# Baryon interactions from lattice QCD with physical masses
## -- S=-3 sector: XiSigma & XiLambda-XiSigma --

Noriyoshi Ishii for HAL QCD Coll.

# Introduction

- Experimental determination of hyperon potentials is one of the most important topics in J-PARC.

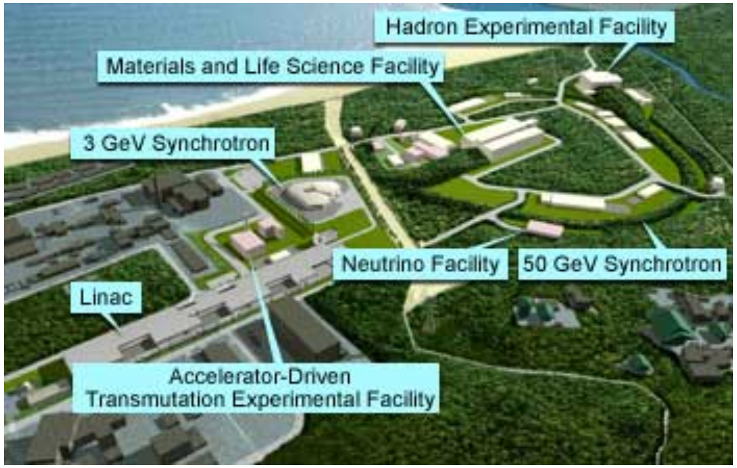
- They are mainly interested in S=-1 and -2 sectors. (Experiments get harder as |S| increases)

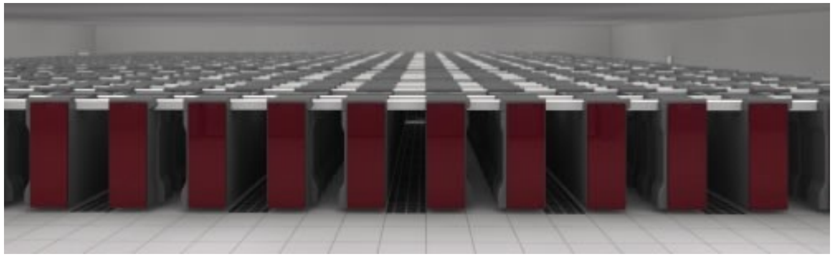
- On the lattice, determination of hyperon potentials become easier for increasing |S|. (Statistical noise reduces)

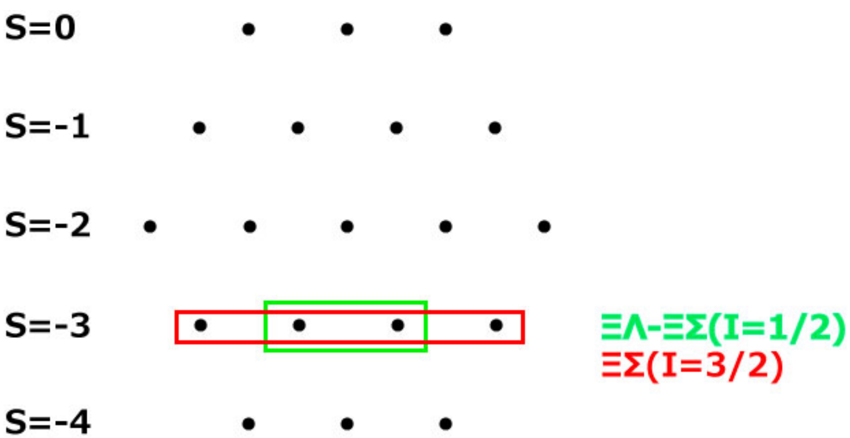
- In this talk, we give a result of hyperon potentials for S=-3 sector by using the physical point gauge confs. generated by K computer at AICS.

**J-PARC**

Exploration of multi-strangeness world





# Setup

In this talk, we use

- "phys. pt." gauge configs. on $96^4$ lattice generated by K computer (AICS)
- 1/a = 2.3 GeV, L= 8.2 fm
- 200 gauge confs.
- 20(source points) * 4(rotation)
- bin size = 10

- hadron mass:
  m(pion) = 145 MeV,
  m(N) = 950 MeV,
  m(Lambda)= 1125 MeV
  m(Sigma) = 1207 MeV
  m(Xi) = 1337 MeV

# XiSigma (I=3/2)

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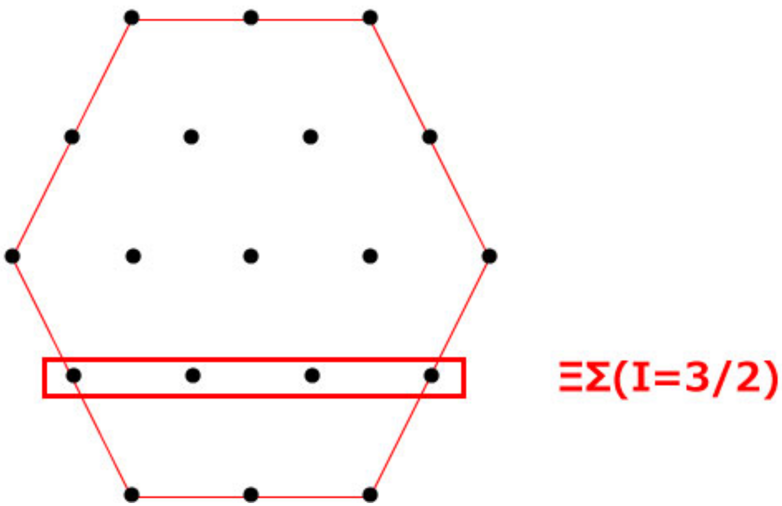
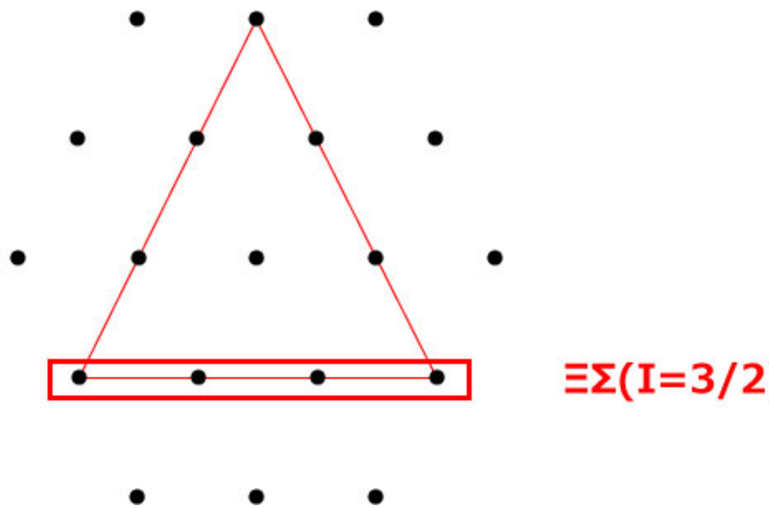
◆Total spin singlet

◆ flavor SU(3) limit
**27** irrep.
(same as NN, dineutron)




◆Total spin triplet

◆flavor SU(3) limit
**10*** irrep.
(same as NN, deuteron)



# Time-dependent Schrödinger-like eq. for unequal mass system



- We define **R-correlator**

$$R(\vec{x}-\vec{y},t) \equiv e^{(m_\Xi+m_\Sigma)t} \left\langle 0 \middle| T\left[\Xi(\vec{x},t)\Sigma(\vec{y},t)\cdot\overline{\Xi\Sigma}(t=0)\right] \middle| 0 \right\rangle$$

$$= \sum_n \psi_{k_n}(\vec{x}-\vec{y}) \cdot \exp\left(-(E_n - m_\Xi - m_\Sigma)t\right) \cdot a_n$$

where $\psi_{k_n}(\vec{x}-\vec{y}) \equiv \left\langle 0 \middle| \Xi(\vec{x})\Sigma(\vec{y}) \middle| n \right\rangle$

- Identity satisfied by two-particle energy in C.M. frame

$$k^2 E^2 = \frac{1}{4}\left(E^2 - (m_\Xi + m_\Sigma)^2\right)\left(E^2 - (m_\Xi - m_\Sigma)^2\right)$$

where $E \equiv \sqrt{m_\Xi^2 + k^2} + \sqrt{m_\Sigma^2 + k^2}$

- Schrödinger eq. satisfied by E-indep. HAL QCD potential

$$\left(\frac{\nabla^2}{2\mu} - \frac{k_n^2}{2\mu}\right)\psi_{k_n}(\vec{r}) = \int d^3r' V(\vec{r},\vec{r}')\psi_{k_n}(\vec{r}')$$

with $\mu \equiv \dfrac{m_\Xi m_\Sigma}{m_\Xi + m_\Sigma}$

➔ R-correlator satisfies **time-dependent Schrödinger-like equation**

$$\left(\frac{\nabla^2}{2\mu}D_t^2 + \frac{1}{8\mu}\left(D_t^2 - (m_\Xi + m_\Sigma)^2\right)\left(D_t^2 - (m_\Xi - m_\Sigma)^2\right)\right)R(\vec{r},t) = \int d^3r' V(\vec{r},\vec{r}')D_t^2 R(\vec{r}',t)$$

$$D_t \equiv \partial_t - m_\Xi - m_\Sigma$$

which enables us to determine the potential without ground state saturation.

