Decay constants f_B and f_{B_s} and quark masses m_b and m_c from HISQ Simulations

Javad Komijani
(Fermilab Lattice, MILC and TUMQCD Collaborations)

Technische Universität München

34th International Symposium on Lattice Field Theory
24-30 July 2016

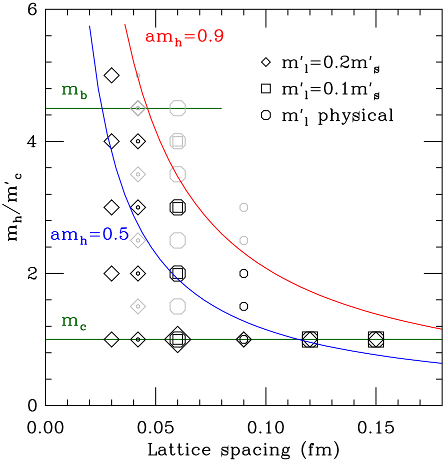
Fermilab Lattice, MILC and TUMQCD Collaborations

A. Bazavov	N. Brambilla	C. Bernard	C. Bouchard
C. DeTar	D. Du	A.X. El-Khadra	E.D. Freeland
E. Gamiz	Steven Gottlieb	U.M. Heller	J. K.
A.S. Kronfeld	J. Laiho	P.B. Mackenzie	C. Monahan
E.T. Neil	J.N. Simone	R.L. Sugar	D. Toussaint
A. Vairo	R.S. Van de Water	R. Zhou	

Motivation and Goal

- Test SM and look for signs of new physics using heavy-light systems
- Need for precise determination
  - Decay constants $\rightarrow$ CKM matrix elements
  - m_b and m_c $\rightarrow$ Higgs boson branching ratios
- Lattice QCD allows us to calculate the decay constants and masses of heavy-light systems for various choices of heavy and light quark mass
- EFTs can be used to perform a combine fit of quark-mass and lattice-spacing dependence to control the systematic error of extrapolation

2+1+1 HISQ Ensembles and Heavy Quark Masses



- 24 Ensembles:
  - 6 Lattice spacings
  - Several sea masses
- Two-point propagators with various light and heavy masses
  - light valence: m_l to m_s
  - heavy valence: m_c to m_b
- am_h: valence heavy mass
- am'_c: simulated sea charm mass
- Drop some m_h/m'_c points to avoid large discretization errors ($am_h < 0.9$)

Constructing Fit Function for Decay Constants

- The fit function can be schematically written as

$$\Phi_{H_q} = C\ (1 + \mathsf{SET})\ (1 + \mathsf{HQET})\ (1 + \mathsf{HMrAS}\chi\mathsf{PT}) \left(\frac{m'_c}{m_c}\right)^{3/27} \tilde{\Phi}_0$$

- These terms correspond to different effective field theories

Symanzik Effective Theory (SET)

$$c_1\alpha_s\,(a\Lambda)^2 + c_2(a\Lambda)^4 + c_3\alpha_s\,(am_h)^2 + c_4(am_h)^4 + c_5\alpha_s\,(am_h)^4$$

Wilson coefficient C

$$\left[\alpha_s(M_{H_s})\right]^{-6/25}\left(1+\mathcal{O}(\alpha_s)\right)$$

HQET (and integrating out sea-charm)

$$k_1\frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}} + k_2\left(\frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}}\right)^2 + k'_1\frac{m_c}{m'_c}$$

Integrating out charm quark

$$\frac{\Lambda^{(3)}_{\mathrm{QCD}}(m'_c)}{\Lambda^{(3)}_{\mathrm{QCD}}(m_c)} \approx \left(\frac{m'_c}{m_c}\right)^{2/27}$$

HMrASχPT at NLO

Chiral-Logs

$$+L_{\mathsf{v}}m_{\mathsf{v}} + L_{\mathsf{s}}(2m_l + m_s) + L_a a^2$$

Chiral logarithms contain effects of

- Taste splittings in “pion” masses & new logs.
- Hyperfine and flavor splittings
- Finite lattice volume

