Landau gauge gluon vertices from Lattice QCD

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July 29, 2016

Outline

1. Introduction and Motivation
2. Results
   - Three gluon vertex
3. Conclusions and outlook

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Green's functions

- Green's functions summarize the dynamics of the theory
  - QCD: information on confinement and chiral symmetry breaking
- *n*-point complete Green's functions

$$G^{(n)}(x_1, \ldots, x_n) = \langle 0| T\left(\phi(x_1) \cdots \phi(x_n)\right) |0\rangle,$$

  - decomposition in terms of one particle irreducible (1PI) functions $\Gamma^{(n)}$
  - access to form factors that define $\Gamma^{(n)}$
- lattice approach allows for first principles determination of the complete Green's functions of QCD

Three gluon vertex

- momentum space three point function
- allow to measure e.g. strong coupling constant
- fundamental role in the structure of Dyson-Schwinger equations
  - DSE studies predict that some form factors associated with the three gluon 1PI change sign in the infrared region — **zero crossing**
  - zero crossing observed in continuum aproaches, SU(2) 3d lattice simulations, and very recently in SU(3) 4d lattice simulations
  - DSE predicts a momentum scale $\sim 130 - 200$ MeV

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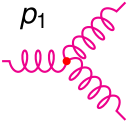
Three point complete Green's function

$$\langle A^{a_1}_{\mu_1}(p_1)\, A^{a_2}_{\mu_2}(p_2)\, A^{a_3}_{\mu_3}(p_3)\rangle =$$
$$V\,\delta(p_1+p_2+p_3)\; G^{a_1 a_2 a_3}_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3)$$



$$G^{a_1 a_2 a_3}_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3) =$$
$$D^{a_1 b_1}_{\mu_1 \nu_1}(p_1)\; D^{a_2 b_2}_{\mu_2 \nu_2}(p_2)\; D^{a_3 b_3}_{\mu_3 \nu_3}(p_3)\,\Gamma^{b_1 b_2 b_3}_{\nu_1 \nu_2 \nu_3}(p_1, p_2, p_3)$$

- color structure:

$$\Gamma^{a_1 a_2 a_3}_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3) = f_{a_1 a_2 a_3}\; \Gamma_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3)$$

Three gluon vertex

- Bose symmetry requires vertex to be symmetric under interchange of any pair (p_i, a_i, μ_i)
  - $\Gamma_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3)$ must be antisymmetric
- $\Gamma_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3)$ in the continuum: requires six Lorentz invariant form factors
  - two associated to the transverse component $\Gamma^{(t)}$
  - remaining associated to the longitudinal $\Gamma^{(l)}$.

J. S. Ball, T.-W. Chiu, Phys. Rev. **D22**, 2550 (1980)

Lattice setup

- Wilson gauge action, $\beta = 6.0$
  - 64^4, 2000 configurations
  - 80^4, 279 configurations
- rotation to the Landau gauge: FFT-SD method
- gluon field

$$a g_0 A_\mu(x + a\hat{e}_\mu) = \frac{U_\mu(x) - U^\dagger(x)}{2 i g_0} - \frac{\mathrm{Tr}\left[U_\mu(x) - U^\dagger(x)\right]}{6 i g_0}$$

in momentum space

$$A_\mu(\hat{p}) = \sum_x e^{-i\hat{p}(x + a\hat{e}_\mu)} A_\mu(x + a\hat{e}_\mu) \quad , \quad \hat{p}_\mu = \frac{2\,\pi\, n_\mu}{a\, L_\mu}$$

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Lattice setup

- tree level improved momentum

$$p_\mu = \frac{2}{a} \sin\left(\frac{a\,\hat{p}_\mu}{2}\right)$$

- accessing the 1PI three gluon vertex from the lattice

$$G_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) = \mathrm{Tr}\;\langle A_{\mu_1}(p_1)\,A_{\mu_2}(p_2)\,A_{\mu_3}(p_3)\rangle =$$
$$= V\delta(p_1+p_2+p_3)\frac{N_c(N_c^2-1)}{4}\;D(p_1^2)\,D(p_2^2)\,D(p_3^2)$$
$$P_{\mu_1\nu_1}(p_1)\,P_{\mu_2\nu_2}(p_2)\,P_{\mu_3\nu_3}(p_3)\;\Gamma_{\nu_1\nu_2\nu_3}(p_1, p_2, p_3)$$

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Case study: one vanishing momentum $p_2 = 0$

- used in the first lattice study

B. Allés et al, Nucl. Phys. B**502**, 325 (1997)

$$G_{\mu_1\mu_2\mu_3}(p,0,-p) = V\frac{N_c(N_c^2-1)}{4}\left[D(p^2)\right]^2 D(0)\frac{\Gamma(p^2)}{3}\; p_{\mu_2}\; T_{\mu_1\mu_3}(p)$$

$$\Gamma(p^2) = 2\left[A(p^2,p^2;0) + p^2\, C(p^2,p^2;0)\right]$$

$$G_{\mu\,\alpha\,\mu}(p,0,-p)\,p_\alpha = V\frac{N_c(N_c^2-1)}{4}\;\left[D(p^2)\right]^2 D(0)\;\;\Gamma(p^2)\;\;p^2$$