# However

In this talk,
because the 4-th time derivative is numerically unstable so far,
we use a non-relativistically approximated version:

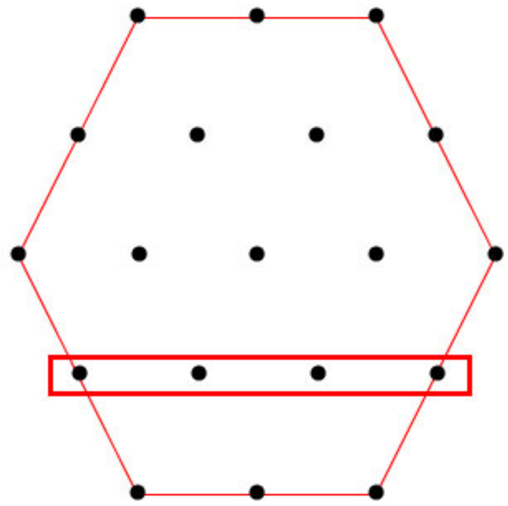
$$\left(\frac{\nabla^2}{2\mu}-\frac{\partial}{\partial t}\right)R(\vec{r},t)\simeq\int d^3r'V(\vec{r},\vec{r}')R(\vec{r}',t)$$

# XiSigma (I=3/2) spin singlet



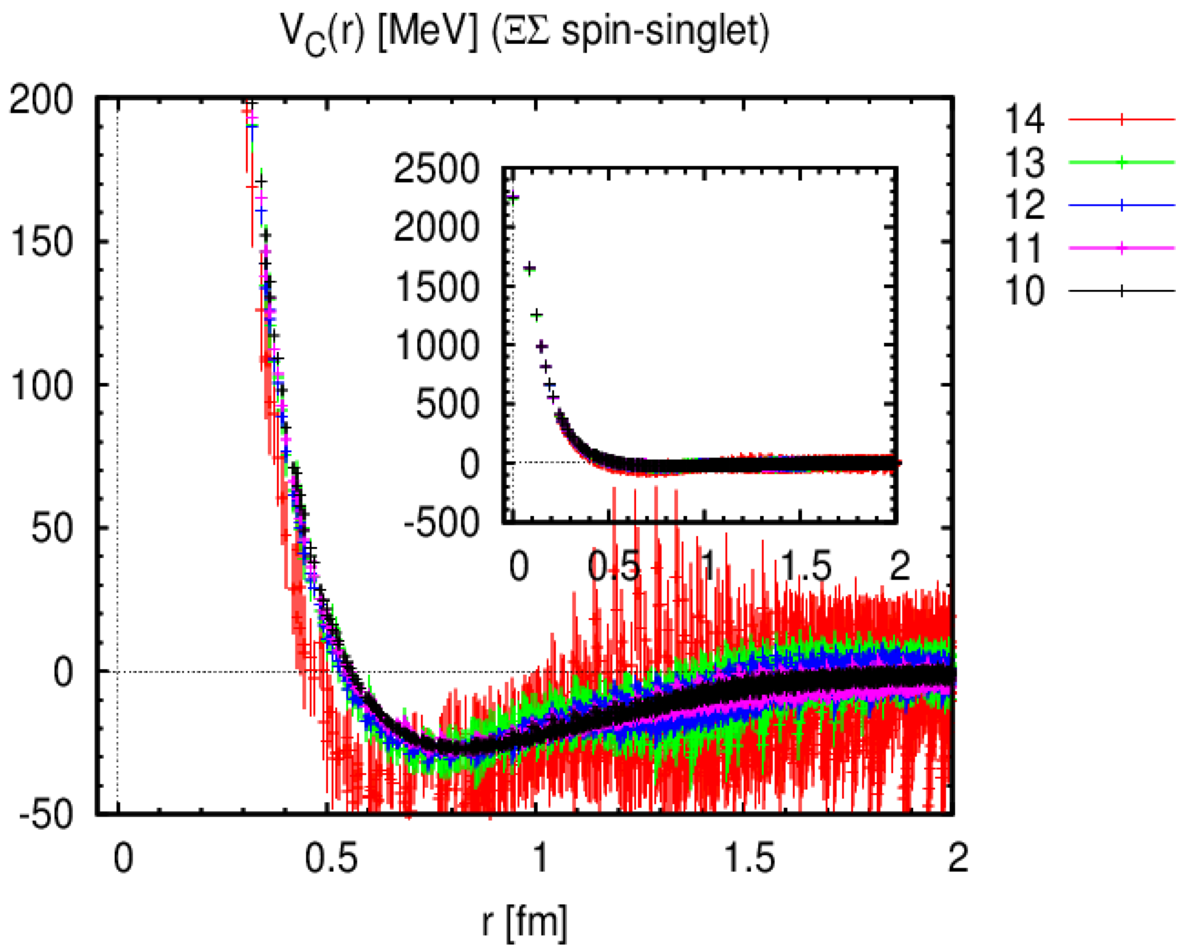
## ◆Total spin singlet

◆ flavor SU(3) limit
**27** irrep.
（same as NN, dineutron）



# XiSigma(I=3/2, spin singlet)



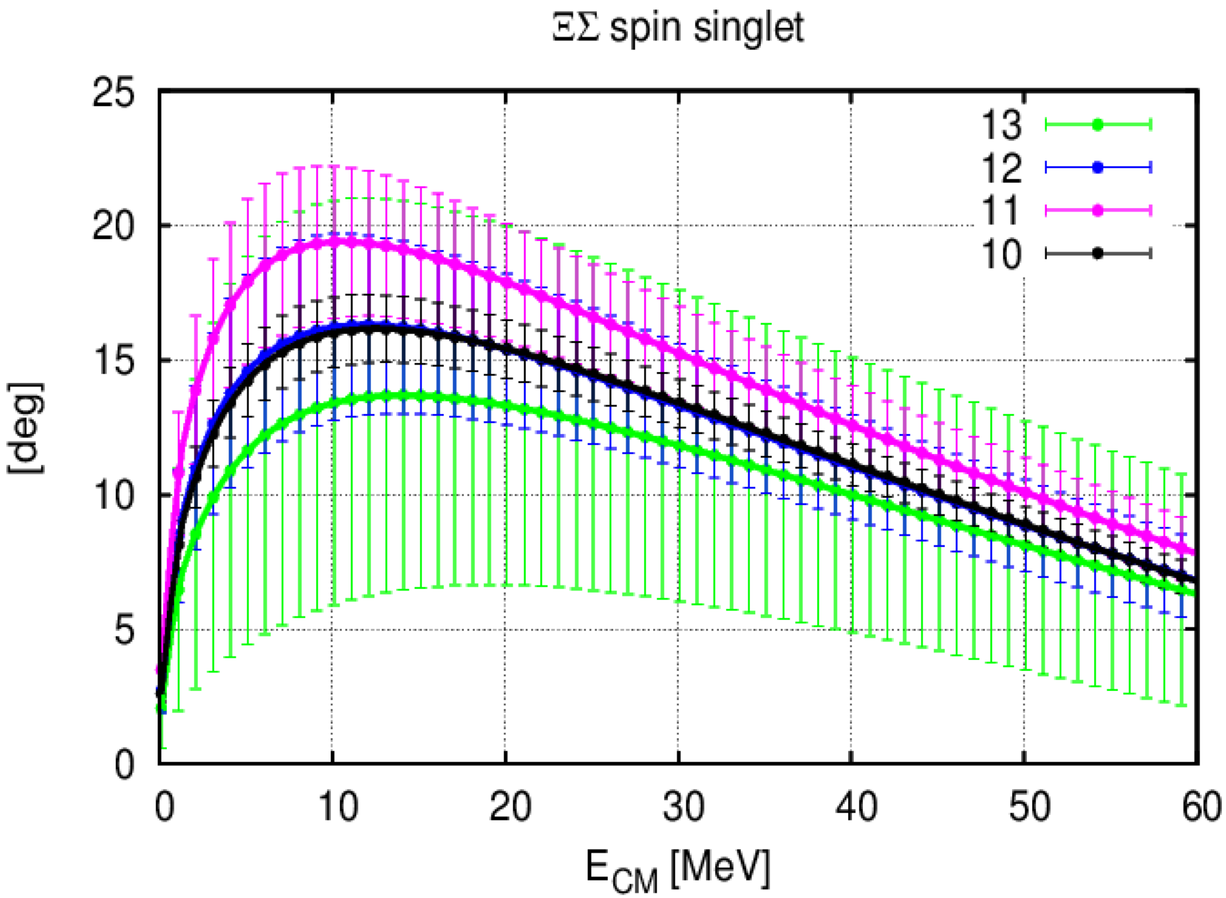



- 1/a ～2300 MeV, L ～8.2 fm
- $m_{pion}$～145 MeV
- 200 gauge confs. are used.
- Binsize = 10
- 20 source points * 4 rotations
- Point sink and wall source

