Electromagnetic Form Factors through Parity-Expanded Variational Analysis

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Introduction

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- Existing parity projection techniques are vulnerable to opposite parity contaminations at nonzero momentum
- We propose the Parity Expanded Variational Analysis (PEVA) technique to resolve this issue

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- Call states that transform positively under parity in their rest frame "positive parity states" (B^+)
- Call states that transform negatively under parity in their rest frame "negative parity states" (B^-)

Conventional analysis

- Conventional baryon operators $\{\chi^i\}$ couple to states of both parities

$$\langle\Omega|\chi^i|B^+\rangle = \lambda_i^{B^+}\sqrt{\frac{m_{B^+}}{E_{B^+}}}\,u_{B^+}(p,s)$$

$$\langle\Omega|\chi^i|B^-\rangle = \lambda_i^{B^-}\sqrt{\frac{m_{B^-}}{E_{B^-}}}\,\gamma_5\,u_{B^-}(p,s)$$

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- Form correlation matrix

$$\mathcal{G}_{ij}(\mathbf{p};\ t) := \sum_{\mathbf{x}} \mathrm{e}^{\mathrm{i}\mathbf{p}\cdot\mathbf{x}}\ \langle\Omega|\chi^i(x)\,\overline{\chi}^j(0)|\Omega\rangle$$

Parity projection

- Introduce $\Gamma_{\pm} = (\gamma_4 \pm \mathbb{I})/2$ and define $G_{ij}(\Gamma_{\pm};\mathbf{p};\ t) := \mathrm{tr}\left(\Gamma_{\pm}\,\mathcal{G}_{ij}(\mathbf{p};\ t)\right)$

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- Can analyse states of each parity independently

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- Better to use variational analysis to remove all contaminating states simultaneously

- Terms in unprojected correlation matrix have Dirac structure

$$\begin{pmatrix} E_{B^\pm}(\mathbf{p}) \pm m_{B^\pm} & -\sigma_k p_k \\ \sigma_k p_k & -\left(E_{B^\pm}(\mathbf{p}) \mp m_{B^\pm}\right) \end{pmatrix}$$

Parity-Expanded Variational Analysis (PEVA)

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$$\Gamma_{\mathbf{p}} = \frac{1}{4}(\mathbb{I} + \gamma_4)(\mathbb{I} - i\gamma_5\gamma_k\hat{p}_k)$$

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- Terms in unprojected correlation matrix have Dirac structure

$$\begin{pmatrix} E_{B^\pm}(\mathbf{p}) \pm m_{B^\pm} & -\sigma_k p_k \\ \sigma_k p_k & -(E_{B^\pm}(\mathbf{p}) \mp m_{B^\pm}) \end{pmatrix}$$

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- Both couple to states with consistent Dirac structure $\Gamma_{\mathbf{p}} u_B(p, s)$

Lattice results

- Second lightest PACS-CS $(2+1)$-flavour full-QCD ensemble
  - $32^3 \times 64$ lattices
  - $a = 0.0951(14)\,\mathrm{fm}$ by Sommer parameter
  - $\kappa_{u,d} = 0.1377$, corresponding to $m_\pi = 280(5)\,\mathrm{MeV}$

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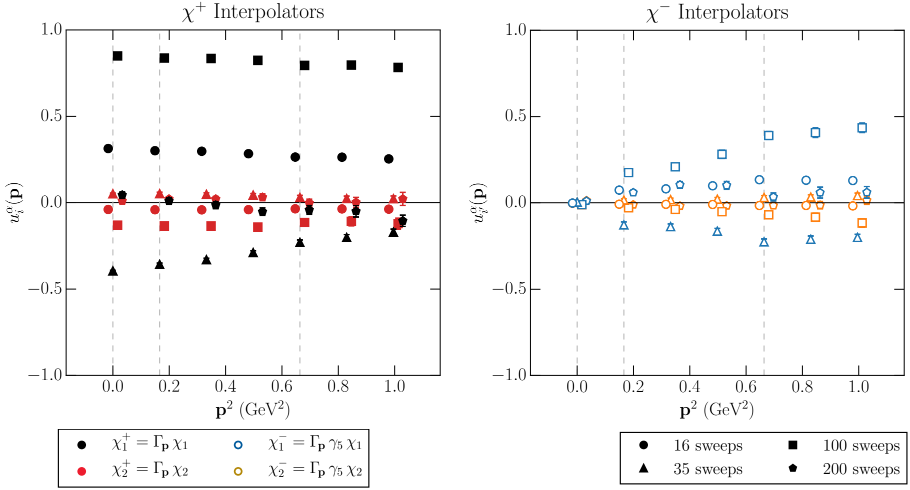
- Using conventional spin-1/2 nucleon operators

$$\chi_1 = \epsilon^{abc}\,[u^{a\top}(C\gamma_5)\,d^b]\,u^c$$
$$\chi_2 = \epsilon^{abc}\,[u^{a\top}(C)\,d^b]\,\gamma_5\,u^c$$

