Triple-gluon and quark-gluon vertex from lattice QCD in Landau gauge

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Overview	in collaboration with
1) Motivation	
2) Triple-Gluon	Balduf (HU Berlin)
3) Quark-Gluon	Kızılersü & Williams (Adelaide U), Oliveira & Silva (Coimbra U), Skullerud (NUIM, Maynooth)

Motivation

Research in hadron physics

- Successful but not restricted to lattice QCD
- Other nonperturbative frameworks exist (for better or for worse)
  - Bound-state equations / Dyson-Schwinger equations
  - Functional Renormalization group (FRG) equation (aka Wetterich equation)

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- Bethe-Salpether equations: Mesonic systems ($q\bar{q}$)
- Faddeev/ quark-diquark equations: Baryonic systems (qqq)
- no restriction to Euclidean metric (makes it simpler)
  - for lattice QCD Euclidean metric mandatory
    (can calculate static quantities: masses, etc. or equilibrium properties)

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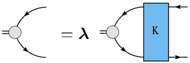
    (can calculate static quantities: masses, etc. or equilibrium properties)
- **Input**: nonperturbative n-point Green's functions (in a gauge)
  - typically taken from numerical solutions of their Dyson-Schwinger equations
  - Note: Greens function enter in a certain gauge, but physical content obtained from BSEs (masses, decay constants) is gauge independent
- **Main problem**: truncation of system of equations required

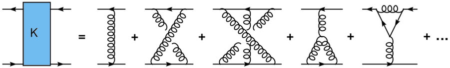
Motivation: Meson-BSE as an example



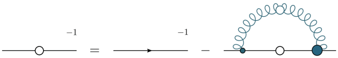
Meson-BSE (meson = two-particle bound state)

$$\Gamma(P,p) = \int_q^\Lambda \mathcal{K}_{\alpha\gamma,\delta\beta}(p,q,P) \left\{ S(\underbrace{q+\sigma P}_{q_+}) \; \Gamma(P,q) \; S(\underbrace{q+(\sigma-1)P}_{-q_-}) \right\}_{\gamma\delta}$$

Scattering Kernel $\mathcal{K}$



Quark Propagator $\mathcal{S}$ (DSE)



Observables [nice review: Eichmann et al., 1606.09602]

- Masses
  (Reduction to an eigenvalue equation for fixed J^{PC}. Masses: $\lambda(P^2 = -M_i^2) = 1$)
- Form factors
  (Build electrom. current from BS-amplitude $\Gamma(q, P^2 = -M^2)$ (solution) and the full quark propagator and quark-photon vertex; and project on tensor structure)
- . . .

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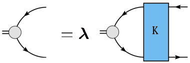

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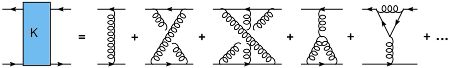

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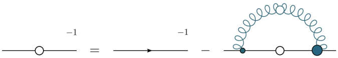
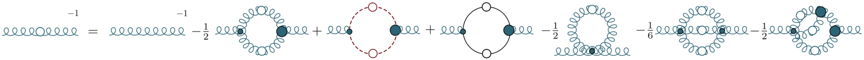
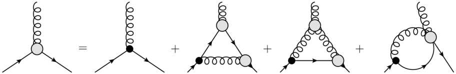


Gluon propagator DSE



Quark-Gluon-Vertex DSE

(infinite tower of equations)



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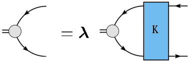

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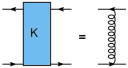

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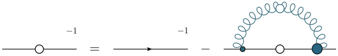
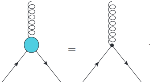


Truncation, e.g. "Rainbow-Ladder"

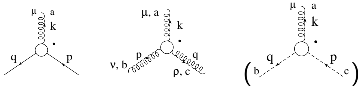


- Simplest truncation, preserves chiral symmetry and Goldstone pion, agrees with PT
- Leading structure of quark-gluon vertex $\Gamma^a_\mu(p,k) = \sum_{i=1}^{14} f_i P_i \simeq \gamma_\mu \Gamma(k^2) t^a$
- Effective coupling (UV known) $$\alpha(k^2) = \frac{Z_{1f}}{Z_2^2}\frac{g^2}{4\pi} Z(k^2)\Gamma(k^2)$$
- Only leading term of system. expansion → Improvements are needed

Motivation: Input from lattice QCD

Greens function from lattice QCD

- Nonperturbative structure of n-point functions in a certain gauge (Landau gauge) are needed to improve truncations / cross-check results
- Lattice QCD can provide these nonperturbative + untruncated
  - 2-point: quark, gluon (and ghost) propagators 
  - 3-point: quark–anti-quark–gluon, 3-gluon, (ghost-ghost-gluon)
  - 4-point: 4-gluon vertex, . . .
  - 5-point: . . .



