Progress update: Nucleon structure with $N_f = 2+1$ Wilson fermions

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Outline

Introduction

- Measurements and ensemble parameters
- Truncated solver method

Preliminary analyses of isovector g_A, $G_E(Q^2)$ and $G_M(Q^2)$

- Excited states
- Dipole and z-expansion

Outlook

Introduction

We examine the spacelike isovector Sachs electromagnetic form factors

$$G_{\mathrm{E}}(Q^2) = F_1(Q^2) - \frac{Q^2}{2m_N^2} F_2(Q^2) \quad \text{and} \quad G_{\mathrm{M}}(Q^2) = F_1(Q^2) + F_2(Q^2)$$

and the isovector axial coupling, $g_{\mathrm{A}} \equiv G_{\mathrm{A}}(0)$ where

$$\langle N(\boldsymbol{p}', s')|\bar{\psi}\gamma_\mu\psi|N(\boldsymbol{p}, s)\rangle = \bar{u}(\boldsymbol{p}', s')\left[\gamma_\mu F_1(Q^2) + i\frac{\sigma_{\mu\nu}q_\nu}{2m_N} F_2(Q^2)\right] u(\boldsymbol{p}, s),$$

$$\langle N(\boldsymbol{p}', s')|\bar{\psi}\gamma_5\gamma_\mu\psi|N(\boldsymbol{p}, s)\rangle = \bar{u}(\boldsymbol{p}', s')\left[\gamma_5\gamma_\mu G_{\mathrm{A}}(Q^2) + \frac{\gamma_5 q_\mu}{2m_N} G_{\mathrm{P}}(Q^2)\right] u(\boldsymbol{p}, s).$$

These constitute difficult observables in particular due to

- excited state-contamination $\rightsquigarrow$
- signal-to-noise problem $\rightsquigarrow$
- other systematics probed by exploring parameter space $(a, m_{\mathrm{PS}}, m_{\mathrm{PS}}L)$

Lattice estimators

The effective form factors, e.g.

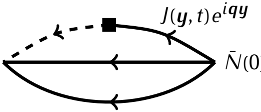
$$G_{\mathrm{E}}^{\mathrm{eff}}(Q^2) = \sqrt{\frac{2E_{Q^2}}{m+E_{Q^2}}}\, R_{V_0}(t,t_s;Q^2), \qquad g_{\mathrm{A}}^{\mathrm{eff}} = -iR_{A_3}(t,t_s;Q^2=0),$$

are defined in terms of the ratios

$$R_J(t,t_s;Q^2) = \frac{C_{3,J}(t,t_s;\boldsymbol{q})}{C_2(t_s;\boldsymbol{q})}\sqrt{\frac{C_2(t_s-t;-\boldsymbol{q})C_2(t,\boldsymbol{0})C_2(t_s;\boldsymbol{0})}{C_2(t_s-t;\boldsymbol{0})C_2(t;-\boldsymbol{q})C_2(t_s;-\boldsymbol{q})}}$$

[Alexandrou et al.]

are computed for fixed $(\boldsymbol{p}'=\boldsymbol{0}, t_s)$ using sequential propagators

$$C_{3,J}(t,t_s;\boldsymbol{q}) = \Gamma\langle \textstyle\sum_{\boldsymbol{x},\boldsymbol{y}} N(\boldsymbol{x},t_s)$$  $$\bar{N}(0)\rangle$$

still contain contributions from excited states at finite t_s
$\Longrightarrow$ try to choose good interpolating operators $N(\mathrm{x},t)$

Parameter space

id	m_π/MeV	a/fm	L/a	$m_\pi L$	N_{meas}	t_s/fm
H102	350	0.086	32	4.9	7988	{1.0, 1.1, 1.3}
H105	280	″	32	3.9	11412	″
*C101	220	″	48	4.7	32416	″
N200	280	0.064	48	4.4	3200	{0.7, 0.8, 1.0, 1.1, 1.2, 1.3}
*D200	200	″	64	4.2	13056	{1.0, 1.1, 1.2, 1.3}
N303	330	0.05	64	4.8	–	{1.0, 1.1, 1.2, 1.3}
J303	280	″	″	4.1	–	″

