The scalar sector of SU(2) gauge theory with \( N_F = 2 \) fundamental flavours

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based on
arXiv:1607.06654

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General interest

- **Facts:**
  - Higgs-like state discovered
  - Well described by the Standard Model!
  - Higgs interaction is the only one not dictated by a gauge principle

- **Gauge theories can feature scalar bound states**

- **Questions:**
  - Can strongly coupled extensions be used to describe a composite Higgs state?
  - Can the scalar or pseudoscalar sector be used in the context if Dark Matter?

- **Here:**
  - Benchmark for spin-0 states
  - Spin-1 states: See T. Janowski talk
SU(2) gauge theory with $N_f=2$ fundamental Dirac flavours.

- SU(2) gauge theory with $N_f=2$ Dirac fermions in the fundamental representation.

\[
\mathcal{L} = -\frac{1}{4} F^{\alpha}_{\mu\nu} F^{\alpha\mu\nu} + i \bar{U} \gamma^\mu D_\mu U + i \bar{D} \gamma^\mu D_\mu D + \frac{m}{2} Q^T (-i \sigma^2) C E Q + \frac{m}{2} \left( Q^T (-i \sigma^2) C E Q \right)^\dagger
\]

- Pseudo-real irrep of SU(2): **global flavour symmetry is upgraded to SU(4)**:

\[
Q \equiv \begin{pmatrix}
U_L \\
D_L \\
\tilde{U}_L \\
\tilde{D}_L
\end{pmatrix} \equiv \begin{pmatrix}
U_L \\
D_L \\
-i \sigma_2 C \bar{u}_R^T \\
-i \sigma_2 C \bar{d}_R^T
\end{pmatrix}, \quad E = \begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0
\end{pmatrix}
\]

- Infinitesimal SU(4) transformation:

\[
Q \rightarrow \left( 1 + i \sum_{n=1}^{15} \alpha^n T^n \right)
\]

- Generators that leaves the Lagrangian invariant satisfy:

\[
ET^n + T^{nT} E = 0
\]

- Chiral symmetry breaking pattern: $SU(4) \rightarrow Sp(4)$ \quad ($\sim SO(6) \rightarrow SO(5)$)

5 Goldstone bosons
EW embedding

\[ Q_L = (U_L, D_L) : \text{SU}(2)_L \text{ doublet with hypercharge } 0 \]
\[ \tilde{U}_L, \tilde{D}_L : \text{SU}(2)_L \text{ singlet with hypercharge } \pm 1/2 \]

Two interesting alignments of the condensate:

\[ \Sigma_H \equiv E = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} : \text{break EW symmetry} \]
\[ \Sigma_B \equiv \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix} : \text{does not break EW} \]

General case:

\[ \Sigma_0 = \cos\theta \Sigma_B + \sin\theta \Sigma_H \]

Two limit cases:

\[ \theta = 0 : \text{EW does not break : composite Higgs limit} \]
\[ \theta = \pi/2 : \text{EW breaks + DM candidate : technicolor limit} \]

at LO: \[ m_W^2 = 2 g (F_{PS} \sin \theta)^2 \]

The Higgs is a linear combination of a GB and of the 0^+ state

General remarks

- **QCD:**
  - $m_\sigma / f_{PS} \sim 5$ and width is large
  - dependence on the quark mass?

- **SU(2) + 2 Fundamental:**
  - $\sigma$ is not light because of symmetry reasons
  - $N_c$ dependence is not known

- Technical issues:
  - Rigorous treatment of resonances is challenging
  - Disconnectected contributions are noisy

- after EW embedding:
  - physical Higgs is a mixture of a GB and of the $\sigma$
  - Scalar state receives large contributions from top loops.

Lattice techniques & results
The setup

- Plaquette action + Wilson Fermions
- Several volumes $V = L^3 \times T$
- 4 lattice spacings : $a$
- Several fermion masses $m_f$
- Intermediate scale setting $w_0$ from the Gradient flow
  
  S. Borsanyi et. al., JHEP 09 (2012) 010, [arXiv:1203.4469]

- Non-perturbative renormalization
- Scale setting : $F_{PS} \sin \Theta = 246 \text{ GeV}$
- HiRep code

Correlators

- Interpolating fields:
  - scalar iso-singlet, $\sigma$
  - scalar iso-vector, $a_0$
  - pseudoscalar, $\eta'$
  
  \[ J = \bar{u}u + \bar{d}d \]
  \[ J = \bar{u}u - \bar{d}d \]
  \[ J = \bar{u}\gamma_5 u + \bar{d}\gamma_5 d \]

- Correlator: $C_{2pts}(t) = \sum_{\vec{x}} \langle J(t, \vec{x})\bar{J}(0) \rangle$

- After integration over the fermions ($\sigma$ case):
  
  \[ C_{2pts}(t) = \sum_{\vec{x}} \text{tr}\{S(x, 0)S(0, x)\} + N_f \sum_{\vec{x}} \text{tr}\{S(x, x)\} \text{tr}\{S(0, 0)\} \]

- Effective mass
  \[ \frac{C(t-a)}{C(t)} = \frac{e^{-m_{\text{eff}}(t)(t-a)} + e^{-m_{\text{eff}}(t)(T-(t-a))}}{e^{-m_{\text{eff}}(t)t} + e^{-m_{\text{eff}}(t)(T-t)}} \]

- Stochastic estimation based on 4-volume sources

- $N_{\text{src}}=64$ is enough for all our ensembles.

results on 3 new states of the theory: $\sigma, a_0, \eta'$
Most chiral run: $\beta=2.0$, $m=-0.958$, $V=32^4$

- Clear plateau

- $\sigma$ is stable in our setup!

- $a_0$ above 3 GBs threshold

- 2200 configurations

- $m_\sigma \sim m_{PS}$!

No need to consider yet the resonance analysis for the $\sigma$
\[ w_0^X m_X = w_0^X m_X^X + A(w_0^X m_{ps})^2 + B(w_0^X m_{ps})^4 + C \frac{\omega}{w_0} \]

σ resonance: 0(0+)

- σ stable in our simulation
- cut-off effects negligible
- Global fit
- \( m_\sigma = 19.2(10.8) \) F_{PS}
Global fit including 4 lattice spacings

Only data below 3 \times m_{PS} threshold included

large cut-off effects?

m=16.7(4.9) F_{PS}
Effective mass: pseudoscalar channel

- Most chiral run: $\beta=2.0$, $m=-0.958$, $V=32^4$
- $\eta'$ signal
- 2200 configurations
- Reasonable plateau

Contribution from the disconnected diagrams is sizable only for our lightest masses
• 2 lattice spacings

• $m_\eta > m_{ps}$ only at our lightest fermion masses.

• cut-off effect negligible

• $m_\eta = 12.8(4.7) \, F_{PS}$

$$w_0^X m_X = w_0^X m_X^X + A (w_0^X m_{ps})^2 + B (w_0^X m_{ps})^4 + C \frac{a}{w_0}$$
Conclusions

• Non-perturbative dynamics of gauge theories is explored
• $\text{SU}(2)_c + N_f = 2$: UV completion for a CH/TC scenario
• Signals are observed
• $\sigma$ is stable in our range of mass
• Extrapolation: decay threshold effects are neglected
• Benchmark results in the spin-0 sector:
  ★ $m_{a0}/F_{PS} = 16.7(4.9)$
  ★ $m_{\sigma}/F_{PS} = 19.2(10.8)$
  ★ $m_{\eta}/F_{PS} = 12.8(4.7)$
• Surprise: $m_{\sigma} \sim m_{PS}$ on a large range of mass