Non-Perturbative Renormalization of Nucleon Charges with Automated Perturbative Subtraction

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Introduction

- The axial, scalar and tensor charges of the nucleon encode important information about nucleon structure
- Scalar and tensor charges hard to probe experimentally, good theoretical predictions needed
- Charges need to be renormalized
- Perturbative renormalization may not be sufficient: non-perturbative renormalization required, but hard
- Here: demonstrate usefulness of automated lattice perturbation theory in subtracting lattice artifacts

Theoretical setup

- Use RI'-MOM scheme [Martinelli et al., 1994] in Landau gauge:

$$\mathrm{tr}_{CD}\left[S_R^{-1}(p)S_{\mathrm{free}}(p)\right]\Big|_{p^2=\mu^2} = 12,$$
$$\mathrm{tr}_{CD}\left[\langle p|O_R|p\rangle\langle p|O_0|p\rangle_{\mathrm{free}}^{-1}\right]\Big|_{p^2=\mu^2} = 12.$$

- Multiplicative renormalization:

$$S_R(p) = Z_q S_0(p),$$
$$O_R = Z_O O_0.$$

- Renormalization factors given by

$$Z_q = \frac{1}{12}\,\mathrm{tr}_{CD}\left[S_0^{-1}(p)S_{\mathrm{free}}(p)\right]\Bigg|_{p^2=\mu^2}$$
$$Z_O = \frac{12 Z_q}{\mathrm{tr}_{CD}\left[\Lambda_O(p)\Lambda_O^{\mathrm{free}}(p)^{-1}\right]\Big|_{p^2=\mu^2}}$$

where

$$\Lambda_O(p) = S_0^{-1}(p)G_O(p)S_0^{-1}(p)$$

Calculational setup

- Fix to Landau gauge by minimizing

$$W(U) = \sum_x \sum_\mu \mathrm{tr}\left[U_\mu^\dagger(x) + U_\mu(x)\right]$$

- Use momentum sources [Göckeler et al., 1998] to compute $S(y|p) = D_{yx}^{-1}\mathrm{e}^{ip\cdot x}$, whence for $O(x) = \overline{u}(x)\Gamma_O d(x)$

$$S(p) = \left\langle \frac{1}{V}\sum_x \mathrm{e}^{-ip\cdot x} S(x|p) \right\rangle$$

$$G_O(p) = \left\langle \frac{1}{V}\sum_x \gamma_5 S(x|p)^\dagger \gamma_5 \Gamma_O S(x|p) \right\rangle$$

- Use diagonal momenta $p = (\mu, \mu, \mu, \mu)$ to reduce O(4) violations with twisted boundary conditions $\psi(x + L_\nu e_\nu) = \mathrm{e}^{i\theta_\nu}\psi(x)$ allowing access to intermediate momenta
- Reduce O(4) violations by averaging over H(4) irreps [Göckeler et al., 2010]

$$\mathrm{tr}_{CD}\left[\Lambda_O(p)\Lambda_O^{free}(p)\right] \mapsto \frac{1}{K}\sum_{l=1}^{K} \mathrm{tr}_{CD}\left[\Lambda_O^l(p)\Lambda_O^{l,\mathrm{free}}(p)\right]$$

Measurements

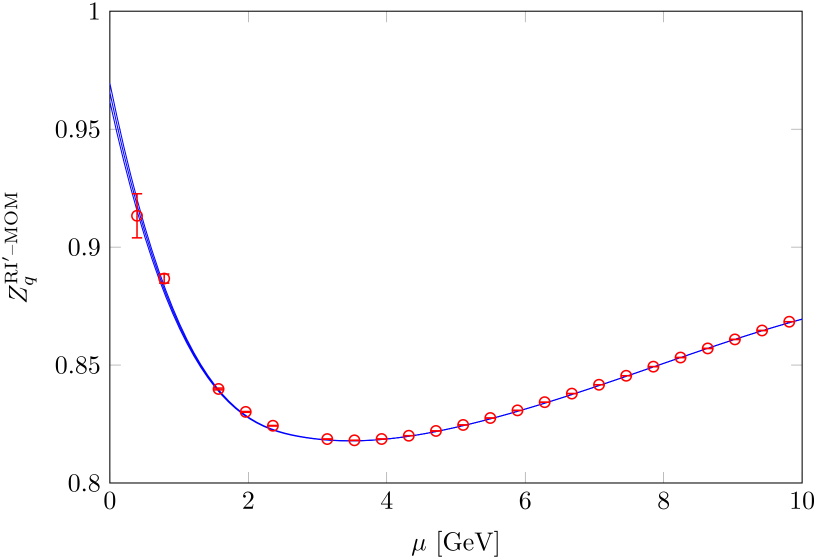
Name	β	a [fm]	Volume	m_π [MeV]
A3	5.2	0.0755	64×32^3	473
A4				364
A5				316
B6			96×48^3	268
E5	5.3	0.0658	64×32^3	457
F6			96×48^3	324
F7				277
G8			128×64^3	193
N5	5.5	0.0486	96×48^3	429
N6				331
O7			128×64^3	261