Ensembles generated using the CLS effort used in this work

- $N_f = 2+1$ flavours of $O(a)$-improved Wilson clover fermion. [Bruno et al.]
- Open boundary conditions in time combat poor scaling of autocorrelation of topological charge as $a \to 0$. [Lüscher, Schäfer]
- Twisted-mass regulator guards against exceptional configurations. [Lüscher, Palombi]

Truncated solver method

Efficient measurements using the truncated solver method: [Bali, 0910.3970; Shintani, 1402.0244]

$$\langle \mathcal{O} \rangle = \underbrace{\langle \mathcal{O}_{\mathrm{LP},N} \rangle}_{\text{cheap}} + \underbrace{\langle \mathcal{O} \rangle - \langle \mathcal{O}_{\mathrm{LP},1} \rangle}_{\text{expensive}}$$

The contribution of the bias correction $\langle \mathcal{O} \rangle - \langle \mathcal{O}_{\mathrm{LP},1} \rangle$ to the variance is small if

$$1 - \mathrm{Corr}(\mathcal{O}, \mathcal{O}_{\mathrm{LP},1}) \ll \frac{N}{2} \qquad \Longrightarrow \qquad \text{tune } N_{\mathrm{iter}}$$

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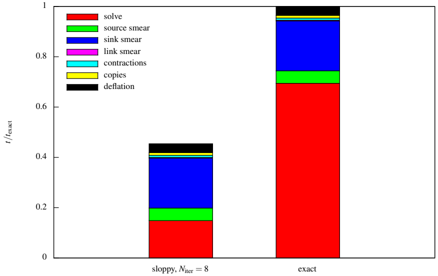

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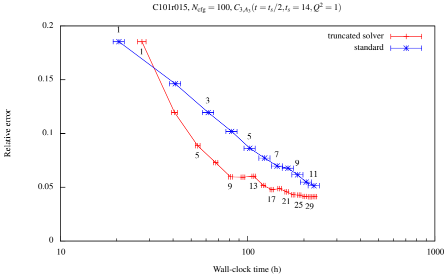

Asymptotic matrix elements 1: Two-state fits

Two-state fit

$$G_X^{\mathrm{eff}}(t, t_s, Q^2) = \widehat{G_X}(Q^2) + c_{1,X}(Q^2)e^{-\Delta t} + c_{2,X}(Q^2)e^{-\Delta'(t_s - t)}$$

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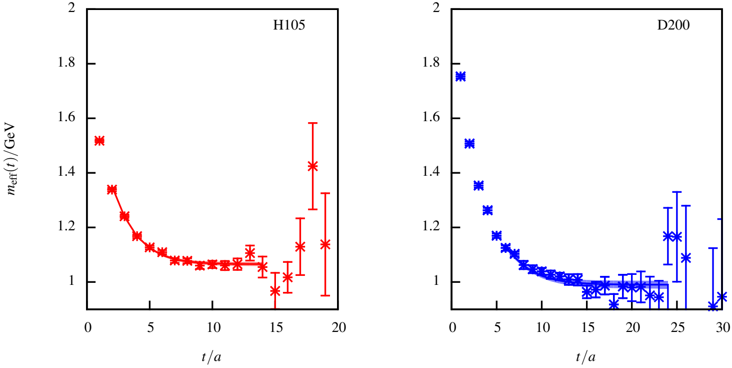

