The coupled channel approach to the ΛcN - ΣcN system in lattice QCD

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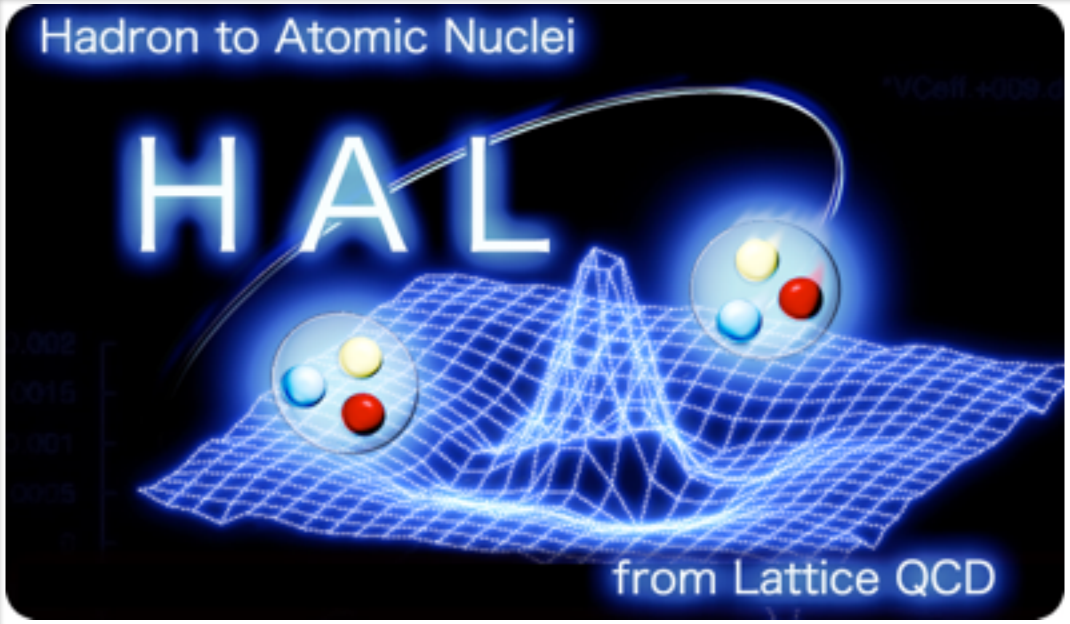
for **HAL QCD Collaboration**



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Introduction

2-baryon bound state — **Di-baryon**

Deuteron is known di-baryon so far.

→ **Are there any other di-baryons?**

- 2-baryon system including heavy quarks has more chance to have bound state.
  - — Small kinetic energies are expected.
  - — Small repulsive core are expected within the CMI ($\sim 1/m_Q$).

Introduction

The pion exchange contribution is important to form **deuteron**.

NN ($J^P=1^+$) → ΣcN ($J^P=1^+$)

Since the pion exchange is allowed in ΣcN ($J^P=1^+$) channel, the bound state may form. Y. R. Liu, M. Oka, Phys. Rev. D85, 014015 (2012).

No experimental data for ΣcN scattering.

We extract the baryon interaction using **HAL QCD method**

Outline

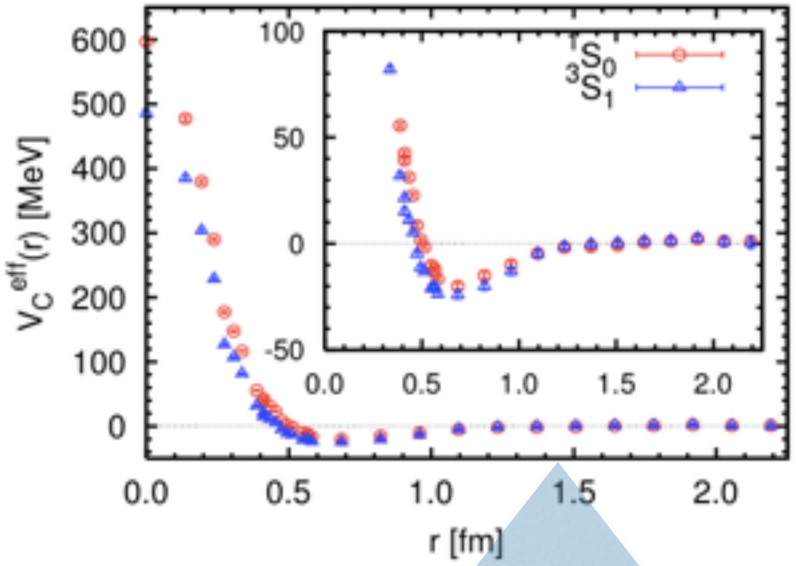
- Introduction
- HAL QCD method
- Lattice simulation setup
- Numerical results
- Summary & Conclusion

HAL QCD method

S. Aoki, T. Hatsuda, N. Ishii,
Prog. Theor. Phys., 123 (2010).

Define the **energy-independent non-local potential**
through the **Schrödinger-type equation**

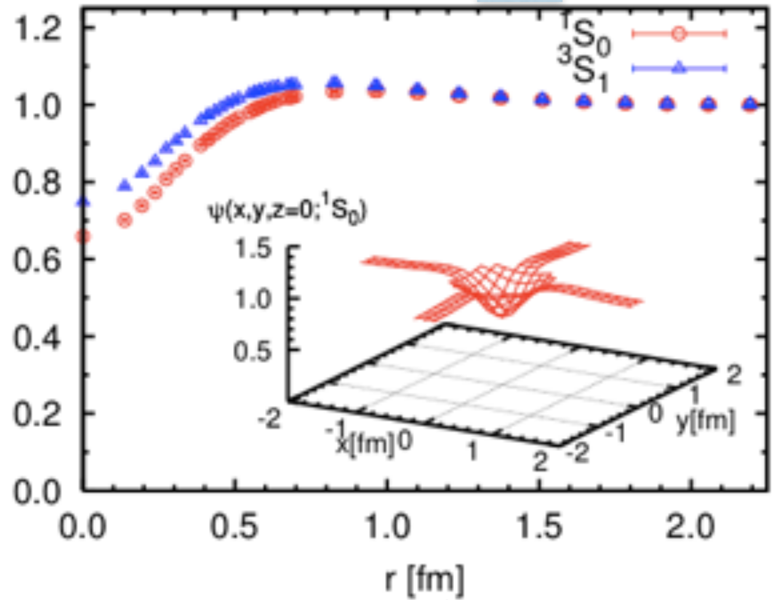
$$(E_n - H_0)\,\underline{\psi_n(\vec{r})} = \int d^3r'\,U(\vec{r},\vec{r}\,')\,\underline{\psi_n(\vec{r}\,')}$$



from **NBS wave function**

$$\underline{\psi^{(W)}_{\alpha\beta}}(\vec{r})e^{-Wt} = \sum_{\vec{x}}\langle 0|T\{B^{(1)}_\alpha(\vec{r}+\vec{x},t)B^{(2)}_\beta(\vec{x},t)\}|B^{(1)}B^{(2)},W\rangle$$

$$\begin{aligned} G_{\alpha\beta}(\vec{r},t-t_0) &= \sum_{\vec{x}}\langle 0|T\{B^{(1)}_\alpha(\vec{r}+\vec{x},t)B^{(2)}_\beta(\vec{x},t)\}\overline{\mathcal{J}^{(1,2)}}(t_0)|0\rangle \\ &= \sum_n A_n\psi^{(W_n)}_{\alpha\beta}(\vec{r})e^{-W_n(t-t_0)} + \cdots \\ &\overset{t\to\infty}{\rightarrow} A_0\underline{\psi^{(W_0)}_{\alpha\beta}}(\vec{r})e^{-W_0(t-t_0)} \end{aligned}$$



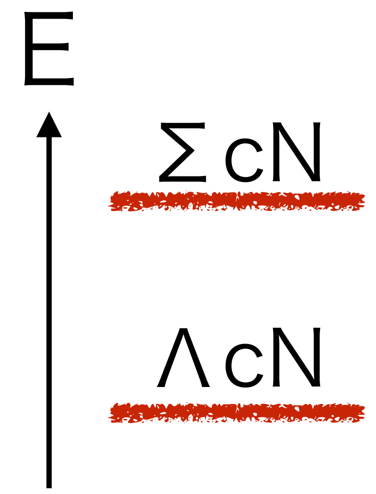
HAL QCD method

S. Aoki, T. Hatsuda, N. Ishii,
Prog. Theor. Phys., 123 (2010).

Since the ΛcN is lowest state of this I = 1/2 ($J^P=1^+$) channel, we have to solve the coupled channel Schrödinger equation to extract the ΣcN interaction.

$$\left(E_n^{(1)} - H_0^{(1)}\right)\psi_n^{(1)}(\vec{r}) = \int d^3r' U_{11}(\vec{r},\vec{r}\,')\psi_n^{(1)}(\vec{r}\,') + \int d^3r' U_{12}(\vec{r},\vec{r}\,')\psi_n^{(2)}(\vec{r}\,')$$

$$\left(E_n^{(2)} - H_0^{(2)}\right)\psi_n^{(2)}(\vec{r}) = \int d^3r' U_{21}(\vec{r},\vec{r}\,')\psi_n^{(1)}(\vec{r}\,') + \int d^3r' U_{22}(\vec{r},\vec{r}\,')\psi_n^{(2)}(\vec{r}\,')$$



+ Velocity expansion

In the low energy state,
LO term of the potential
is significant.

$$U(\vec{r},\vec{r'}) = V(\vec{r},\vec{v})\delta^3(\vec{r}-\vec{r'})$$

$$V(\vec{r},\vec{v}) = \underbrace{V_0(r) + V_\sigma(r)\vec{\sigma_1}\cdot\vec{\sigma_2} + V_T(r)S_{12}}_{\text{LO}} + \underbrace{V_{LS}(r)\vec{L}\cdot\vec{S}}_{\text{NLO}} + \mathcal{O}(v^2)$$

HAL QCD method

S. Aoki, T. Hatsuda, N. Ishii,
Prog. Theor. Phys., 123 (2010).

We use **Time-dependent HAL QCD method**
to construct the potential using all elastic states.