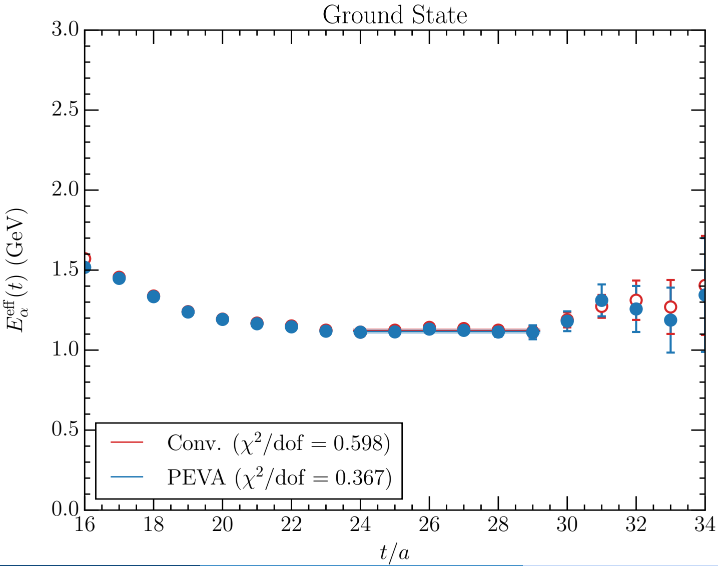
- Apply 16, 35, 100 and 200 sweeps of gauge invariant gaussian smearing in creating the propagators



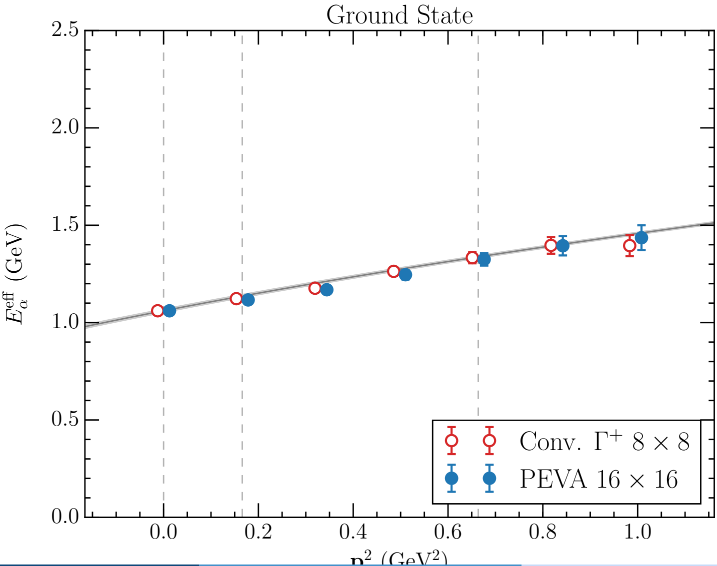
χ^+ Interpolators
χ^- Interpolators
$u_i^\alpha(\mathbf{p})$
$u_{i'}^\alpha(\mathbf{p})$
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0
$\mathbf{p}^2$ (GeV^2)
$\chi_1^+ = \Gamma_\mathbf{p}\,\chi_1$
$\chi_2^+ = \Gamma_\mathbf{p}\,\chi_2$
$\chi_1^- = \Gamma_\mathbf{p}\,\gamma_5\,\chi_1$
$\chi_2^- = \Gamma_\mathbf{p}\,\gamma_5\,\chi_2$
16 sweeps
35 sweeps
100 sweeps
200 sweeps