- Already: 2-point functions are used as input to DSE studies

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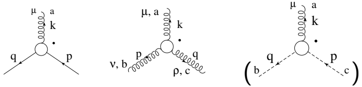
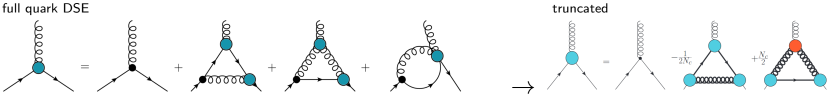


- Already: 2-point functions are used as input to DSE studies

Most desired 3-point functions (quenched + unquenched)

- Quark-gluon Vertex and Triple-Gluon Vertex
- Improved truncations of quark-DSE



Parameters of our gauge field ensembles

$N_f = 2$ and $N_f = 0$ ensembles

β	κ	$L_s^3 \times L_t$	a [fm]	m_π [GeV^2]	#config
5.20	0.13596	$32^3 \times 64$	0.08	280	900
5.29	0.13620	$32^3 \times 64$	0.07	422	900
5.29	0.13632	$32^3 \times 64$	0.07	290	908
5.29	0.13632	$64^3 \times 64$	0.07	290	750
5.29	0.13640	$64^3 \times 64$	0.07	150	400
5.40	0.13647	$32^3 \times 64$	0.06	430	900
6.16	—	$32^3 \times 64$	0.07	—	1000
5.70	—	$48^3 \times 96$	0.17	—	1000
5.60	—	$72^3 \times 72$	0.22	—	699

Allows to study

- quenched vs. unquenched, quark mass dependence
- discretization and volume effects
- infrared behavior, i.e., $|p| \approx 0.1 \ldots 1$ GeV

Acknowledgements

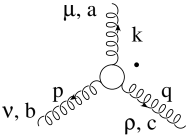
- $N_f = 2$ configurations provided by RQCD collaboration (Regensburg)
- Gauge-fixing and calculation of propagators at the HLRN (Germany)

Results for

Triple-gluon vertex in Landau gauge

(in collaboration with MSc. P. Balduf)

Triple-Gluon-Vertex in Landau gauge



$$\boldsymbol{\Gamma_{\mu\nu\lambda}(p,q)} = \sum_{i=1,\ldots,14} f_i(p,q)\, P^{(i)}_{\mu\nu\lambda}(p,q)$$

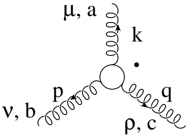
- Perturbation theory: f_i known up to three-loop order (Gracey)
- Nonperturbative structure mostly unknown (few DSE and lattice results)
- Most relevant ingredient for improved truncations of quark-DSE

DSE results

- Blum et al., PRD89(2014)061703 (improved truncation)
- Eichmann et al., PRD89(2014)105014 (full transverse form)
- ...

$$P^{(1)} = \delta_{\mu\nu} p_\lambda, \quad P^{(2)} = \delta_{\nu\lambda} p_\mu, \ \ldots, \quad P^{(5)} = \delta_{\nu\lambda} q_\mu, \ \ldots, \quad P^{(9)} = \frac{1}{\mu^2} q_\mu p_\nu q_\lambda, \ \ldots$$

Triple-Gluon-Vertex in Landau gauge



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Lattice results (deviation from tree-level)

- Cucchieri et al., [PRD77(2008)094510], quenched SU(2) Yang-Mills
  - "zero-crossing" at small momenta (2d, 3d)
- Athenodorou et al. [1607.01278], quenched SU(3) data (4d)
  - "zero-crossing" for $p^2 < 0.03\,\mathrm{GeV}^2$ for symmetric momentum setup
- Duarte et al. [1607.03831], quenched SU(3) data (4d)
  - "zero-crossing" for $p^2 \sim 0.05\,\mathrm{GeV}^2$ for $p = -q$

$$P^{(1)} = \delta_{\mu\nu} p_\lambda, \quad P^{(2)} = \delta_{\nu\lambda} p_\mu, \ \ldots, \quad P^{(5)} = \delta_{\nu\lambda} q_\mu, \ \ldots, \quad P^{(9)} = \frac{1}{\mu^2} q_\mu p_\nu q_\lambda, \ \ldots$$