◆ repulsive core surrounded by an attraction

◆Qualitative behavior is similar to NN
（flavor SU(3) limit：**27** irrep.）

# XiSigma (I=3/2, spin singlet) phase shift




- attractive
- no bound state

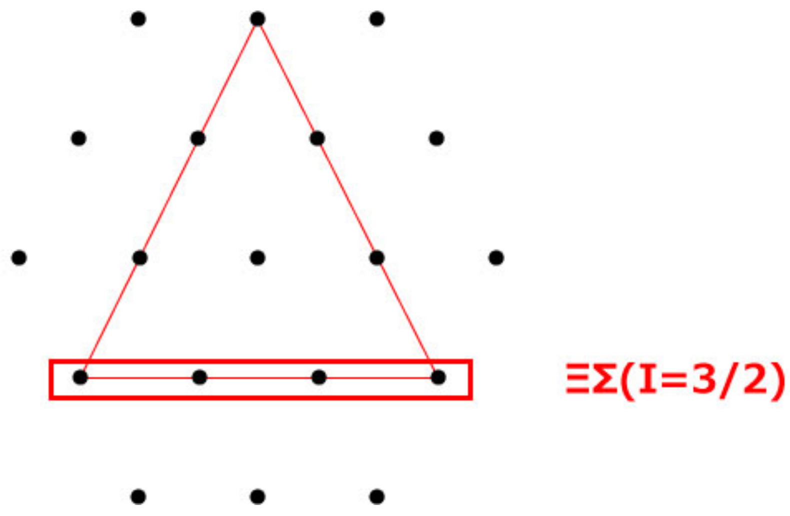
- Qualitatively similar to NN
  （flavor SU(3) limit：**27** irrep. same as dineutron）

# XiSigma (I=3/2) spin triplet



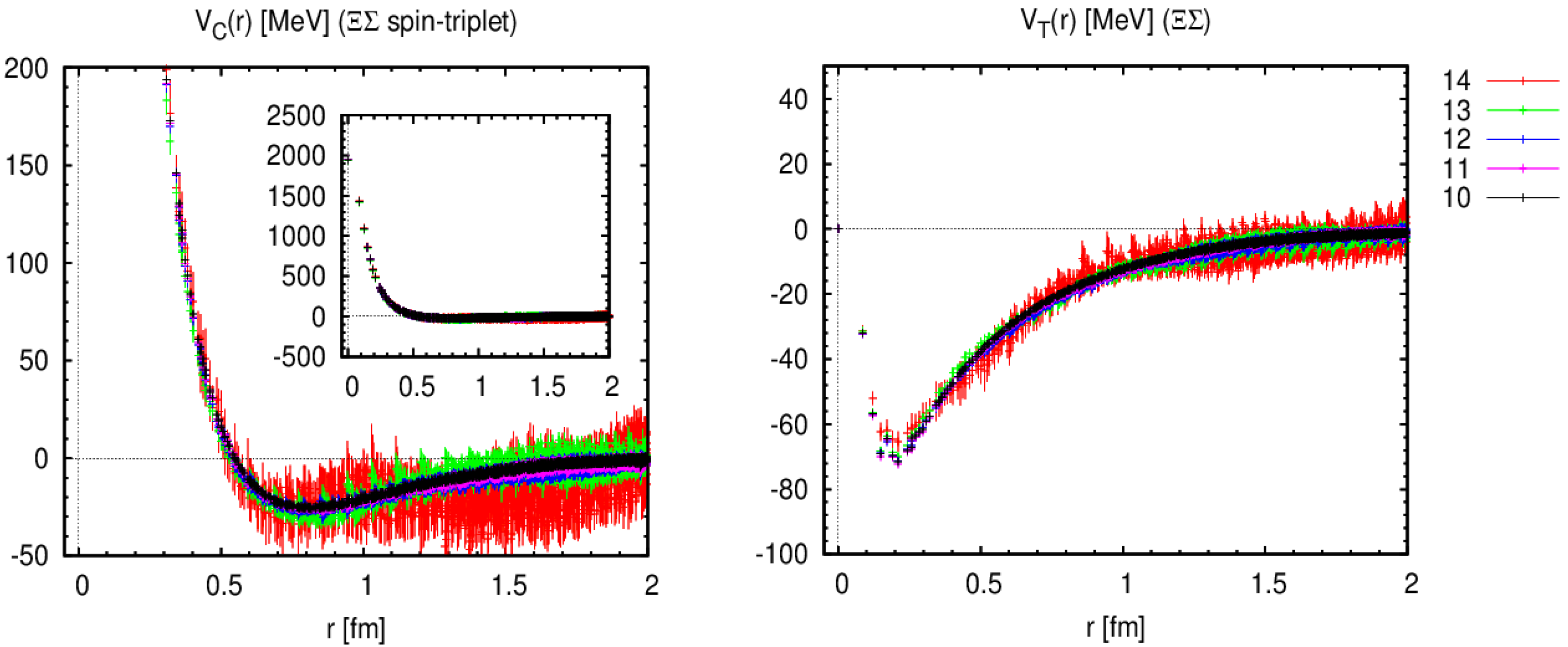
## ◆Total spin triplet

◆flavor SU(3) limit
**10*** irrep.
(same as NN, deuteron)



# XiSigma(I=3/2, spin triplet)






- Qualitative behaviors of central and tensor forces are the same as NN

- flavor SU(3) limit : **10*** irrep. (same as deutron)

- 1/a ～2300 MeV, L ～8.2 fm
- $m_{pion}$～145 MeV
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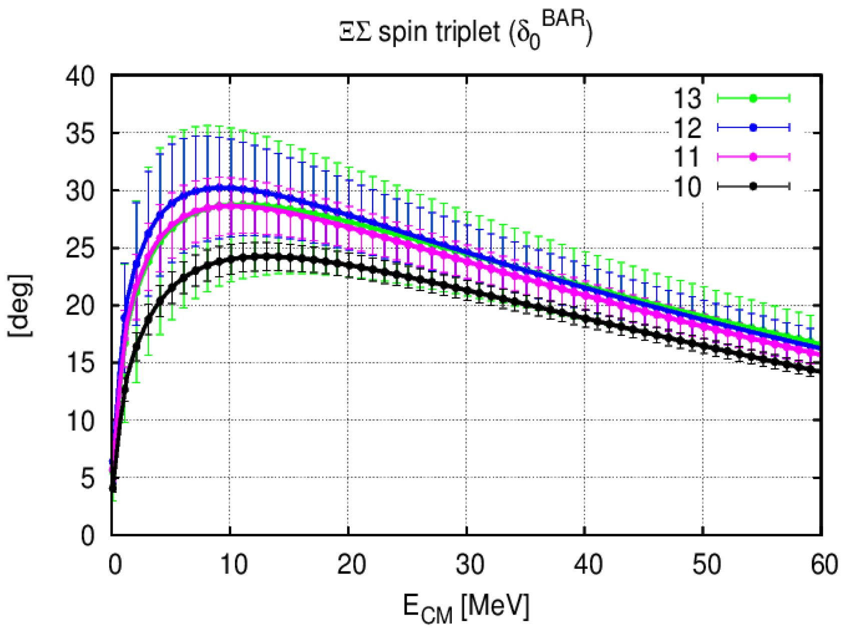
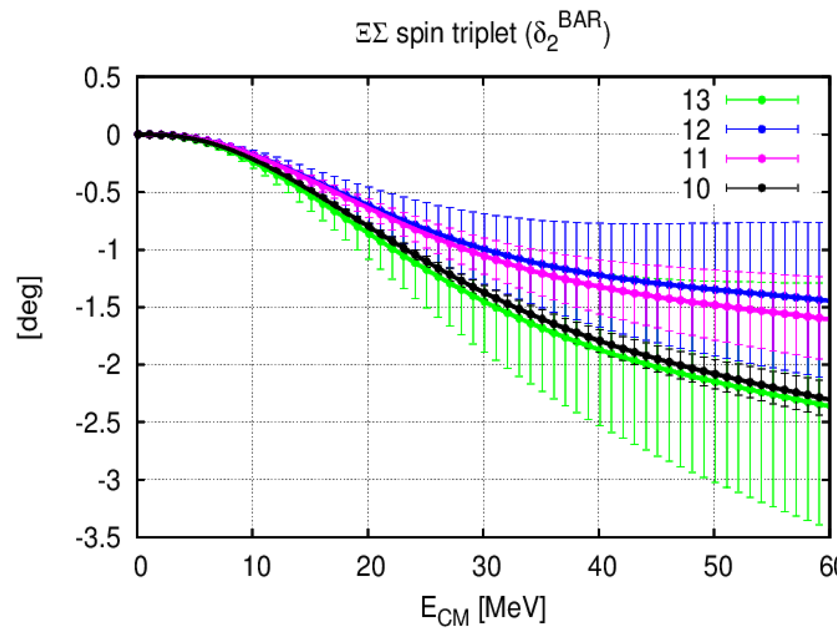
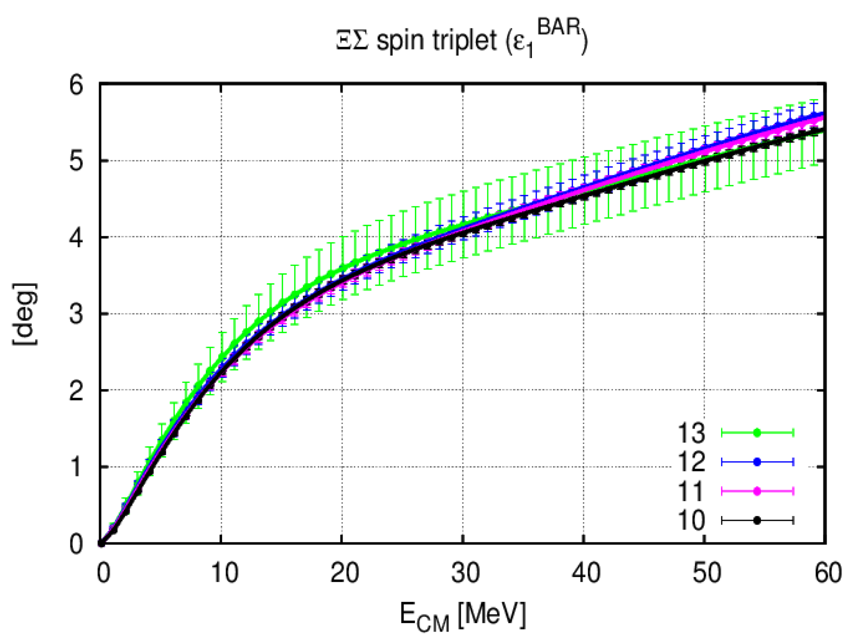
# XiSigma(I=3/2, spin triplet) phase shift






