Landau gauge gluon vertices from Lattice QCD

Anthony Duarte, Orlando Oliveira, Paulo Silva

CFisUC, Department of Physics, University of Coimbra, Portugal

July 29, 2016
Outline

1. Introduction and Motivation

2. Results
   - Three gluon vertex

3. Conclusions and outlook
Green’s functions

- Green’s functions summarize the dynamics of the theory
  - QCD: information on confinement and chiral symmetry breaking

- $n$-point complete Green’s functions

\[ G^{(n)}(x_1, \ldots, x_n) = \langle 0 | T (\phi(x_1) \cdots \phi(x_n)) | 0 \rangle, \]

- decomposition in terms of one particle irreducible (1PI) functions $\Gamma^{(n)}$
- access to form factors that define $\Gamma^{(n)}$

- lattice approach allows for first principles determination of the complete Green’s functions of QCD
Three gluon vertex

- momentum space three point function
- allow to measure e.g. strong coupling constant
- fundamental role in the structure of Dyson-Schwinger equations
  - DSE studies predict that some form factors associated with the three gluon 1PI change sign in the infrared region — zero crossing
  - zero crossing observed in continuum approaches, SU(2) 3d lattice simulations, and very recently in SU(3) 4d lattice simulations
  - DSE predicts a momentum scale $\sim 130 - 200$ MeV
Three point complete Green’s function

\[ \langle A_{\mu_1}^{a_1}(p_1) A_{\mu_2}^{a_2}(p_2) A_{\mu_3}^{a_3}(p_3) \rangle = V \delta(p_1 + p_2 + p_3) G_{\mu_1 \mu_2 \mu_3}^{a_1 a_2 a_3}(p_1, p_2, p_3) \]

\[ G_{\mu_1 \mu_2 \mu_3}^{a_1 a_2 a_3}(p_1, p_2, p_3) = D_{\mu_1 \nu_1}^{a_1 b_1}(p_1) D_{\mu_2 \nu_2}^{a_2 b_2}(p_2) D_{\mu_3 \nu_3}^{a_3 b_3}(p_3) \Gamma_{\nu_1 \nu_2 \nu_3}^{b_1 b_2 b_3}(p_1, p_2, p_3) \]

• color structure:

\[ \Gamma_{\mu_1 \mu_2 \mu_3}^{a_1 a_2 a_3}(p_1, p_2, p_3) = f_{a_1 a_2 a_3} \Gamma_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3) \]
Three gluon vertex

Bose symmetry requires vertex to be symmetric under interchange of any pair \((p_i, a_i, \mu_i)\)

\[ \Gamma_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) \] must be antisymmetric

\[ \Gamma_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) \] in the continuum: requires six Lorentz invariant form factors

- two associated to the transverse component \(\Gamma^{(t)}\)
- remaining associated to the longitudinal \(\Gamma^{(l)}\).

**Lattice setup**

- Wilson gauge action, $\beta = 6.0$
  - $64^4$, 2000 configurations
  - $80^4$, 279 configurations
- Rotation to the Landau gauge: FFT-SD method
- Gluon field

\[ a g_0 A_\mu(x + a \hat{e}_\mu) = \frac{U_\mu(x) - U_\mu^\dagger(x)}{2i g_0} - \frac{\text{Tr} \left[ U_\mu(x) - U_\mu^\dagger(x) \right]}{6i g_0} \]

In momentum space

\[ A_\mu(\hat{p}) = \sum_x e^{-i \hat{p}(x + a \hat{e}_\mu)} A_\mu(x + a \hat{e}_\mu), \quad \hat{p}_\mu = \frac{2 \pi n_\mu}{a L_\mu} \]
Lattice setup

- tree level improved momentum

\[ p_\mu = \frac{2}{a} \sin \left( \frac{a \hat{p}_\mu}{2} \right) \]

- accessing the 1PI three gluon vertex from the lattice

\[
G_{\mu_1 \mu_2 \mu_3} (p_1, p_2, p_3) = \text{Tr} \left( A_{\mu_1} (p_1) A_{\mu_2} (p_2) A_{\mu_3} (p_3) \right) = \\
= V \delta(p_1 + p_2 + p_3) \frac{N_c (N_c^2 - 1)}{4} D(p_1^2) D(p_2^2) D(p_3^2) \\
P_{\mu_1 \nu_1} (p_1) P_{\mu_2 \nu_2} (p_2) P_{\mu_3 \nu_3} (p_3) \Gamma_{\nu_1 \nu_2 \nu_3} (p_1, p_2, p_3)
\]
Case study: one vanishing momentum $p_2 = 0$

