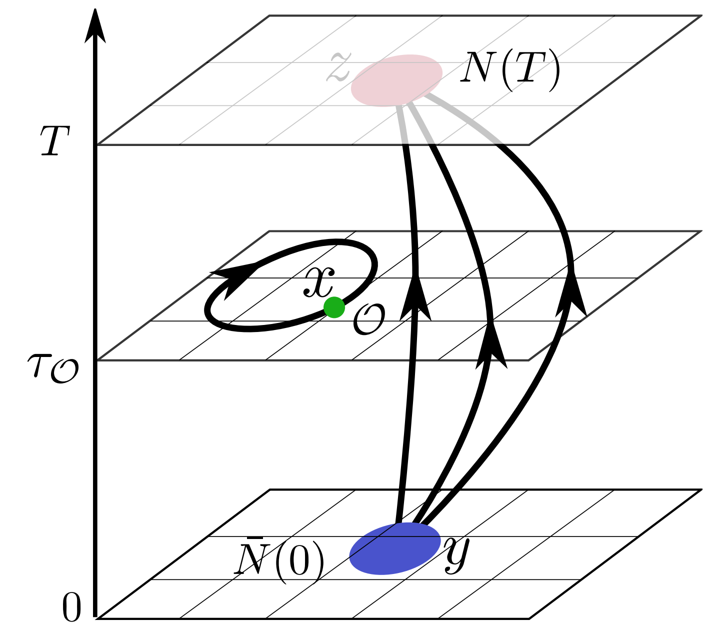
- Need to evaluate disconnected diagrams and operator improvement term
- Experiments hint at GEp/GMp intersection at Q^2=8 GeV^2

Disconnected Quark Loops

- Stochastic evaluation: $\begin{cases} \xi(x) = \text{ random } Z_2\text{-vector} \\ E\big[\xi^\dagger(x)\xi(y)\big] = \delta_{x,y} \end{cases}$

$$\sum_x e^{iqx} \not{D}^{-1}(x,x) \approx \frac{1}{N_{MC}} \sum_i^{N_{MC}} \xi^\dagger_{(i)} \big(e^{iqx} \not{D}^{-1} \xi_{(i)}\big)$$

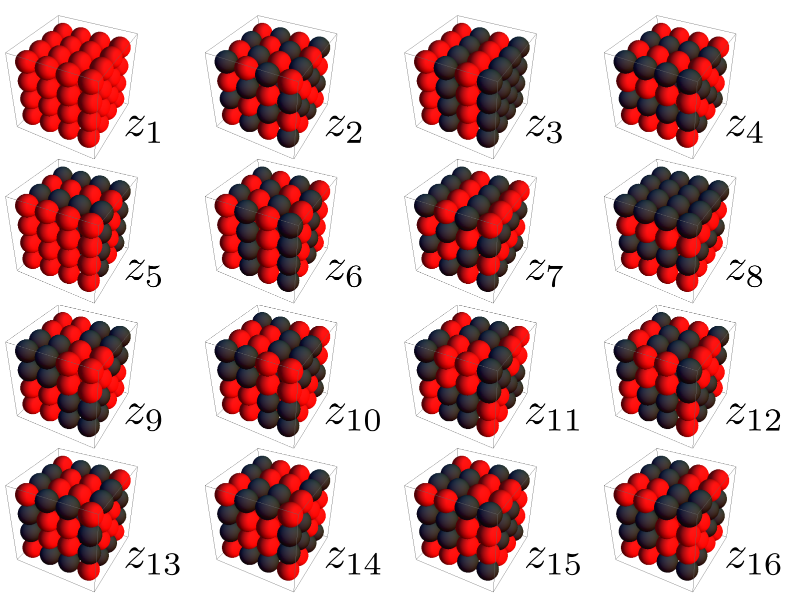
$$\mathrm{Var}\Big(\sum_x \not{D}^{-1}(x,x)\Big) \sim \frac{1}{N_{MC}} \quad \text{(contributions from } \not{D}^{-1}(x \neq y)\text{)}$$



- Exploit $\not{D}^{-1}(x,y)$ FALLOFF to reduce $\sum_{x\neq y} |\not{D}^{-1}(x,y)|^2$