Use 20 configurations on each ensemble.

Chiral extrapolation

- Interpolate in μ on each ensemble, using cubic splines, to get $Z(\beta, m_\pi; \mu)$ for arbitrary μ

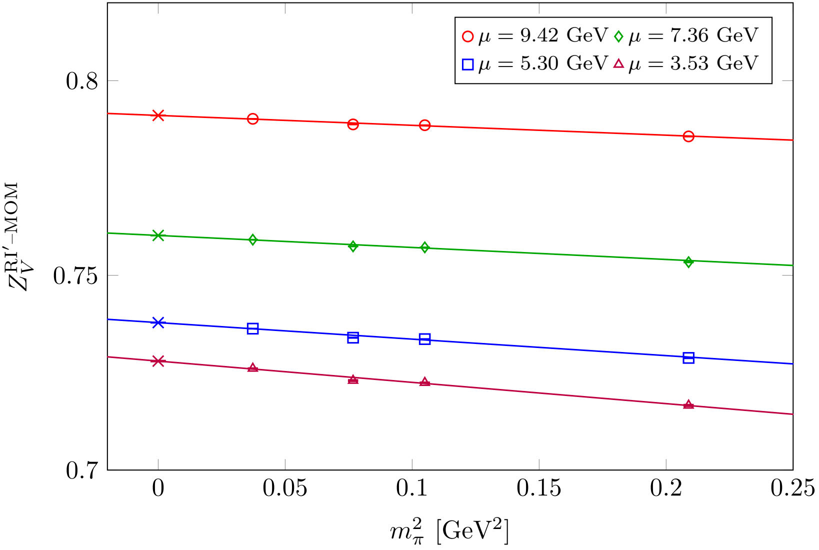
Chiral extrapolation



Chiral extrapolation

- Interpolate in μ on each ensemble, using cubic splines, to get $Z(\beta, m_\pi; \mu)$ for arbitrary μ
- Chirally extrapolate for fixed values of μ at each value of β, using a linear $(am_\pi)^2$ dependence, to get $Z(\beta, 0; \mu)$

Chiral extrapolation



RGI conversion

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- Convert to $\overline{\mathrm{MS}}$ using 3-loop continuum perturbation theory [Gracey 2003]

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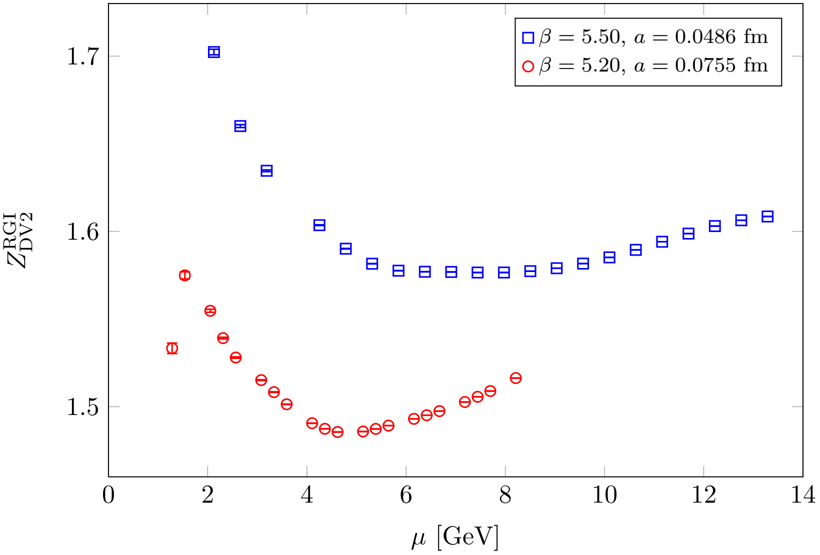
- Convert to RGI using 3-loop $\overline{\mathrm{MS}}$ β- and γ-functions [Vermaseren, Larin, Ritbergen, 1997]