N. Ishii et al [HAL QCD Coll.], PLB712 (2012) 437.

$$(E_n - H_0)\,\psi_n(\vec{r}) = \int d^3r' U(\vec{r},\vec{r}\,')\psi_n(\vec{r}\,')$$

at $m_{B^{(1)}} \neq m_{B^{(2)}}$

$$E_n = (\Delta W) + \frac{1}{8\mu}(\Delta W)^2\left[1+\delta^2\right] + \mathcal{O}\left((\Delta W)^3\right)$$

$$\delta \equiv \frac{m_{B^{(1)}} - m_{B^{(2)}}}{m_{B^{(1)}} + m_{B^{(2)}}}$$

At the difference mass system,
we take the approximation
up to $(\Delta W)^3$

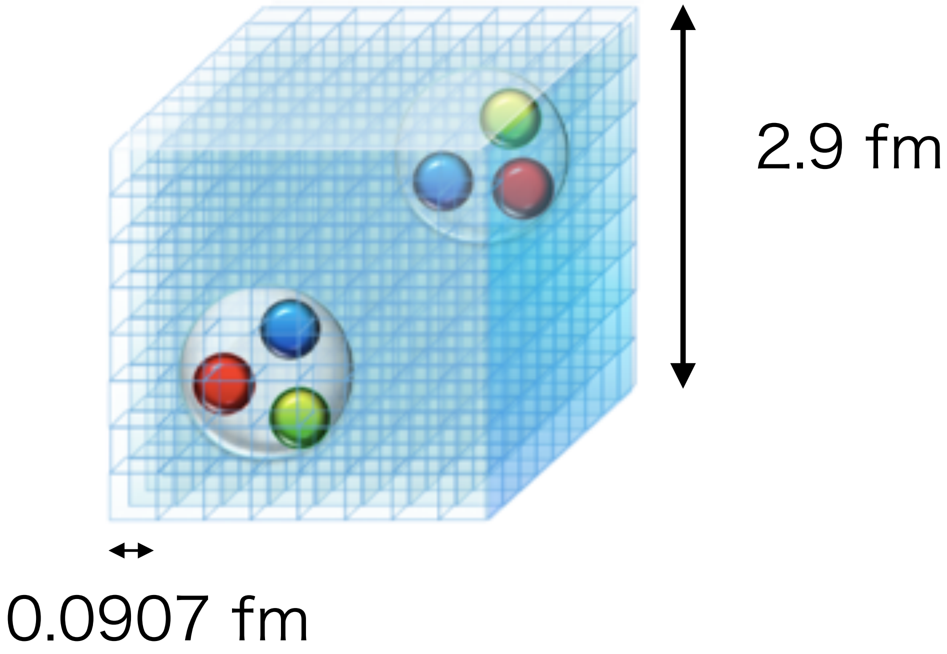
$$\left(-\frac{\partial}{\partial t} + \left[\frac{1+\delta^2}{8\mu}\right]\frac{\partial^2}{\partial t^2} - H_0\right) R(\vec{r},t) = \int d^3r' U(\vec{r},\vec{r}\,')R(\vec{r}\,',t)$$

$$R_{\alpha\beta}(\vec{r},t) = \frac{G_{\alpha\beta}(\vec{r},t)}{e^{-(m_{B^{(1)}}+m_{B^{(2)}})t}} = \sum_n A_n \psi_{\alpha\beta}(\vec{r},t)e^{-\Delta W_n t} + \cdots$$

$$\Delta W \equiv \sqrt{k_n^2 + m_{B^{(1)}}^2} + \sqrt{k_n^2 + m_{B^{(2)}}^2} - (m_{B^{(1)}} + m_{B^{(2)}})$$

Lattice QCD setup

Nf=2+1 full QCD configurations generated by the PACS-CS Coll.



PACS-CS Collaboration:

S. Aoki, et al., Phys. Rev. D79 (2009) 034503

- Iwasaki gauge action
- O(a) improved Wilson-clover quark action
- a ~ 0.09 fm, L ~ 3 fm (32^3 x 64)

m_π = 700, 570, 410 MeV

For charm quark, we use

Relativistic Heavy Quark (RHQ) action

to reduce the leading O((ma)^n) discretization error.

Same parameters set as in Namekawa, et al. is employed.

Namekawa, et al.,

Phys. Rev. D84 (2011) 074505

BG/Q @ KEK

Each hadron masses we calculated (MeV).

π	700	570	410
N	1578(6)	1392(6)	1221(10)
Λc	2689(4)	2552(6)	2434(6)
Σc	2783(6)	2674(9)	2568(10)

Numerical Results

ΛcN-ΣcN Coupled channel

effective $J^P=1^+$ potential

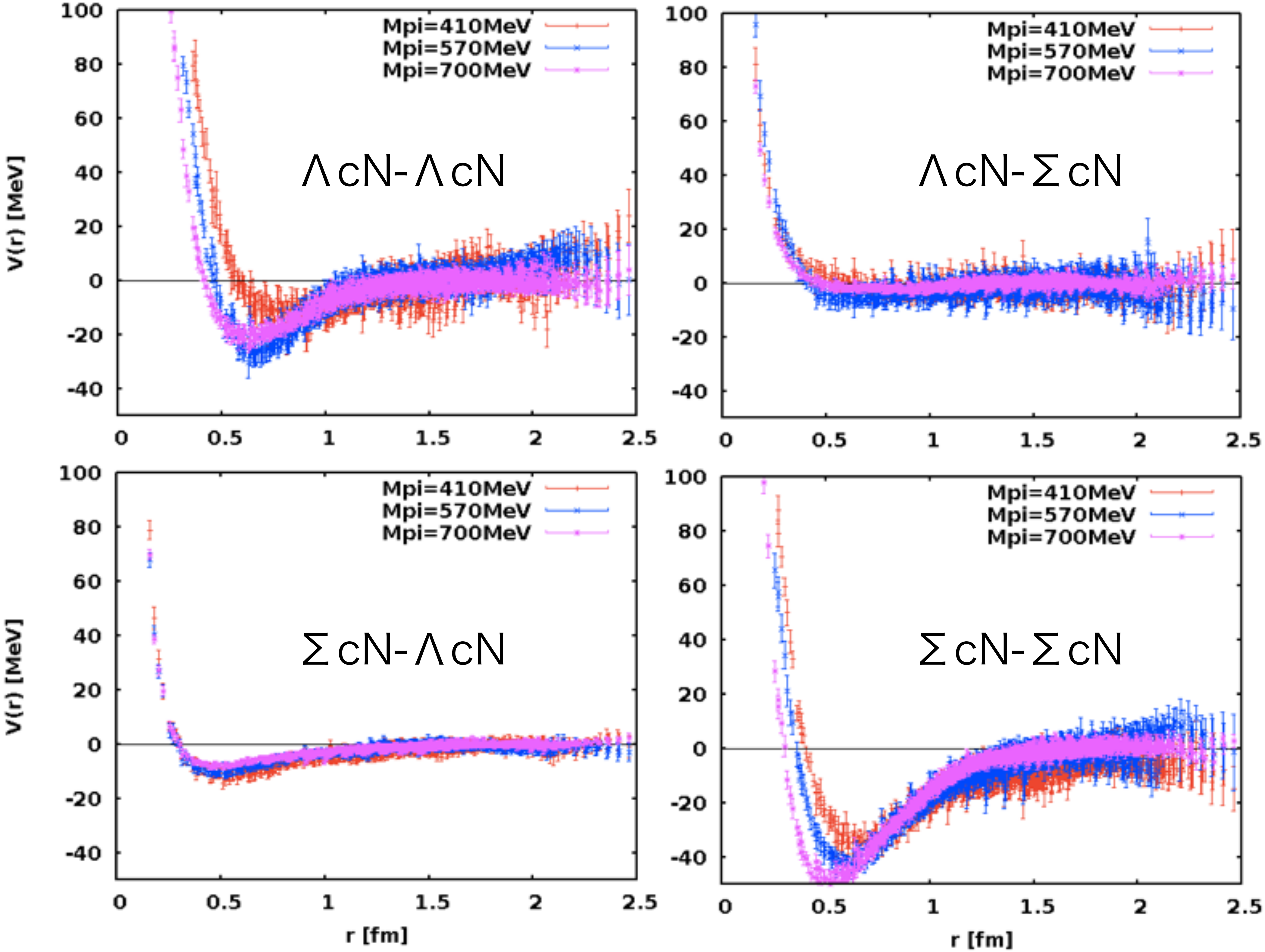
$$\left(E_n^{(1)} - H_0^{(1)}\right) \psi_n^{(1)}(\vec{r}) = V_{11}^{eff}(\vec{r})\psi_n^{(1)}(\vec{r}) + V_{12}^{eff}(\vec{r})\psi_n^{(2)}(\vec{r})$$

$$\left(E_n^{(2)} - H_0^{(2)}\right) \psi_n^{(2)}(\vec{r}) = V_{21}^{eff}(\vec{r})\psi_n^{(1)}(\vec{r}) + V_{22}^{eff}(\vec{r})\psi_n^{(2)}(\vec{r})$$

