Calculation of Quark Condensates and Chirality using Improved Staggered Fermions

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Introduction

- We study quark condensates using improved staggered fermions.
- We search zero modes of the Dirac operator to subtract out their contributions from the quark condensate, in which they give simple poles in the chiral limit.
- We also identify the quantum numbers of those zero modes using spectral flow method in the staggered fermion formalism.

Quark Condensate for Staggered Fermions

- quark condensate :

$$\langle \bar\psi\psi \rangle = -\frac{1}{VN_t}\left\langle \mathrm{Tr}\,\frac{1}{D_s+m}\right\rangle_U \tag{1}$$

  - D_s : staggered Dirac operator, m : quark mass
  - V : lattice volume, N_t : number of tastes, U : gauge field,
  - trace is over space-time, color, taste

- $D_s^\dagger = -D_s$: eigenvalues are purely imaginary or zero.

$$D_s f_\lambda(x) = i\lambda f_\lambda(x) \tag{2}$$

- $\varepsilon D_s = -D_s\varepsilon$ for $\varepsilon = [\gamma_5 \otimes \xi_5]$: nonzero eigenvalues exist as $\pm$ pairs.
- In terms of the eigenvalues,

$$\langle \bar\psi\psi \rangle = -\frac{1}{VN_t}\sum_\lambda \frac{1}{i\lambda+m} \quad \text{simple pole as } m\to 0 \tag{3}$$

$$= -\frac{1}{VN_t}\sum_{\lambda>0}\frac{2m}{\lambda^2+m^2} - \boxed{\frac{\tilde n_+ + \tilde n_-}{VN_t\, m}} \tag{4}$$

where $\tilde n_\pm$ are the numbers of zero modes with $P_\pm = \frac{1\pm\varepsilon}{2}$ projection.

- To subtract out the simple poles, we need to find the zero modes of D_s.
- One can show that the total number of the zero modes of D_s is equal to that of the zero modes of the continuum Dirac operator D. ([1])

$$\tilde n_+ + \tilde n_- = n_+ + n_- \tag{5}$$

where n_+ and n_- are the number of right-handed and left-handed zero modes of D, respectively.

Spectral Flow

- Consider a *Hermitian* operator H_s:

$$H_s \equiv -iD_s + \mu[\gamma_5 \otimes \mathbb{1}] \tag{6}$$

- Eigenvalues of H_s are real: $H_s f_s(x,\mu) = \lambda_s(\mu) f_s(x,\mu)$.

$$\lambda_s(\mu) = \langle f_s(\mu)|H_s|f_s(\mu)\rangle \tag{7}$$

- $\lambda_s(\mu=0) = \lambda$ and $f_s(x,\mu=0) = f_\lambda(x)$ for λ and f_λ in Eq. (2).
- Taking derivative with respect to μ,

$$\lambda_s'(\mu) = \langle f_s(\mu)|[\gamma_5\otimes\mathbb{1}]|f_s(\mu)\rangle \tag{8}$$

- We define the chirality of the zero modes by

$$\mathrm{sign}\left(\lim_{\lambda\to 0}\left[\lim_{\mu\to 0}\lambda_s'(\mu)\right]\right) = \pm 1\,, \tag{9}$$

- We can determine the chirality by **spectral flow** method.

Chirality of Zero Modes

- $Z = \pm 1$: quantum number of $\varepsilon = [\gamma_5 \otimes \xi_5]$
  $X = \pm 1$: quantum number of $[\gamma_5 \otimes \mathbb{1}]$
  $Y = \pm 1$: quantum number of $[\mathbb{1} \otimes \xi_5]$

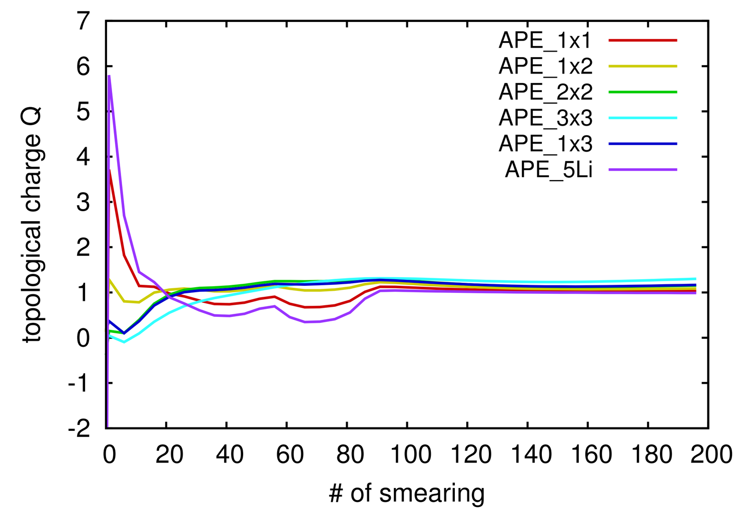
n_{XY}	X	Y	$Z = XY$
n_{++}	$+1$	$+1$	$+1$
n_{+-}	$+1$	-1	-1
n_{-+}	-1	$+1$	-1
n_{--}	-1	-1	$+1$