$$Z^{\mathrm{RGI}}(a) = \Delta Z^{\overline{\mathrm{MS}}}(\mu) Z^{\overline{\mathrm{MS}}}_{\mathrm{RI'-MOM}}(\mu) Z^{\mathrm{RI'-MOM}}(a, \mu)$$

RGI conversion



Perturbative subtraction

- Lattice artifacts can be sizeable at large μ
- Subtract lattice artifacts at $\mathrm{O}(g^2)$ using lattice perturbation theory for

$$\begin{aligned} Z^{\mathrm{RI'-MOM}}(\mu, a) &= 1 + g^2 F(\mu, a) \\ &= 1 + g^2 \left[\gamma_0 \log(\mu a) + C + \mathrm{O}(\mu^2 a^2)\right] \end{aligned}$$

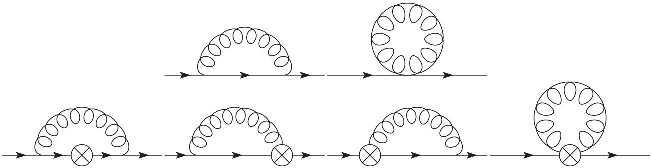
- Lattice artifacts given by

$$D(\mu, a) = g^2 \left[F(\mu, a) - (\gamma_0 \log(\mu a) + C)\right]$$

where γ_0 known analytically, and C known analytically (or fittable from $F(\mu, a) - \gamma_0 \log(\mu a)$ as $a \to 0$)

Perturbative subtraction

- Using the HiPPy/HPsrc packages [Hart, Horgan, GvH, 2009] separate the Feynman rules and Feynman diagrams
- Code diagrams once in an operator- and action-independent fashion to calculate for in principle arbitrary operators
- Can also switch gauge (Wilson, Symanzik) and fermion (clover, smeared) actions easily



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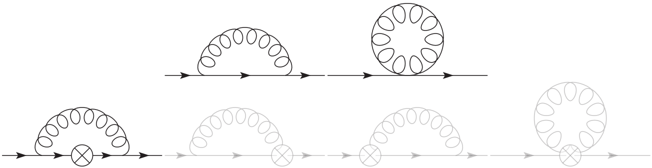
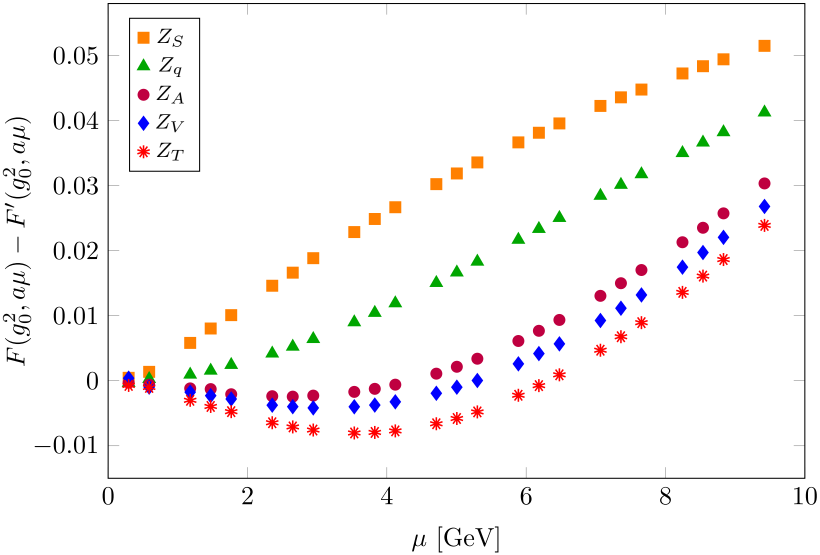
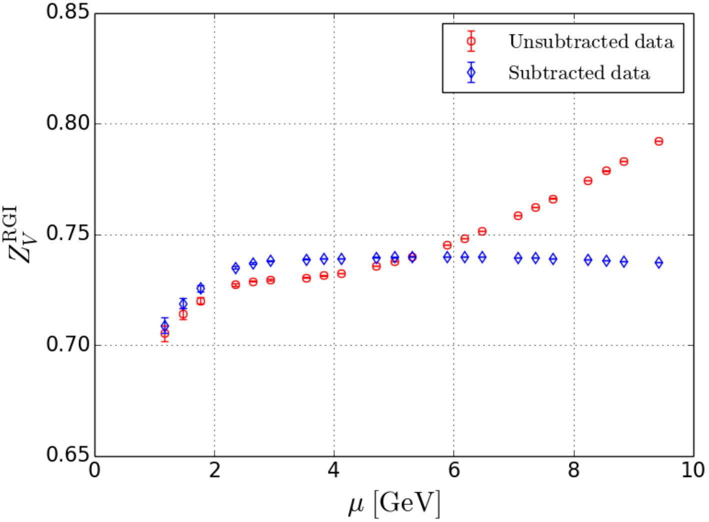