$\psi^{(1)} \equiv$ ΛcN

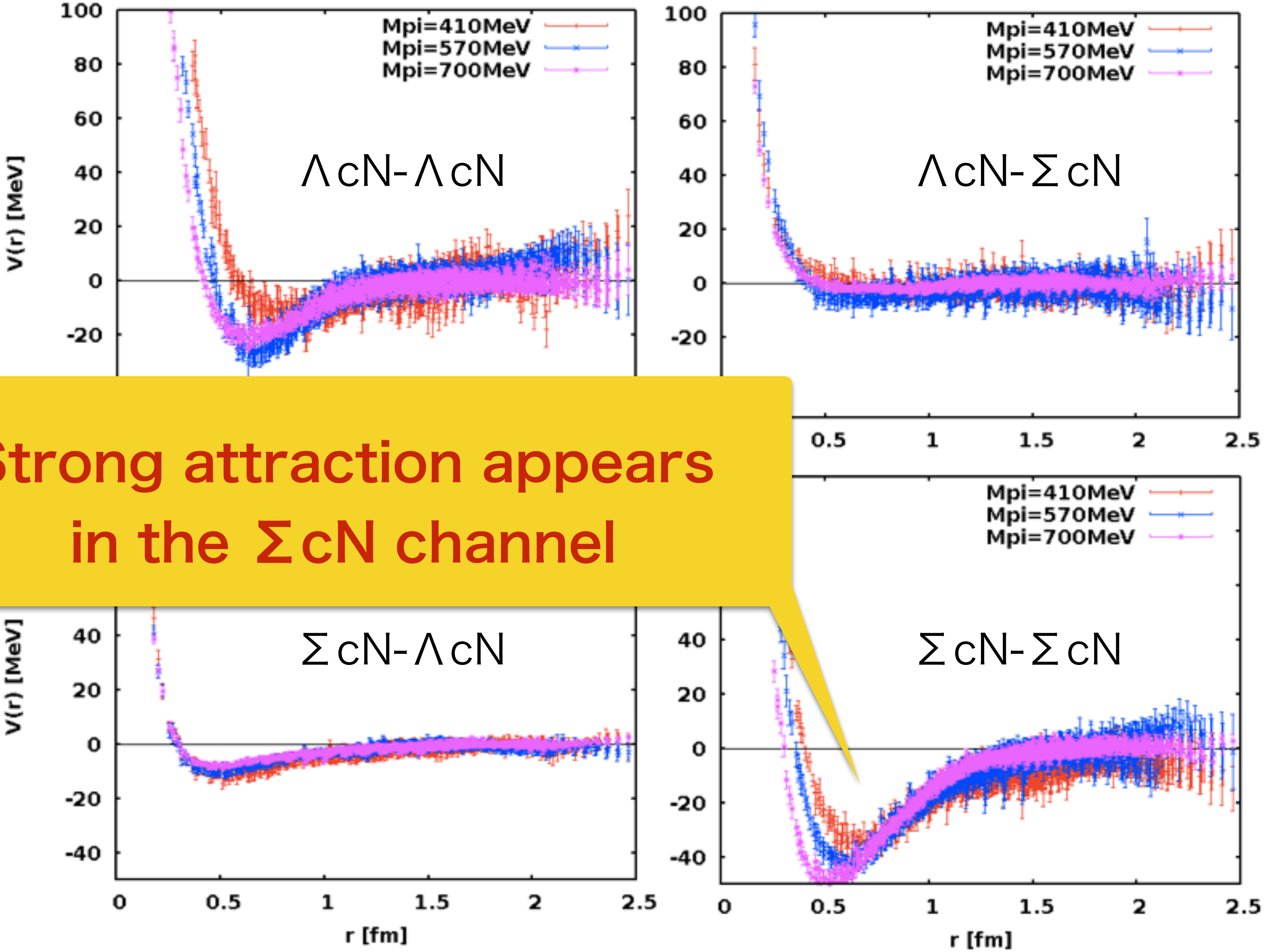
$\psi^{(2)} \equiv$ ΣcN

Numerical Results



Numerical Results

Mpi=410MeV
Mpi=570MeV
Mpi=700MeV

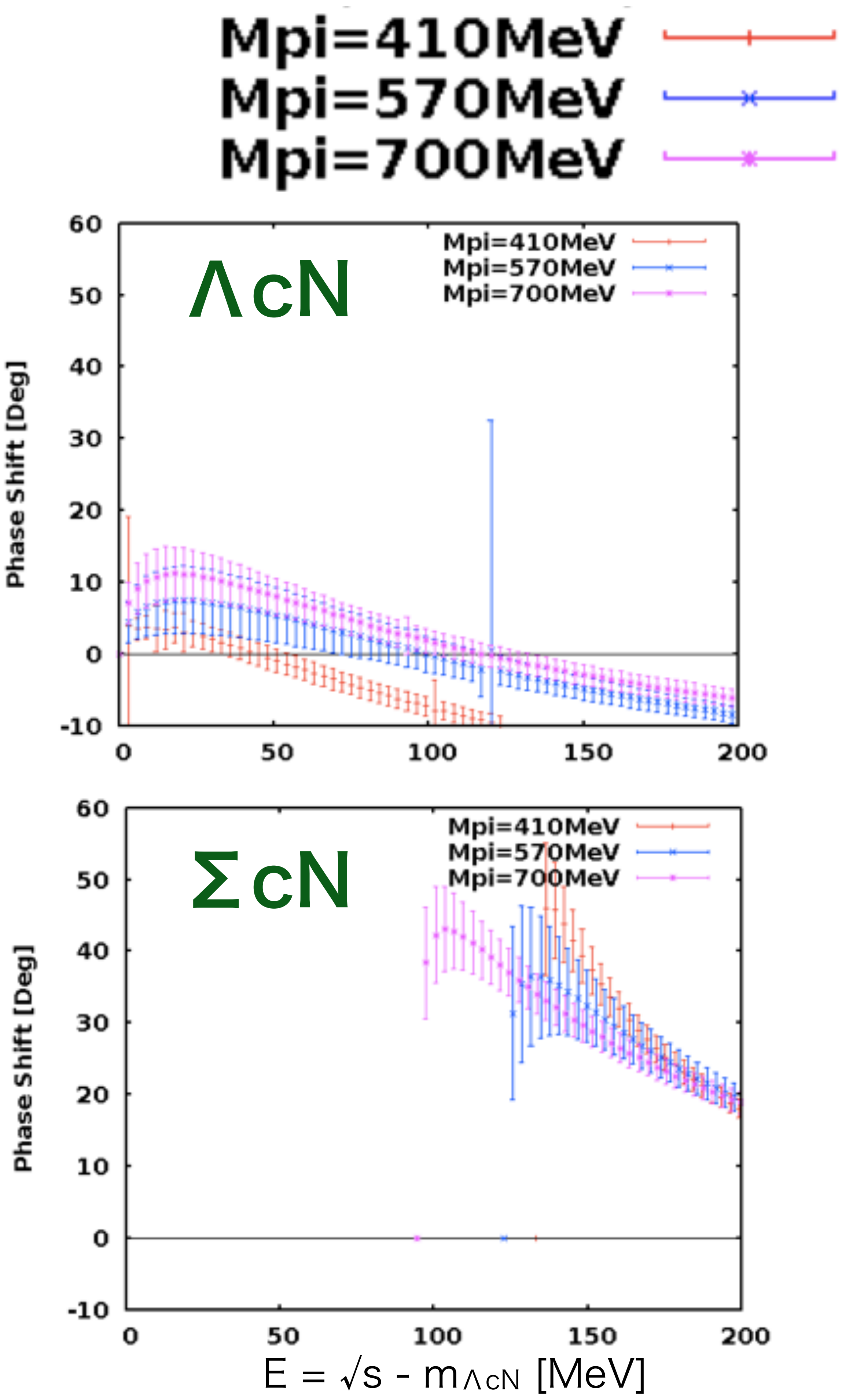


Strong attraction appears in the ΣcN channel

Phase shift

In order to calculate the phase shift, we solve the 2x2 coupled channel Schrödinger equation with HAL QCD potential.

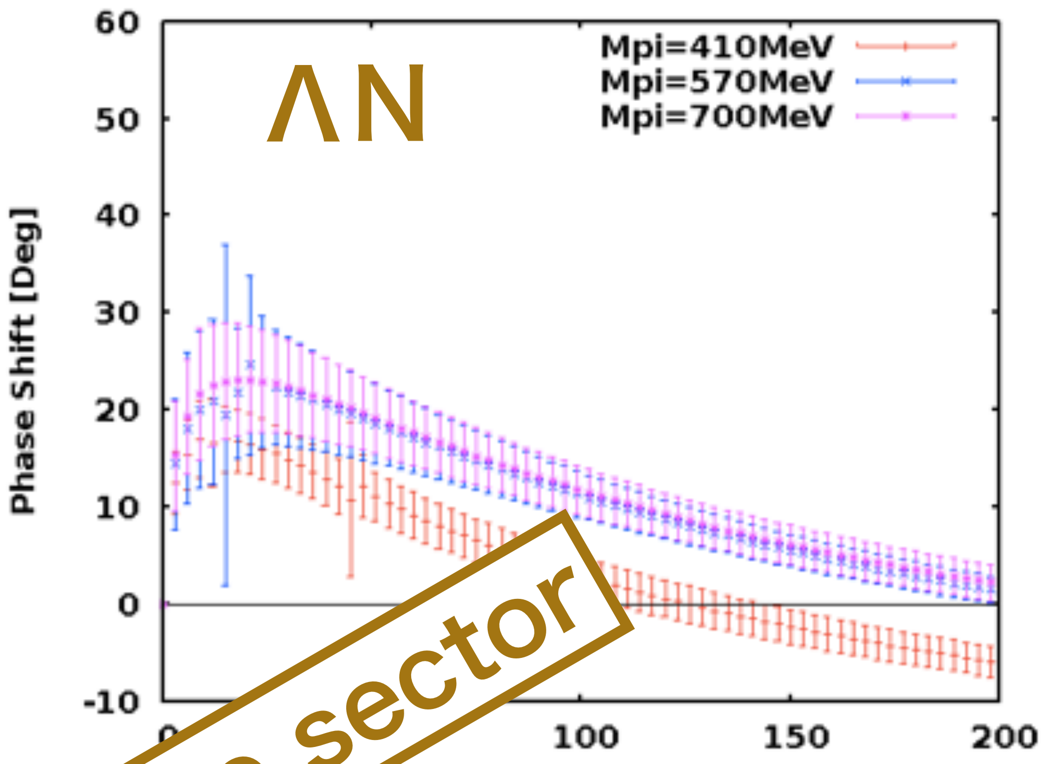
- Phase shift shows strong attraction in ΣcN channel.
- Quark mass dependence is not so strong.