Hierarchical probing method [K.Orginos, A.Stathopoulos, '13] :
In sum over $N=2^{nd+1}$ 3D(4D) Hadamard vectors,
near-(x,y) terms cancel:

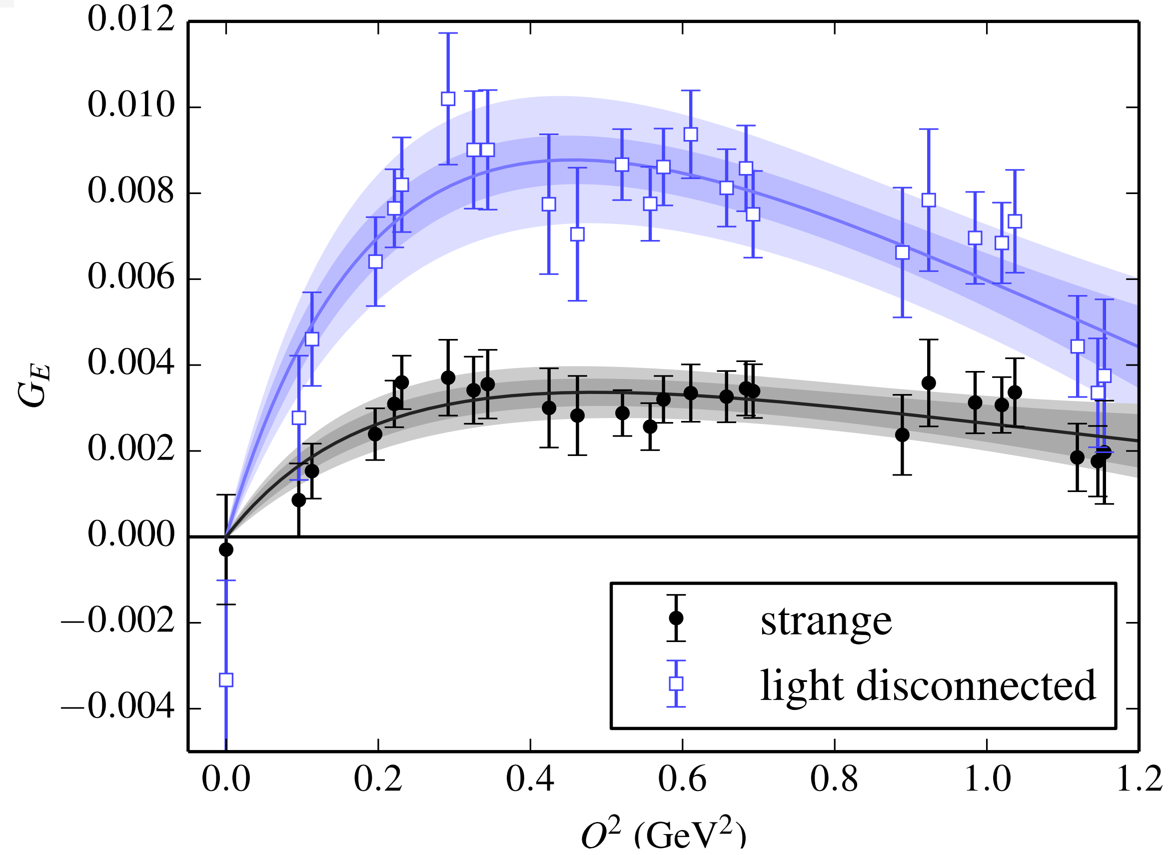
$$\frac{1}{N}\sum_i z_i(x) z_i(y)^\dagger = \begin{cases} 0, & 1 \le |x-y| \le 2^k, \\ 1, & x = y \text{ or } 2^k < |x-y| \end{cases}$$

