OSAKA UNIVERSITY

Relaxation time of the fermions in the magnetic field (I)

- the case for relativistic fermions -

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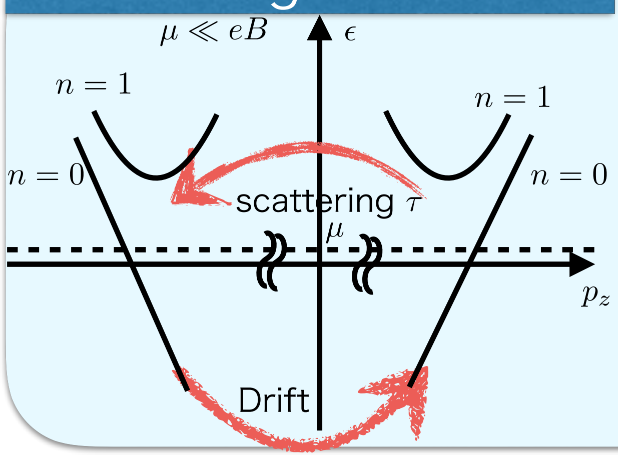
Chiral Magnetic Effect



Chiral fermion + Magnetic field ➡ Anomaly related charge transport

[H. B. Nielsen, M. Ninomiya, 1983]

Scattering ⟷ balance ⟷ Drift

τ determines the size of CME.

There is no explicit calculation for τ including relativistic effect.

cf.) NR[P. N. Argyres, E. N. Adams, 1956]

Goal: τ for relativistic fermions in $eB\to\infty, T=0$

1. Electric Current and Relaxation Time

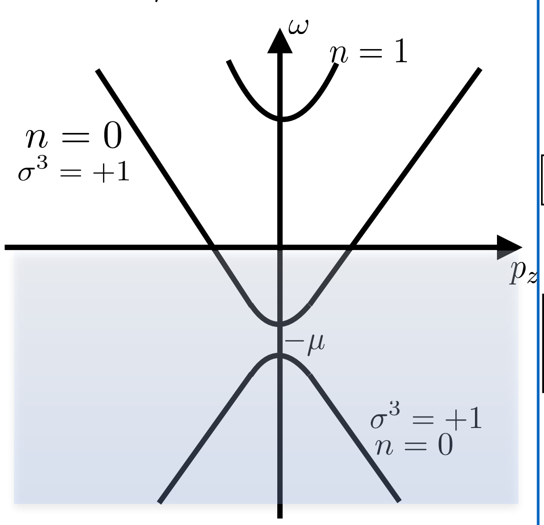
Start from the thermodynamical equiliblium:

$$f_0(p)=\frac{1}{1+\exp\left(\frac{\omega}{kT}\right)}$$

In the small E, $f=f_0+\delta f$

E accelerate the electron

$$\left(\frac{\partial f}{\partial t}\right)_{\rm drift}=eE\frac{\partial f}{\partial p_z}$$



Electrons scattered back.

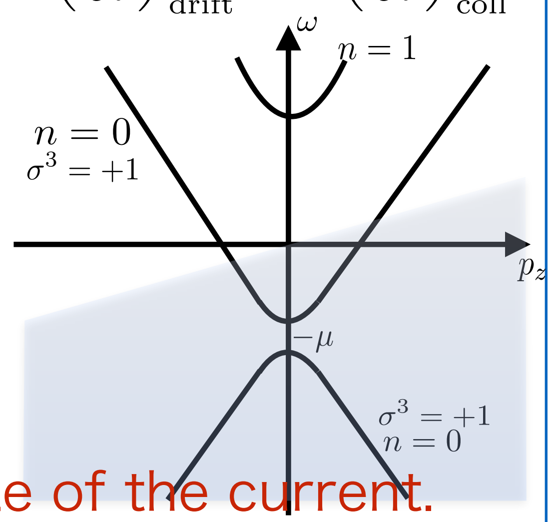
Relaxation time app. :

$$\left(\frac{\partial f}{\partial t}\right)_{\rm coll}=-\frac{1}{\tau}\delta f$$

Static Boltzmann equation: $\left(\frac{\partial f}{\partial t}\right)_{\rm drift}=-\left(\frac{\partial f}{\partial t}\right)_{\rm coll}$

Solution, lowest in E:

$$f(\mathbf{p})=f_0(p)+eE\tau\frac{\partial f_0}{\partial p_z}$$



T=0

Current density:

$$J=\frac{1}{6\pi^2}e^2E\frac{\mu^2}{v_F}\tau$$

τ determines the size of the current.