Lattice calculation: nonperturbative deviation from tree-level

- Gauge-fix all gauge field ensembles to Landau gauge

$$U_{x\mu} \to U^g_{x\mu} = g_x U_{x\mu} g^\dagger_{x+\mu} \quad \text{with} \quad \nabla^{bwd}_\mu A^a_{x\mu}[U^g] = 0$$

- Gluon field: $A^a_\mu(p) = \sum_x e^{ipx} A^a_\mu(x)$ with $A^a_\mu(x) := 2\,\mathfrak{Im}\,\mathrm{Tr}\,T^a U_{x\mu}$
- Triple-gluon Green's function (implicit color sum)

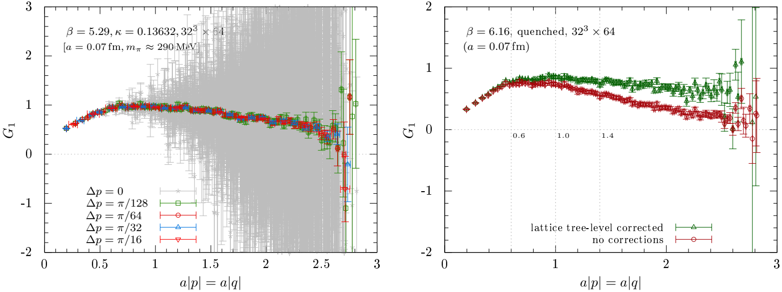
$$G_{\mu\nu\rho}(p,q,p-q) = \langle A_\mu(p) A_\nu(q) A_\rho(-p-q)\rangle_U$$

- Gluon propagator $D_{\mu\nu}(p) = \langle A_\mu(p) A_\nu(-p)\rangle_U$
- Momenta: all pairs of nearly diagonal $|p| = |q|$
- Average data for equal $a|p| = a|q|$ and nearby momenta

Projection on lattice tree-level form

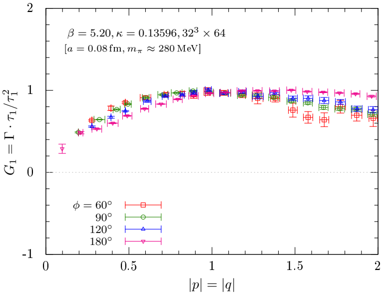
$$G_1(p,q) = \frac{\Gamma^{(0)}_{\mu\nu\rho}}{\Gamma^{(0)}_{\mu\nu\rho}} \frac{G_{\mu\nu\rho}(p,q,p-q)}{D_{\mu\lambda}(p) D_{\nu\sigma}(q) D_{\rho\omega}(p-q)\Gamma^{(0)}_{\lambda\sigma\omega}}$$

Effect of binning and tree-level improvement



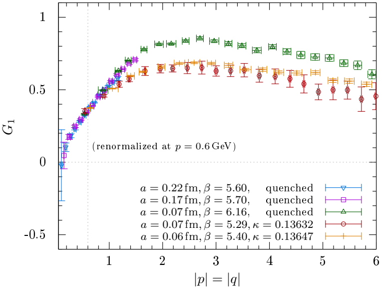
- Achieve much reduced statistical noise per configuration through binning
- Lattice tree-level improvement relevant only for larger ap_μ (expected)
- Deviations moderate if large momenta are not of interest

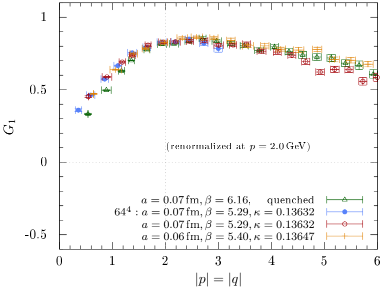
1) Angular dependence



- Consider $|p| = |q|$ and $\phi = \angle(p, q)$
- Visible but small angular dependence in all lattice data
- small $|p|$: $\phi = 180°$ data increase less strong than $\phi = 60°$ data
- large $|p|$: $\phi = 180°$ data falls less strong than $\phi = 60°$ data

2) Quenched versus unquenched ($N_f = 2$)



