Chiral condensate from OPE of the overlap quark propagator

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Outline

- Motivation
- Lattice set up
- Preliminary results
- Summary

Motivation

- The quark chiral condensate $\langle\bar{\psi}\psi\rangle$ is one of the two LECs in leading order χPT.

$$\langle\bar{u}u\rangle = -\Sigma = -BF^2.$$

- It is an order parameter for chiral symmetry breaking.
- It is an input parameter in QCD sum rules.

Motivation

- The quark chiral condensate $\langle\bar{\psi}\psi\rangle$ is one of the two LECs in leading order χPT.

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- It is an order parameter for chiral symmetry breaking.
- It is an input parameter in QCD sum rules.
- Many ways on the lattice are being used to determine its value.
    - Chiral extrapolation of pseudoscalar meson masses and decay constants.
    - Finite-size study of correlators in the ϵ-regime.
    - Comparing the distribution of the low-lying eigenvalues of the Dirac operator to RMT predictions in a given topological sector.
    - $\cdots$

FLAG, arXiv:1607.00299 [hep-lat]

What we are doing

- Fitting lattice data of quark propagator in Landau gauge to OPE formula to extract $\langle\bar{\psi}\psi\rangle$.
- The quark propagator in momentum space $S_q(p)$ can be written as

$$S_q(p) = \frac{S(p^2)}{p^2} + \frac{\not{p}}{p^2} V(p^2).$$

- The renormalized scalar form factor $S(p^2, \mu)$ has an OPE of the form

$$S(p^2, \mu) = S_0(p^2, \mu) m_q(\mu) + \frac{C_{m_q^3}}{p^2} m_q^3(\mu) + \frac{C_{m_q A^2}}{p^2} \langle m_q A^2 \rangle + \frac{C_{\bar{\psi}\psi}}{p^2} \langle\bar{\psi}\psi\rangle$$

- Operators up to mass dimension three are included

$$A^2 \equiv A_\mu^a A^{a\mu}, m^2, m^3, mA^2, \bar{\psi}\psi.$$

[Chetyrkin & Maier, JHEP01(2010)092]

Formulae

- The pure perturbative contribution $S_0(p^2, \mu)$ was given in [Chetyrkin & Retey, NPB 2000] at 3-loop level.
- The Wilson coefficients $C_?$ in the $\overline{\mathrm{MS}}$ scheme were given in [Chetyrkin & Maier, JHEP01(2010)092] also at 3-loop level.
- The quark propagator $S_q(p)$ can be calculated on the lattice.

$$Z_q(\mu, a) S_q(p, a) = S_q^R(p, \mu), \quad \frac{1}{12} \mathrm{Tr} S_q^R(p, \mu) = \frac{S(p^2, \mu)}{p^2}$$

- A fitting window $\Lambda^2_{\mathrm{QCD}} \ll p^2 \ll (\pi/a)^2$ is needed.
- Fittings to Wilson twisted mass quark propagators were investigated in [Burger et al., PRD2013] with $N_f = 2$.

Lattice setup

Table: Parameters of configurations with 2+1 flavor dynamical domain wall fermions (RBC-UKQCD). [Aoki et al., PRD 2011]

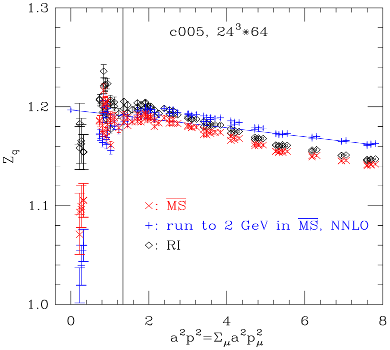
$1/a$(GeV)	label	am_{sea}	volume	N_{conf}
1.75(4)	c005	0.005/0.04	$24^3 \times 64$	92
	c01	0.01/0.04	$24^3 \times 64$	88
	c02	0.02/0.04	$24^3 \times 64$	138

- The lattice spacing is from [Yang, ZL et al., PRD 2015]
- Overlap valence quark mass $am_q = 0.0062, 0.0089, 0.0102, 0.0135, 0.0172, 0.0243, 0.0365, 0.0489$
- The pion masses range from about 220 MeV to 600 MeV.
- Point source quark propagators. Statistical errors are from Jackknife.

Quark field renormalization

- The RI/MOM scheme was used to calculate the renormalization constants of quark bilinear operators for our lattice setup. [ZL et al., 1312.7628[hep-lat], PRD]
- $Z_q^{\mathrm{RI}}(p^2=\mu^2)$ was already obtained ($\psi_R = Z_q^{1/2}\psi$).
- Now it is converted to $Z_q^{\overline{\mathrm{MS}}}$ by using the 3-loop conversion ratio given in [Chetyrkin & Retey NPB(2000)].
- Then we run it to $\mu_0 = 2$ GeV from all available initial scale p^2 in the $\overline{\mathrm{MS}}$ scheme (3-loop).
- Finally a linear extrapolation in $a^2p^2(>5)$ is done to reduce $\mathcal{O}(a^2p^2)$ discretization effects.
- To reduce Lorentz noninvariant discretization errors, the momenta used are close to the diagonal line: $p^{[4]}/(p^2)^2 < 0.32, \quad p^{[n]} = \sum_\mu p_\mu^n$.