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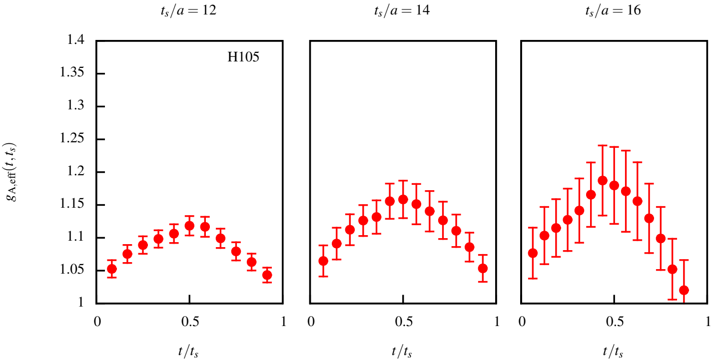

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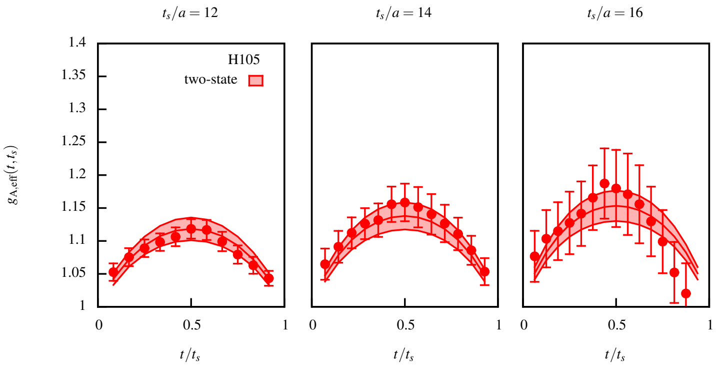

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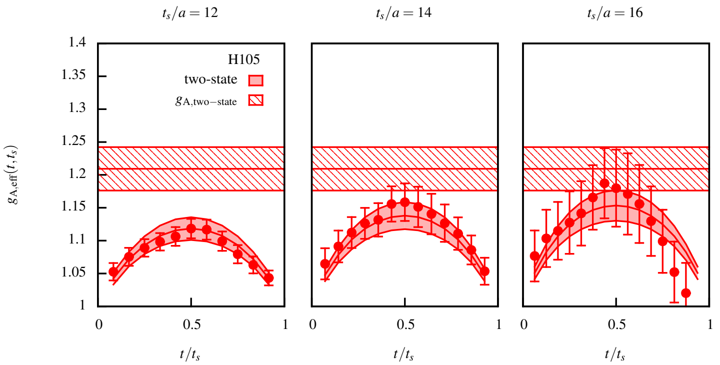

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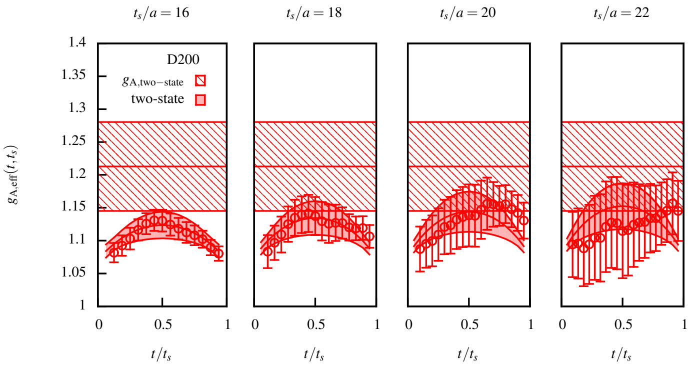

Asymptotic matrix elements 2: Summation method

Summed operator insertions method

$$S_X(t_s;Q^2)/a \equiv \sum_{t=a}^{t_s-a} G_X^{\text{eff}}(t,t_s;Q^2) = c_X(Q^2) + t_s\widehat{G_X}(Q^2)$$

