# Further Study of BRST-Symmetry Breaking on the Lattice

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## 1. BRST-Symmetry Breaking

The so-called **minimal Landau gauge** in Yang-Mills (YM) theories [1] is obtained by restricting the functional integral to the **first Gribov region** $\Omega$, given by the set of transverse gauge configurations for which the **Faddeev-Popov (FP) matrix** $\mathcal{M}$ is non-negative. On the lattice, this gauge fixing is implemented by a minimization procedure, without the need to consider the addition of a **non-local horizon-function** term $\gamma^4 S_h$ to the (Landau-gauge) action, as done in the **Gribov-Zwanziger (GZ)** approach in the continuum [2]. In the **GZ** approach, the resulting (nonlocal) action may be localized by introducing the **auxiliary fields** $\phi_\mu^{ab}(x)$ and $\omega_\nu^{cd}(y)$, yielding $S_{GZ} = S_{YM} + S_{gf} + S_{aux} + S_\gamma$. Here, $S_{YM}$ is the usual four-dimensional **YM action**, $S_{gf}$ is the covariant-gauge-fixing term,

$$S_{aux} = \int d^4x \left[ \overline{\phi}_\mu^{ac} \partial_\nu \left( D_\nu^{ab} \phi_\mu^{bc} \right) - \overline{\omega}_\mu^{ac} \partial_\nu \left( D_\nu^{ab} \omega_\mu^{bc} \right) - g_0 \left( \partial_\nu \overline{\omega}_\mu^{ac} \right) f^{abd} D_\nu^{be} \eta^e \phi_\mu^{dc} \right] ,$$

which is necessary to localize the **horizon function**, and

$$S_\gamma = \int d^4x \left[ \gamma^2 D_\mu^{ab} \left( \phi_\mu^{ab} + \overline{\phi}_\mu^{ab} \right) - 4 \left( N_c^2 - 1 \right) \gamma^4 \right] ,$$

which allows one to fix the $\gamma$ parameter through the so-called **horizon condition**. Also, one can define for these fields a nilpotent **BRST transformation** $s$ [3], which is a simple extension of the usual **perturbative BRST transformation** that leaves $S_{YM} + S_{gf}$ invariant. However, in the **GZ** case, this local BRST symmetry is **broken** by terms proportional to a power of the **Gribov parameter** $\gamma$. Since a nonzero value of $\gamma$ is related to the restriction of the functional integration to $\Omega$, it is somewhat natural to expect a **breaking of the perturbative BRST symmetry**, as a direct consequence of the **nonperturbative gauge-fixing**. The above interpretation is supported by the introduction [4] of a nilpotent **nonperturbative BRST transformation** $s_\gamma$, which leaves the local **GZ** action invariant. The new symmetry is a simple modification of the usual BRST transformation $s$, by adding (for some of the fields) a **nonlocal term** proportional to a power of the Gribov parameter $\gamma$.

## 2. The Bose-Ghost Propagator

The Gribov parameter $\gamma$ is **not** introduced explicitly on the **lattice**, since in this case the restriction of gauge-configuration space to the region $\Omega$ is achieved by numerical minimization. Nevertheless, the **breaking** of the **perturbative BRST symmetry** induced by the **GZ** action may be investigated by the lattice computation of suitable observables, such as the so-called **Bose-ghost propagator**

$$Q_{\mu\nu}^{abcd}(x,y) = \langle s(\phi_\mu^{ab}(x) \overline{\omega}_\nu^{cd}(y)) \rangle = \langle \omega_\mu^{ab}(x) \overline{\omega}_\nu^{cd}(y) + \phi_\mu^{ab}(x) \overline{\phi}_\nu^{cd}(y) \rangle .$$