Quark field Renormalization



ensemble	c02	c01	c005
$Z_q^{\overline{\rm MS}}(2\ \text{GeV})$	1.2022(20)	1.2095(33)	1.1968(19)

Table: $Z_q^{\overline{\rm MS}}(2\ \text{GeV})$ on the $24^3 \times 64$ lattices. Statistical errors only.

Scalar form factor in the chiral limit

- In the quark massless limit,

$$S(p^2,\mu) = S_0(p^2,\mu)m_q(\mu) + \frac{C_{m_q^3}}{p^2} m_q^3(\mu) + \frac{C_{m_q A^2}}{p^2}\langle m_q A^2\rangle + \frac{C_{\bar\psi\psi}}{p^2}\langle\bar\psi\psi\rangle$$

$$\Longrightarrow S(p^2,\mu) = \frac{C_{\bar\psi\psi}}{p^2}\langle\bar\psi\psi\rangle(\mu).$$

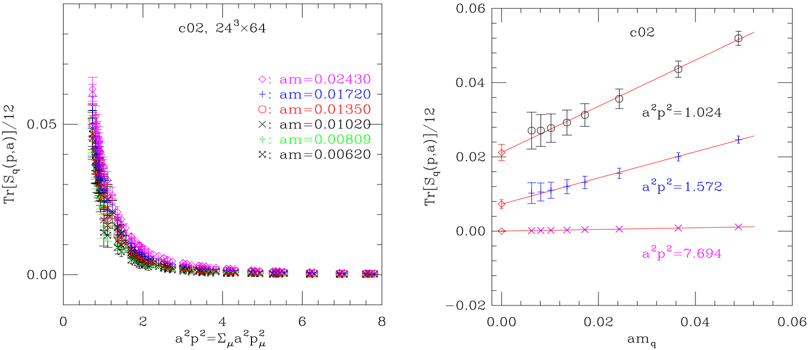
- Thus we can fit the lattice data with

$$\frac{1}{12}\mathrm{Tr} S_q^R(p,\mu) = \frac{C_{\bar\psi\psi}}{(p^2)^2}\langle\bar\psi\psi\rangle.$$

- In lattice units and take into account $\mathcal{O}(a^2p^2)$ effects,

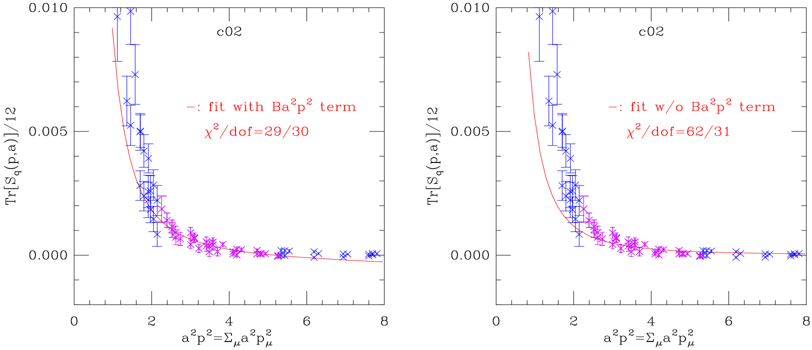
$$\frac{1}{12}\mathrm{Tr}\frac{S_q(p,a)}{a} = \frac{C_{\bar\psi\psi}}{Z_q(\mu,a)(a^2p^2)^2} a^3\langle\bar\psi\psi\rangle + Ba^2p^2.$$

Scalar form factor in the chiral limit



- $ap = (2\pi k_i/L, (2k_4+1)\pi/T)$, $k_\mu = -6, -5, ..., 6$. $p^{[4]}/(p^2)^2 < 0.32$
- The linear extrapolations in am_q at all a^2p^2 have good χ^2/dof.
- Similarly on the other ensembles.

Fitting of the scalar form factor



- $a^2p^2 \in [2.2, 5.3]$. The Ba^2p^2 term reduces χ^2/dof significantly.
- Using lattice spacing $1/a = 1.75(4)$ GeV, on ensemble c02 one gets

$$\langle\bar{\psi}\psi\rangle^{\overline{\text{MS}}}(2\text{ GeV}) = -(277(10)\text{ MeV})^3.$$

Dependence on the fitting range

- $a^2p^2 \in [2.2, 5.3]$ corresponds to $p^2 \in [6.7, 16.2]$ GeV^2. χ^2/dof$= 29/30$.

Table: $\langle\bar{\psi}\psi\rangle^{\overline{\mathrm{MS}}}$(2 GeV) from different fitting ranges.