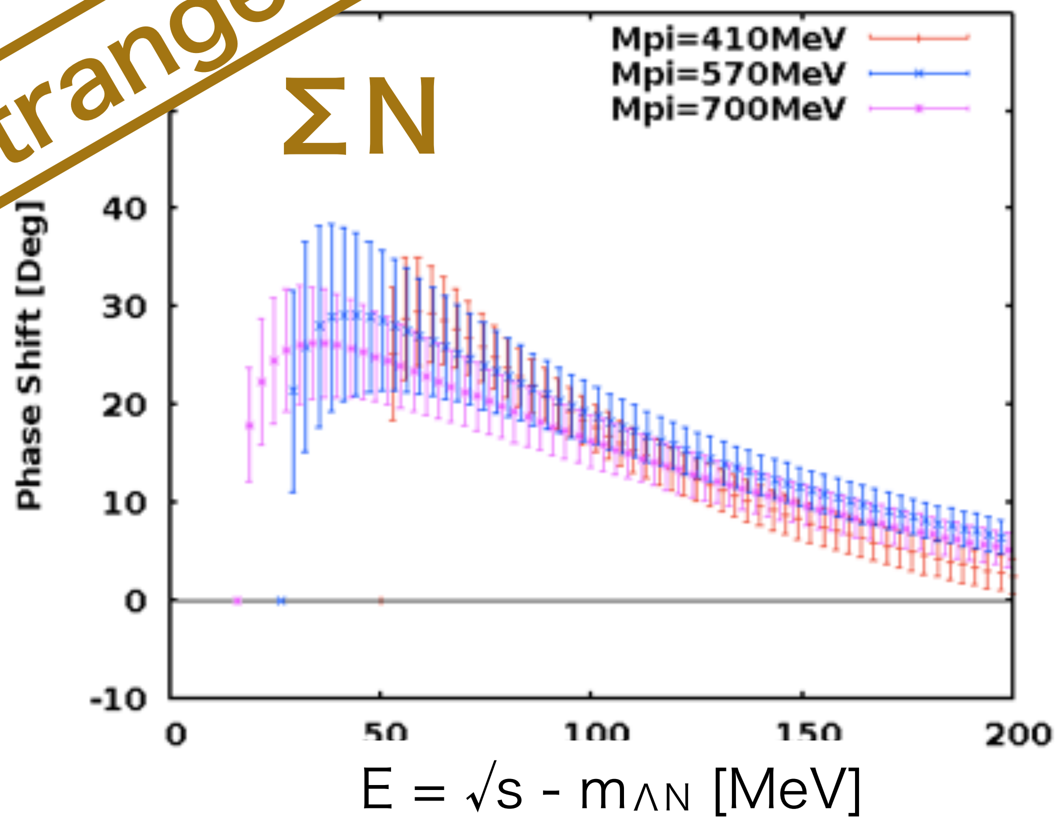


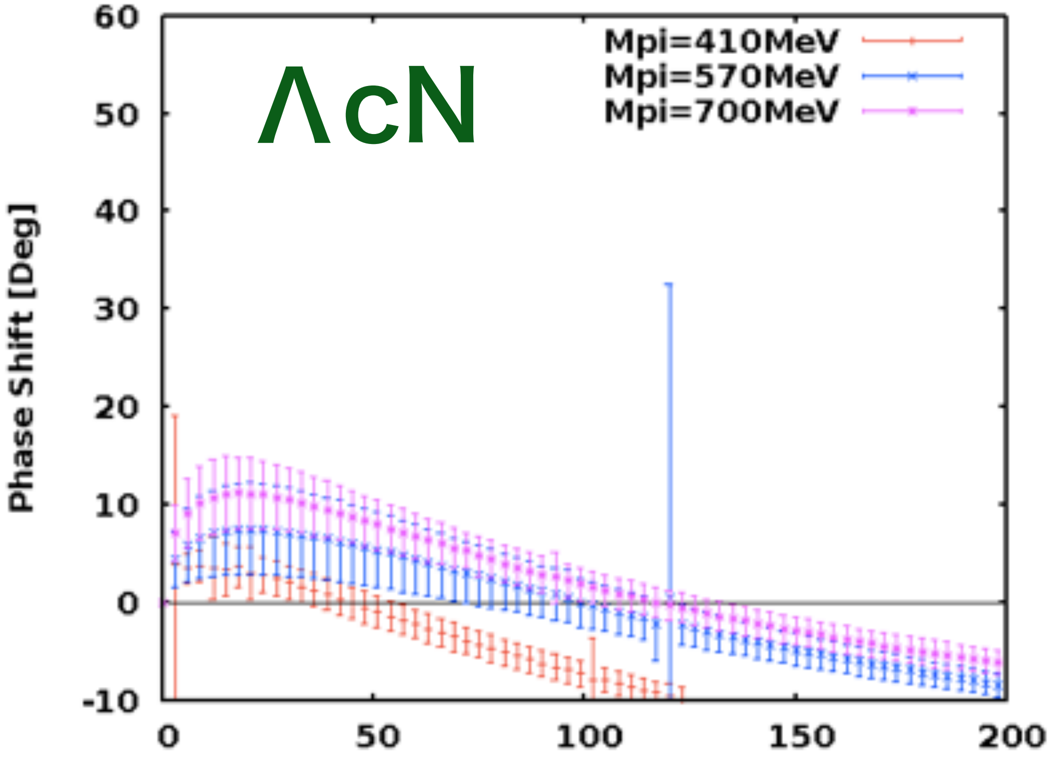
Phase shift

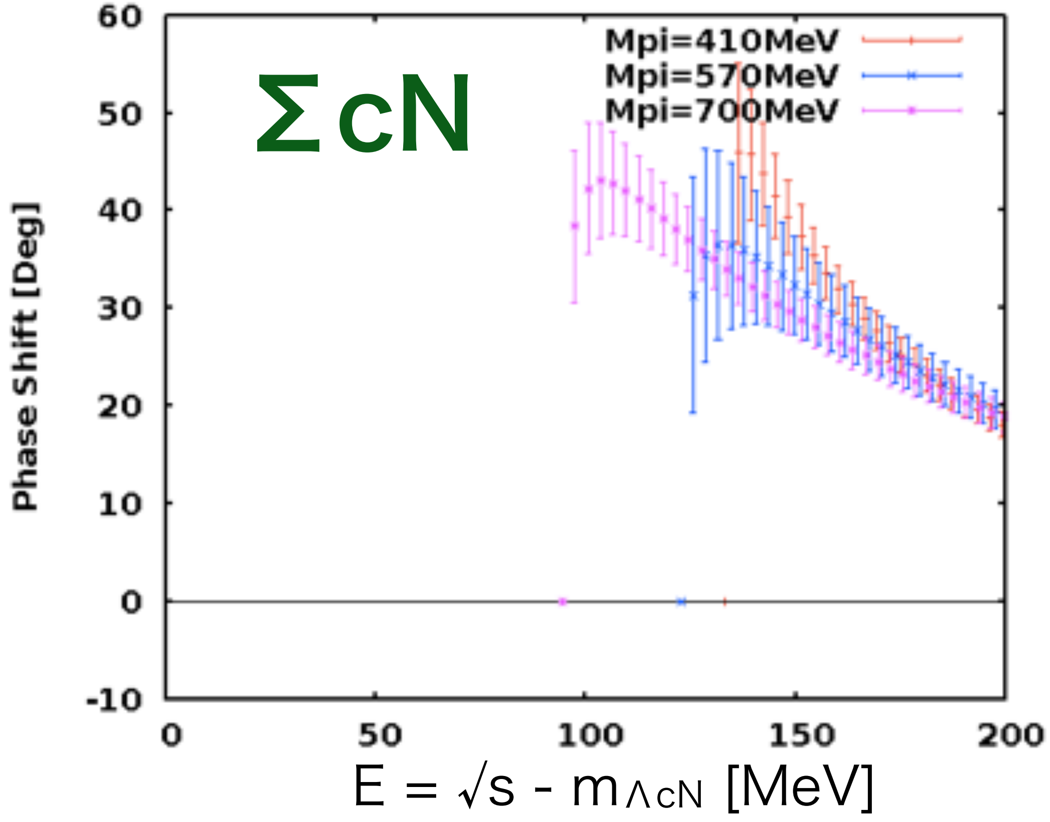
Mpi=410MeV
Mpi=570MeV
Mpi=700MeV

Strange sector

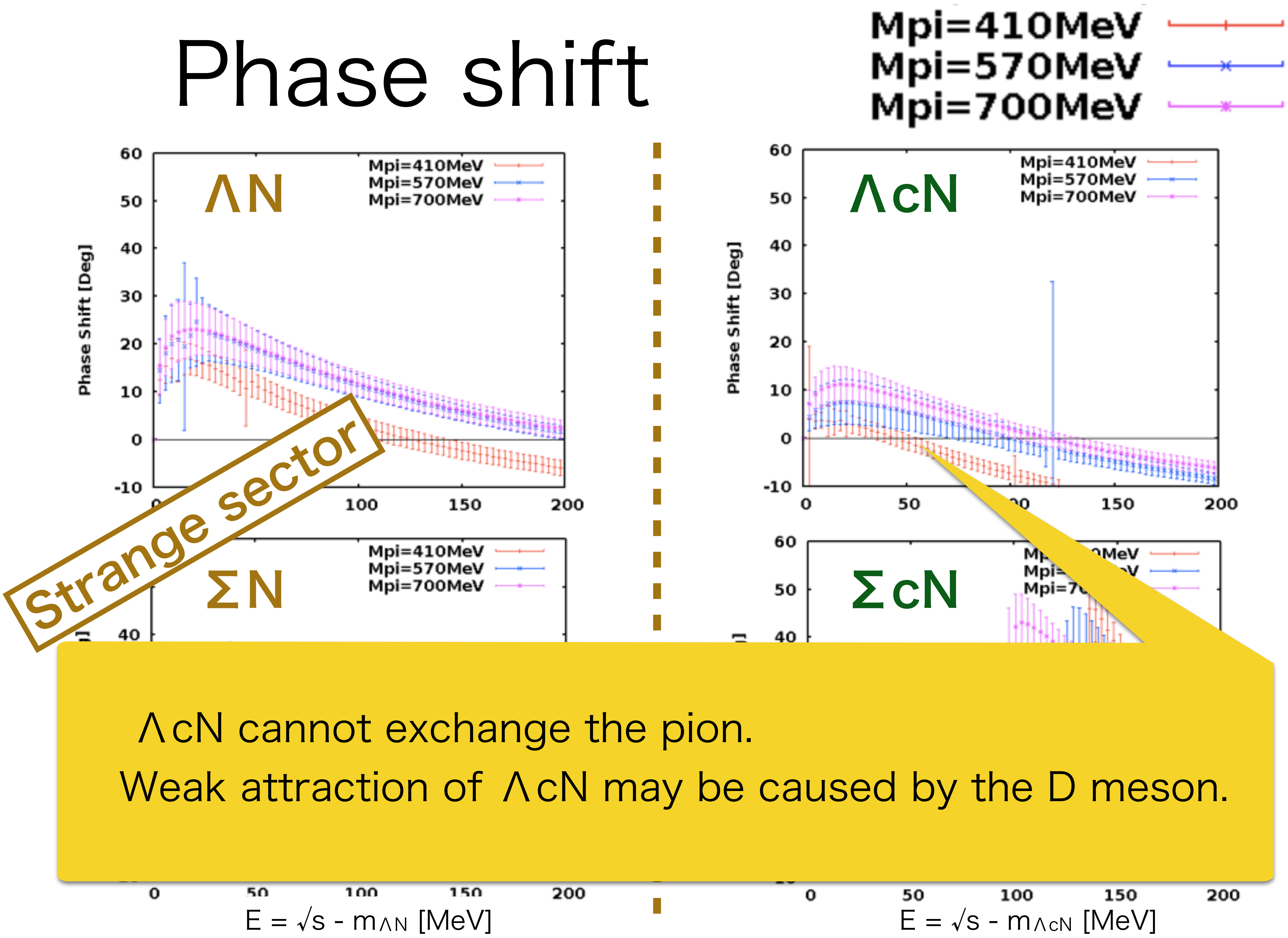








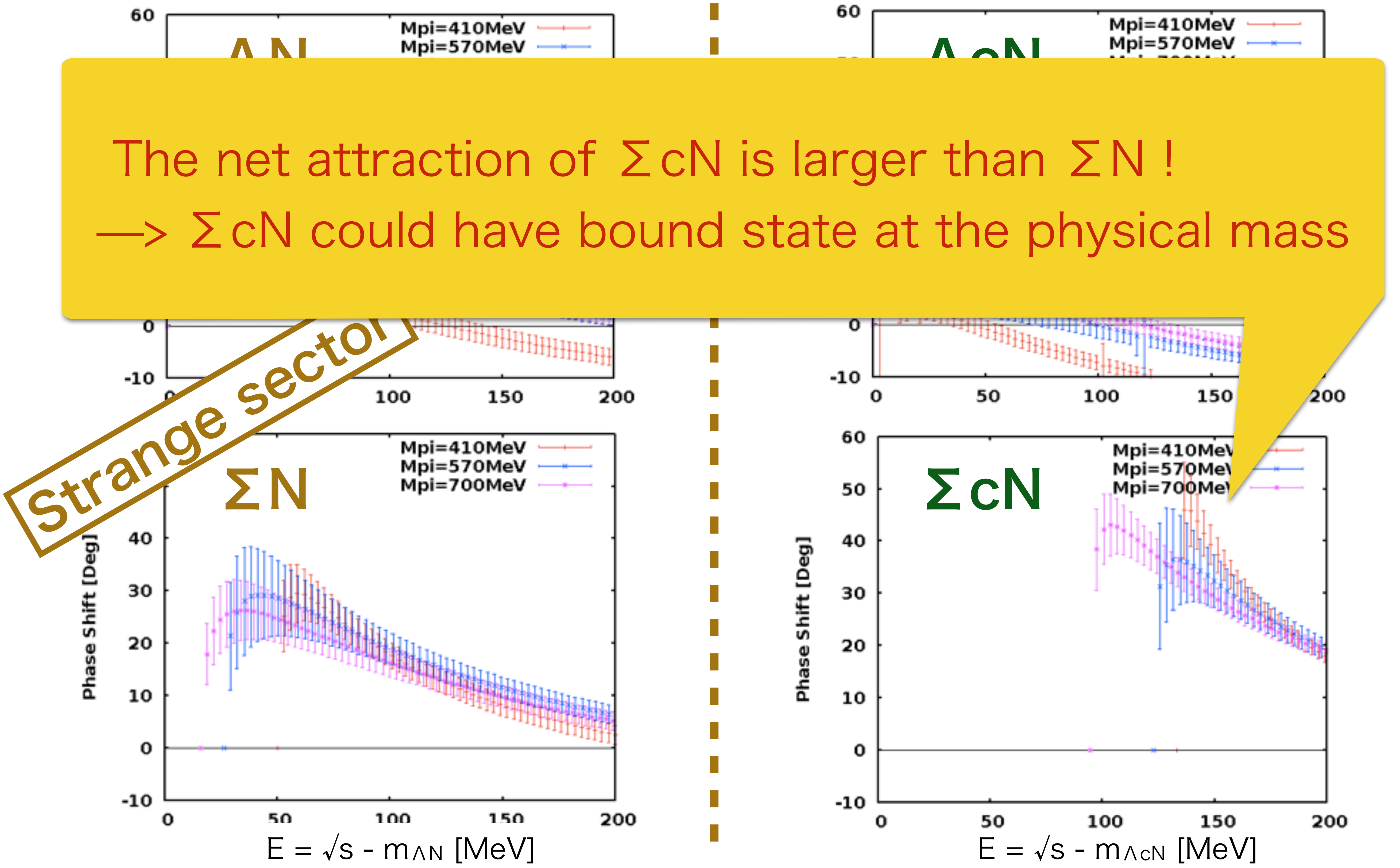
Phase shift



ΛcN cannot exchange the pion.

Weak attraction of ΛcN may be caused by the D meson.

Phase shift



