Relaxation time of the fermions in magnetic field (II)

- away from the strong magnetic field limit –

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1. Motivation

Chiral Magnetic effect (CME) $J \propto \mathbf{E} \cdot \mathbf{B}$

Magnetic field B // Electric field E → Anomaly induced current

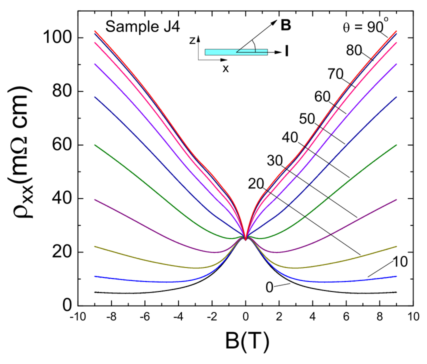
Theory prediction

Weyl-Semimetals or Semi-conductors

- Agryres-Adams Phys. Rev. 104,1956
- Nielsen- Ninomiya Phys. Lett. 130B, 1983

QGP at heavy-ion collision

- Fukushima, Kharzeev, Warringa, Phys. Rev. D 78, 073033 (2008)

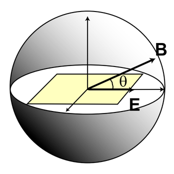


Experimental Observation

Weyl-semimetal, Dirac-semimetal

- Huang et al, arXiv:1503.01304
- Xiong et al, arXiv:1503.08179

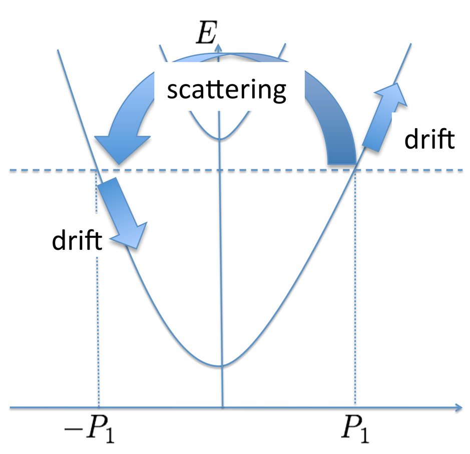
Detailed theoretical study is needed



Magneto-resistance :

drift ⟷ scattering

interplay



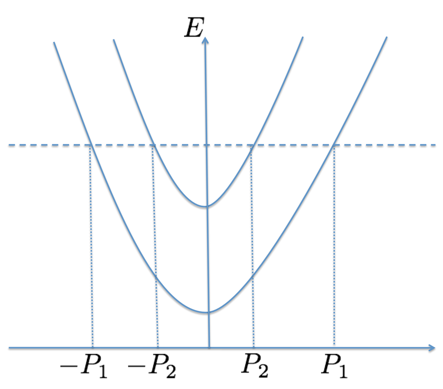
Scattering time determines
The Size of CME effect

However, calculation of the scattering time is usually made in the strong magnetic field limit

Calculation away from the strong magnetic field limit is needed.

2. Goal

We extend the calculation of the scattering time away from the strong magnetic field limit
Where more than one Landau-level contributes



This work

- Zero temperature
- Only scattering with impurities

3. Basics of transport theory

Boltzmann equation

$$\frac{\partial}{\partial t} f(n, P_y, P_z, t) - eE\frac{\partial}{\partial P_z} f(n, P_y, P_z, t) = \left(\frac{\partial}{\partial t} f(n, P_y, P_z)\right)_{\text{collision}}$$

$f(n, P_y, P_z, t)$: probability distribution function

Assuming small deviation from equilibrium $f = f_0 + \delta f$,

The collision term is defined as

$$\left(\frac{\partial f}{\partial t}\right)_{\text{collision}} = -\sum_{n'} \int d^2P' W(n, \mathbf{P} \to n', \mathbf{P}')(\delta f(n, \mathbf{P}, t) - \delta f(n', \mathbf{P}', t))$$

W : transition probability per unit time

Relaxation time approximation

$$\left(\frac{\partial f(n, \mathbf{P}, t)}{\partial t}\right)_{\text{colllison}} \approx -\frac{\delta f(n, \mathbf{P}, t)}{\tau(n, \mathbf{P})}$$

➡ Static solution of the Boltzmann equation

$$\delta f(n, \mathbf{P}) = \tau(n, \mathbf{P}) eE \frac{\partial}{\partial P_z} f_0(n, \mathbf{P})$$