$a^2p^2 \in$	$p^2 \in$/GeV^2	χ^2/dof	$(\langle\bar{\psi}\psi\rangle)^{1/3}$/MeV
[2.2, 5.5]	[6.7, 16.8]	1.28	−270(10)
[2.2, 5.3]	[6.7, 16.2]	0.97	−277(10)
[2.2, 5.1]	[6.7, 15.6]	1.02	−276(10)
[2.2, 4.9]	[6.7, 15.0]	1.06	−276(10)
[2.2, 4.7]	[6.7, 14.4]	1.07	−283(11)
[2.6, 5.3]	[8.0, 16.2]	1.03	−271(11)
[2.4, 5.3]	[7.4, 16.2]	0.98	−274(10)
[2.0, 5.3]	[6.1, 16.2]	1.06	−278(9)
[1.8, 5.3]	[5.5, 16.2]	1.20	−285(9)

Truncation effects

- The evaluations of α_s and $C_{\bar{\psi}\psi}$ are truncated at the same n-loop ($n = 1, 2, 3$).
- $\alpha_s^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})$ is obtained by using $\Lambda_{\mathrm{QCD}}^{\overline{\mathrm{MS}}} = 339(10)$ MeV for 3 flavors. [PDG2012]
- Fitting range: $a^2p^2 \in [2.2, 5.3]$.

n	χ^2/dof	$(\langle\bar{\psi}\psi\rangle)^{1/3}/\mathrm{MeV}$
1	0.98	−263(9)
2	0.98	−292(10)
3	0.97	−277(10)

Results from ensemble c01

- $a^2p^2 \in [1.8, 3.8]$ corresponds to $p^2 \in [5.5, 11.6]$ GeV^2.
  χ^2/dof$= 25/26$.

Table: $\langle\bar{\psi}\psi\rangle^{\overline{\rm MS}}$(2 GeV) from different fitting ranges on ensemble c01.

$a^2p^2 \in$	$p^2 \in$/GeV^2	χ^2/dof	$(\langle\bar{\psi}\psi\rangle)^{1/3}$/MeV
$[1.8, 3.8]$	$[5.5, 11.6]$	0.95	$-277(11)$
$[1.8, 3.6]$	$[5.5, 11.0]$	0.99	$-278(12)$
$[1.8, 3.4]$	$[5.5, 10.4]$	1.08	$-273(13)$
$[2.2, 3.8]$	$[6.7, 11.6]$	0.80	$-279(17)$
$[2.0, 3.8]$	$[6.1, 11.6]$	0.71	$-273(14)$
$[1.6, 3.8]$	$[4.9, 11.6]$	1.31	$-297(9)$

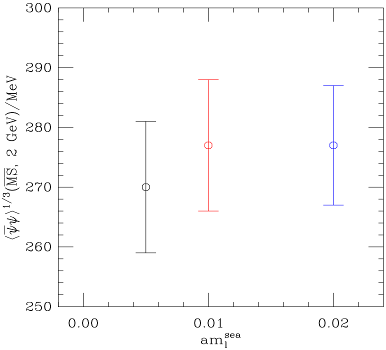
Results from ensemble c005

- $a^2p^2 \in [1.4, 2.4]$ corresponds to $p^2 \in [4.3, 7.4]$ GeV^2. χ^2/dof$= 22/17$.

Table: $\langle\bar{\psi}\psi\rangle^{\overline{\rm MS}}$(2 GeV) from different fitting ranges on ensemble c005.

$a^2p^2 \in$	$p^2 \in$/GeV^2	χ^2/dof	$(\langle\bar{\psi}\psi\rangle)^{1/3}$/MeV
[1.4, 2.6]	[4.3, 8.0]	1.40	−280(10)
[1.4, 2.4]	[4.3, 7.4]	1.30	−270(11)
[1.4, 2.2]	[4.3, 6.7]	1.38	−281(15)
[1.7, 2.6]	[5.2, 8.0]	1.34	−267(13)
[1.5, 2.6]	[4.6, 8.0]	1.37	−273(12)
[1.3, 2.6]	[4.0, 8.0]	1.35	−283(9)

Light sea quark mass dependence



- The sea quark mass dependence seems not big in our data.
- A constant fit gives $\langle\bar{\psi}\psi\rangle^{\overline{\rm MS}}(2\ \text{GeV}) =\sim -(275\ \text{MeV})^3$.

Summary

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- A fitting window in momentum seems to exist to give a stable result for $\langle\bar{\psi}\psi\rangle$.
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- The analysis is done on three ensembles with different sea quark masses. We are increasing some of the statistics...
- Hope to do an analysis on a finer lattice...
- Fitting of the vector form factor of the quark propagator...

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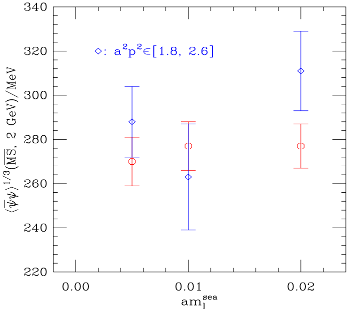
Thank you for your attention!

Extra Slides

Fitting in a same momentum range



- Require a same fitting range $a^2p^2 \in [1.8, 2.6]$ on all three ensembles. dof=11

ensemble	χ^2/dof	$(\langle\bar{\psi}\psi\rangle)^{1/3}$/MeV
c02	1.36	$-311(18)$
c01	1.51	$-263(24)$
c005	1.45	$-288(16)$