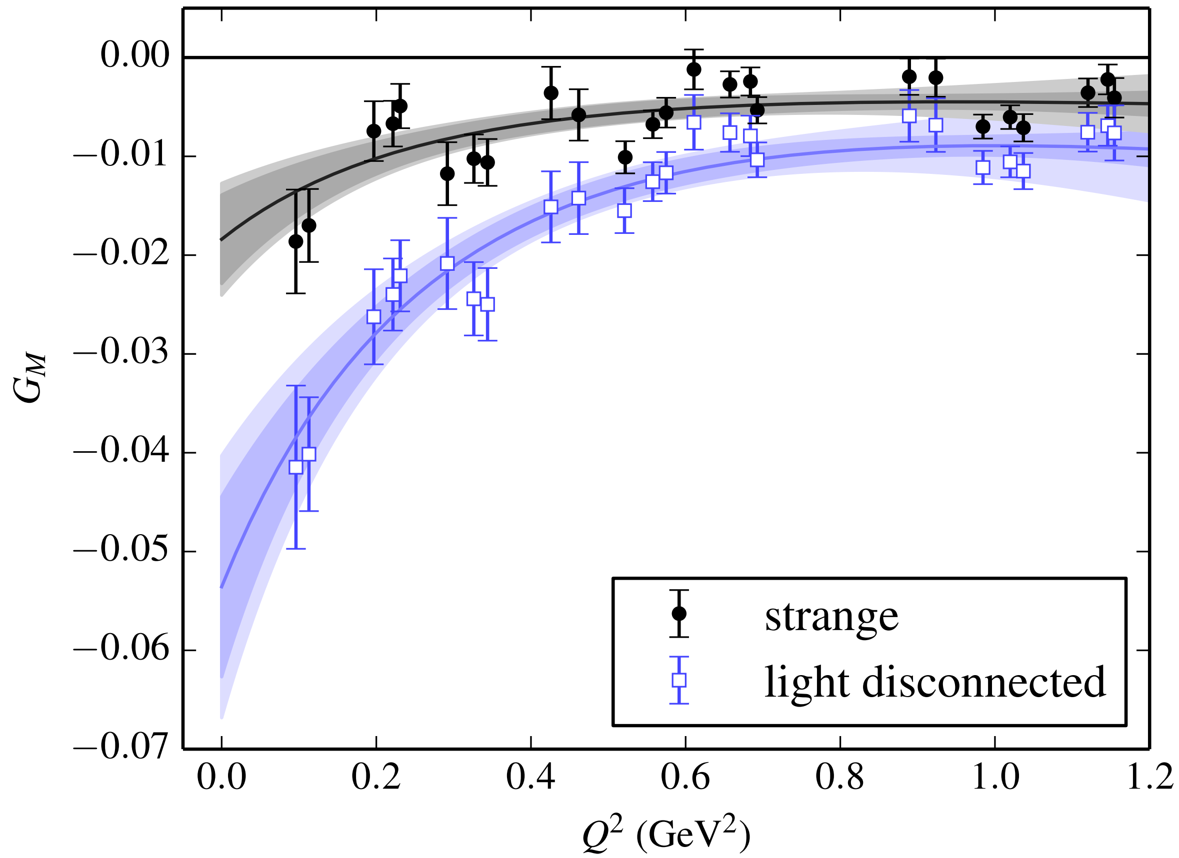


- Further decrease variance by deflating low-lying, long-range modes

[See A.Gambhir's talk "Algorithms for disconnected diagrams", Wed 11:50 - 12:10 in <<Algorithms and Machines>>] :

