Chiral Magnetic Effect

Chiral fermion + Magnetic field $\rightarrow$ Anomaly related charge transport

[Chiral Magnetic Effect Diagram]

Scattering $\rightarrow$ Balance $\rightarrow$ Drift

$T$ determines the size of CME.

There is no explicit calculation for $T$ including relativistic effect.

cf. NR[P. N. Argyres, E. N. Adams, 1956]

Goal: $T$ for relativistic fermions in $eB \to \infty, T = 0$

1. Electric Current and Relaxation Time

Start from the thermodynamical equilibrium:

$$f_0(p) = \frac{1}{1 + \exp \left( \frac{\mu - \epsilon}{k_B T} \right)}$$

In the small $E$, $f = f_0 + \delta f$ $E$ accelerate the electron

$$\left( \frac{\partial f}{\partial t} \right)_{\text{drift}} = eE \frac{\partial f}{\partial p_z}$$

Electrons scattered back.

Relaxation time app.: $\left( \frac{\partial f}{\partial t} \right)_{\text{coll}} = -\frac{1}{T} \delta f$

Static Boltzmann equation:

$$\left( \frac{\partial f}{\partial t} \right)_{\text{drift}} = -\left( \frac{\partial f}{\partial t} \right)_{\text{coll}}$$

Solution, lowest in $E$:

$$f(p) = f_0(p) + eE_T \frac{\partial f_0}{\partial p_z}$$

Current density:

$$J = \frac{1}{e} \frac{e^2}{6\pi^2} E \mu^2 \tau$$

$T$ determines the size of the current.

$T$ in terms of transition rate: $W(p \to p')$

$$\frac{1}{\tau} = \sum_{p'} \frac{p_z - p_z'}{p_z} W(p \to p')$$

In $T=0$ case, phonon scattering is negligible.

Consider only ionized impurity scattering:

$$v(r) = \frac{e^2}{\kappa} e^{-r/\kappa}$$

2. Our Study of Relaxation Time

Dirac equation: $[i\gamma \partial_0 - m + \gamma^0 \gamma^i D_i + m \gamma^0] \Phi = 0$

Solution is expressed as

$$(x|\psi) = \Psi(x) = [i\partial_0 + \mu - i\gamma^0 \gamma^i D_i + m \gamma^0] \Phi$$

Auxiliary function $\Phi$ is a solution of the equation

$$[i\partial_0 + \mu + i\gamma^0 \gamma^i D_i - m \gamma^0] [i\partial_0 + \mu - i\gamma^0 \gamma^i D_i + m \gamma^0] \Phi = 0$$

Equation of the harmonic oscillator type

Energy levels are given by the Landau levels:

$$\omega_{n,s^3}(p_z) = -\mu \pm \sqrt{2eB \left( n + \frac{1}{2} \right) + p_z^2 + m^2 - eB \sigma^3}$$

In $eB \to \infty$ limit, only $n = 0, \sigma^3 = +$ mode contributes to scattering.

$$\Phi_0 = N_0 e^{-v_y y - v_z z} \exp \left[ -\frac{1}{2} eB \left( x - \frac{p_y}{eB} \right)^2 \right]$$

$N_0$ is determined by normalization condition:

$$|N_{0,+}(k_z)|^2 = \frac{1}{4(\omega(k_z) + \mu)(\omega(k_z) + \mu + m)}$$

In first order of perturbation transition rate:

$$W(p \to p') = 2\pi |\langle \psi_{p'} | \psi_p \rangle|^2 \delta(\omega(p_z') - \omega(p_z))$$

$$\frac{1}{\tau} = \frac{N_0 I}{4} \left( \frac{4\pi e^2}{\kappa} \right)^2 \frac{m^2}{|p_z|} \frac{1}{\sqrt{\mu^2 - m^2}} \frac{1}{\sqrt{\mu^2 - m^2} + 1/r_s^2} \times \frac{[(\omega(p_z) + \mu + m)^2 - p_z'^2]^2}{4(\omega(p_z) + \mu)^2 (\omega(p_z) + \mu + m)^2 \sqrt{4k_z^2 + 1/r_s^2}}$$

Result

Around Fermi point: $|k_z| \approx \sqrt{\mu^2 - m^2}$

$$\frac{1}{\tau_0} = \frac{N_0 I}{4} \left( \frac{4\pi e^2}{\kappa} \right)^2 \frac{m^2}{\sqrt{\mu^2 - m^2} \sqrt{\mu^2 - m^2} + 1/r_s^2}$$

We derived the interpolating formula between relativistic and non-relativistic relaxation time.