Results



Z_S
Z_q
Z_A
Z_V
Z_T
0.05
0.04
0.03
0.02
0.01
0
−0.01
$F(g_0^2, a\mu) - F'(g_0^2, a\mu)$
0
2
4
6
8
10
μ [GeV]

Results



0.85
0.80
0.75
0.70
0.65
0
2
4
6
8
10
Z_V^{RGI}
μ [GeV]
Unsubtracted data
Subtracted data

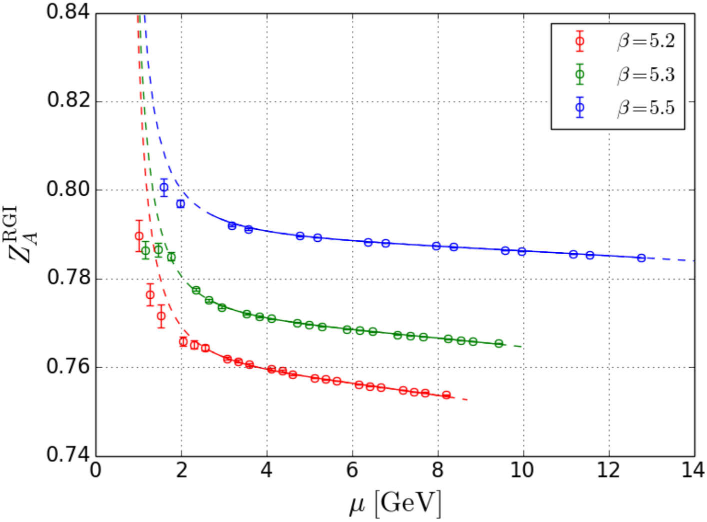
Final fits

- Account for matching to three-loop order, and for remaining lattice artifacts by fitting

$$Z^{\mathrm{RGI,sub}}(a,\mu) = Z^{\mathrm{RGI}}(\beta)\left\{1 + d_1\left[g^{\overline{\mathrm{MS}}}(\mu)\right]^8\right\} + d_2(\beta)(a\mu)^2\Delta Z^{\overline{\mathrm{MS}}}(\mu) Z^{\overline{\mathrm{MS}}}_{\mathrm{RI'-MOM}}$$