- HMrASχPT at NLO is not sufficient for current precision of lattice data

HMrASχPT at NLO

Chiral-Logs

$+L_{\mathsf{v}} m_{\mathsf{v}} + L_{\mathsf{s}}(2m_l + m_s) + L_a a^2$

$+$ NNLO ($+$NNNLO) analytic terms

m_{v}^2, $(2m_l + m_s)m_{\mathsf{v}}$, $(2m_l + m_s)^2$, $(2m_l^2 + m_s^2)$

m_{v}^3, ~~6 cubic terms depending on sea masses~~

- HMrASχPT at NLO is not sufficient for current precision of lattice data

HMrASχPT at NLO

Chiral-Logs

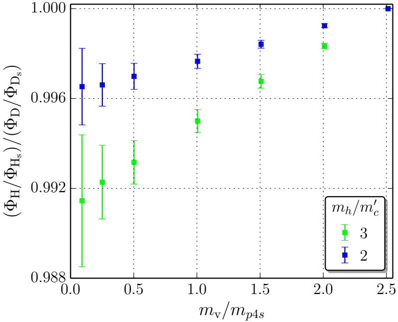
$+L_{\mathsf{v}} m_{\mathsf{v}} + L_{\mathsf{s}}(2m_l + m_s) + L_a a^2$

$+$ NNLO ($+$NNNLO) analytic terms

m_{v}^2, $(2m_l + m_s)m_{\mathsf{v}}$, $(2m_l + m_s)^2$, $(2m_l^2 + m_s^2)$

m_{v}^3, ~~6 cubic terms depending on sea masses~~

- L_{v}, L_{s} and other LECs have $\mathcal{O}(1/m_h)$ corrections



$p4s$ unites: $m_{p4s} = 0.4 m_s$

- Construct double-ratio in which leading-order terms in SET, HQET, HMrASχPT cancel to look for higher-order terms that depend upon both light- and heavy-quark masses
- LECs must have dependence on m_h

$$L_{\mathsf{v}} \to L_{\mathsf{v}} + L_{\mathsf{v}'} x + L_{\mathsf{v}''} x^2$$
$$L_{\mathsf{s}} \to L_{\mathsf{s}} + L_{\mathsf{s}'} x + L_{\mathsf{s}''} x^2$$
$$L_{\mathsf{vv}} \to L_{\mathsf{vv}} + L_{\mathsf{vv}'} x$$
$$x = \left(\frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}} - \frac{\Lambda_{\mathrm{HQET}}}{M_{D_s}}\right)$$

Description of a "Central" Fit

- Set $g_\pi = 0.45$
  (free parameter in alternative fits)

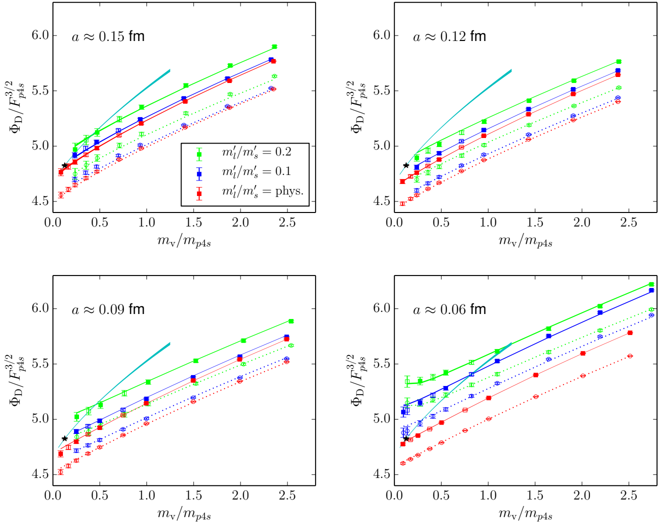
$$g_\pi = \begin{cases} < 0.53, & \text{B system} \\ 0.53(8), & \text{D system} \end{cases}$$

- 23 parameters
- Combined correlated, multidimensional fit to 328 data points
  - 6 lattice spacings
  - several sea-quark masses
  - several valence-quark masses

    m_h/m'_c: 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5 and 5 (if $am_h < 0.9$)

    m_v/m'_s: 1, 0.8, 0.6, 0.4, 0.2, [0.1 and 0.04]

Snapshot: Dependence on a and m_{v}



P-value $= 0.95$
Filled Symbols: included
Open Symbols: excluded

Upper points: $m_h = m'_c$

Lower points: $m_h = 0.9 m'_c$
(illustrated for sanity check of extrapolation, not included in the fits)