$\widehat{G_X}$ has $O(e^{-\Delta t_s})$ corrections

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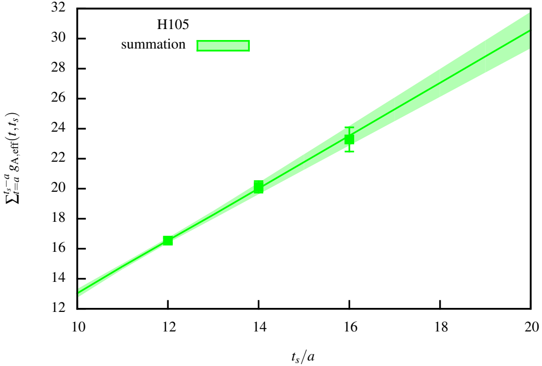

Asymptotic matrix elements 3: Many-states for g_A

Many two-particle states

- Finite-volume $N\pi$ levels are close to non-interacting for typical parameters
  [Hansen, Fri. 14.20]
- Although $E_{N\pi} > E_{N\pi\pi}$, amplitudes enhanced by L^3
- LO ChPT analysis predicts $\langle N|A_\mu|N(\boldsymbol{p})\pi(-\boldsymbol{p})\rangle$ slowly varying with $\boldsymbol{k}$
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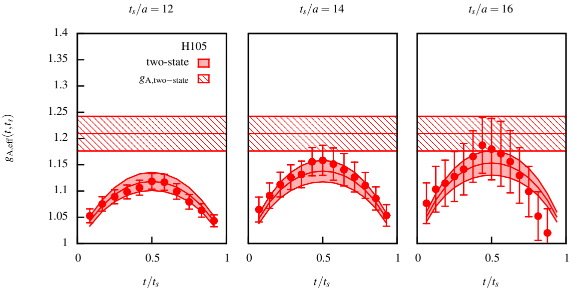

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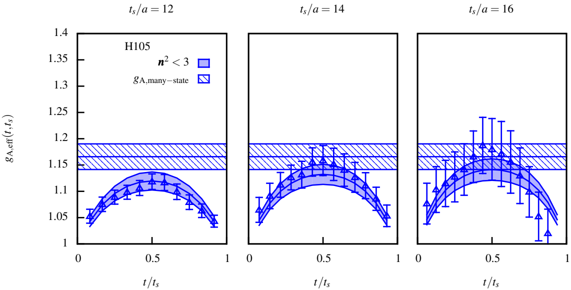

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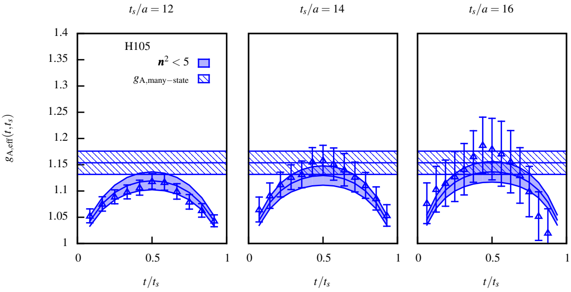

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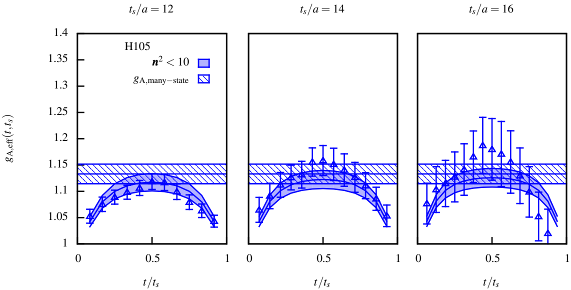

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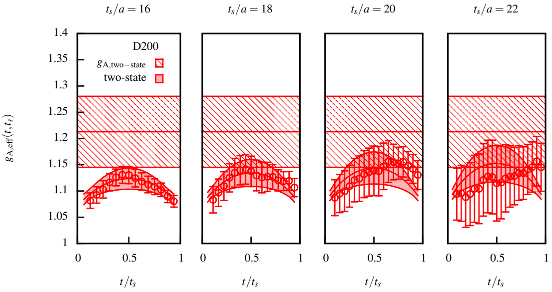