$\tau(n, \mathbf{P})$ Determined from the definition of the collision term

$$-\frac{\delta f(n, \mathbf{P})}{\tau(n, \mathbf{P})} = -\sum_{n'} \int d^2\mathbf{P}' W(n, \mathbf{P} \to n', \mathbf{P}')(\delta f(n, \mathbf{P}) - \delta f(n, \mathbf{P}))$$

where $\delta f(n, \mathbf{P}) = \tau(n, \mathbf{P}) eE \frac{\partial \epsilon}{\partial P_z} \frac{\partial f_0(\epsilon)}{\partial \epsilon}$

⬇

Equation for the scattering time

$$P_z = \sum_{n'} \int d^2\mathbf{P}' W(n, \mathbf{P} \to n', \mathbf{P}')(\tau(n', \mathbf{P}') P'_z - \tau(n, \mathbf{P}) P_z)$$

What is the solution of this equation ?

We assume $\tau(n, P_y, P_z)$ is independent of P_y

At T=0, 4 set of states at the fermi level contributes (labeled by I=1,2,3,4)

From Fermi Golden Rule

$$W(n, \mathbf{P} \to n', \mathbf{P}') \equiv \delta(\epsilon - \epsilon') \hat{W}(n, \mathbf{P} \to n', \mathbf{P}')$$

Integrating over P'_z and defining

$$w_{IJ} = \int dP'_y \hat{W}(n_I, P_y, P_I \to n_J, P'_y, P_J)$$

we obtain

$$P_I = \sum_{J=1}^{4} w_{IJ} (\tau_I P_I - \tau_J P_J) \frac{\mu}{v|P_J|}$$

Normalization condition:

f and f_0 are normalized to unity

$$\sum_n \int d^2\mathbf{P} \delta f(n, \mathbf{P}) = 0$$

➡ $\tau_1 + \tau_2 - \tau_3 - \tau_4 = 0$

Explicit form of the coupled equations for scattering times

$$P_1 = \frac{\mu}{v}\left[\tau_1 \left\{(w_{12} + w_{13})\frac{P_1}{P_2} + w_{14}\right\} - \tau_2 w_{12} + \tau_3 w_{13} + \tau_4 w_{14}\right]$$

$$P_2 = \frac{\mu}{v}\left[-\tau_1 w_{12} + \tau_2 \left\{(w_{12} + w_{13})\frac{P_2}{P_1} + w_{23}\right\} + \tau_3 w_{23} + \tau_4 w_{13}\right]$$

$$-P_2 = \frac{\mu}{v}\left[-\tau_1 w_{13} - \tau_2 w_{23} + \tau_3 \left\{-(w_{12} + w_{13})\frac{P_2}{P_1} - w_{23}\right\} + \tau_4 w_{12}\right]$$

$$-P_1 = \frac{\mu}{v}\left[-\tau_1 w_{14} - \tau_2 w_{13} + \tau_3 w_{12} + \tau_4 \left\{-(w_{12} + w_{13})\frac{P_1}{P_2} - w_{14}\right\}\right]$$

Solution to the equation for scattering time

$$\tau_1 = \tau_4, \ \tau_2 = \tau_3$$

$$\tau_1 = \frac{\mu}{v} \frac{(w_{23}P_1 + w_{12}P_2)P_1P_2}{2(w_{12}w_{13} + w_{14}w_{23})P_1P_2 + (w_{12} + w_{13})(w_{23}P_1^2 + w_{14}P_2^2)}$$

$$\tau_2 = \frac{(w_{12}P_1 + w_{14}P_2)P_1P_2}{2(w_{12}w_{13} + w_{14}w_{23})P_1P_2 + (w_{12} + w_{13})(w_{23}P_1^2 + w_{14}P_2^2)}$$

As B becomes small, n=1 Landau-level starts to contribute
a jump of scattering time (and current) can ocurr.

$$\tau_1 = \frac{\mu}{2v} \frac{P_1}{w_{14}} \to \frac{\mu}{v} \frac{P_1 P_2}{2w_{14}P_2 + (w_{12} + w_{13})P_1}$$

Interesting signal for CME

5. Summary

1. We studied the relaxation time for CME away from the strong magnetic limit.
2. We derived the coupled equations for the relaxation time when Landau level with n=1 contributes.
3. Requiring the normalization condition, we obtained solution to the equation.

Future plan:

- Numerical evaluation for the scattering time