Tuning of hopping parameters in Oktay-Kronfeld action for heavy quarks on the $N_f = 2+1+1$ MILC HISQ ensemble

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Introduction

- We determine hopping parameters of the Oktay-Kronfeld (OK) action for charm and bottom quarks.
- We compute the masses of pseudoscalar (vector) mesons $B_s^{(*)}$, $D_s^{(*)}$; the valence light quark is simulated with HISQ action. We also monitor the inconsistency parameter and the hyperfine splitting.
- We discuss about the computational cost for using OK action rather than using the Fermilab action (clover).

Simulation Details

- We simulate on the $N_f = 2+1+1$ MILC HISQ gauge ensembles:

a(fm)	$N_s^3 \times N_t$	am_l, am_s, am_c	u_0	N_{conf}	N_{tsrc}
0.12	$24^3 \times 64$	$0.0102, 0.0509, 0.635$	0.86372	500	1

- For light valence (strange) HISQ propagator, we use QUDA conjugate gradient (CG) inverter.

source / sink	strange mass (am_0)	ϵ
point	0.0509	-0.0017468

- We use a tadpole improved OK action for heavy valence (charm and bottom) quarks. The propagator is generated by an optimized BiCGStab inverter using CUDA. To tune the kappa, we try 2 and 3 different kappa values for charm and bottom regions, respectively.

source / sink	κ_b	κ_c
covariant gaussian smearing: $r_0 = 5$, source iters=60	0.042, 0.041, 0.039	0.049, 0.048

- The production is done on the Seoul National University GPU cluster (DAVID) with GTX Titan Black / Titan X.

Meson Correlators & Dispersion Relation

We compute meson correlators: $C(t,\mathbf{p}) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \langle O^\dagger(t,\mathbf{x}) O(0,\mathbf{x})\rangle$ with 11 momenta $a\mathbf{p} = (2\pi/N_L)\mathbf{n}; \mathbf{n}^2 \le 10$. Here $O(t,\mathbf{x})$ is the meson interpolating operator. Fit function is :

$$f(t) = Ae^{-Et}\left(1-(-1)^t r e^{-\Delta E t}\right) + Ae^{-E(T-t)}\left(1-(-1)^t r e^{-\Delta E(T-t)}\right) \quad (1)$$

with 4 fitting parameters: A ground state energy and amplitude (E, A), an amplitude ratio $(r = A^p/A)$ and energy difference $(\Delta E = E^p - E)$ where the superscript p stands for a staggered parity partner state. We use full covariance matrix without Bayesian prior with fit range $t \in [8,15]$. For heavy-heavy, there is no staggered parity partners, and we use fit range $t \in [15,20]$. And then we use the fit function of the ground state energy E from the meson dispersion relation for a small $\mathbf{p}$:

$$E(\mathbf{p}) = M_1 + \frac{\mathbf{p}^2}{2M_2} - \frac{(\mathbf{p}^2)^2}{8M_4^3} - \frac{a^3 W_4}{6}\sum_{i=1}^{3} p_i^4 \quad (2)$$

with 4 fitting parameters: M_1 (rest mass), M_2 (kinetic mass), M_4 and W_4. We do the full covariance fit with the 11 momenta as a fit range.