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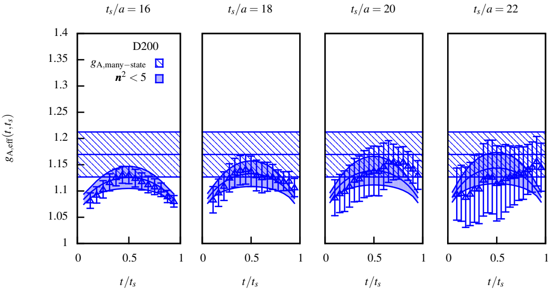
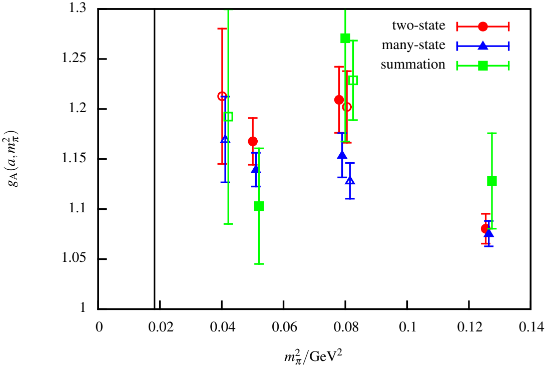
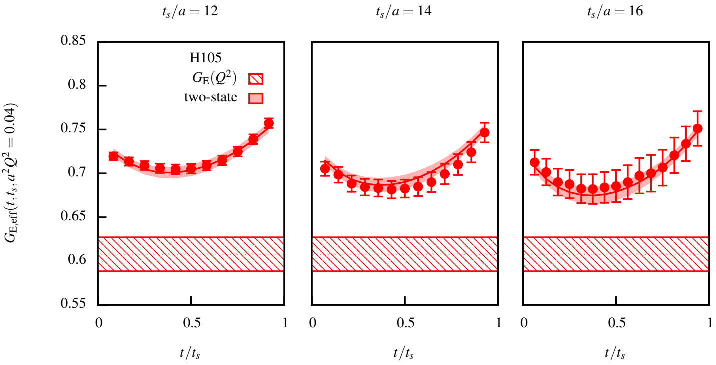


g_A summary



- Too narrow a range of t_s to constrain summation method
- Excited state systematics need to be quantified

Asymptotic matrix elements: Two-state method



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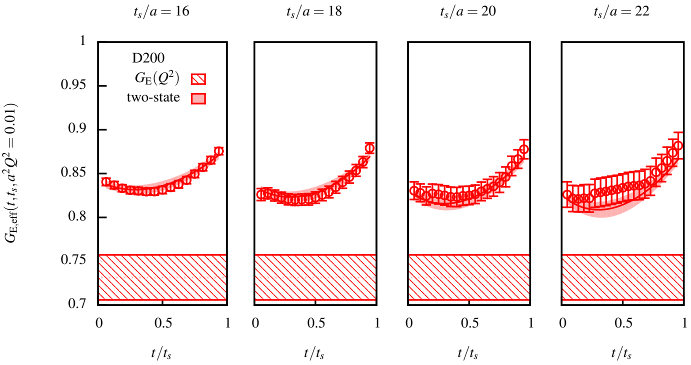

Extracting the electric and magnetic radii

- Dipole ansatz

$$F_X(Q^2) = \frac{\widehat{F_X}(0)}{1 + Q^2/\widehat{M_X}^2} \qquad \widehat{r_X}^2 = \frac{12}{\widehat{M_X}^2} \qquad ⚠$$

- z-expansion

[Hill, 1008.4619]

$$F_X(Q^2) = \sum_{n=1}^{n_{\max}} a_n z(Q^2)^n \qquad z(Q^2) = \frac{\sqrt{4m_\pi^2 + Q^2} - 2m_\pi}{\sqrt{4m_\pi^2 + Q^2} + 2m_\pi}$$