Effective energy

Ground state - $\mathbf{p}^2 \simeq 0.166\,\mathrm{GeV}^2$

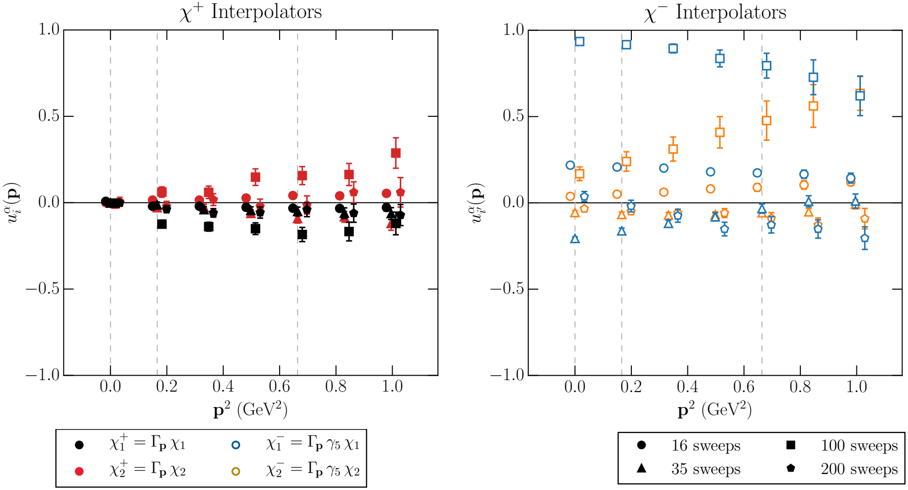


Effective energy
Ground state



Eigenvector components

First negative parity excitation



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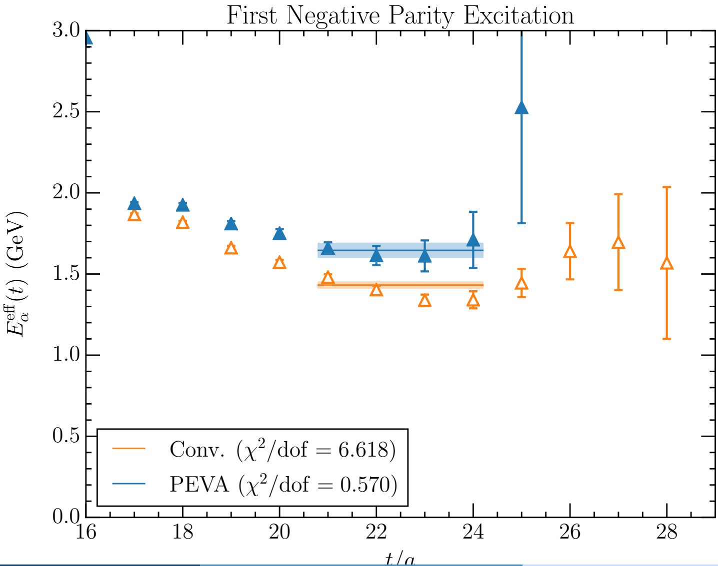

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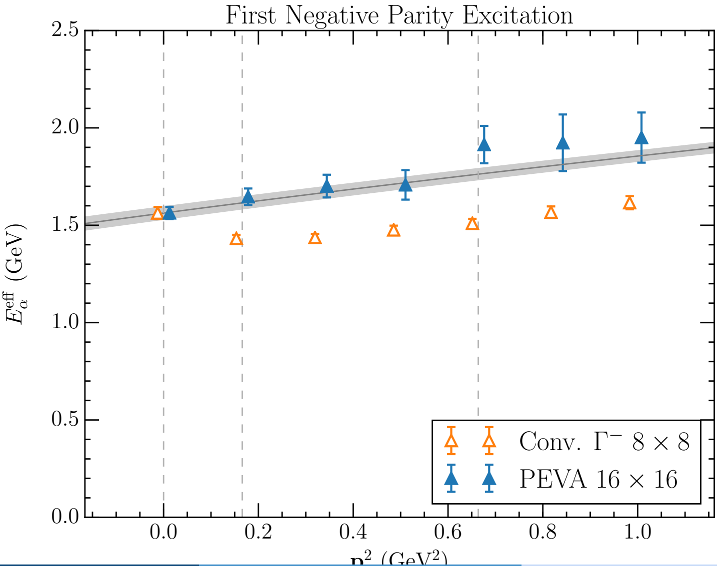

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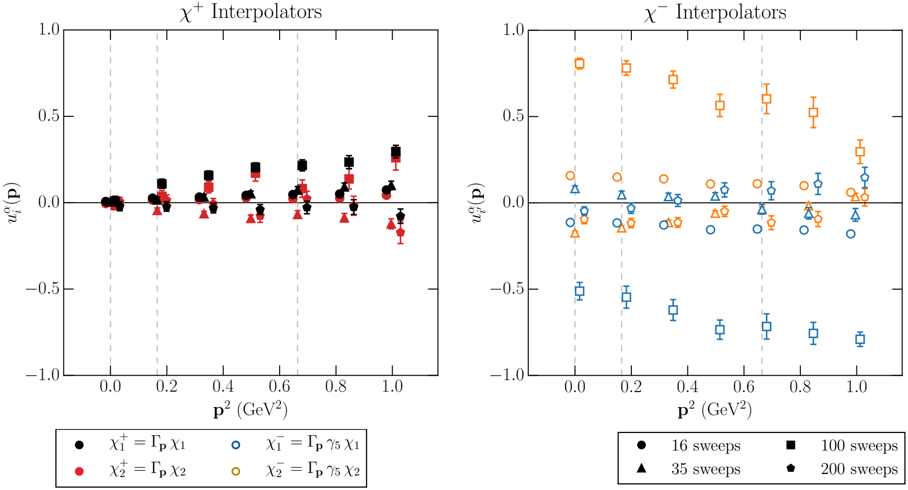

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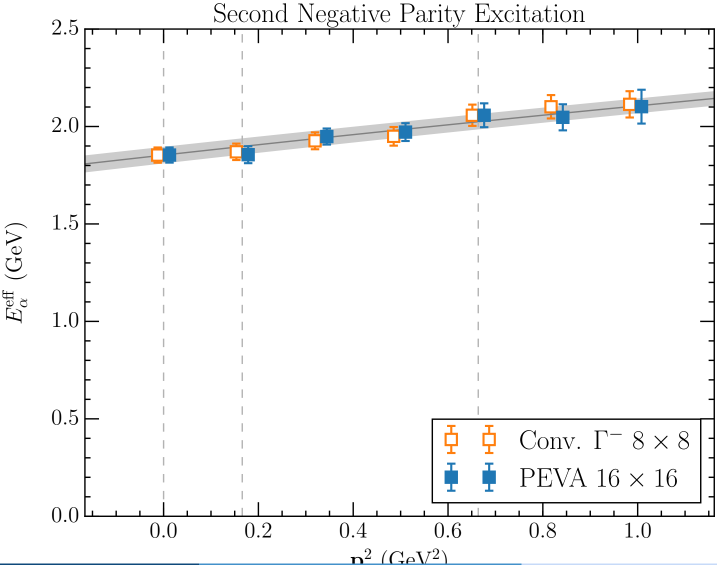