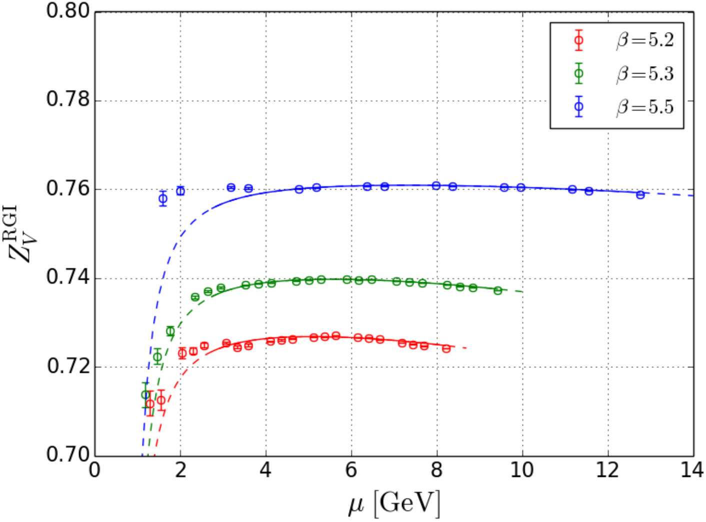
- d_1 is independent of β $\rightsquigarrow$ perform a combined fit accross all lattice spacings
- Fit window should fulfil $\Lambda^{\overline{\mathrm{MS}}} \ll \mu \ll a^{-1}$
- Take $\mu_{\mathrm{min}} = 3$ GeV, $a\mu_{\mathrm{max}} = 2.75$ relying on perturbative subtraction of leading artifacts
- Study systematic errors of final result by varying
    - chiral extrapolation (quadratic, including $\mathrm{e}^{-m_\pi L}$ term),
    - value of $a\Lambda^{\overline{\mathrm{MS}}}$,
    - fitting window for final fit.

Results



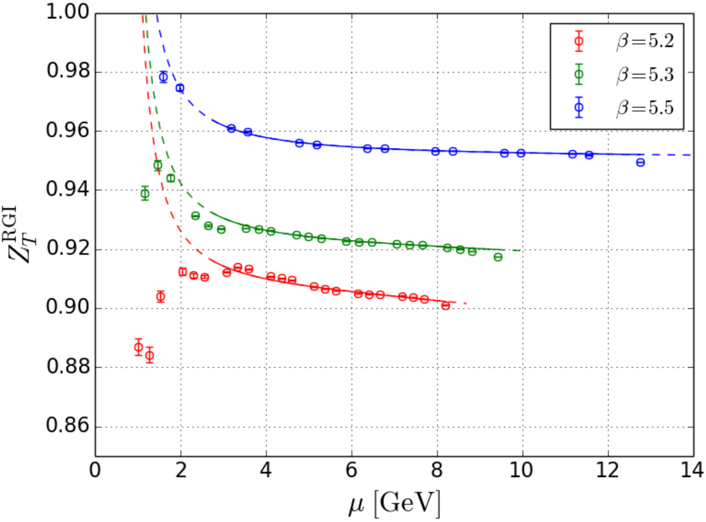
0.84
0.82
0.80
0.78
0.76
0.74
0
2
4
6
8
10
12
14
Z_A^{RGI}
μ [GeV]
β=5.2
β=5.3
β=5.5

Results



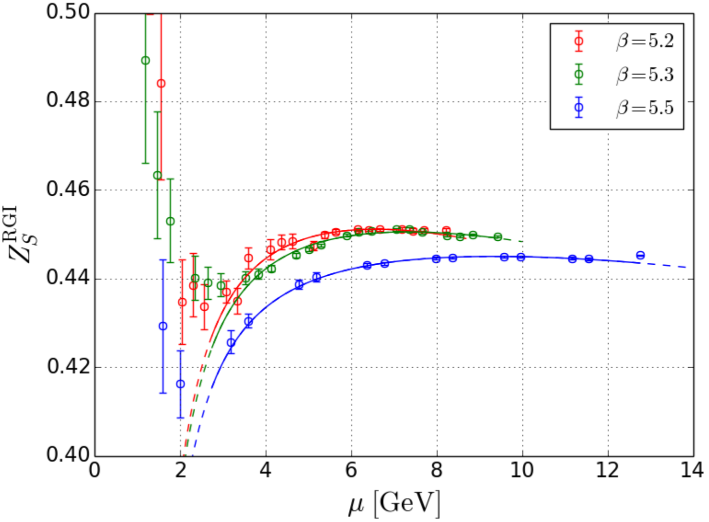
$\beta=5.2$
$\beta=5.3$
$\beta=5.5$
0.80
0.78
0.76
0.74
0.72
0.70
Z_V^{RGI}
0
2
4
6
8
10
12
14
μ [GeV]