- ◆attractive
- ◆no bound state
- ◆Qualitatively similar to NN
- ◆ flavor SU(3) limit : **10*** irrep. (same as deutron)

# XiLambda-XiSigma coupled channel (I=1/2)

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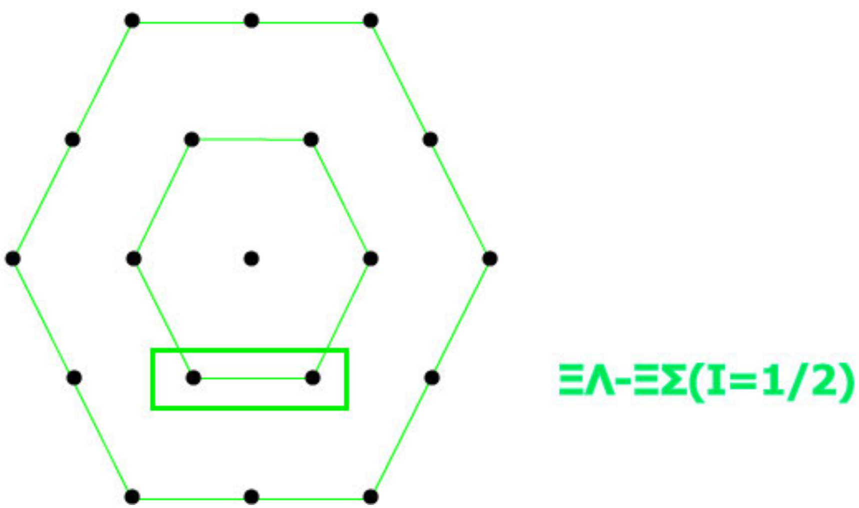
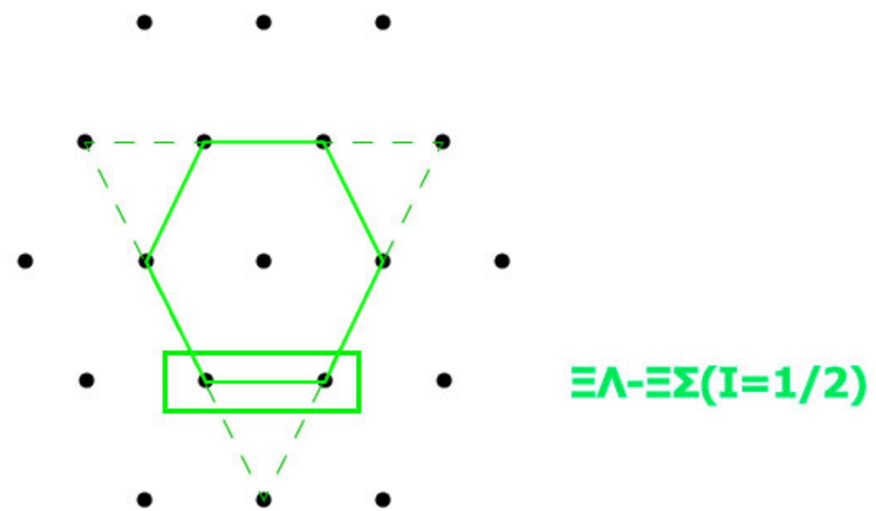
◆spin singlet

◆flavor SU(3) limit :
mixture of of **27** & **8s** irreps.
(cf. 27 irrep. contains NN)




◆spin triplet

◆flavor SU(3) limit :
mixture of **10** & **8a** irreps.
(nothing to do with NN)



# Coupled channel generalization of time-dep. Schrödinger-like eq.



- Similar argument as before leads
  to the **coupled channel generalization** of the **time-dep. Schrödinger-like eq.**

$$\begin{bmatrix} \mathcal{D}_{\Xi\Lambda} R_{\Xi\Lambda}(\vec{r},t;\mathcal{J}) \\ \mathcal{D}_{\Xi\Sigma} R_{\Xi\Sigma}(\vec{r},t;\mathcal{J}) \end{bmatrix} = \int d^3r' \begin{bmatrix} V_{\Xi\Lambda;\Xi\Lambda}(\vec{r},\vec{r}\,') & \zeta^{t} V_{\Xi\Lambda;\Xi\Sigma}(\vec{r},\vec{r}\,') \\ \zeta^{-t} V_{\Xi\Sigma;\Xi\Lambda}(\vec{r},\vec{r}\,') & V_{\Xi\Sigma;\Xi\Sigma}(\vec{r},\vec{r}\,') \end{bmatrix} \cdot \begin{bmatrix} D^2_{t;\Xi\Lambda} R_{\Xi\Lambda}(\vec{r}\,',t;\mathcal{J}) \\ D^2_{t;\Xi\Sigma} R_{\Xi\Sigma}(\vec{r}\,',t;\mathcal{J}) \end{bmatrix}$$

where (VERY COMPLICATED !)

$$R_{\Xi\Lambda}(\vec{x}-\vec{y},t;\mathcal{J}) \equiv Z_{\Xi}^{-1/2} Z_{\Lambda}^{-1/2} e^{(m_\Xi+m_\Lambda)t} \left\langle 0 \middle| T\left[\Xi(\vec{x},t)\Lambda(\vec{y},t)\cdot\mathcal{J}(t=0)\right] \middle| 0 \right\rangle$$

$$R_{\Xi\Sigma}(\vec{x}-\vec{y},t;\mathcal{J}) \equiv Z_{\Xi}^{-1/2} Z_{\Sigma}^{-1/2} e^{(m_\Xi+m_\Sigma)t} \left\langle 0 \middle| T\left[\Xi(\vec{x},t)\Sigma(\vec{y},t)\cdot\mathcal{J}(t=0)\right] \middle| 0 \right\rangle$$

$$\zeta \equiv \exp\left(m_\Sigma - m_\Lambda\right)$$

$$\mathcal{D}_{\Xi\Lambda} \equiv \frac{\nabla^2}{2\mu_{\Xi\Lambda}} D^2_{t;\Xi\Lambda} + \frac{1}{8\mu_{\Xi\Lambda}} \left(D^2_{t;\Xi\Lambda} - (m_\Xi+m_\Lambda)^2\right)\left(D^2_{t;\Xi\Lambda} - (m_\Xi-m_\Lambda)^2\right)$$

$$\mathcal{D}_{\Xi\Sigma} \equiv \frac{\nabla^2}{2\mu_{\Xi\Sigma}} D^2_{t;\Xi\Sigma} + \frac{1}{8\mu_{\Xi\Sigma}} \left(D^2_{t;\Xi\Sigma} - (m_\Xi+m_\Sigma)^2\right)\left(D^2_{t;\Xi\Sigma} - (m_\Xi-m_\Sigma)^2\right)$$

$$D_{t;\Xi\Lambda} \equiv \partial_t - m_\Xi - m_\Lambda$$

$$D_{t;\Xi\Sigma} \equiv \partial_t - m_\Xi - m_\Sigma$$

$$\mu_{\Xi\Lambda} \equiv \frac{1}{1/m_\Xi + 1/m_\Lambda}$$

$$\mu_{\Xi\Sigma} \equiv \frac{1}{1/m_\Xi + 1/m_\Sigma}$$

It enables us to obtain the potentials without single state saturation.