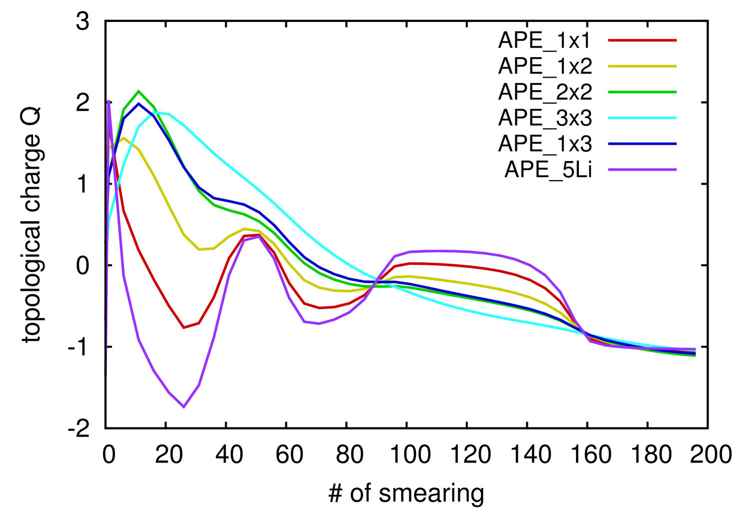
n_{XY} is the number of zero modes with X and Y quantum numbers

- We will measure the entries of this table by spectral flow method.

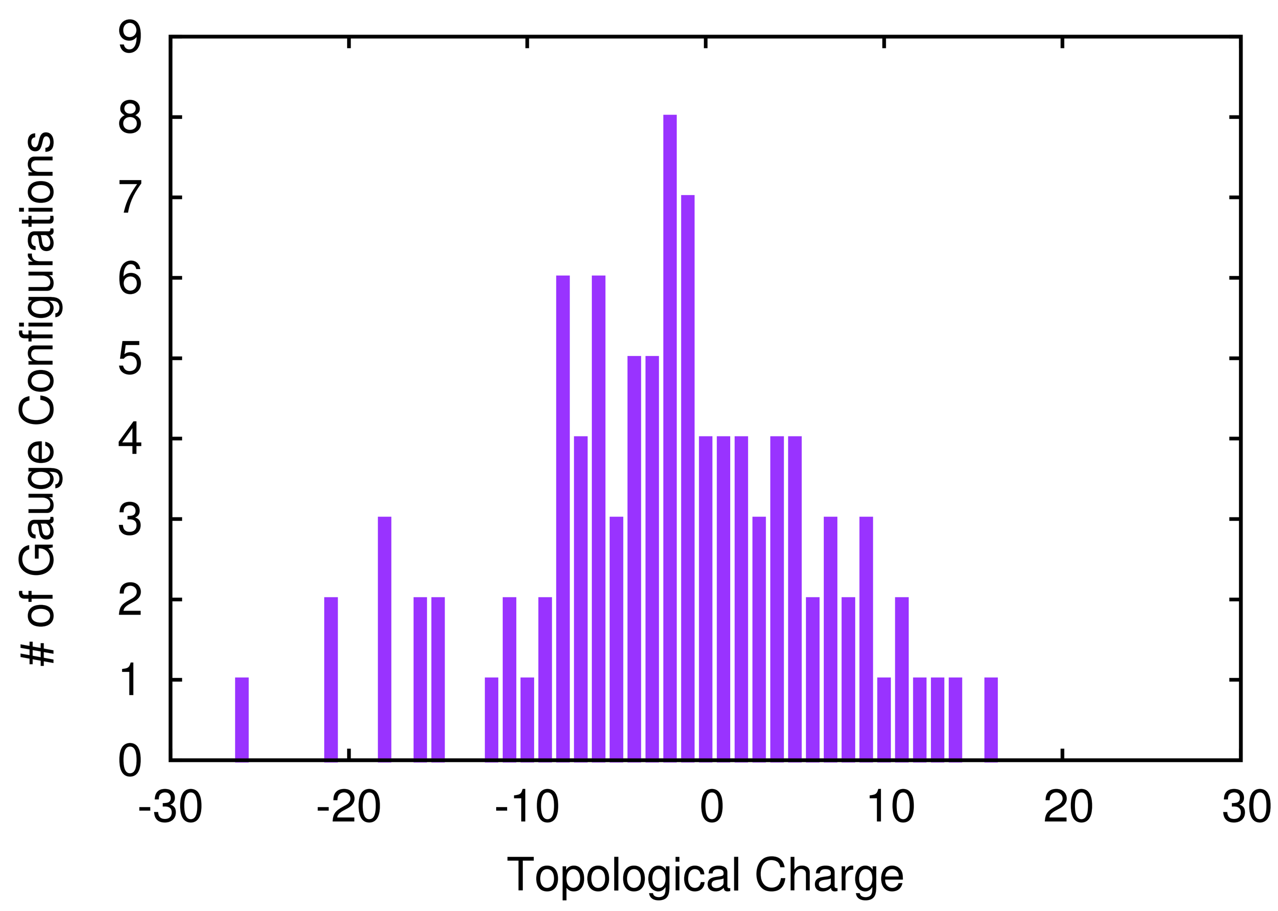
Topological Charge

- $Q = \frac{1}{32\pi^2}\int d^4x\, \varepsilon_{\mu\nu\rho\sigma}\,\mathrm{tr}\,[F_{\mu\nu}(x)F_{\rho\sigma}(x)] = n_+ - n_-$
- can be measured from the gauge field configuration with proper smoothing. We used APE smearing with the parameter $\alpha = 0.45$ ([2]) (In plots, APE_MxN means that $F_{\mu\nu}$ is calculated from $M\times N$ rectangular clover leaves, and 5Li means 5 Loop improvement in [3].)





- topological charge distribution for a $20^3 \times 64$ MILC asqtad lattice



- We can compare the results with those by the spectral flow method

References

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[3] P. de Forcrand, M. Garcia Perez, and I.-O. Stamatescu *Nucl. Phys.* **B499** (1997) 409–449, [hep-lat/9701012].