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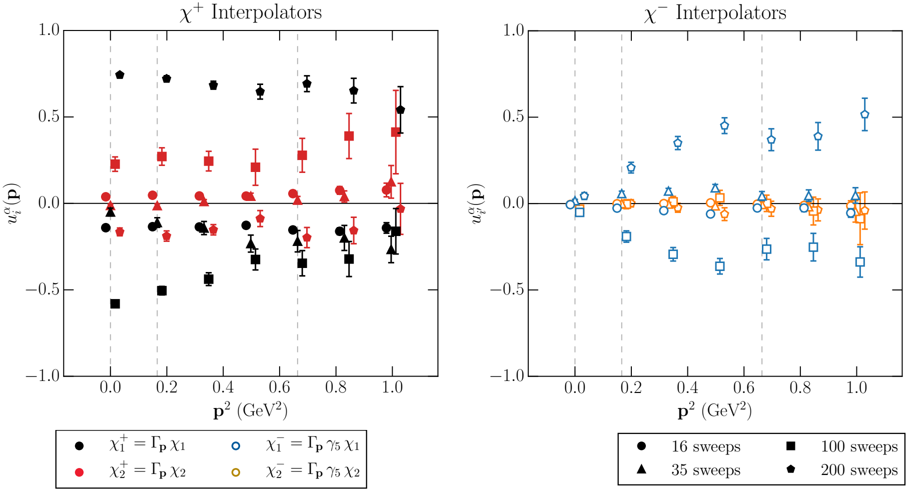

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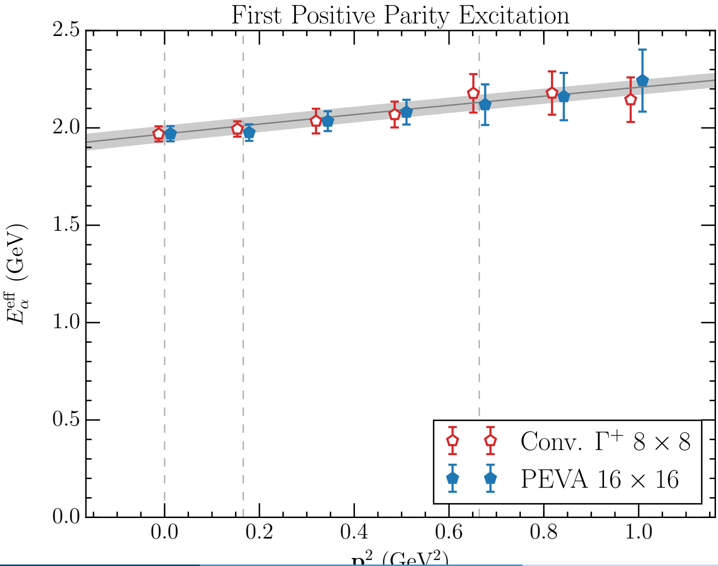
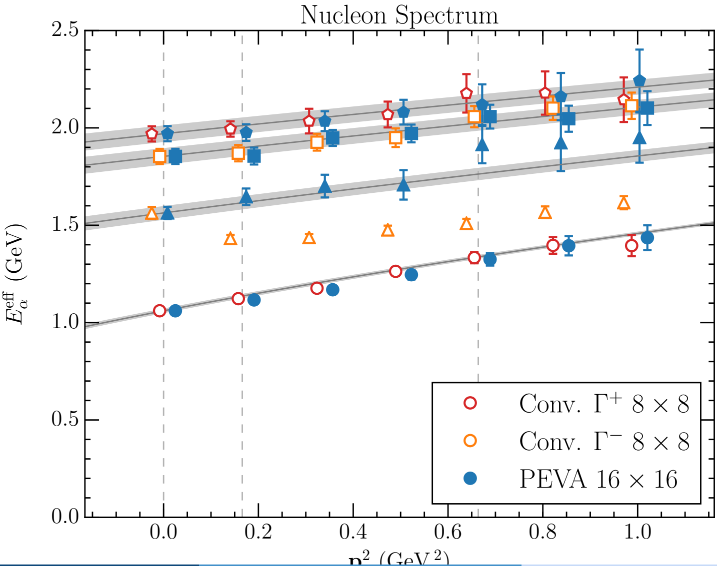