Since this quantity is **BRST-exact**, with respect to the usual perturbative BRST transformation $s$, it should be **zero** for a BRST-invariant theory but it does not necessarily vanish if the **BRST symmetry** $s$ is broken. On the lattice, however, one does not have direct access to the auxiliary fields $(\overline{\phi}_\mu^{ac}, \phi_\mu^{ac})$ and $(\overline{\omega}_\mu^{ac}, \omega_\mu^{ac})$. Nevertheless, since these fields enter the continuum action at most quadratically, we can **integrate them out exactly**, obtaining for the Bose-ghost propagator an expression that is suitable for **lattice simulations**. This yields

$$Q_{\mu\nu}^{abcd}(x-y) = \gamma^4 \left\langle R_\mu^{ab}(x) R_\nu^{cd}(y) \right\rangle , \tag{1}$$

where

$$R_\mu^{ac}(x) = \int d^4z \, (\mathcal{M}^{-1})^{ae}(x,z) \, B_\mu^{ec}(z)$$

and $B_\mu^{ec}(z)$ is given by the **covariant derivative** $D_\mu^{ec}(z)$. One can also note that, at the classical level, the **total derivatives** $\partial_\mu(\phi_\mu^{aa} + \overline{\phi}_\mu^{aa})$ in the action $S_\gamma$ can be neglected [3, 5]. In this case the expression for $B_\mu^{ec}(z)$ simplifies to

$$B_\mu^{ec}(z) = g_0 \, f^{ebc} A_\mu^b(z) , \tag{2}$$

as in Ref. [5]. Let us stress that, in both cases, the expression for $Q_{\mu\nu}^{abcd}(x-y)$ in Eq. (1) depends only on the **gauge field** $A_\mu^b(z)$ and can be evaluated on the lattice. In fact, all auxiliary fields have been integrated out.

## 3. Numerical Simulations

The first **numerical evaluation** of the **Bose-ghost propagator** in **minimal Landau gauge** was presented —for the $SU(2)$ case in four space-time dimensions— in Ref. [6]. In particular, we evaluated the scalar function $Q(k^2)$ defined [for the $SU(N_c)$ **gauge group**] through the relation

$$Q^{ac}(k) \equiv Q_{\mu\mu}^{abcb}(k) \equiv \delta^{ac} N_c P_{\mu\mu}(k) \, Q(k^2) ,$$

where $P_{\mu\nu}(k)$ is the usual **transverse projector**. This calculation has been extended in Ref. [7], where we have investigated the approach to the **infinite-volume** and **continuum limits** by considering **four** different values of the **lattice coupling** $\beta$ and different physical volumes, ranging from about $(3.366\,fm)^4$ to $(13.462\,fm)^4$. We find **no significant finite-volume effects** in the data. As for **discretization effects**, we observe **small such effects** for the **coarser lattices**, especially in the **IR** region. We also tested three different discretizations for the **sources** $B_\mu^{bc}(x)$, used in the inversion of the **FP** matrix $\mathcal{M}$, and find that the data are fairly independent of the chosen **lattice discretization** of these sources. Our results concerning the **BRST symmetry-breaking** and the form of the Bose-ghost propagator are similar to the previous analysis [6], i.e. we find a $1/p^6$ behavior at **large momenta** and a a **double-pole singularity** at **small momenta**, in agreement with the **one-loop analysis** carried out in Ref. [8].