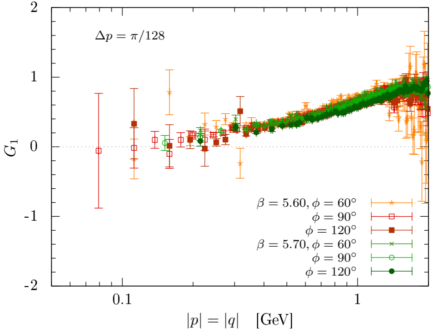


- Where to renormalize ?
- Different slope at small momenta ($N_f = 0$ vs. $N_f = 2$)
- Consistent with findings from Williams et al., PRD93(2016)034026
- Unquenching effect cleary visible but small depending on ren. point

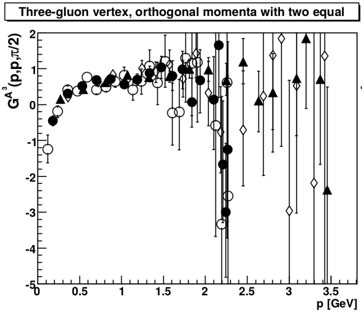
3) "Zero crossing"

considered here for $N_f = 0$

1. Angular dependence
2. Quenched vs. unquenched
3. "Zero-Crossing"



Cucchieri, Maas, Mendes (2008)
3-dim $SU(2)$ YM theory



- Is "zero-crossing" feature of 2- and 3-dim YM theory?
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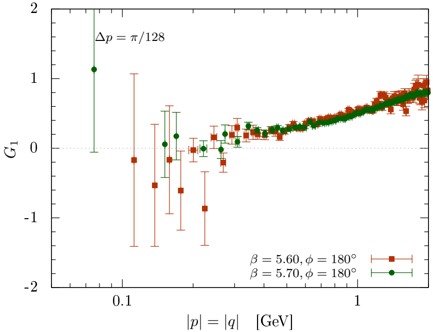

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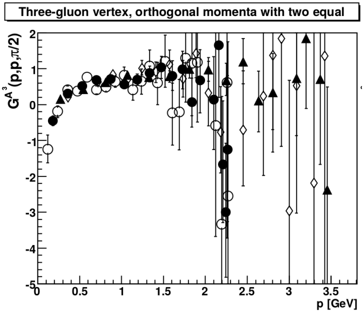

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Bose-symmetric transverse tensor structure

Eichmann et al., PRD89(2014)105014

Tensor structure of triple-gluon vertex

$$\Gamma_{\mu\nu\lambda}(p,q) = \sum_{i=1}^{14} f_i(p,q) P^{(i)}_{\mu\nu\lambda}(p,q)$$

Tensor structure of transversely projected triple-gluon vertex

$$\Gamma^T_{\mu\nu\rho}(p,q) = \sum_{i=1}^{4} F_i(\mathcal{S}_0, \mathcal{S}_1, \mathcal{S}_2)\, \tau^{\mu\nu\rho}_{i\perp}(p_1, p_2, p_3)$$

with the Lorentz invariants $[p_1 = p, p_2 = q, p_3 = -(p+q)]$

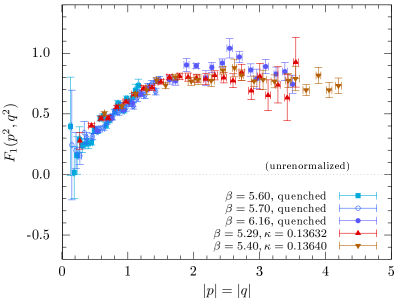
$$\mathcal{S}_0 \equiv \mathcal{S}_0(p_1,p_2,p_3) = \frac{1}{6}\left(p_1^2 + p_2^2 + p_3^2\right)$$

$$\mathcal{S}_1 \equiv \mathcal{S}_1(p_1,p_2,p_3) = a^2 + s^2 \in [0,1] \qquad \text{with} \quad a \equiv \sqrt{3}\,\frac{p_2^2 - p_1^2}{6\mathcal{S}_0}$$

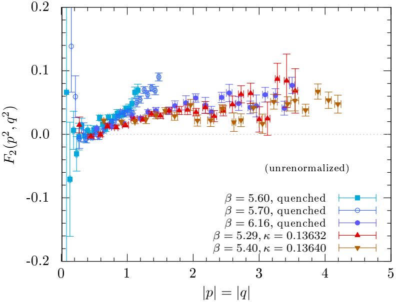
$$\mathcal{S}_2 \equiv \mathcal{S}_2(p_1,p_2,p_3) = s(3a^2 - s^2) \in [-1,1] \qquad \text{and} \quad s \equiv \frac{p_1^2 + p_2^2 - 2p_3^2}{6\mathcal{S}_0}$$