Effective energy

Nucleon spectrum



Form Factors

- Construct three point correlator for a single energy eigenstate

$$G^{\mu}_{ij}(\mathbf{p}', \mathbf{p}; \Gamma; t_2, t_1) := \mathrm{tr}\left[\Gamma \sum_{\mathbf{x},\mathbf{y}} \mathrm{e}^{\mathrm{i}\mathbf{p}\cdot\mathbf{x}}\, \mathrm{e}^{\mathrm{i}(\mathbf{p}'-\mathbf{p})\cdot\mathbf{y}}\, \langle\Omega|\chi^{i}(x)\, J^{\mu}(y)\, \overline{\chi}^{j}(0)|\Omega\rangle\right]$$

$$G^{\mu}_{\alpha}(\mathbf{p}', \mathbf{p}; \Gamma; t_2, t_1) := v^{\alpha}_{i}(\mathbf{p}')\, G^{\mu}_{ij}(\mathbf{p}', \mathbf{p}; \Gamma; t_2, t_1)\, u^{\alpha}_{j}(\mathbf{p})$$

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- Define ratio

$$R^{\mu}_{\alpha}(\mathbf{p}',\, \mathbf{p};\, t_1) := \sqrt{\frac{G^{\mu}_{\alpha}(\mathbf{p}',\, \mathbf{p};\, \Gamma;\, t_2,\, t_1)\, G^{\mu}_{\alpha}(\mathbf{p},\, \mathbf{p}';\, \Gamma;\, t_2,\, t_1)}{G_{\alpha}(\mathbf{p}';\, t_2)\, G_{\alpha}(\mathbf{p};\, t_2)}}$$

Form Factors

- Construct three point correlator for a single energy eigenstate

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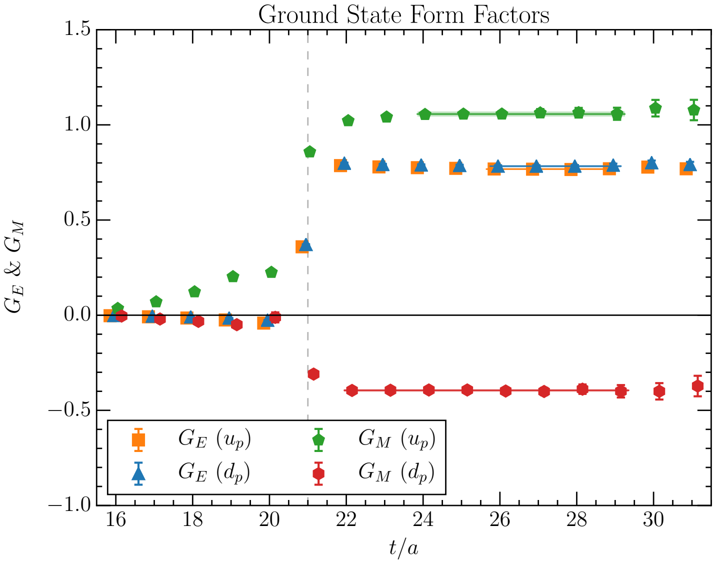
- Extract $G_E(Q^2)$ and $G_M(Q^2)$ from ratio



Ground State Form Factors
G_E & G_M
t/a
1.5
1.0
0.5
0.0
−0.5
−1.0
16
18
20
22
24
26
28
30
$G_E\ (u_p)$
$G_E\ (d_p)$
$G_M\ (u_p)$
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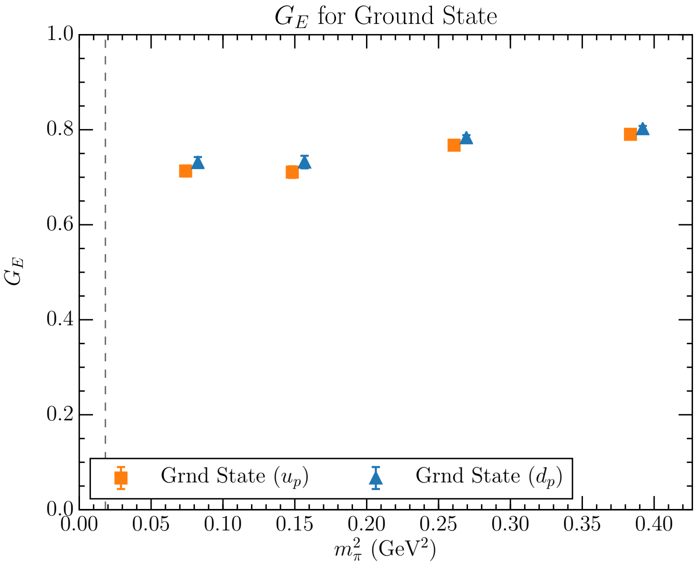
Form Factors