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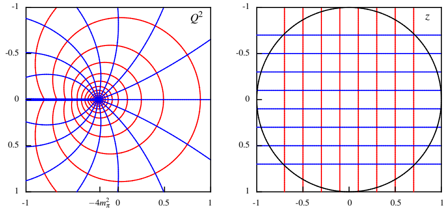

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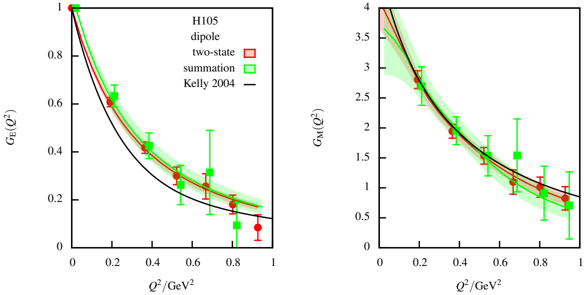

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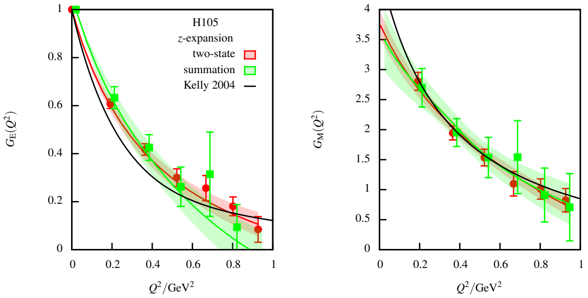

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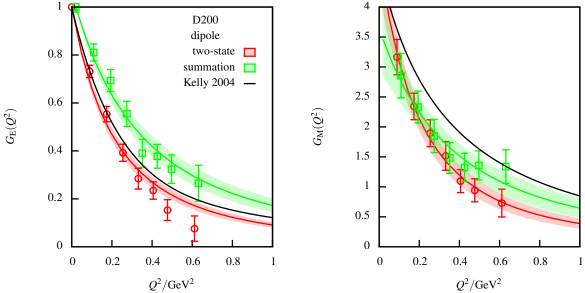

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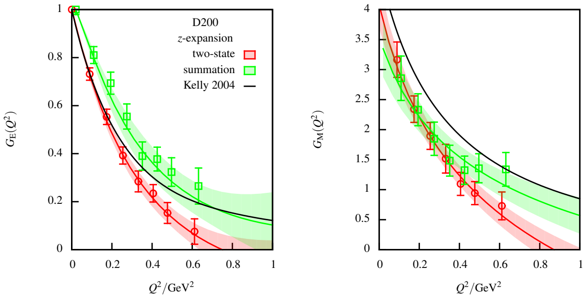
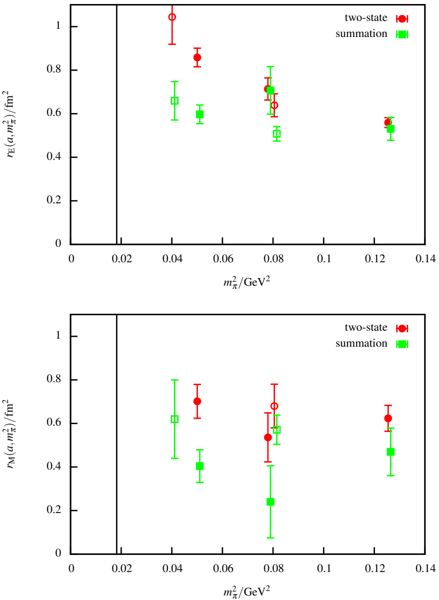


r_E and r_M summary



Outlook

Ongoing work

- Quantify systematics due to excited-state contamination
- Full O(a)-improvement
- Extrapolation to the physical point
- Other couplings g_S and g_T

Future work

- Application of many new technologies exhibited at this conference