Results



1.00
0.98
0.96
0.94
0.92
0.90
0.88
0.86
Z_T^{RGI}
0
2
4
6
8
10
12
14
μ [GeV]
$\beta = 5.2$
$\beta = 5.3$
$\beta = 5.5$

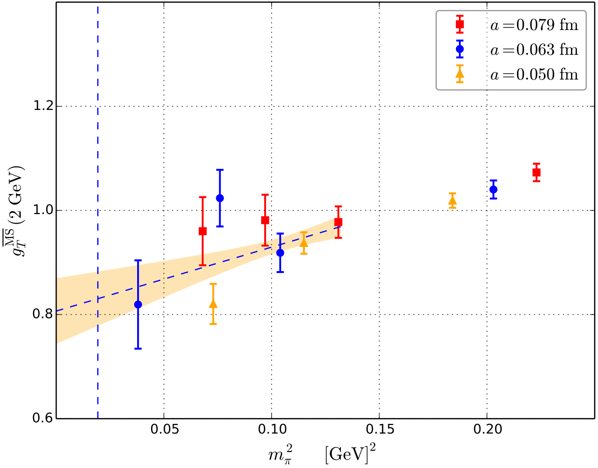
Results



0.50
0.48
0.46
0.44
0.42
0.40
Z_S^{RGI}
0
2
4
6
8
10
12
14
μ [GeV]
β=5.2
β=5.3
β=5.5

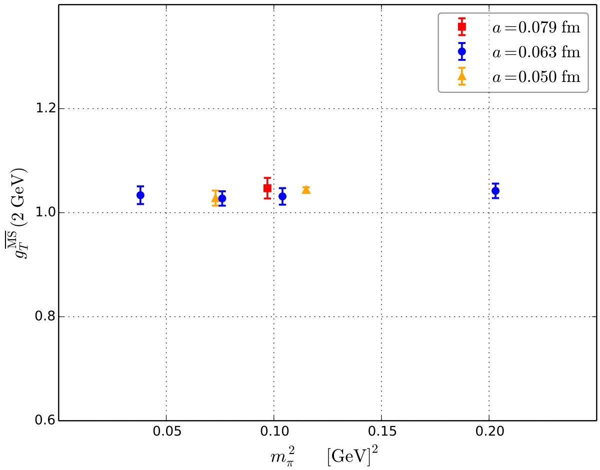
Nucleon tensor charge

PRELIMINARY



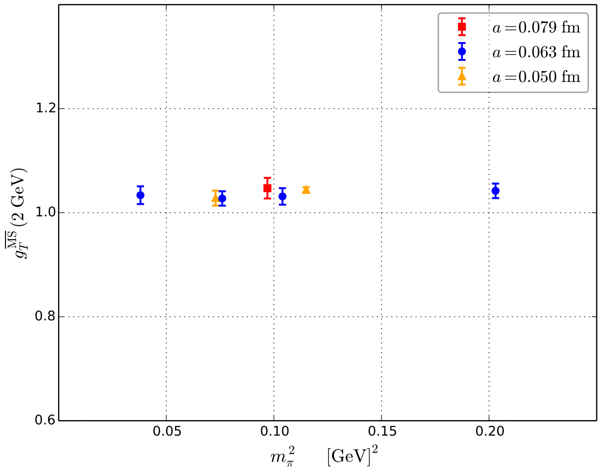
Nucleon tensor charge with AMA

arXiv:1605.00564



Nucleon tensor charge

Excited states? Statistics?



Summary

- Implemented NPR using the RI'-MOM scheme on the CLS $N_\mathrm{f} = 2$ ensembles for local quark bilinears
- Automated perturbative subtraction allows easy adaptation to different operators and actions
- In particular, application to $N_\mathrm{f} = 2 + 1$ CLS ensembles poses no major difficulties (for periodic boundary conditions)
- Results for nucleon charges (and form factors) forthcoming
- Intend to treat derivative operators for $\langle x \rangle$ and related observables in much the same way

The end

Thank you for your attention

Backup Slides

– **BACKUP** –

Quark field renormalization