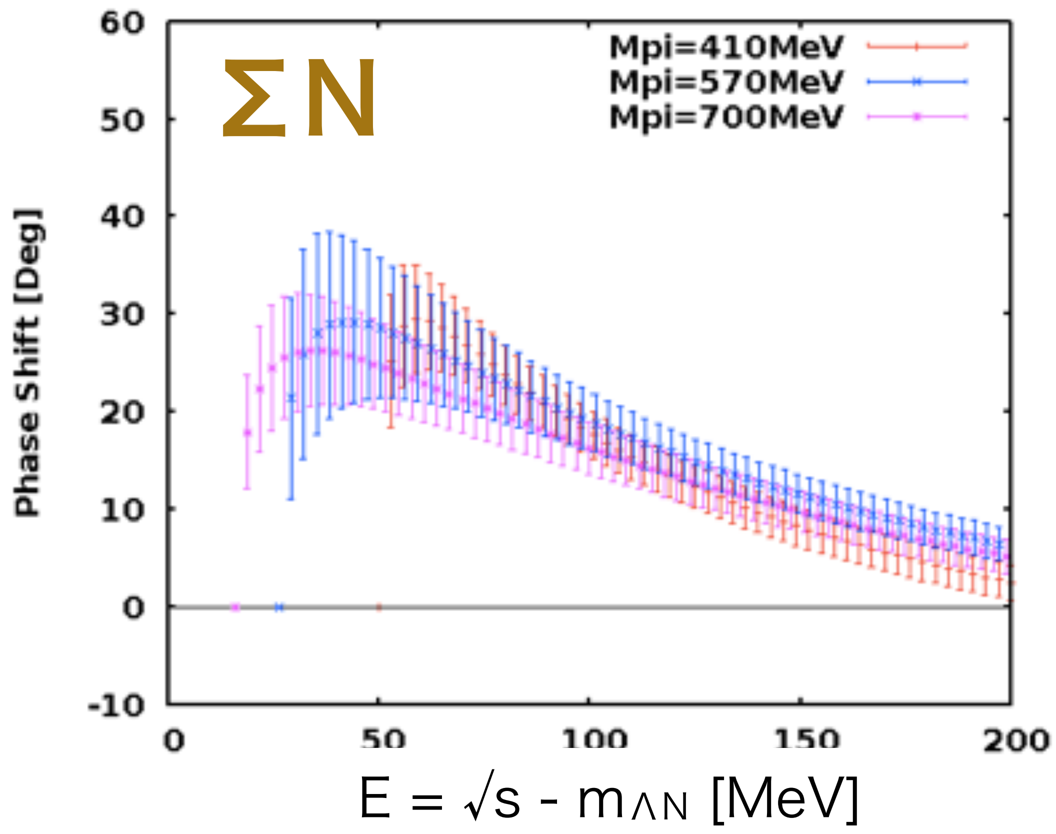
Strange sector

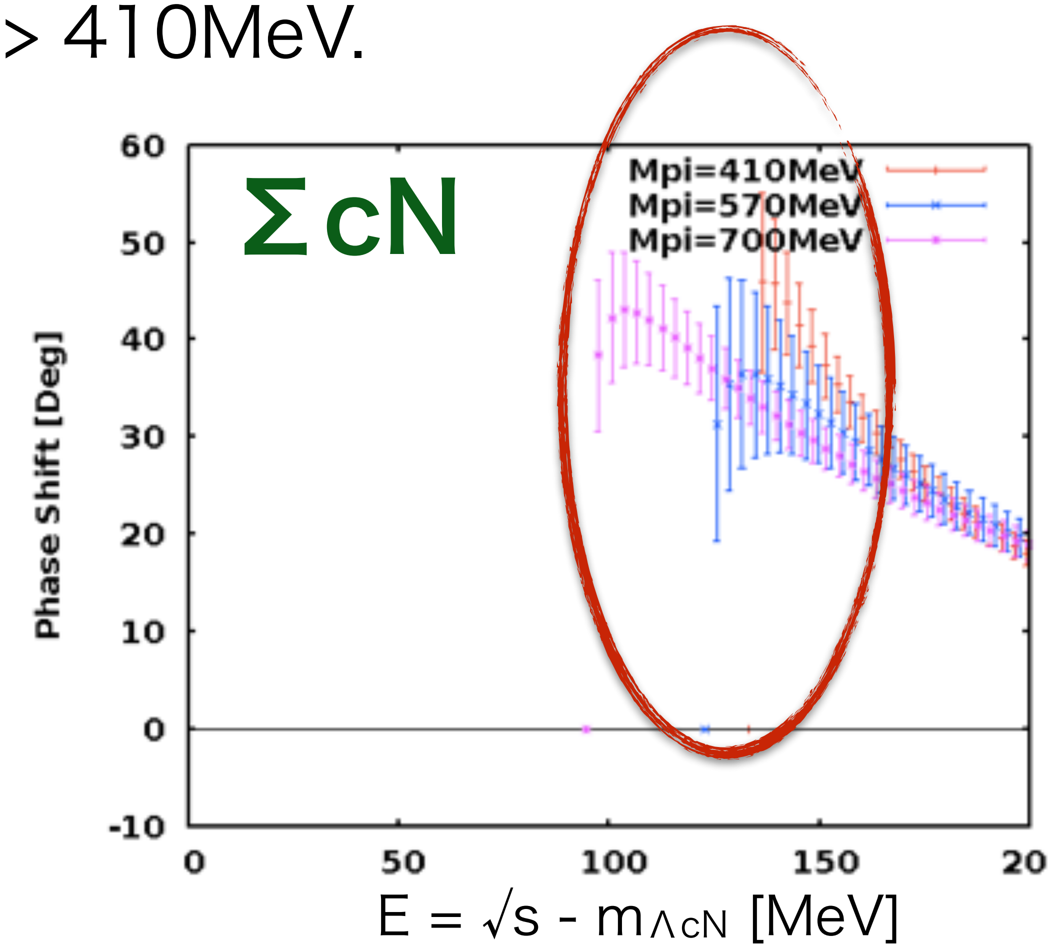
The net attraction of ΣcN is larger than ΣN !

—> ΣcN could have bound state at the physical mass

Summary & conclusion

- We investigate ΣcN interaction **by the HAL QCD method extended to the coupled channel potential**.
- ΣcN phase shift indicates the strong attractive but there is no bound state at least $m_\pi > 410$MeV.