Disconnected Nucleon FF's for up to ~1 GeV^2





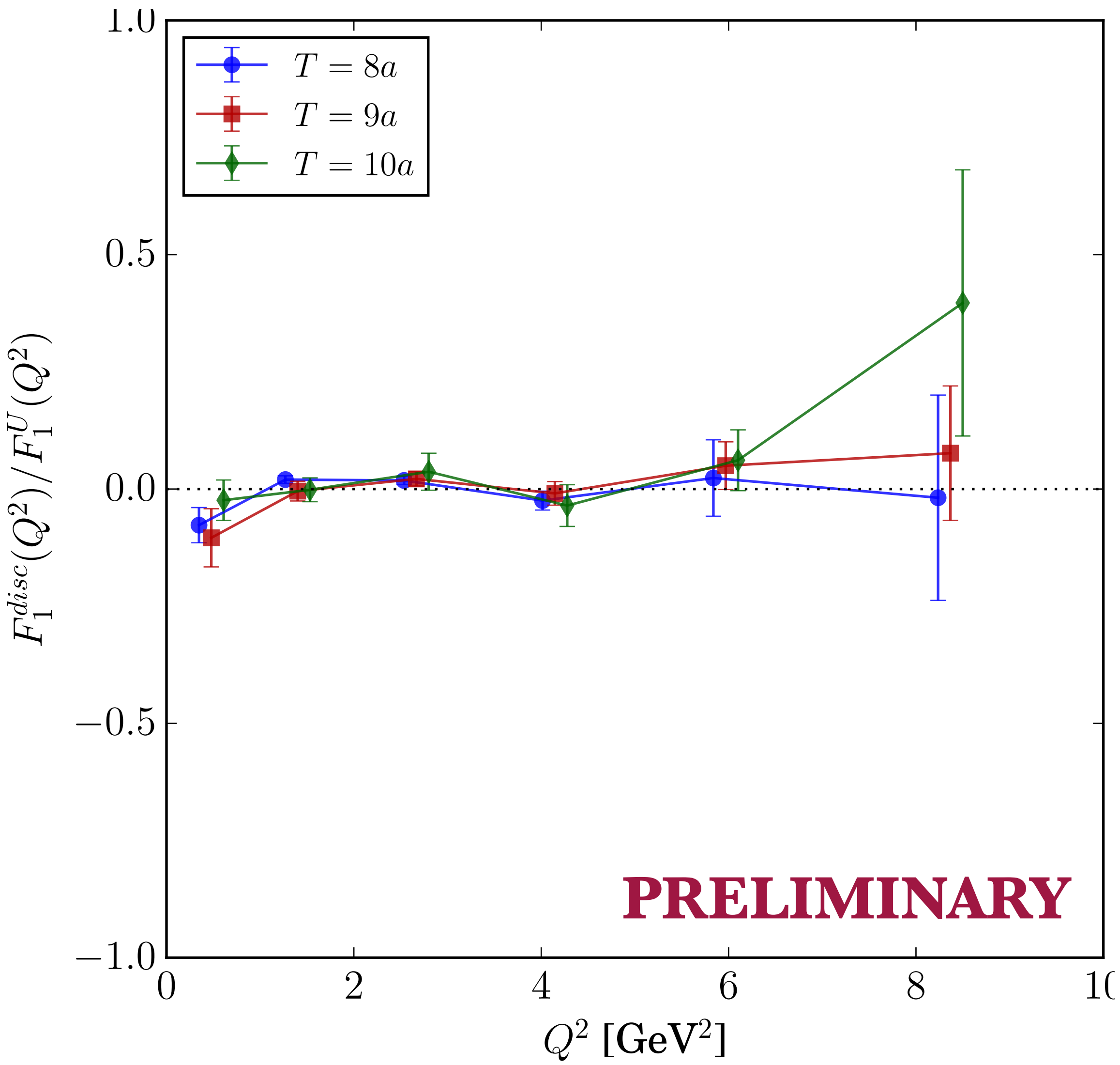
[J. Green, S. Meinel, et al (LHPc); PRD92:031501]