τ in terms of transition rate: $W(p\to p')$

$$\frac{1}{\tau}=\sum_{\mathbf{p}'}\frac{p_z-p_z'}{p_z}W(p\to p')$$

In T=0 case, phonon scattering is negligible.

Consider only ionized impurity scattering:

$$v(\mathbf{r})=\frac{e^2}{\kappa}\frac{e^{-r/r_s}}{r}$$

2. Our Study of Relaxation Time

Dirac equation: $\left[i\not{D}-m+\mu\gamma^0\right]|\psi\rangle=0$

Solution is expressed as

$$\langle x|\psi\rangle=\psi(x)=\left[i\partial_0+\mu-i\gamma^0\gamma^iD_i+m\gamma^0\right]\Phi$$

Auxiliary function Φ is a solution of the equation

$$\left[i\partial_0+\mu+i\gamma^0\gamma^iD_i-m\gamma^0\right]\left[i\partial_0+\mu-i\gamma^0\gamma^iD_i+m\gamma^0\right]\Phi=0$$

$A_2=Bx$

$$\left[-\partial_x^2+(eB)^2\left(x-\frac{p_y}{eB}\right)^2+p_z^2+m^2-eB\sigma^3\right]\Phi=(\omega+\mu)^2\Phi$$

Equation of the harmonic oscillator type

Energy levels are given by the Landau levels:

$$\omega_{n,\sigma^3}(p_z)=-\mu\pm\sqrt{2eB\left(n+\frac{1}{2}\right)+p_z^2+m^2-eB\sigma^3}$$

In $eB\to\infty$ limit,

only $n=0,\sigma^3=+$ mode contributes to scattering.

$$\Phi_0=N_0e^{-ip_yy-ip_zz}\exp\left[-\frac{1}{2}eB\left(x-\frac{p_y}{eB}\right)^2\right]\chi$$

N_0 is determined by normalization condition:

$$\langle\psi_{\mathbf{p}'}|\psi_{\mathbf{p}}\rangle=(2\pi)^2\delta(p_y-p_y')\delta(p_z-p_z')$$

$$|N_{0,+}(k_z)|^2=\frac{1}{4\left(\omega(k_z)+\mu\right)\left(\omega(k_z)+\mu+m\right)}$$

In first order of perturbation transition rate:

$$W(p\to p')=2\pi|\langle\psi_{\mathbf{p}'}|v|\psi_{\mathbf{p}}\rangle|^2\delta(\omega(p_z')-\omega(p_z))$$

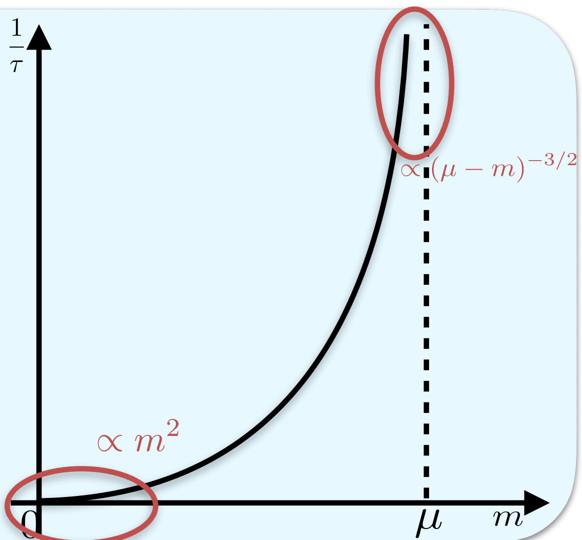
$eB\to\infty$

$$\frac{1}{\tau}=\frac{L_xN_I}{4}\left(\frac{4\pi e^2}{\kappa}\right)^2\frac{\sqrt{p_z^2+m^2}}{|p_z|}$$
$$\times\frac{\left[\left(\omega(p_z)+\mu+m\right)^2-p_z^2\right]^2}{4\left(\omega(p_z)+\mu\right)^2\left(\omega(p_z)+\mu+m\right)^2}\frac{1}{\sqrt{4k_z^2+1/r_s^2}}.$$

Result

Around Fermi point: $|k_z|\approx\sqrt{\mu^2-m^2}$

$$\frac{1}{\tau_0}=\frac{L_xN_I}{4}\left(\frac{4\pi e^2}{\kappa}\right)^2\frac{m^2}{\sqrt{\mu^2-m^2}}\frac{1}{\sqrt{4(\mu^2-m^2)+1/r_s^2}}$$



We derived the interpolating formula between relativistic and non-relativistic relaxation time.