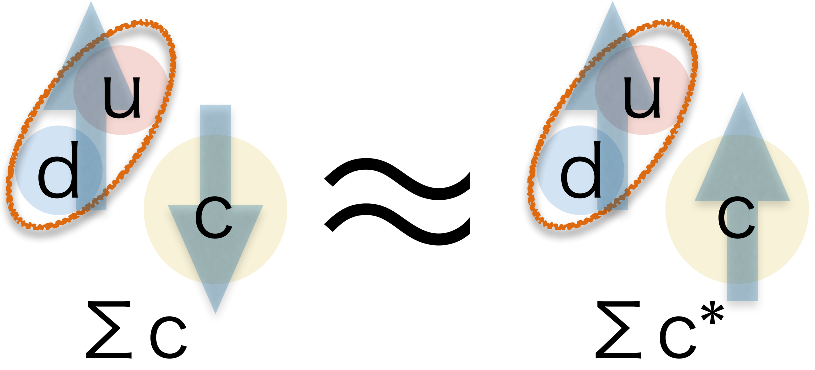




Future prospects

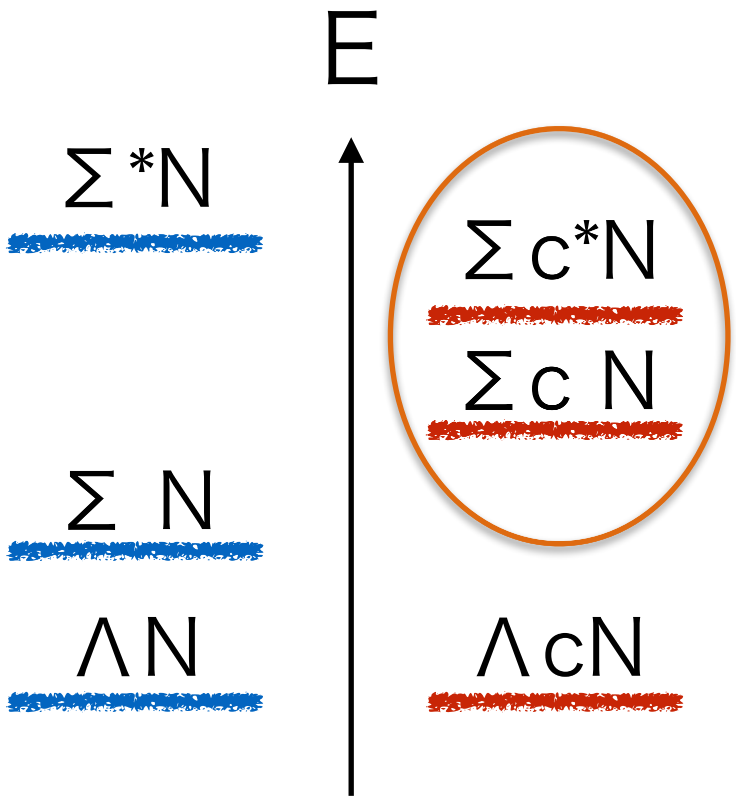
- Calculation at **physical mass**.

Heavy quark spin symmetry

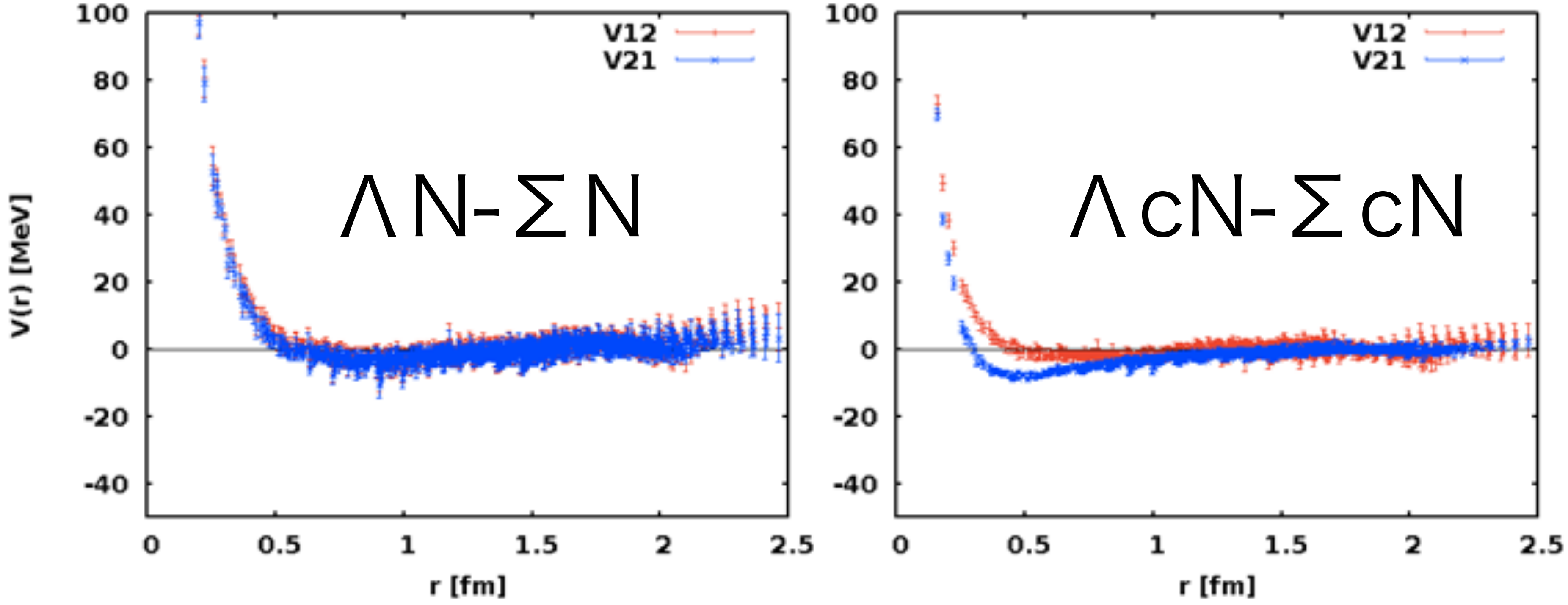


The spin flip amplitude of heavy quark **is suppressed**.

The mass splitting between ΣcN-Σc*N **become smaller.**



Off-diagonal components of potential



ΛcN-ΣcN off-diagonal component is non-hermite due to small mass splitting between Σc-Σc*

Summary & conclusion

- We investigate ΣcN interaction **by the HAL QCD method extended to the coupled channel potential**.

- ΣcN phase shift indicates the strong attraction although no bound state is found (at least $m_\pi > 410$MeV).

Future prospects

- Calculation at **physical mass**.
- Calculation of the **ΛcN-ΣcN-Σc*N 3x3 coupled channel potential** to investigate the effect of ΣcN-Σc*N transition.

Backup

Introduction

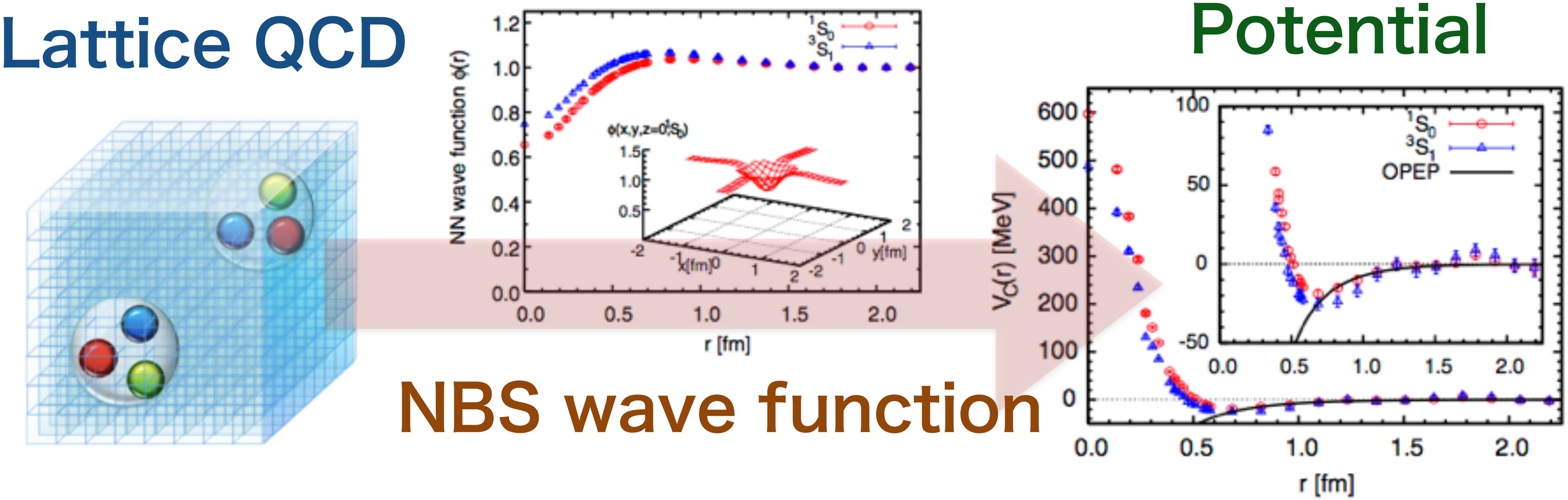
- It is difficult to get experimental data for heavy baryons, but easy to calculate on the lattice.

Lattice data can became **prediction**.

To extract the baryon interaction, we use **HAL QCD method**

Hadron force from lattice QCD

N. Ishii, S. Aoki, T. Hatsuda, Phys.Rev.Lett. 99, 022001 (2007)



Lattice QCD

NBS wave function

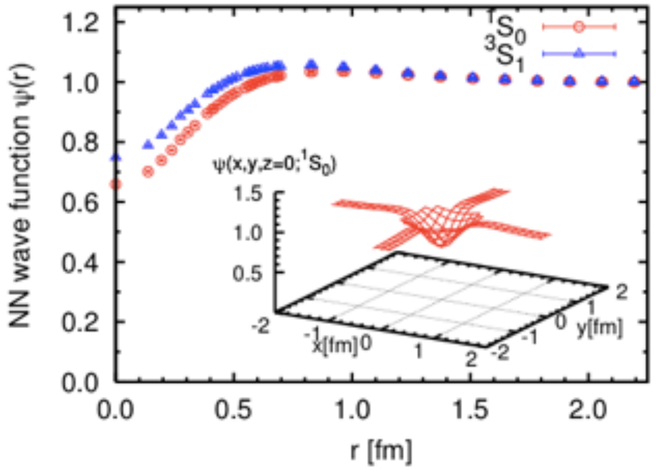
Potential

- Octet Baryon Interactions
- Decaplet Baryon Interactions
- N—Ω、Ω—Ω Interactions
- Meson Interactions, Meson-Baryon Interactions
- Three-body forces
- **Charmed Baryon Interactions** **<- This work !**

HAL QCD method

S. Aoki, T. Hatsuda, N. Ishii,

Prog. Theor. Phys., 123 (2010).