$G_E(Q^2 = 0.15(1)\,\mathrm{GeV}^2)$ for ground state



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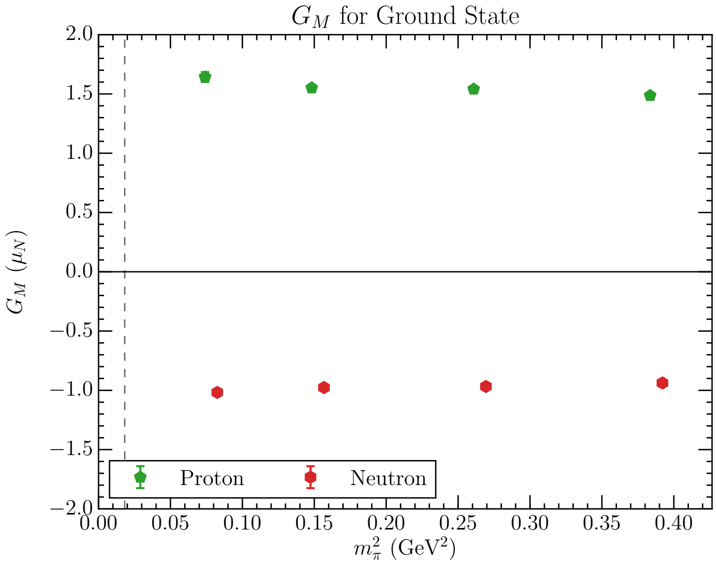
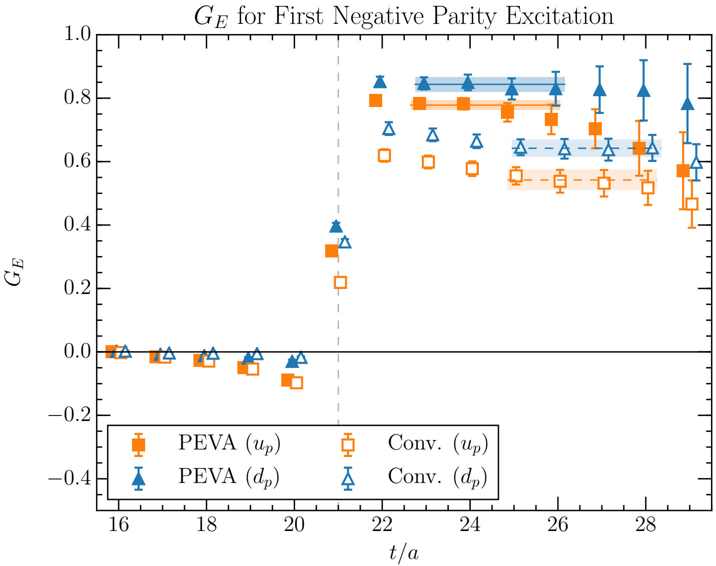


Fits to G_E for first negative parity excitation



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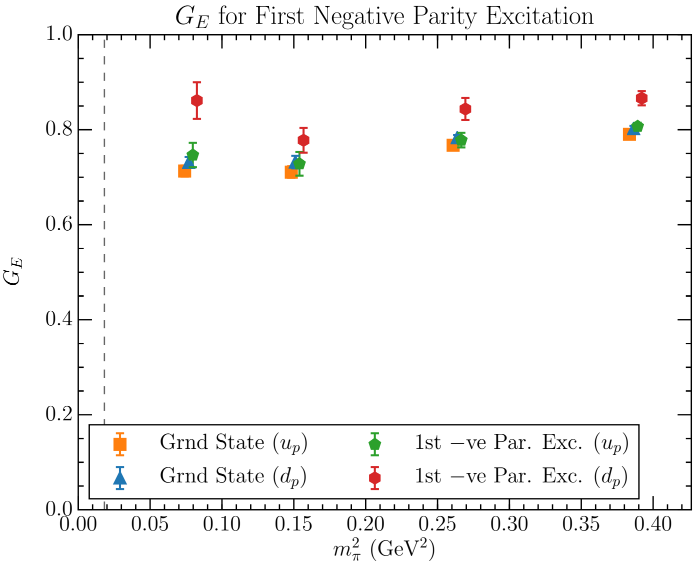