## However

In this talk,
because the 4-th time derivative is numerically unstable so far,
we use a non-relativistically approximated version:

$$\left(\frac{\nabla^2}{2\mu_{\Xi\Lambda}} - \frac{\partial}{\partial t}\right) R_{\Xi\Lambda}(\vec{r},t;\mathcal{J}) \simeq V_{\Xi\Lambda;\Xi\Lambda}(\vec{r}) R_{\Xi\Lambda}(\vec{r}',t;\mathcal{J}) + e^{+(m_\Sigma - m_\Lambda)t} V_{\Xi\Lambda;\Xi\Sigma}(\vec{r}) R_{\Xi\Sigma}(\vec{r}',t;\mathcal{J})$$

$$\left(\frac{\nabla^2}{2\mu_{\Xi\Sigma}} - \frac{\partial}{\partial t}\right) R_{\Xi\Sigma}(\vec{r},t;\mathcal{J}) \simeq e^{-(m_\Sigma - m_\Lambda)t} V_{\Xi\Sigma;\Xi\Lambda}(\vec{r}) R_{\Xi\Lambda}(\vec{r}',t;\mathcal{J}) + V_{\Xi\Sigma;\Xi\Sigma}(\vec{r}) R_{\Xi\Sigma}(\vec{r}',t;\mathcal{J})$$

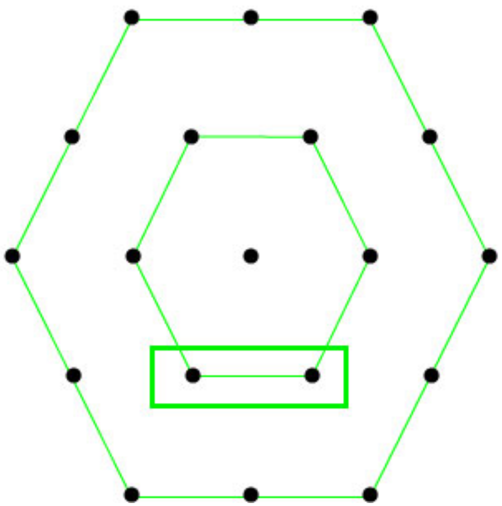
with an approx.

$$Z_\Lambda \simeq Z_\Sigma$$

# XiLambda-XiSigma coupled channel (I=1/2) spin singlet

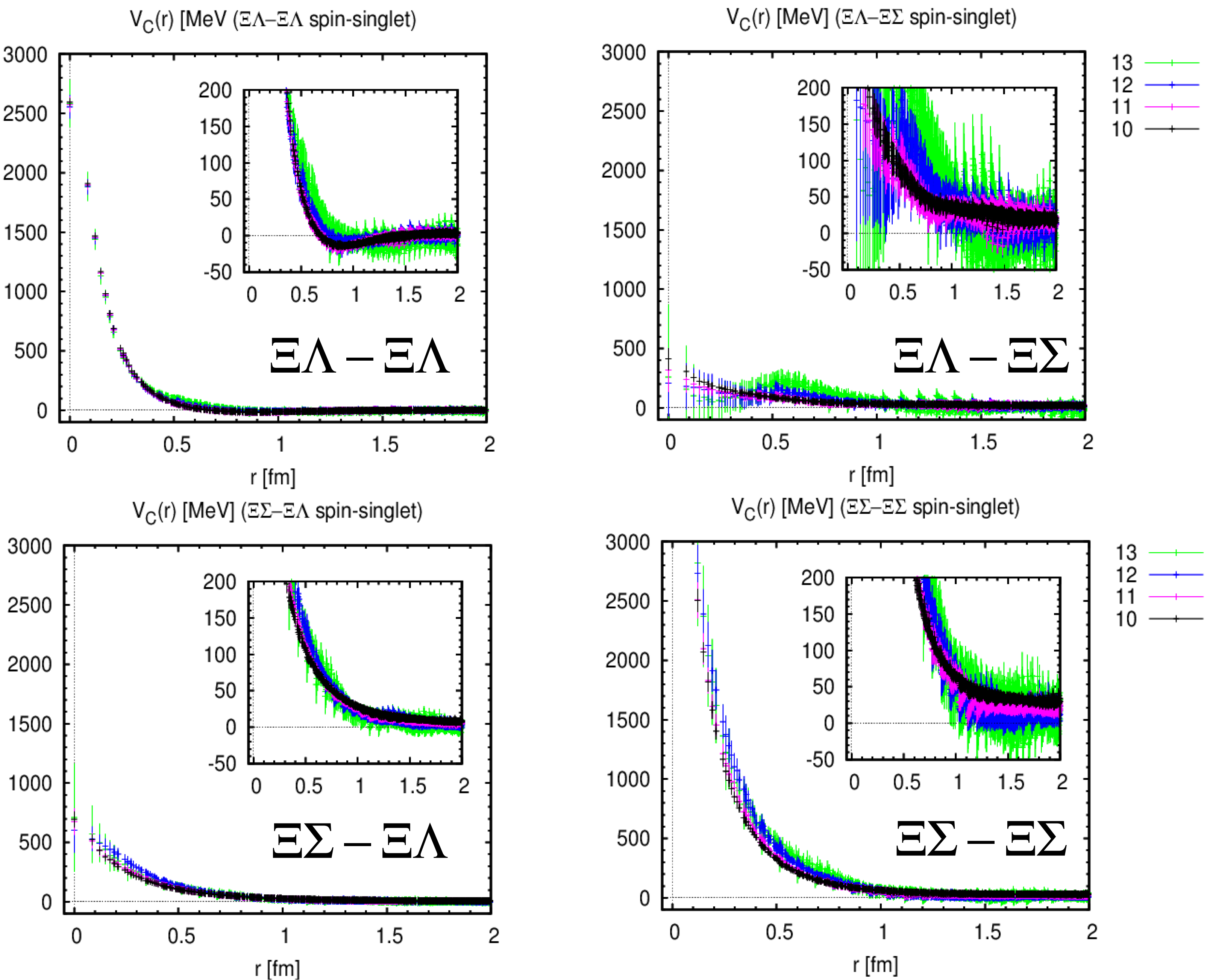
## ◆spin singlet

◆flavor SU(3) limit :
mixture of of **27** & **8s** irreps.
(cf. 27 irrep. contains NN)



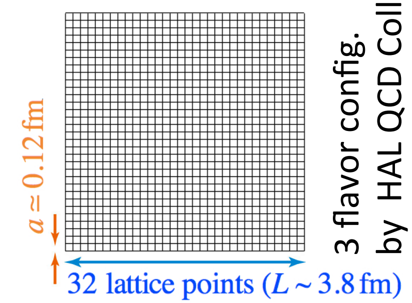
ΞΛ-ΞΣ(I=1/2)

# XiLambda-XiSigma coupled channel(I=1/2) spin singlet



# XiLambda-XiSigma coupled channel (I=1/2)

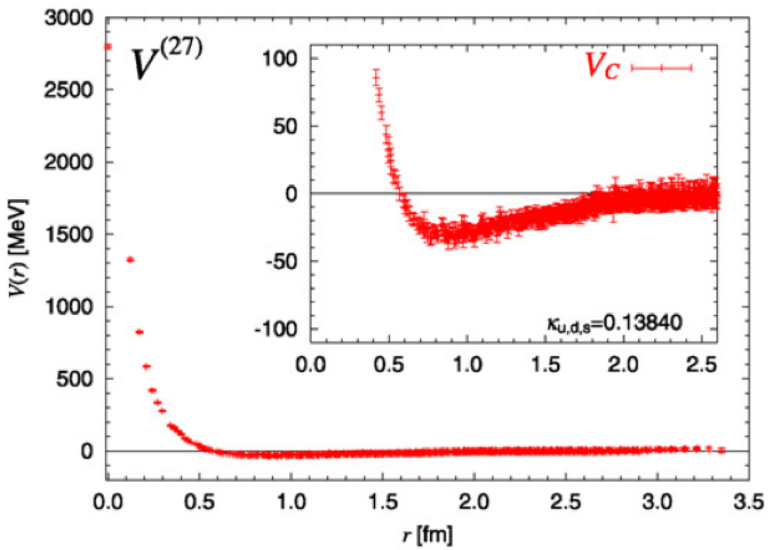
Flavor SU(3) limit to understand qualitative behavior



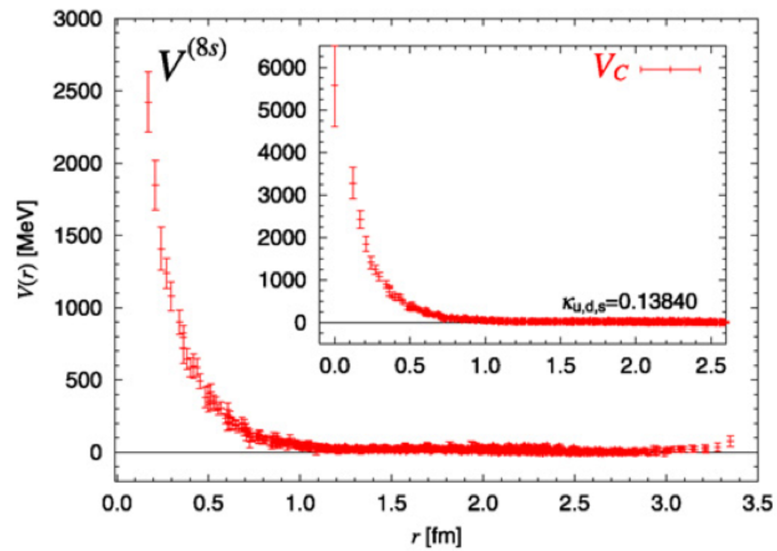

3 flavor config.
by HAL QCD Coll.
m(PS) = 469 MeV
m(B) = 1161 MeV

Potential in flavor SU(3) limit
(T.Inoue et al., NPA881(2012)28)

flavor **27** irrep.




flavor **8S** irrep.