Nambu-Bethe-Salpeter (NBS) wave function

$$\psi^{(W)}_{\alpha\beta}(\vec{r})e^{-Wt} = \sum_{\vec{x}} \langle 0|T\{B^{(1)}_{\alpha}(\vec{r}+\vec{x},t)B^{(2)}_{\beta}(\vec{x},t)|B^{(1)}B^{(2)},W\rangle$$

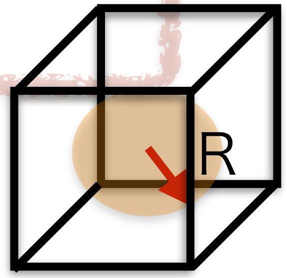
$$(k^2+\vec{\nabla}^2)\,\psi^{(W)}_{\alpha\beta}(\vec{r}) = 0 \qquad (\,|r| > \mathrm{R}\,)$$

$$\psi^{(W)}_{\alpha\beta}(\vec{r}) \propto \frac{\sin\left(kr - \frac{L\pi}{2} + \delta_{LS}(k)\right)}{kr}$$

same behavior with

QM wave function

at large r.



Hadron 4pt function on the Lattice

$$G_{\alpha\beta}(\vec{r},t-t_0) = \sum_{\vec{x}} \langle 0|T\{B^{(1)}_{\alpha}(\vec{r}+\vec{x},t)B^{(2)}_{\beta}(\vec{x},t)\overline{\mathcal{J}^{(1,2)}}(t_0)|0\rangle$$

$$= \sum_{n} A_n \psi^{(W_n)}_{\alpha\beta}(\vec{r})e^{-W_n(t-t_0)} + \cdots \xrightarrow{t\to\infty} A_0\psi^{(W_0)}_{\alpha\beta}(\vec{r})e^{-W_0(t-t_0)}$$

Extract NBS wave function

from **Hadron 4pt function.**

Using NBS wave function, define

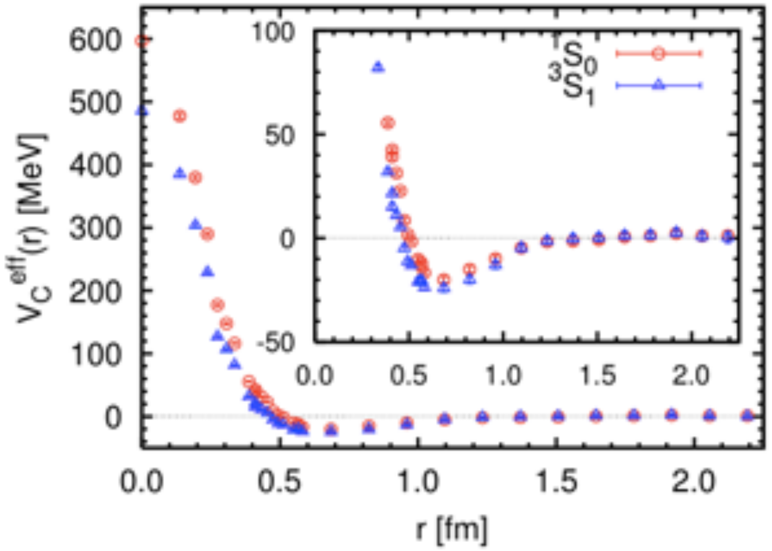
Energy-independent non-local potential

$$(E_n - H_0)\psi_n(\vec{r}) = \int U(\vec{r},\vec{r'})\psi_n(\vec{r'})d^3r'$$

$$U(\vec{r},\vec{r'}) = V(\vec{r},\vec{v})\delta^3(\vec{r}-\vec{r'})$$

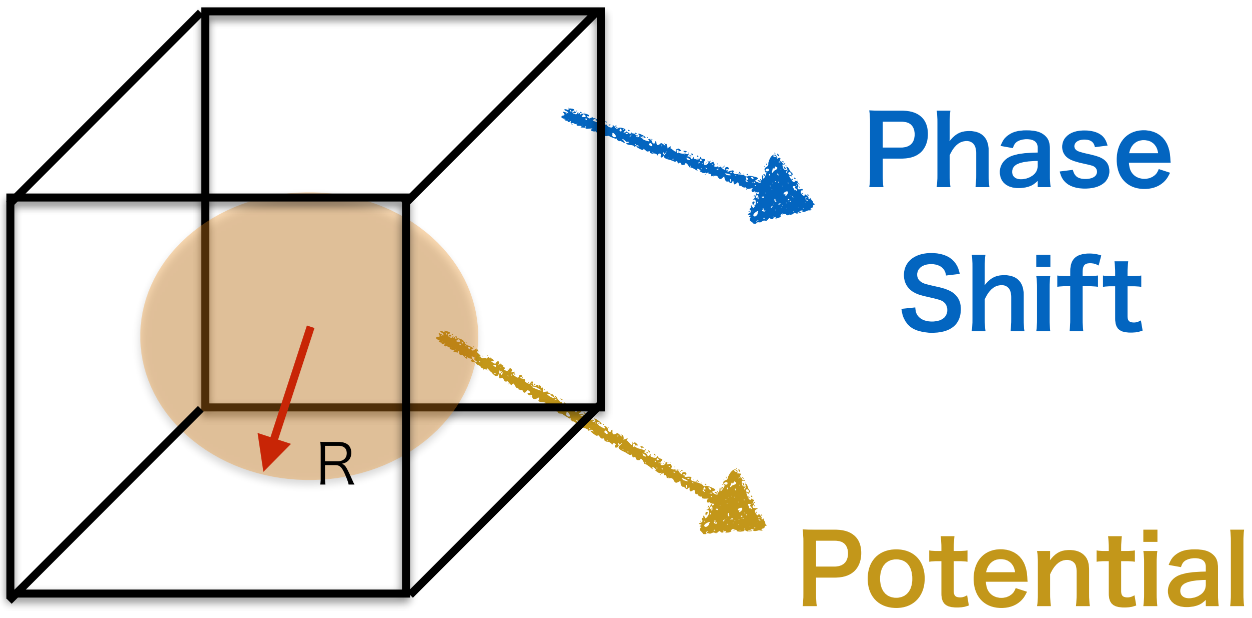
Velocity expansion

$$V(\vec{r},\vec{v}) = \underbrace{V_0(r) + V_\sigma(r)\vec{\sigma_1}\cdot\vec{\sigma_2} + V_T(r)S_{12}}_{\text{LO}} + \underbrace{V_{LS}(r)\vec{L}\cdot\vec{S}}_{\text{NLO}} + \mathcal{O}(v^2)$$



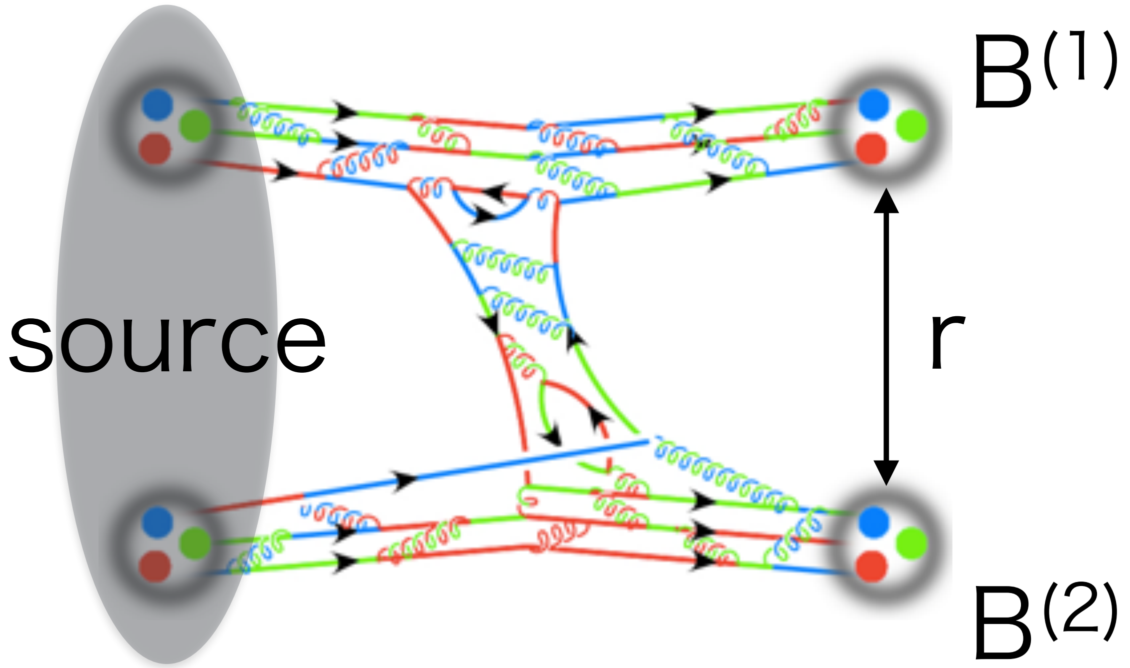
HAL QCD method

S. Aoki, T. Hatsuda, N. Ishii,
Prog. Theor. Phys., 123 (2010).



Phase Shift

Potential



$$\psi^{(W)}_{\alpha\beta}(\vec{r})e^{-Wt} = \sum_{\vec{x}}\langle 0|T\{B^{(1)}_{\alpha}(\vec{r}+\vec{x},t)B^{(2)}_{\beta}(\vec{x},t)|B^{(1)}B^{(2)},W\rangle$$

$$B^{(p)}_{\alpha}(\vec{x}) = \epsilon_{ijk}\left(u_i(\vec{x})C\gamma_5 d_j(\vec{x})\right)u_{k\alpha}(\vec{x})$$

$$C \equiv \gamma_2\gamma_4$$

The behavior at large R

$$(k^2+\vec{\nabla}^2)\,\psi^{(W)}_{\alpha\beta}(\vec{r}) = 0 \qquad (\,|r| > \mathrm{R}\,)$$

$$\psi^{(W)}_{\alpha\beta}(\vec{r}) \propto \frac{\sin\left(kr-\frac{L\pi}{2}+\delta_{LS}(k)\right)}{kr}$$

is the same with **QM wave function**

The potential is constructed so as to reproduce **the QM phase shift**.

HAL QCD method

S. Aoki, T. Hatsuda, N. Ishii,
Prog. Theor. Phys., 123 (2010).

When we extract the NBS wave function from the hadron-4pt correlation function, we use **Time-dependent HAL QCD method**.