- used in the first lattice study

\begin{equation*}
G_{\mu_1 \mu_2 \mu_3}(p, 0, -p) = V \frac{N_c(N_c^2 - 1)}{4} \left[ D(p^2) \right]^2 D(0) \frac{\Gamma(p^2)}{3} p_{\mu_2} T_{\mu_1 \mu_3}(p)
\end{equation*}

\begin{equation*}
\Gamma(p^2) = 2 \left[ A(p^2, p^2; 0) + p^2 C(p^2, p^2; 0) \right]
\end{equation*}

\begin{equation*}
G_{\mu \alpha \mu}(p, 0, -p) p_{\alpha} = V \frac{N_c(N_c^2 - 1)}{4} \left[ D(p^2) \right]^2 D(0) \Gamma(p^2) p^2
\end{equation*}


Lattice 2016
A word about statistical errors on $\Gamma(p^2)$

- measurement of $\Gamma(p^2)$ requires to compute the ratio
  
  $$G_{\mu\alpha\mu}(p, 0, -p)p_\alpha / [D(p^2)]^2 \frac{D(0)}{D^2(p^2)}$$

- large statistical fluctuations at high momenta:

  $$\left[ \Delta \Gamma(p^2) \right]^2 = \frac{1}{[D(p^2)]^4} \left\{ \left[ \frac{\Delta G_{\mu\alpha\mu} p_\alpha}{D(0)} \right]^2 + 2 \Delta D(p^2) \frac{G_{\mu\alpha\mu} p_\alpha}{D(p^2) D(0)} \right\}^2$$

  + $$\left[ 2 \Delta D(0) \frac{G_{\mu\alpha\mu} p_\alpha}{[D(0)]^2} \right]^2$$

  for large momenta:

  $$D(p^2) \sim 1/p^2$$

  $$\Delta \Gamma(p^2) \sim p^4$$
Outline

1. Introduction and Motivation
2. Results
   - Three gluon vertex
3. Conclusions and outlook
Gluon propagator

Both lattices

64^4, different momenta

no differences between different types of momenta
Three gluon vertex

\[ \Gamma(p^2)p^2 \]

will not consider momenta of type \((n_x 0 0 0)\)
Three gluon vertex

\[ \Gamma(p^2) \]

- \( \Gamma(p^2) = -0.80(37) \) at \( p = 216 \text{ MeV} \)
- compatible with zero only within to 2.2 \( \sigma \)
- \( \Gamma(p = 270 \text{ MeV}) = 0.171(73) \) for 64\(^4\)
- \( \Gamma(p = 264 \text{ MeV}) = 0.58(43) \) for 80\(^4\)
- zero crossing for \( p \lesssim 250 \text{ MeV} \)
- compatible with earlier lattice results
Introduction and Motivation

Results

Conclusions and outlook

Three gluon vertex

**Γ(p^2) at high momenta**

\[ \Gamma_{UV}(p^2) = [D(p^2)]^2 D(0) \Gamma(p^2) p^2 \]

- D and Γ at high momenta:

\[ \frac{Z \left[ \ln \left( \frac{p^2}{\mu^2} \right) \right]^{\gamma}}{p^2} \]

- gluon (\(\gamma_{2g} = -13/22\))
- 3g 1PI (\(\gamma_{3g} = 17/44\))

\[ \Gamma_{UV}(p^2) = \frac{Z}{p^2} \left[ \ln \left( \frac{p^2}{\mu^2} \right) \right]^{\gamma_{3g} - 2\gamma} \]

- \(\gamma' = \gamma_{3g} + 2\gamma_{2g} = -35/44\)
- tree level: \(\Gamma_{UV}(p^2) \sim \frac{1}{p^2}\)
Conclusions

- Computation of the three gluon complete Green’s function on the lattice
  - particular kinematical configuration $p_2 = 0$
  - two different lattice volumes: $(6.5 \text{ fm})^4$ and $(8.2 \text{ fm})^4$ ($a = 0.102 \text{ fm}$)
- Form factor $\Gamma(p^2)$ exhibits zero crossing for $p \sim 250 \text{ MeV}$

  Cucchieri, Maas, Mendes, Phys. Rev. D77, 094510 (2008) [SU(2), 3d]
  Athenodorou, Binosi, Boucaud, De Soto, Papavassiliou, Rodríguez-Quintero, Zafeiropoulos,
  arXiv:1607.01278 [SU(3), 4d]

- For high momenta: lattice data compatible with prediction of renormalisation group improved perturbation theory
Outlook

- Three gluon vertex
  - explore other momentum configurations
- Four gluon vertex

Computing time provided by the Laboratory for Advanced Computing at the University of Coimbra and by PRACE projects COIMBRALATT (DECI-9) and COIMBRALATT2 (DECI-12). Work of P. J. Silva partially supported by FCT under Contract No. SFRH/BPD/40998/2007.