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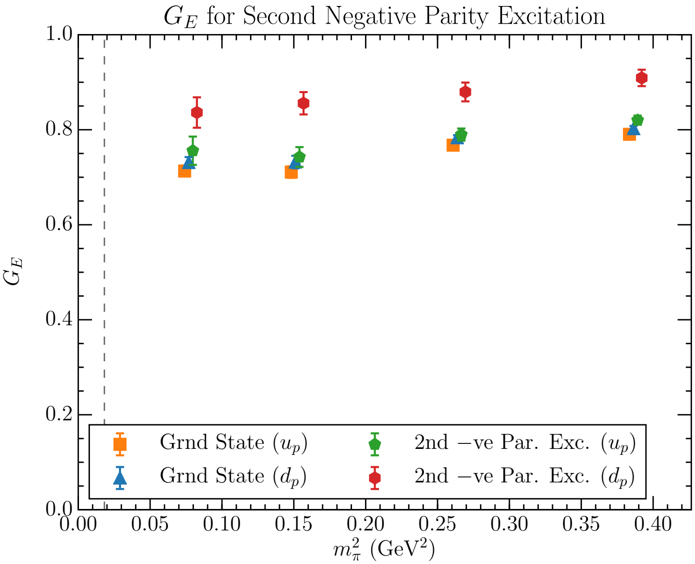
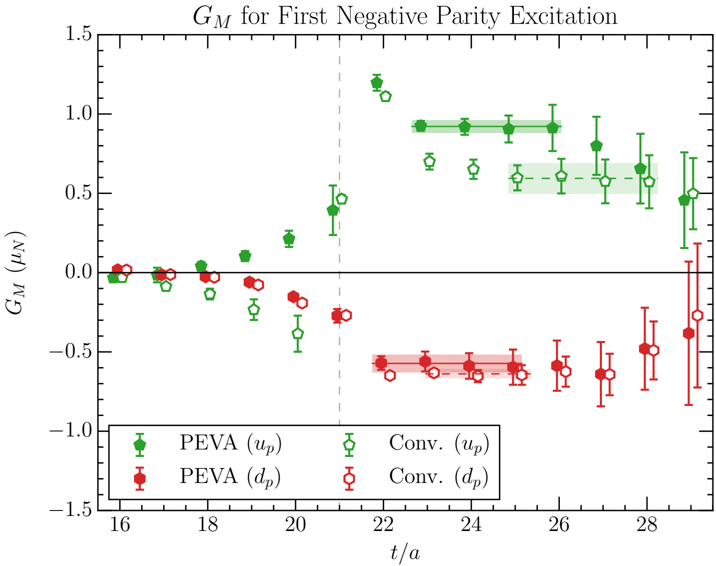


Form Factors

Fits to G_M for first negative parity excitation



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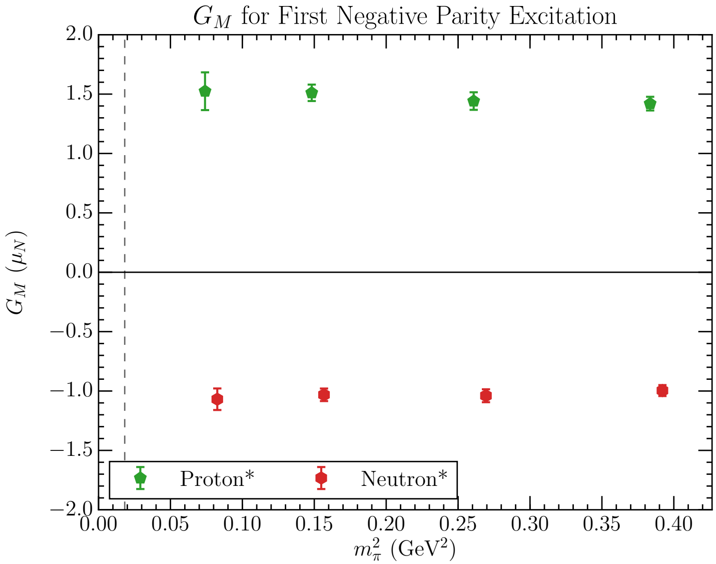

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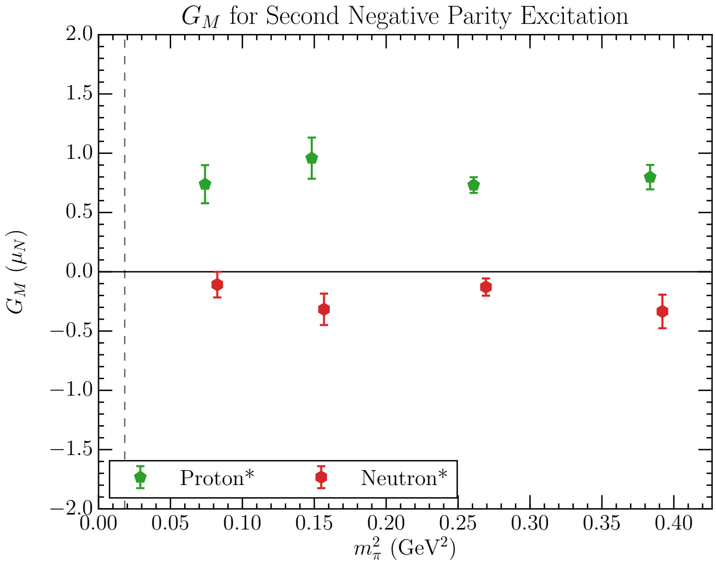