N. Ishii et al [HAL QCD Coll.], PLB712 (2012) 437.

$$R_{\alpha\beta}(\vec{r},t) = \frac{G_{\alpha\beta}(\vec{r},t)}{e^{-(m_{B^{(1)}}+m_{B^{(2)}})t}} = \sum_n A_n \psi_{\alpha\beta}(\vec{r},t) e^{-\Delta W_n t} + \cdots$$

$$\Delta W \equiv \sqrt{k_n^2 + m_{B^{(1)}}^2} + \sqrt{k_n^2 + m_{B^{(2)}}^2} - (m_{B^{(1)}} + m_{B^{(2)})}$$

$$(E_n - H_0)\,\psi_n(\vec{r}) = \int d^3r' U(\vec{r},\vec{r}\,')\psi_n(\vec{r}\,')$$

at $m_{B^{(1)}} = m_{B^{(2)}} \equiv m$

$$E_n = (\Delta W) + \frac{1}{4m}(\Delta W)^2$$

Construct the potential
using all elastic states.

$$\left(-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2} - H_0\right) R(\vec{r},t) = \int d^3r' U(\vec{r},\vec{r}\,')R(\vec{r}\,',t)$$

The ground state saturation
is not necessary.

Energy independent, non-local potential

$$(E_n - H_0)\psi_n(\vec{r}) = \int U(\vec{r},\vec{r}')\psi_n(\vec{r}')d^3r'$$

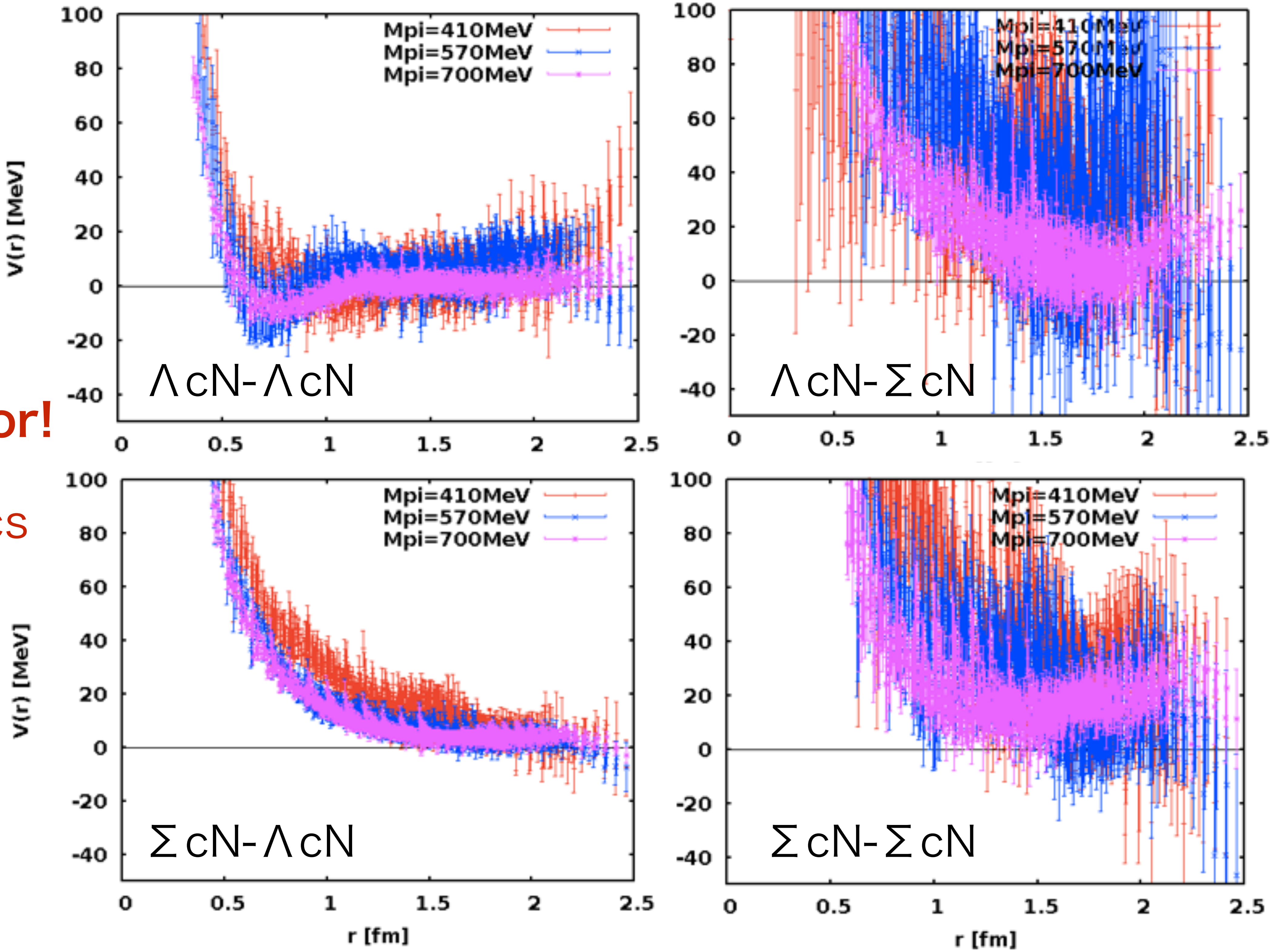
Below the threshold of inelastic scattering,
(The energy of threshold is defined by E_{n_c})

$$U(\vec{r},\vec{r'}) = \sum_{nn'}^{n_c} K_n(\vec{r})\mathcal{N}^{-1}_{nn'}\psi^*_{n'}(\vec{r'})$$

$$\begin{aligned}
&\int U(\vec{r},\vec{r'})\psi_n(\vec{r'})d^3r' \\
&= \int \left[\sum_{n'n''}^{n_c} K_{n'}(\vec{r})\mathcal{N}^{-1}_{n'n''}\psi^*_{n''}(\vec{r'})\right]\psi_n(\vec{r'})d^3r' \\
&= \sum_{n'n''}^{n_c} K_{n'}(\vec{r})\mathcal{N}^{-1}_{n'n''}\int \psi^*_{n''}(\vec{r'})\psi_n(\vec{r'})d^3r' \\
&= \sum_{n'n''}^{n_c} K_{n'}(\vec{r})\mathcal{N}^{-1}_{n'n''}\mathcal{N}_{n''n} \\
&= \sum_{n'}^{n_c} K_{n'}(\vec{r})\delta_{n'n} \\
&= K_n(\vec{r}) = (E_n - H_0)\psi_n(\vec{r})
\end{aligned}$$

$$\left(\begin{array}{l}
K_n(\vec{r}) = (E_n - H_0)\psi_n(\vec{r}) \\
\\
N_{nn'} = \int \psi^*_n(\vec{r})\psi_{n'}(\vec{r})d^3r
\end{array}\right)$$

Results $J^P = 0^+$ case



Too large error!

More statistics are needed.

ΛcN channel

[T.M, for HAL QCD Collaboration]

- ΛcN single channel

— Short range repulsive core

— Mid range attractive pocket

— No bound state at least

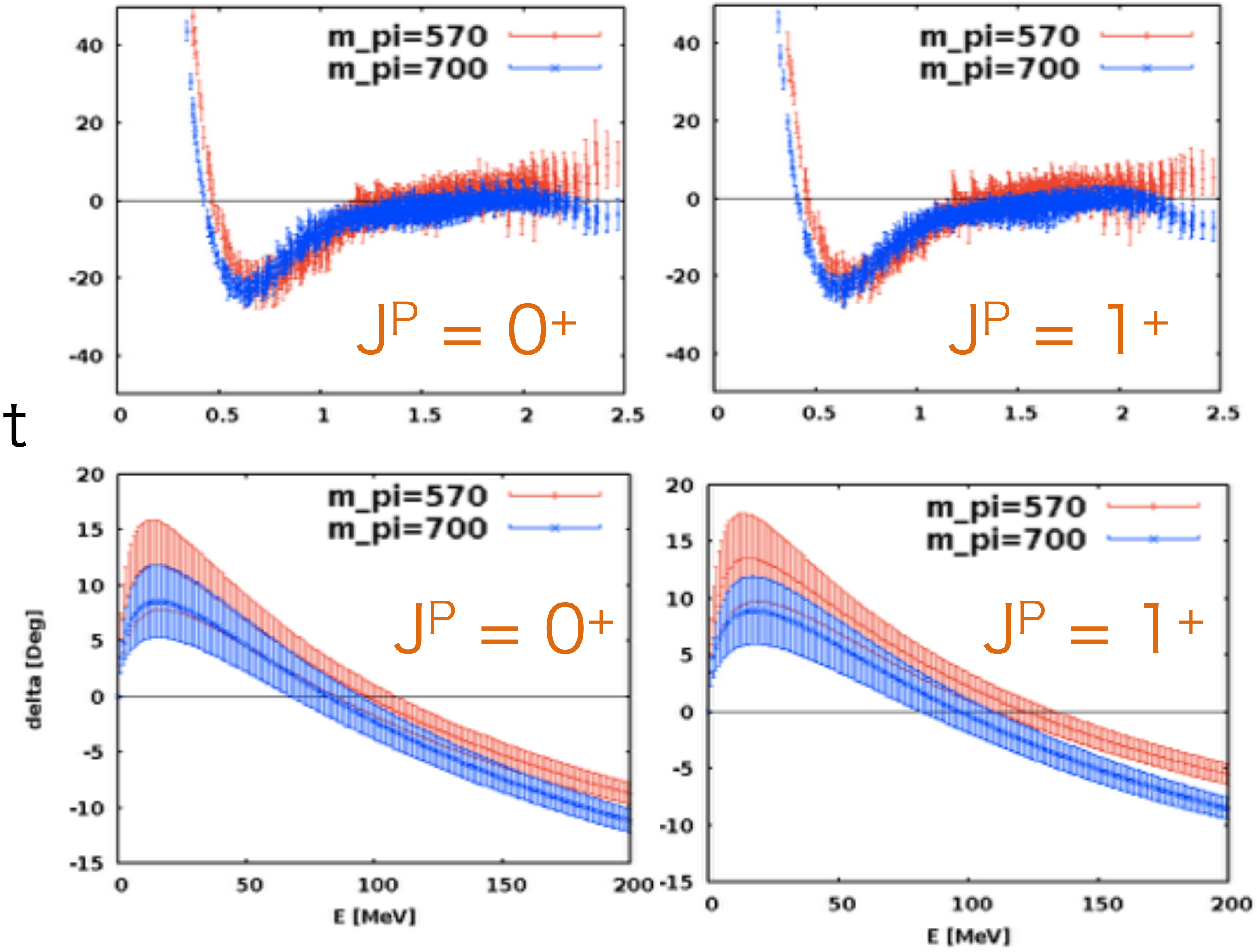
m_π > 570 MeV.

The pion exchange

ΛcN channel : absent

ΣcN channel : present

ΣcN channel could be more attractive.



	$J^P = 0^+$	$J^P = 1^+$
m_π=700MeV:	a = 0.42(13) fm	a = 0.49(15) fm
m_π=570MeV:	a = 0.21(08) fm	a = 0.21(07) fm