## References

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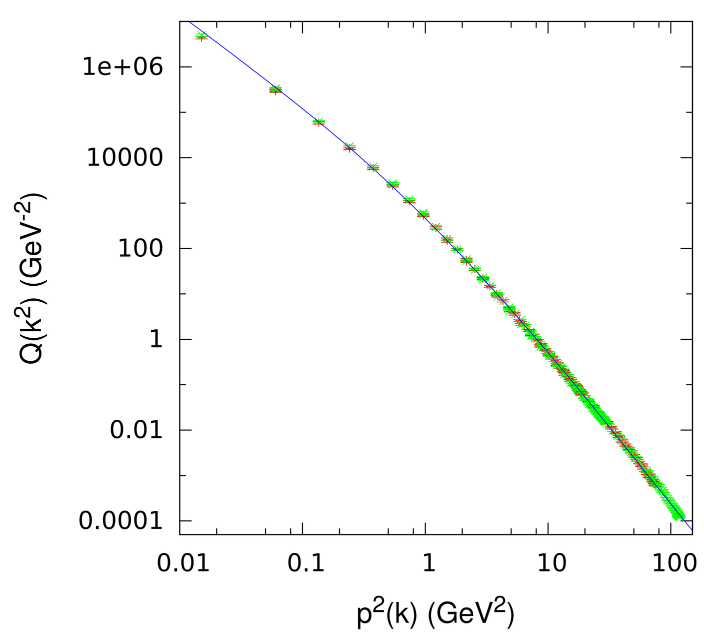
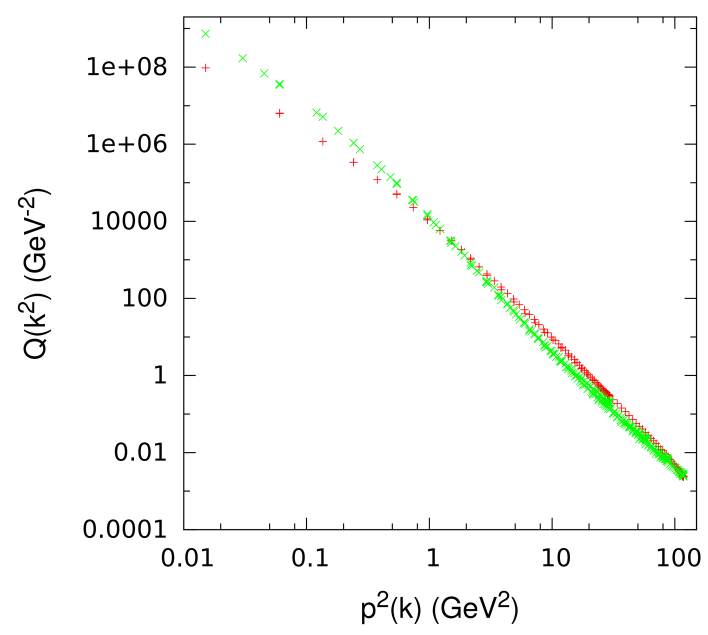
## 4. Results




In the **left plot** we show the **Bose-ghost propagator** $Q(k^2)$ as a function of the **(improved) lattice momentum** squared $p^2(k)$. We plot data for $\beta_2 \approx 2.44$, $V = 96^4$ ($\times$) and $\beta_3 \approx 2.51$, $V = 120^4$ ($*$), after applying a matching procedure [9] to the former set of data. We also plot, for $V = 120^4$, a fit using the **fitting function**

$$f(p^2) = \frac{c}{p^4} \frac{p^2 + s}{p^4 + u^2 p^2 + t^2} ,$$

which can be related [see Eq. (3) below] to a **massive gluon propagator** $D(p^2)$ in combination with an **IR-free FP ghost propagator** $G(p^2) \sim 1/p^2$.

In the **right plot** we show the **Bose-ghost propagator** $Q(k^2)$ ($+$) and the product $g_0^2 G^2(p^2) D(p^2)$ ($\times$) as a function of the **(improved) lattice momentum** squared $p^2(k)$ for the lattice volume $V = 120^4$ at $\beta_3 \approx 2.51$. The result

$$Q(p^2) \sim g_0^2 G^2(p^2) D(p^2) , \tag{3}$$

was obtained in Ref. [5] using a **cluster decomposition**. The data of the **Bose-ghost propagator** have been **rescaled** in order to agree with the data of the product $g_0^2 G^2(p^2) D(p^2)$ at the largest momentum.

For both plots we use the **sources** defined in Eq. (2). Also note the logarithmic scale on both axes.