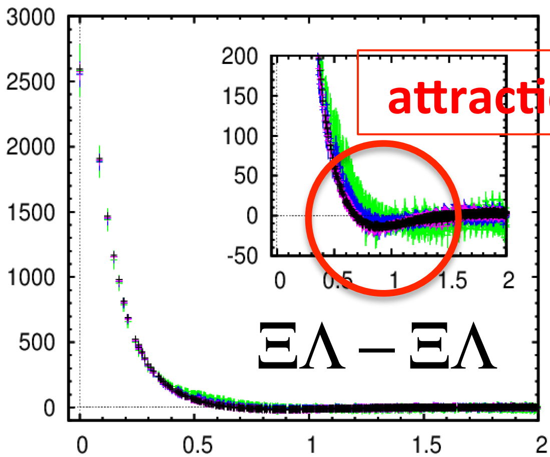

In flavor SU(3) limit,
the coupled channel potential is a linear combination of these two

$$\begin{pmatrix} V_{\Xi\Lambda;\Xi\Lambda} & V_{\Xi\Lambda;\Xi\Sigma} \\ V_{\Xi\Sigma;\Xi\Lambda} & V_{\Xi\Sigma;\Xi\Sigma} \end{pmatrix} = \begin{pmatrix} \frac{9}{10}V^{(27)} + \frac{1}{10}V^{(8s)} & -\frac{3}{10}V^{(27)} + \frac{3}{10}V^{(8s)} \\ -\frac{3}{10}V^{(27)} + \frac{3}{10}V^{(8s)} & \frac{1}{10}V^{(27)} + \frac{9}{10}V^{(8s)} \end{pmatrix}$$

# XiLambda-XiSigma coupled channel (I=1/2) spin singlet

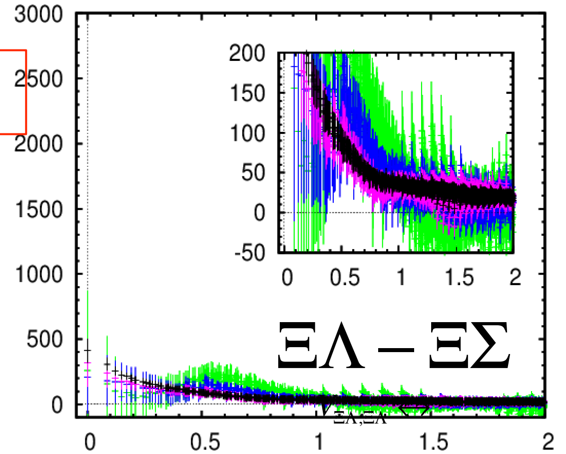
phys. pt. potential

$$V_{\Xi\Lambda;\Xi\Lambda} \leftrightarrow \frac{9}{10}V^{(27)} + \frac{1}{10}V^{(8S)}$$

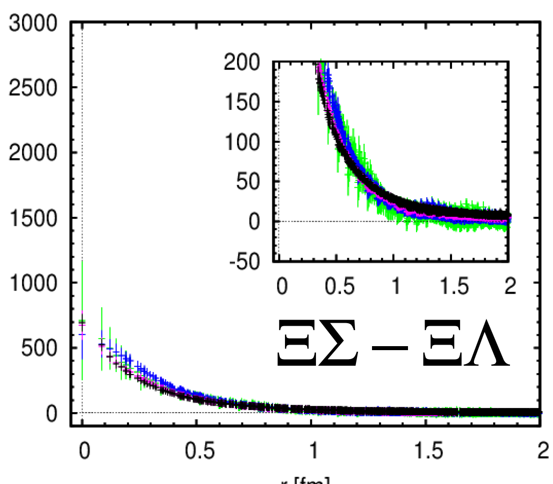



phys. pt. potential

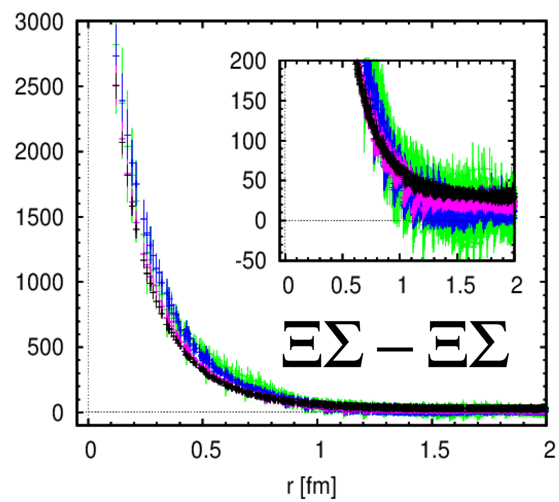
$$V_{\Xi\Lambda;\Xi\Sigma} \leftrightarrow -\frac{3}{10}V^{(27)} + \frac{3}{10}V^{(8S)}$$



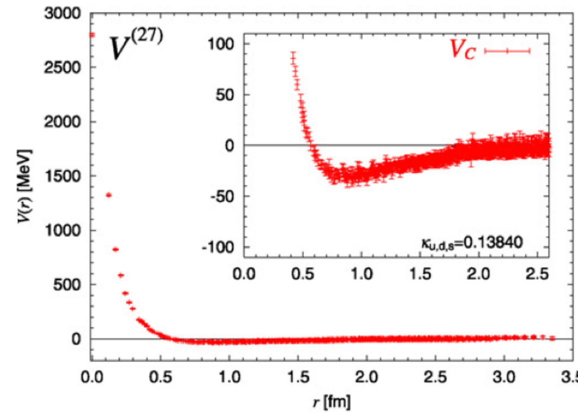

$$V_{\Xi\Sigma;\Xi\Lambda} \leftrightarrow -\frac{3}{10}V^{(27)} + \frac{3}{10}V^{(8S)}$$



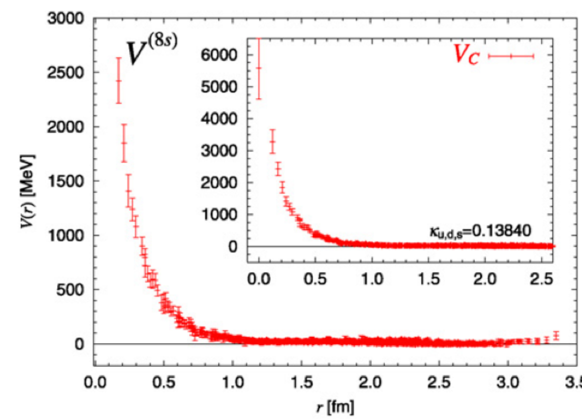

$$V_{\Xi\Sigma;\Xi\Sigma} \leftrightarrow \frac{1}{10}V^{(27)} + \frac{9}{10}V^{(8S)}$$