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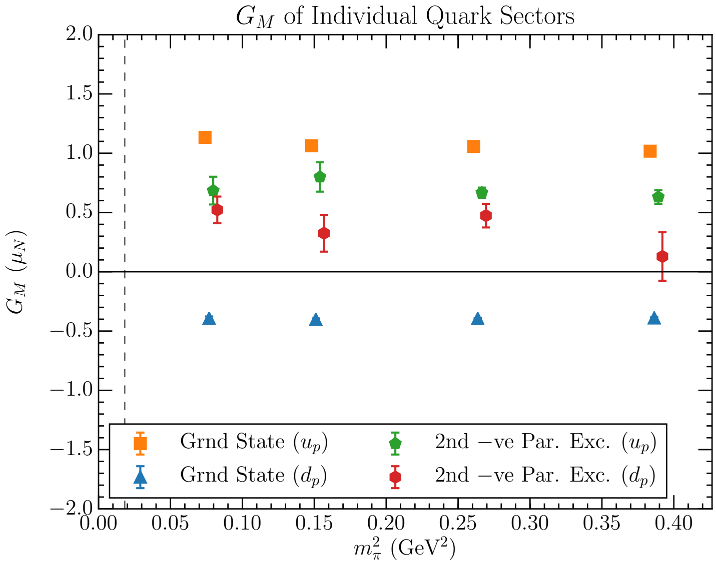
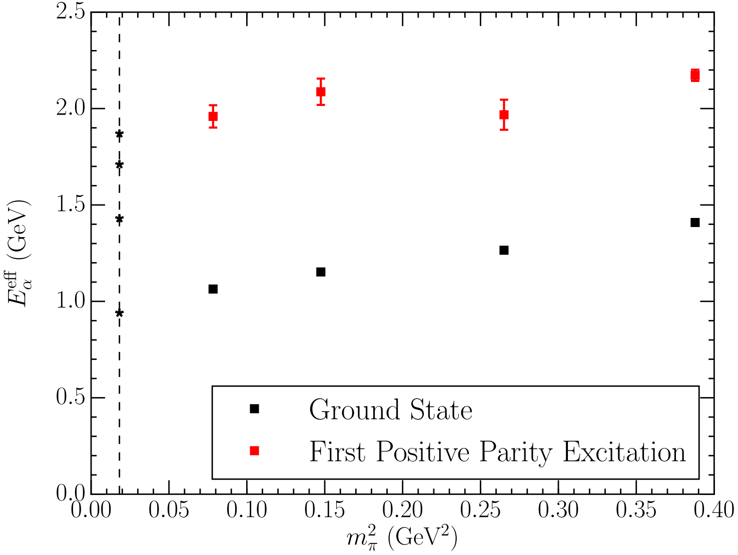


Positive Parity Nucelon Spectrum



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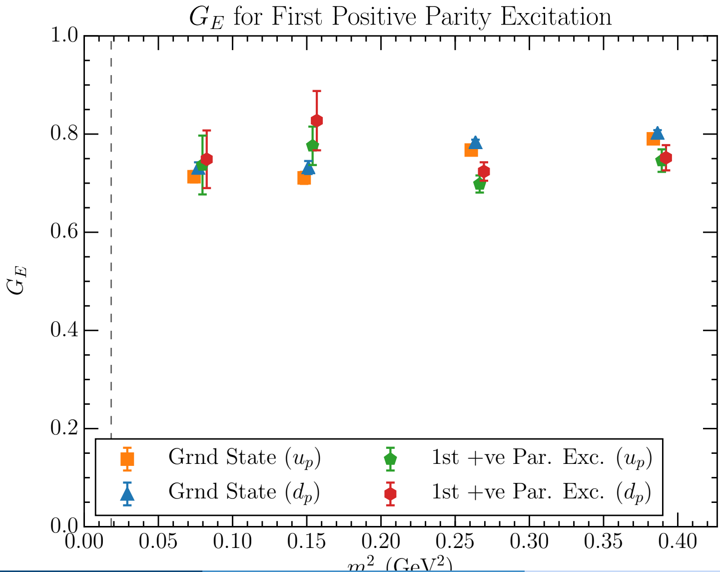

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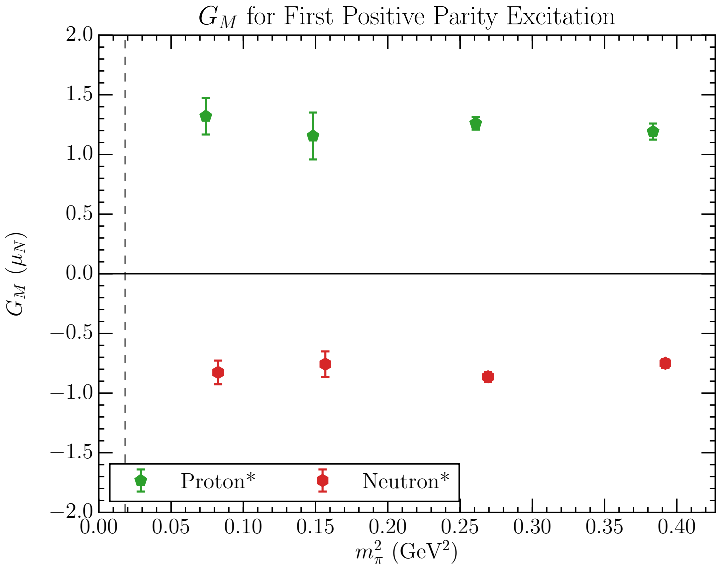

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- Clear effect on two point function for lowest lying negative parity excitation
- Has significant effects on three point functions for excited states
- This is an important step towards making contact with experiment through calculations such as baryon transition moments

More information

"Parity-expanded variational analysis for nonzero momentum"
F. M. Stokes, W. Kamleh, D. B. Leinweber, M. S. Mahbub,
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