Hyperfine Spliting

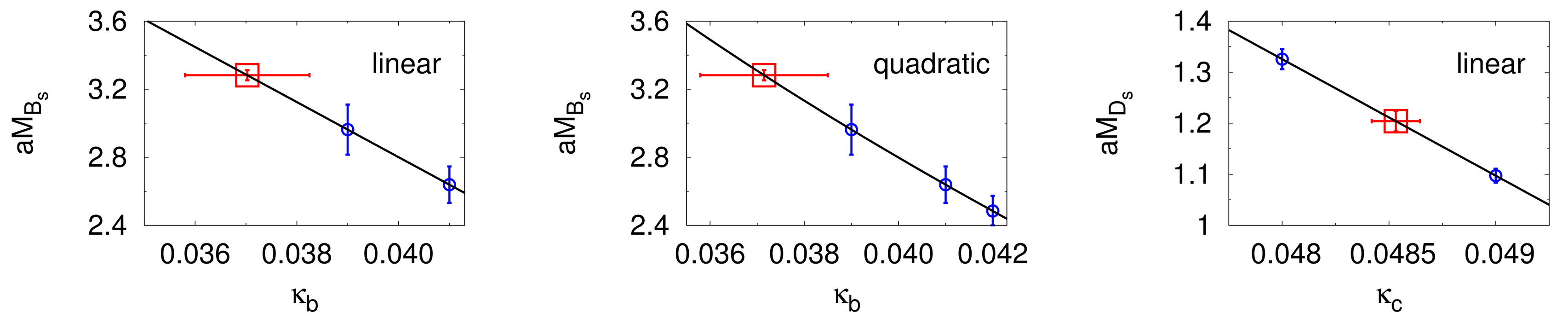
We calculate the hyperfine splittings Δ_1 from rest mass and Δ_2 from kinetic mass. In the continuum limit, we have $\Delta_1 = \Delta_2$. Here we present the $a\Delta_1$ and $a\Delta_2$ with rescaled factor 10^{-3}, and the errors are from a jackknife resampling.

	κ	0.049	0.048	0.042	0.041	0.039
heavy-light	$a\Delta_1$	85(2)	68(2)	30(2)	26(2)	22(2)
	$a\Delta_2$	55(21)	27(25)	-60(70)	-59(78)	-97(98)
quarkonia	$a\Delta_1$	72(1)	57(1)	24(1)	22(0)	18(0)
	$a\Delta_2$	85(40)	96(45)	41(85)	19(90)	6(113)

We have unexpectedly large error in the Δ_2 for bottom quark region, and the errors are especially large for quarkonium.

Kappa tuning

We determine the κ that yields the physical pseudoscalar B_s for κ_b and D_s for κ_c. Here we used $aM_{D_s} = 1.204(20)$ and $aM_{B_s} = 3.282(30)$. We do the linear interpolation/extrapolation by choosing 2 kappa values. For the bottom region, we also tried a quadratic extrapolation. In the following figures, blue circles are the M_2 measured using specific kappas and the y-error is the statistical error from a jackknife resampling.

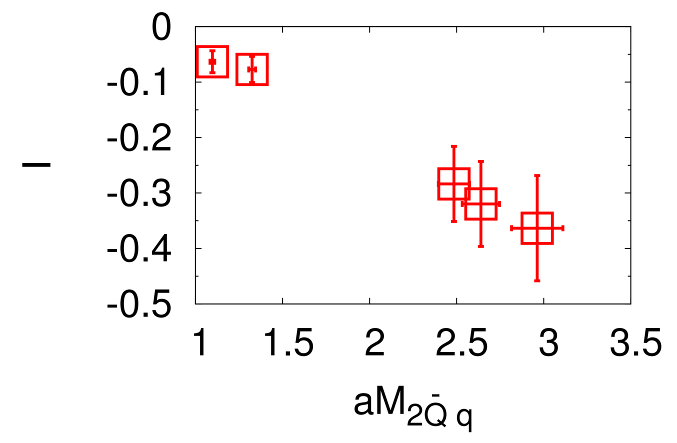


The red square represents a tuned kappa κ where the x-error is the statistical error from a jackknife resampling and the y-error comes from input meson masses. In the table, we present tuned κ values and the second error for κ_b is from the difference between the linear and quadratic extrapolations.