A word about statistical errors on $\Gamma(p^2)$

- measurement of $\Gamma(p^2)$ requires to compute the ratio

$$G_{\mu\alpha\mu}(p,0,-p)p_\alpha / \left[D(p^2)\right]^2 \; D(0)$$

- large statistical fluctuations at high momenta:

$$\left[\Delta\Gamma(p^2)\right]^2 = \frac{1}{[D(p^2)]^4}\left\{ \left[\frac{\Delta G_{\mu\alpha\mu}p_\alpha}{D(0)}\right]^2 + \left[2\,\Delta D(p^2)\;\frac{G_{\mu\alpha\mu}p_\alpha}{D(p^2)\,D(0)}\right]^2 + \left[2\,\Delta D(0)\;\frac{G_{\mu\alpha\mu}p_\alpha}{[D(0)]^2}\right]^2 \right\}$$

- for large momenta:
  - $D(p^2) \sim 1/p^2$
  - $\Delta\Gamma(p^2) \sim p^4$

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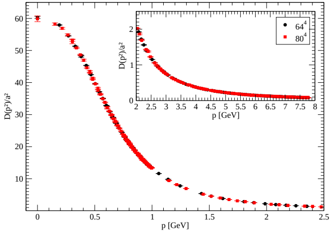
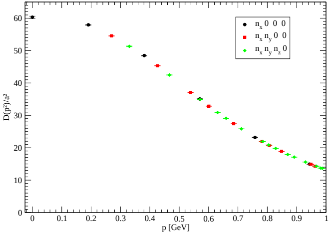
Gluon propagator

Both lattices



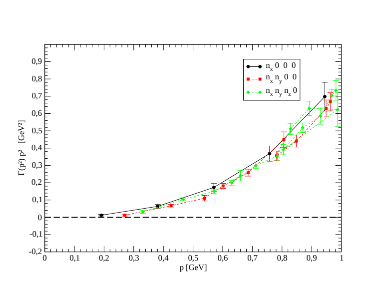
64^4, different momenta



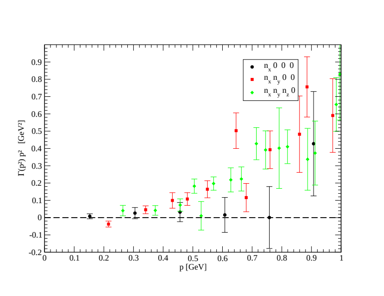
no differences between different types of momenta

$\Gamma(p^2)p^2$

64^4



80^4



- will not consider momenta of type $(n_x\,0\,0\,0)$

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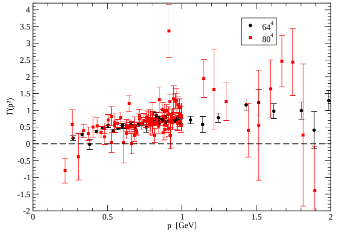
$\Gamma(p^2)$



- $\Gamma(p^2)$ = -0.80(37) at $p = 216$ MeV
- compatible with zero only within to 2.2 σ
- $\Gamma(p = 270 \text{ MeV}) = 0.171(73)$ for 64^4
- $\Gamma(p = 264 \text{ MeV}) = 0.58(43)$ for 80^4
- zero crossing for $p \lesssim 250$ MeV
- compatible with earlier lattice results

$\Gamma(p^2)$ at high momenta

$$\Gamma_{UV}(p^2) = \left[D(p^2)\right]^2 D(0)\,\Gamma(p^2)\,p^2$$

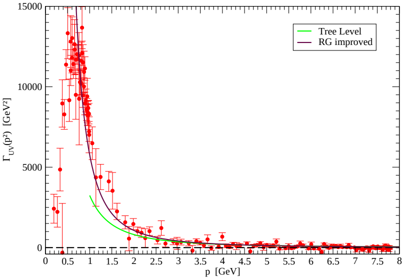
- D and Γ at high momenta:

$$Z\frac{\left[\ln\frac{p^2}{\mu^2}\right]^{\gamma}}{p^2}$$

- gluon ($\gamma_{2g} = -13/22$)
- 3g 1PI ($\gamma_{3g} = 17/44$)

$$\Gamma_{UV}(p^2) = \frac{Z}{p^2}\left[\ln\frac{p^2}{\mu^2}\right]^{\gamma_{3g}-2\gamma}$$

- $\gamma' = \gamma_{3g} + 2\gamma_{2g} = -35/44$
- tree level: $\Gamma_{UV}(p^2) \sim \frac{1}{p^2}$



Conclusions

- Computation of the three gluon complete Green's function on the lattice
  - particular kinematical configuration $p_2 = 0$
  - two different lattice volumes: $(6.5\ \mathrm{fm})^4$ and $(8.2\ \mathrm{fm})^4$ ($a = 0.102$ fm)
- form factor $\Gamma(p^2)$ exhibits zero crossing for $p \sim 250$ MeV

Cucchieri, Maas, Mendes, Phys. Rev. D**77**, 094510 (2008) [SU(2), 3d]

Athenodorou, Binosi, Boucaud, De Soto, Papavassiliou, Rodríguez-Quintero, Zafeiropoulos,

arXiv:1607.01278 [SU(3), 4d]

- for high momenta: lattice data compatible with prediction of renormalisation group improved perturbation theory

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Outlook

- Three gluon vertex
  - explore other momentum configurations
- Four gluon vertex

Computing time provided by the Laboratory for Advanced Computing at the University of Coimbra and by PRACE projects COIMBRALATT (DECI-9) and COIMBRALATT2 (DECI-12). Work of P. J. Silva partially supported by FCT under Contract No. SFRH/BPD/40998/2007.