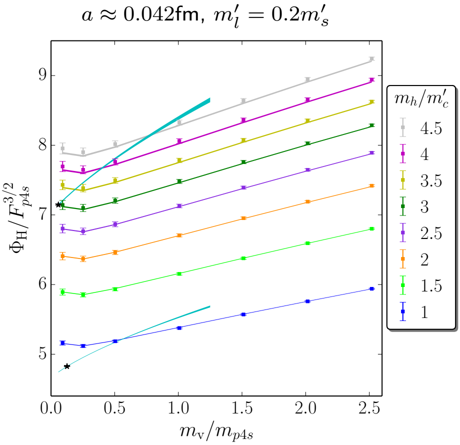
Cyan band:
unitary/continuum
($m_l \approx m_{\mathsf{v}}$, up to difference between m_d and m_l)

Black star:
Φ_{D^+} at continuum

PQ effects at 0.06fm
(when $m_{\mathsf{v}} < m'_l$)

Hessian errors

Snapshot: Dependence on m_h and m_{v}



P-value = 0.95

Upper cyan band:
unitary/continuum, B system
($m_l \approx m_{\mathrm{v}}$, up to difference between m_u and m_l)

Black star: Φ_{B+} at continuum

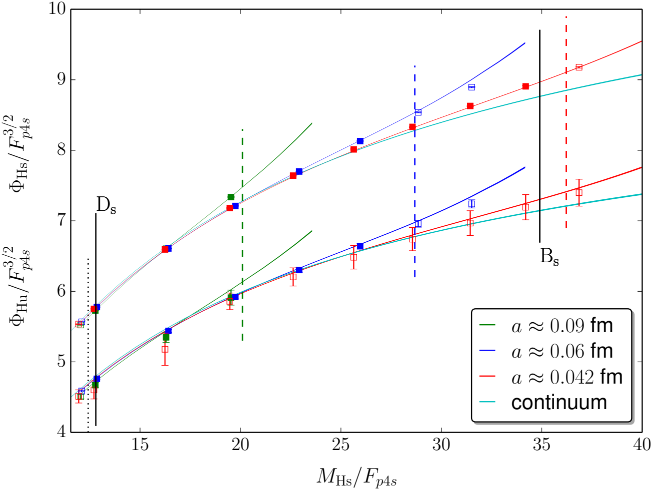
Lower cyan band:
unitary/continuum, D system
($m_l \approx m_{\mathrm{v}}$, up to difference between m_d and m_l)

Black star: Φ_{D+} at continuum

PQ effects (when $m_{\mathrm{v}} < m_l'$)

Hessian errors

Snapshot: Dependence on a and m_h



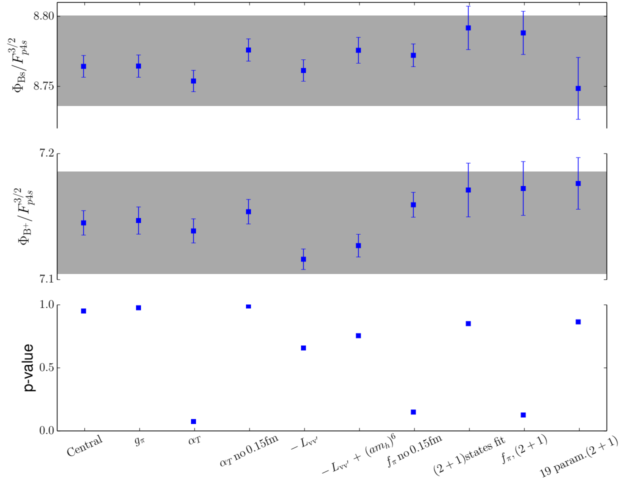
P-value = 0.95
Filled Symbols: included
Open Symbols: excluded

Upper curves: $m_{\rm v} = m_s$

Lower curves: $m_{\rm v} = m_u$
Cyan bands:
extrap. to continuum

Hessian errors

Stability under Different Assumptions



10 choices for chiral fits

8 versions of inputs

Error bars: Hessian errors

Error bands: estimate the full systematic error

Decay-constant Error Budget

- Estimation of systematic error based on Histogram

	f_{B^+}	f_{B_s}
Histograms*	1.1	0.8 MeV
Statistics**	0.4	0.3
Scale(f_π)	0.3	0.4
Total	1.2	1.0 MeV

  * Includes chiral fit, light- and heavy-quark discretization, excited state contamination, FV and EM error
  ** Contains the statistical error in setting the lattice-space and quark mass ratios

- The analysis is closed to be finalized

Part 2: Extracting m_b and m_c

- The same data set can be used to extract heavy quark masses
- We present our strategy to determine m_b and m_c