κ_c	0.04853(11)	linear
κ_b	0.0370(12)(2)	linear / quadratic

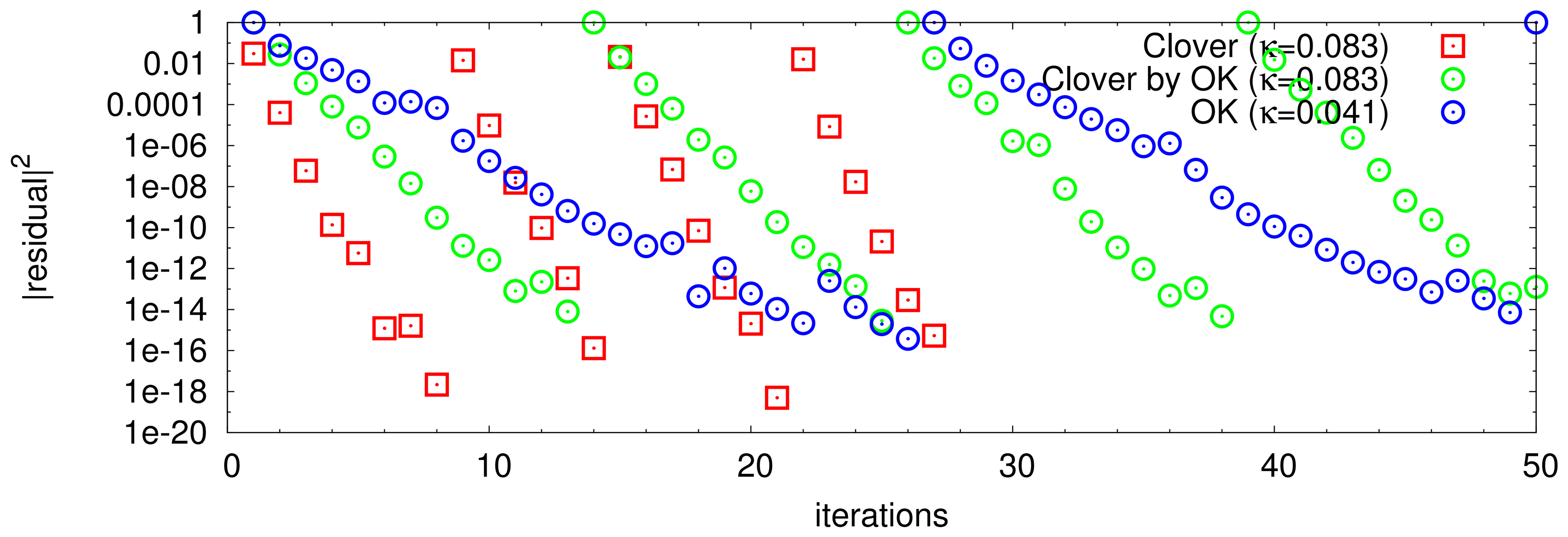
Inconsistency Parameter

We calculate the inconsistency parameter $I = \frac{2\delta M_{\bar{Q}q} - (\delta M_{\bar{Q}Q} + \delta M_{\bar{q}q})}{2M_{2\bar{Q}q}}$ where $\delta M_X = M_{2X} - M_{1X}$, $(X = \bar{Q}q, \bar{Q}Q)$ for pseudoscalar mesons. The error is estimated by jackknife resampling.



Computational Cost of the OK Action

The OK action has 3.8 times more FLOPs than the Clover action. The OK action cannot use the even-odd (EO) preconditioning due to its 2-hop terms in the action, and therefore its Dirac matrix inversion requires 2 times larger FLOPs than the preconditioned one (clover). Also, EO preconditioning improves the condition number of the matrix such that a number of CG iterations decrease. Turning on the c_4 term of the OK action makes a number of CG iterations increased roughly by a factor of 2. In summary, the theoretical limit of the cost factor for the OK action is 3.8(FLOPs)×2(EO-FLOPs)×1.5(EO-Cond.)×1.5~2(c_4)=17~18. In spite of this amount of the additional cost, we can compensate a part of the OK production time using GPUs.



	Clov.	Clov. by OK	OK	OK (GPU)
time (s)	49	520	977	288
iterations	638	1386	2470	2416

Summary and Plan

In this work, we tuned the κ using a $a = 0.12$fm HISQ ensemble but somehow the statistics was low to tune the κ. We will increase the statistics by using another source time slices and tune the κ including the other HISQ ensemble with $a = 0.09$ As a part of the ongoing project, we are also calculating the improved OK action current relevant to $B \to D^* l\nu$ for the calculation of V_{cb}.