**27** irrep. in flavor SU(3) limit



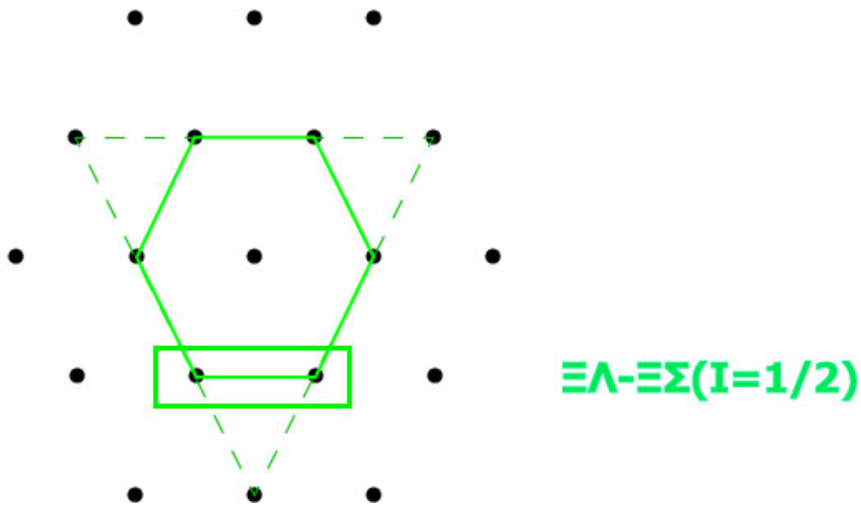

**8S** irrep. in flavor SU(3) limit



# XiLambda-XiSigma coupled channel (I=1/2) spin triplet

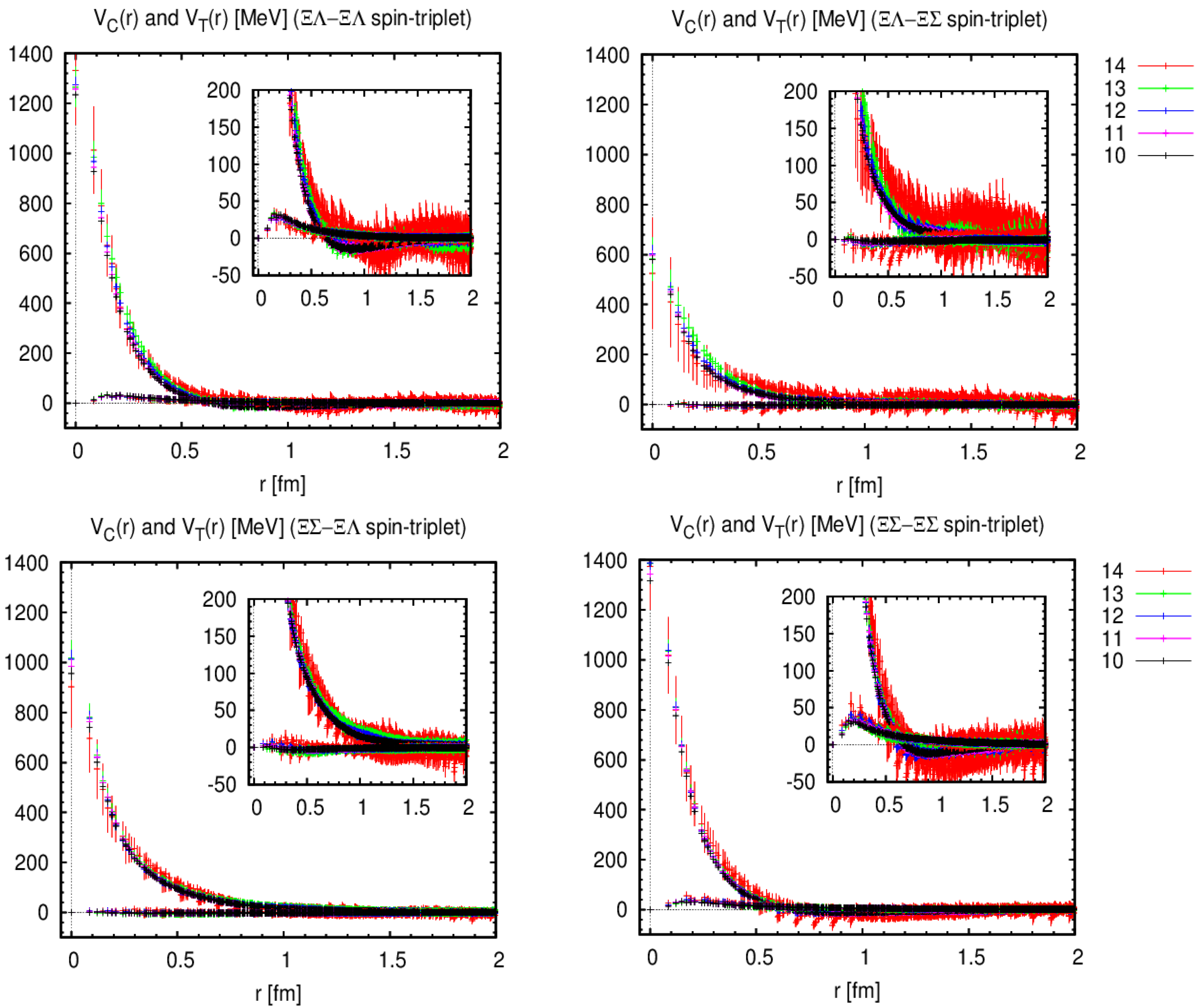
## ◆spin triplet

◆flavor SU(3) limit :
mixture of **10** and **8a** irreps.
(nothing to do with NN)



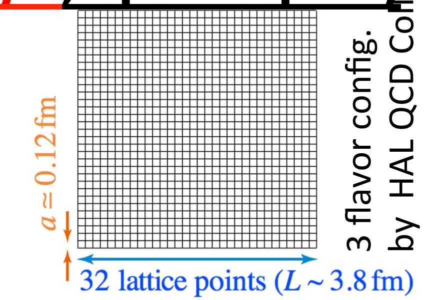
# XiLambda-XiSigma coupled channel potential (I=1/2, spin triplet)





# XiLambda-XiSigma coupled channel potential (I=1/2, spin triplet)

Flavor SU(3) limit to understand qualitative behavior

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Potentials in flavor SU(3) limit
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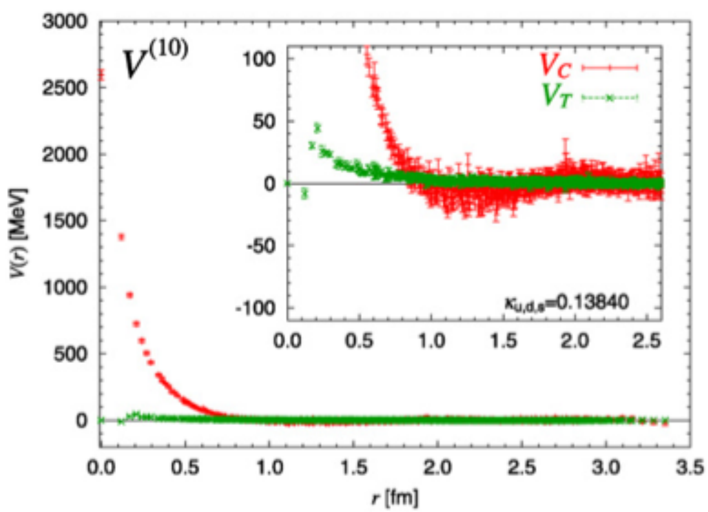
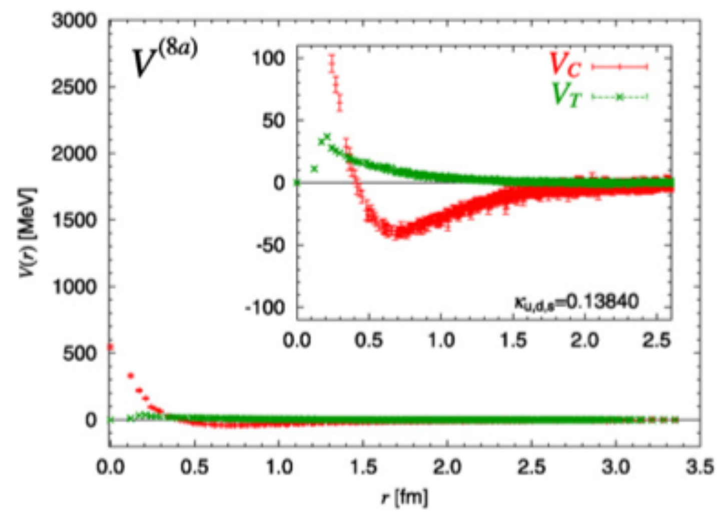
**10** irrep.




**8A** irrep.



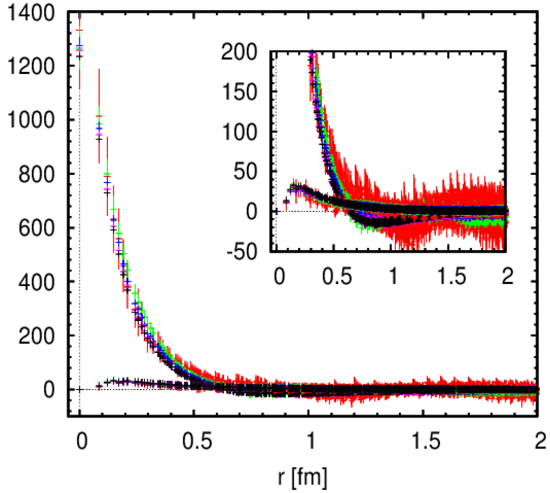

In flavor SU(3) limit, the coupled channel potential is a linear conbinations of the two.

$$\begin{pmatrix} V_{\Xi\Lambda,\Xi\Lambda} & V_{\Xi\Lambda,\Xi\Sigma} \\ V_{\Xi\Sigma,\Xi\Lambda} & V_{\Xi\Sigma,\Xi\Sigma} \end{pmatrix} = \begin{pmatrix} \frac{V_{10}+V_{8A}}{2} & \frac{V_{10}-V_{8A}}{2} \\ \frac{V_{10}-V_{8A}}{2} & \frac{V_{10}+V_{8A}}{2} \end{pmatrix}$$

# XiLambda-XiSigma coupled channel potential (I=1/2, spin triplet)

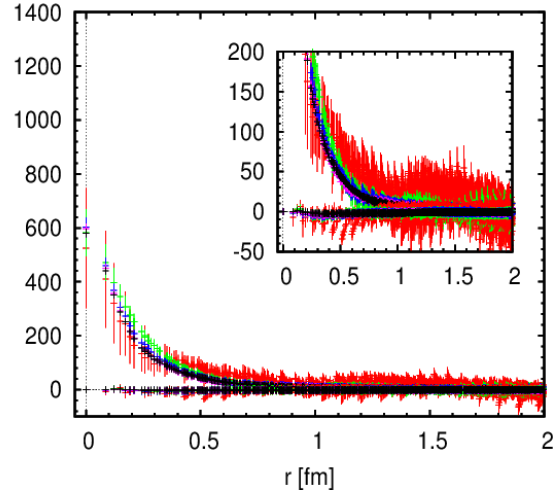
phys. pt. potential

$$V_{\Xi\Lambda;\Xi\Lambda} \leftrightarrow \frac{1}{2}V^{(10)} + \frac{1}{2}V^{(8a)}$$