Heavy-light Meson Mass in HQET

- Expansion of the mass of a heavy-light system in terms of heavy quark mass

$$M_H = m_h + \bar{\Lambda} + \frac{\mu_\pi^2}{2m_h} - \frac{\mu_G^2(m_h)}{2m_h} + \mathcal{O}(1/m_h^2)$$

- $\bar{\Lambda}$: energy of quark and gluons inside the system
- $\mu_\pi^2/2m_h$: kinetic energy of the heavy quark inside the system
- $\mu_G^2(m_h)/2m_h$: hyperfine energy due to heavy quark's spin (μ_G runs)
- Challenges:
  - How apply it to lattice data?
    How to map the bare mass in lattice units to some renormalized mass?
  - What quark mass?
    Renormalon problem in the pole mass

Heavy-light Meson Mass in HQET

- Expansion of the mass of a heavy-light system in terms of heavy quark mass

$$M_H = m_h + \bar{\Lambda} + \frac{\mu_\pi^2}{2m_h} - \frac{\mu_G^2(m_h)}{2m_h} + \mathcal{O}(1/m_h^2)$$

- $\bar{\Lambda}$: energy of quark and gluons inside the system
- $\mu_\pi^2/2m_h$: kinetic energy of the heavy quark inside the system
- $\mu_G^2(m_h)/2m_h$: hyperfine energy due to heavy quark's spin (μ_G runs)
- Challenges:
    - How apply it to lattice data?
      How to map the bare mass in lattice units to some renormalized mass?
    - What quark mass?
      Renormalon problem in the pole mass
      $\rightarrow$ Renormalon-subtracted scheme [Pineda hep-ph/0105008]

Quark Masses on Lattice

- The key relation

$$\lim_{a\to 0} \frac{am_2}{am_1} = \frac{m_2^{\overline{\mathsf{MS}}}(\mu)}{m_1^{\overline{\mathsf{MS}}}(\mu)}$$

- The pole mass on lattice [Mason,et al hep-lat/0510053]

$$\begin{aligned} m^{\text{pole}} &= \frac{am}{a}\left[1 + \alpha_{\mathsf{Lat}}\left(-\frac{2}{\pi}\log(am) + A_{10}\right) + \mathcal{O}(\alpha_{\mathsf{L}}^2)\right] \\ \alpha_{\mathsf{Lat}}^{-1} &= \alpha_{\overline{\mathsf{MS}}}^{-1}(\mu) - 2\beta_0 \ln\left(a\mu/\pi\right) + \text{const.} + \mathcal{O}(\alpha_{\overline{\mathsf{MS}}}) \\ A_{10} &= \text{const.} + \mathcal{O}\left((am)^2\right) \end{aligned}$$

- Ratios

$$\frac{m_2^{\text{pole}}}{m_1^{\text{pole}}} = \frac{am_2}{am_1}\left(1 - \frac{2}{\pi}\alpha_{\overline{\mathsf{MS}}}(\mu)\left[\log\left(\frac{am_2}{am_1}\right) + A_{10}\left((am_2)^2\right) - A_{10}\left((am_1)^2\right)\right] + \mathcal{O}(\alpha^2)\right)$$

- As $a \to 0$, we find the continuum relation of ratios of pole masses and $\overline{\text{MS}}$ masses

Quark Masses on Lattice

- The key relation

$$\lim_{a\to 0} \frac{am_2}{am_1} = \frac{m_2^{\overline{\mathsf{MS}}}(\mu)}{m_1^{\overline{\mathsf{MS}}}(\mu)}$$

- The pole mass on lattice [Mason,et al hep-lat/0510053]

$$\begin{aligned} m^{\text{pole}} &= \frac{am}{a}\left[1+\alpha_{\mathsf{Lat}}\left(-\frac{2}{\pi}\log(am)+A_{10}\right)+\mathcal{O}(\alpha_{\mathsf{L}}^2)\right] \\ \alpha_{\mathsf{Lat}}^{-1} &= \alpha_{\overline{\mathsf{MS}}}^{-1}(\mu)-2\beta_0\ln\left(a\mu/\pi\right)+\text{const.}+\mathcal{O}(\alpha_{\overline{\mathsf{MS}}}) \\ A_{10} &= \text{const.}+\mathcal{O}\left((am)^2\right) \end{aligned}$$

- Ratios

$$\frac{m_2^{\text{pole}}}{m_1^{\text{pole}}} = \frac{am_2}{am_1}\left(1-\frac{2}{\pi}\alpha_{\overline{\mathsf{MS}}}(\mu)\left[\log\left(\frac{am_2}{am_1}\right)+A_{10}\left((am_2)^2\right)-A_{10}\left((am_1)^2\right)\right]+\mathcal{O}(\alpha^2)\right)$$