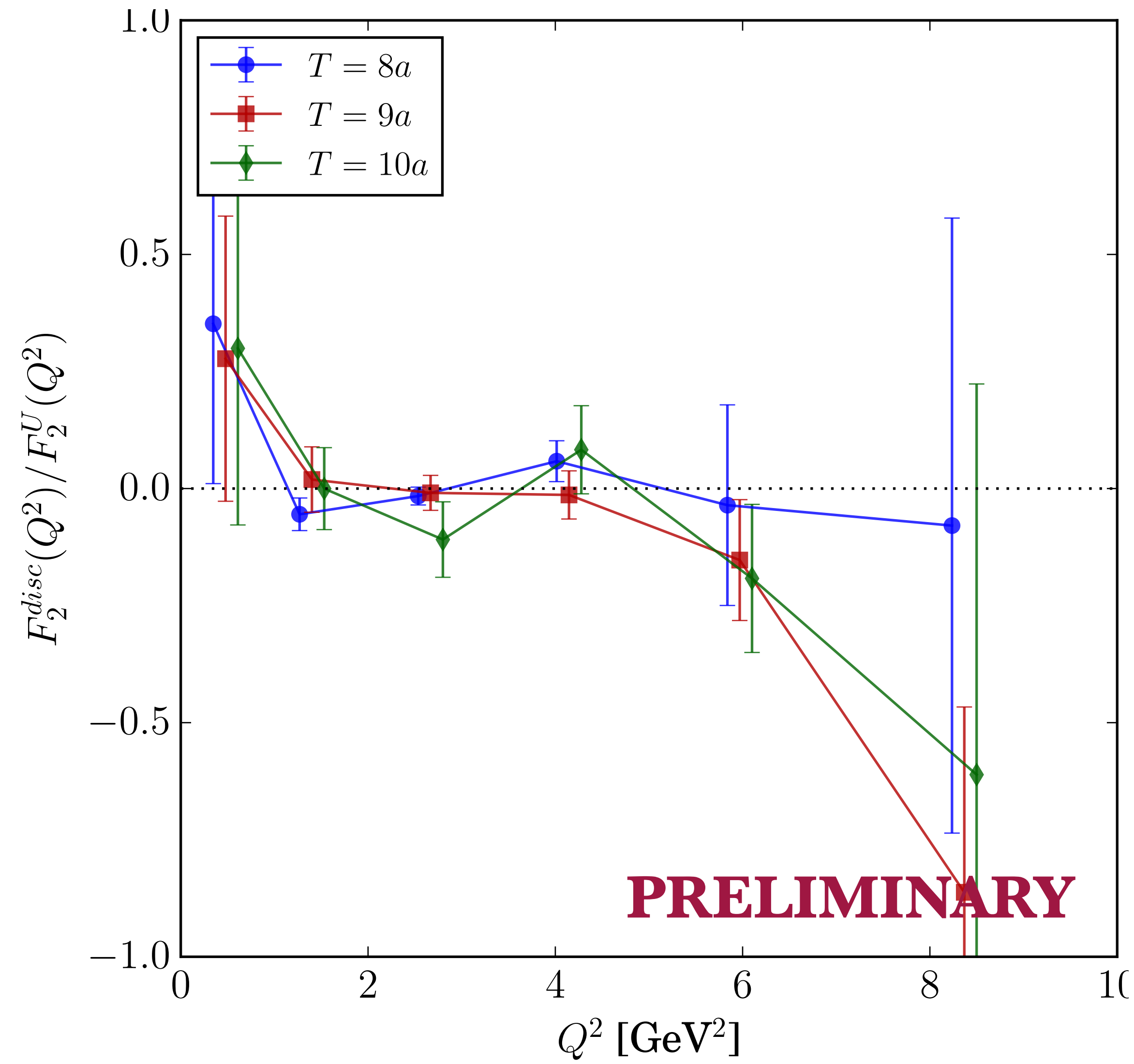
N_f=2+1 dynamical fermions, m_π=319 MeV
(the "coarse" JLab Clover ensemble)

$$|(G_E^{u/d})_{\rm disc}| \lesssim 0.010 \text{ of } |(G_E^{u-d})_{\rm conn}|$$
$$|(G_E^{s})_{\rm disc}| \lesssim 0.005 \text{ of } |(G_E^{u-d})_{\rm conn}|$$

$$|(G_M^{u/d})_{\rm disc}| \lesssim 0.015 \text{ of } |(G_M^{u-d})_{\rm conn}|$$
$$|(G_M^{s})_{\rm disc}| \lesssim 0.005 \text{ of } |(G_M^{u-d})_{\rm conn}|$$

Disconnected Contributions to F_1, F_2





- Ratio of disconnected to connected(U) contributions
- Preliminary analysis (plateau averages), a=0.081 fm ensemble

Summary and Outlook

- First calculation of high-momentum form factors with momentum-boosted smearing
  momentum-boosted smearing is efficient in improving signal for relativistic hadrons on a lattice

- F_2/F_1 scaling agrees qualitatively with perturbative QCD
  agreement is apparently independent of excited states

- The new TMD and PDF programmes on a lattice (Lin, Engelhardt) depend on efficient and reliable evaluation relativistic nucleon matrix elements
  computing form factors is a "benchmark" for studying discretization and excited state effects for relativistic nucleons on a lattice

- More statistics and operator improvement are underway