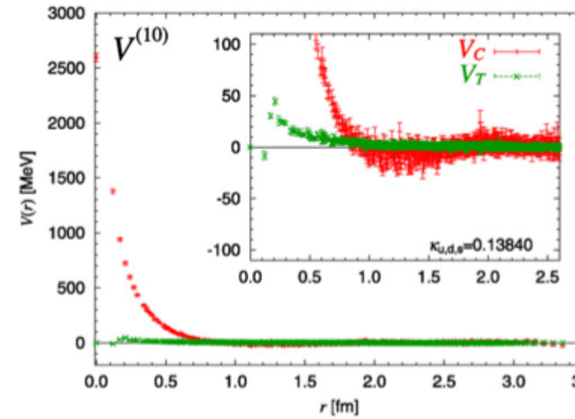



phys. pt. potential

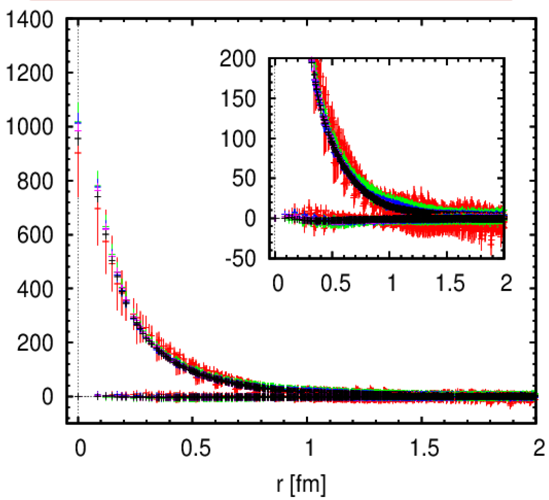
$$V_{\Xi\Lambda;\Xi\Sigma} \leftrightarrow \frac{1}{2}V^{(10)} - \frac{1}{2}V^{(8a)}$$



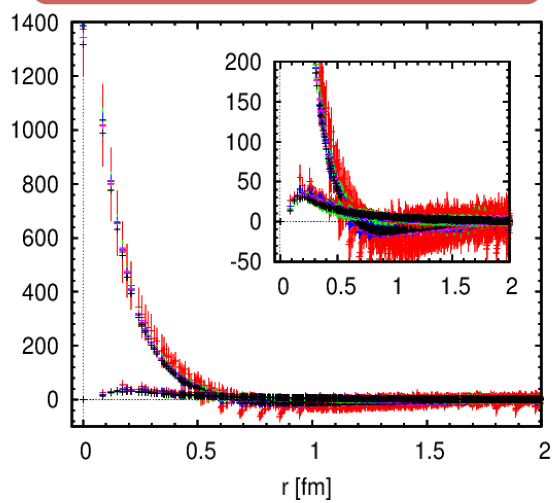

**10** irrep. in flavor SU(3) limit



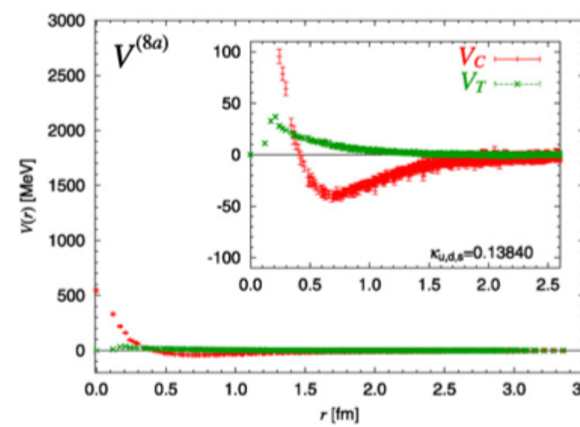

$$V_{\Xi\Lambda;\Xi\Lambda} \leftrightarrow \frac{1}{2}V^{(10)} - \frac{1}{2}V^{(8a)}$$




$$V_{\Xi\Lambda;\Xi\Lambda} \leftrightarrow \frac{1}{2}V^{(10)} + \frac{1}{2}V^{(8a)}$$




**8A** irrep. in flavor SU(3) limit



# Comment

- To derive the time-dependent Schrodinger-like eq.

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) \simeq V(\vec{r},\vec{r}') R(\vec{r}',t)$$

we define the R-correlator to be

$$R(\vec{r},t) \equiv C_{BB}(\vec{r},t) \big/ e^{-2m_B t}$$

$$C_{BB}(\vec{x}-\vec{y},t) \equiv \left\langle 0 \middle| T\left[ B(\vec{x},t) B(\vec{y},t) \cdot \overline{BB(t=0)} \right] \middle| 0 \right\rangle$$

- In the actual numerical calculations, we replace

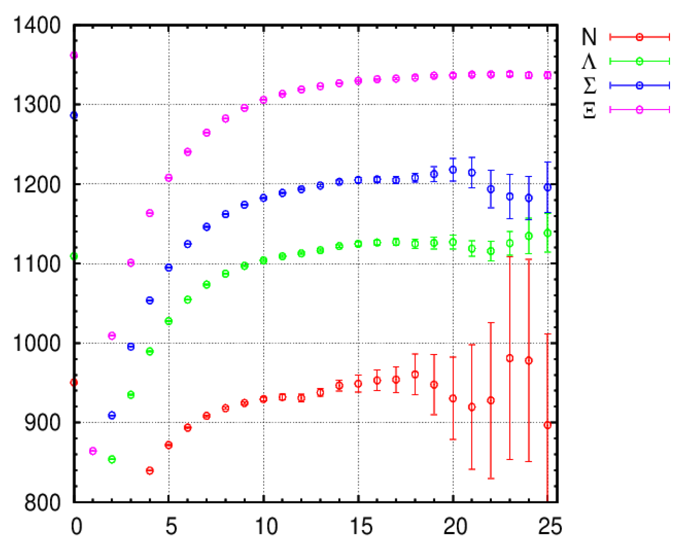
$$R(\vec{r},t) \equiv C_{BB}(\vec{r},t) \big/ C_B(t)^2$$

with $C_B(t) \equiv \left\langle 0 \middle| T\left[ B(t) B(0) \right] \middle| 0 \right\rangle$

to utilize the cancellation of statistical noises.

➔ We need the ground state saturation of single baryon correlators

single baryon effective mass plot




- We have used t = 10-14 because of the statistical reason.

- We desire to go to t = 15 by improving the statistics. (Our plan: x 4.8)

# Summary

- We have presented the results of hyperon potentials and phase shifts for S=-3 sector by phys. pt. gauge configs. (m(pion)=145 MeV)
  - Potentials
    - ✓ ΞΣ(I=3/2) : spin singlet and triplet
    - ✓ ΞΛ-ΞΣ(I=1/2) : spin singlet and triplet
  - Phase shifts
    - ✓ XiSigma(I=3/2): spin singlet and triplet
  - Qualitative behaviors are consistent with flavor SU(3) limit.

- Todo
  - improvement of statistics

    Now : 200 conf x 4 rot x 20 src. pts.

    ⇓ x 4.8

    Future : 400 conf x 4 rot x 48 src. pts.

  - We will use the full time-dependent Schrödinger-like eq.

    (Non-rela. approx. has been used to stabilize the result at the moment)

**backup**

**HALQCD method**

SKIP

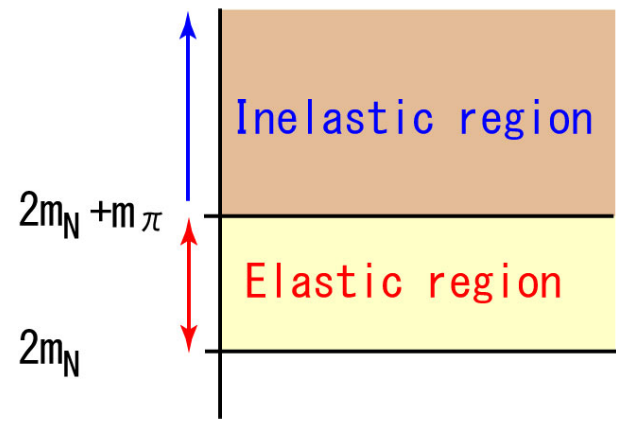
## ◆Proof of existence of E-indep. U(r,r')

### ◆Assumption:

Linear indep. of NBS wave func's for E < Eth.

→ Dual basis exists

$$\int d^3r\, \tilde{\psi}_{\vec{k}'}(\vec{r})\psi_{\vec{k}}(\vec{r}) = (2\pi)^3\delta^3(\vec{k}'-\vec{k})$$




### ◆Proof:

$$K_{\vec{k}}(\vec{r}) \equiv \left(k^2/m_N - H_0\right)\psi_{\vec{k}}(\vec{r})$$

$$K_{\vec{k}}(\vec{r}) = \int \frac{d^3k'}{(2\pi)^3} K_{\vec{k}'}(\vec{r}) \int d^3r'\, \tilde{\psi}_{\vec{k}'}(\vec{r})\psi_{\vec{k}}(\vec{r})$$

$$= \int d^3r' \left\{ \int \frac{d^3k}{(2\pi)^3} K_{\vec{k}'}(\vec{r})\tilde{\psi}_{\vec{k}'}(\vec{r}') \right\} \psi_{\vec{k}}(\vec{r}')$$

$$\left(k^2/m_N - H_0\right)\psi_{\vec{k}}(\vec{r}) = \int d^3r'\, U(\vec{r},\vec{r}')\psi_{\vec{k}}(\vec{r}')$$

$$U(\vec{r},\vec{r}') \equiv \int \frac{d^3k'}{(2\pi)^3} K_{\vec{k}'}(\vec{r})\tilde{\psi}_{\vec{k}'}(\vec{r})$$

**U(r,r') does not depend on E**
because of the integration of k'.