- As $a\to 0$, we find the continuum relation of ratios of pole masses and $\overline{\mathsf{MS}}$ masses
- Consider a fit parameter to capture this lattice artifact as

$$A_{10}\left((am_2)^2\right)-A_{10}\left((am_1)^2\right) \quad \to \quad K\left[(am_2)^2-(am_1)^2\right]$$

Procedure to Map Lattice Mass to Pole Mass

1) At each ensemble calculate the tuned charm mass am_c
2) Consider the continuum relation of the $\overline{\text{MS}}$ and pole mass (3-loop order)

$$\frac{m_h^{\text{pole}}}{m_c^{\text{pole}}} = \frac{m_h(\mu)}{m_c(\mu)}\left[1 - \frac{2\alpha_s(\mu)}{\pi}\log\left(\frac{m_h(\mu)}{m_c(\mu)}\right) + \cdots\right]$$

replace $\frac{m_h(\mu)}{m_c(\mu)}$ with $\frac{am_h}{am_c}$
and add the lattice artifact at $\mathcal{O}\big(\alpha\ (am)^2\big)$ to the above expression

3) Introduce a fit parameter M_c,

$$m_h^{\text{pole}} = M_c\ \frac{m_h^{\text{pole}}}{m_c^{\text{pole}}}$$

4) Subtract the renormalon

- Schematically we have

$$\frac{am_h}{am_c} \to \frac{m_h(\mu)}{m_c(\mu)} \to \frac{m_h^{\text{pole}}}{m_c^{\text{pole}}} \to m_h^{\text{pole}} \to m_h^{\text{RS}}$$

The Fit Function

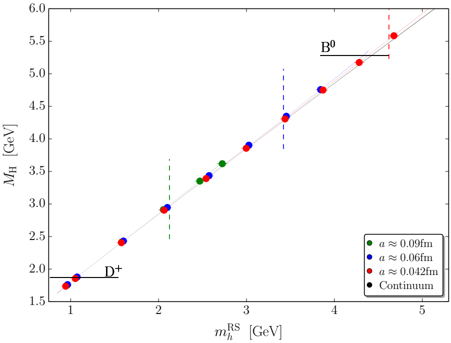
- The fit function in terms of renormalon-subtracted mass

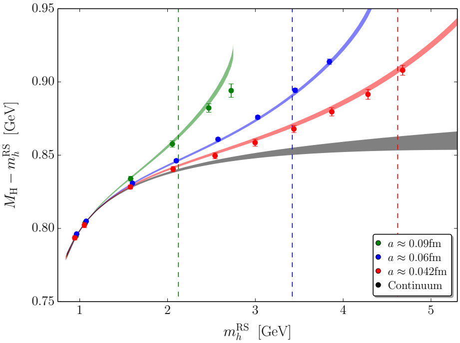
$$M_H = m_h^{\mathrm{RS}} + \bar{\Lambda} + \frac{\mu_\pi^2}{2m_h^{\mathrm{RS}}} - \frac{\mu_G^2(m_h^{\mathrm{RS}})}{2m_h^{\mathrm{RS}}} + \frac{\lambda_3}{(m_h^{\mathrm{RS}})^2}$$

- Add more terms if required

Fit to Lattice Data

- A sample fit to 16 data points with 6 fit parameters
- Data points at three lattice spacings with $am'_c \leq am_h < 0.9$





- Work in progress

Conclusion

- Decay constants
  - We have presented the status of our analysis of f_{B+} and f_{B_s} from a calculation with all HISQ quarks
  - Small errors due to having
    - Physical mass ensembles, especially at 0.045 fm help anchor the chiral analysis
    - 0.03 fm ($m_l' = 0.2 m_s'$) ensemble ($am_b = 0.6$)
- Quark masses
  - HQET description of heavy-light meson masses is used to analyze the lattice data
  - Analysis is in progress

Conclusion

- Decay constants
  - We have presented the status of our analysis of f_{B+} and f_{B_s} from a calculation with all HISQ quarks
  - Small errors due to having
    - Physical mass ensembles, especially at 0.045 fm help anchor the chiral analysis
    - 0.03 fm ($m_l' = 0.2 m_s'$) ensemble ($am_b = 0.6$)
- Quark masses
  - HQET description of heavy-light meson masses is used to analyze the lattice data
  - Analysis is in progress

Thanks for your attention!