First lattice results for transverse tensor structure

F₁



F₂



DSE study of triple-gluon vertex [Eichmann et al. (2014)]

- Leading form factor is F_1
- $F_{i=2,3,4}$ close to zero $\forall$ momenta

Lattice results confirm that behavior

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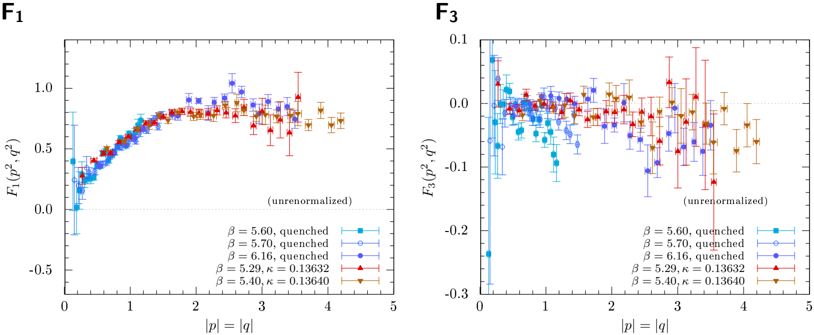

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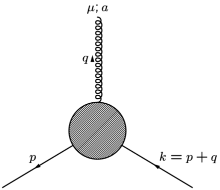


Quark-Gluon Green's function

$$G_\mu^{\bar\psi\psi A} = \Gamma_\lambda^{\bar\psi\psi A}(p,q) \cdot S(p) \cdot D_{\mu\lambda}(q) \cdot S(p+q)$$

- Up to now: only lattice data for quenched QCD

Ball-Chiu parametrization

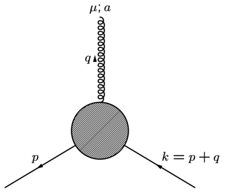
$$\Gamma_\mu^{\bar\psi\psi A}(p,q) = \Gamma_\mu^{ST}(p,q) + \Gamma_\mu^{T}(p,q)$$

with $\Gamma_\mu^{ST}(p,q) = \sum_{i=1\ldots4} \lambda_i(p^2,q^2) L_{i\mu}(p,q)$ satisfies Slavnov-Taylor identities

$\Gamma_\mu^{T}(p,q) = \sum_{i=1\ldots8} \tau_i(p^2,q^2) T_{i\mu}(p,q)$ is transverse ($q_\mu \Gamma_\mu^T = 0$).

$$L_{1\mu}(p,q) = \gamma_\mu, \quad L_{2\mu}(p,q) = -\gamma_\mu(2p_\mu + q_\mu), \ \ldots,$$
$$T_{1\mu}(p,q) = i[p_\mu q^2 - q_\mu(p\cdot q)], \ \ldots$$

Quark-Gluon vertex in Landau gauge



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Lattice calculation

- Calculate (on same gauge-fixed ensembles)
  1. quark and gluon propagators: $S^{ab}_{\alpha\beta}(p)$ and $D^{ab}_{\mu\nu}$
  2. Quark-Antiquark-Gluon Greens functions: $G_\mu^{\bar\psi\psi A}(p,q) = \left\langle A^a_\mu(q) S^{bc}_{\alpha\beta}(q)\right\rangle$
- Amputate: gluon and quark legs and project out tensor structure

$$\Gamma_\lambda^{\bar\psi\psi A}(p,q) = \frac{G_\mu^{\bar\psi\psi A}(p,q)}{D(q)S(p)S(p+q)} = \sum_{i=1\ldots4} \lambda_i(p^2,q^2)L_{i\mu} + \sum_{i=1\ldots8} \tau_i(p^2,q^2)T_{i\mu}$$

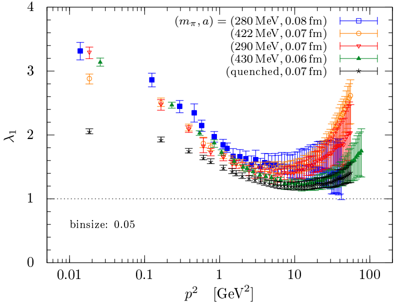
- Averaging over physically equivalent momenta, bin over nearby momenta

Lattice results for a few form factors

Soft-gluon kinematic: $q = 0$

- $\lambda_{i=1,2,3}(p,0) \neq 0$
- $\tau_i(p,0) = 0$

$\boldsymbol{\lambda_1}$



Discretization effects

- Significant for large p^2 even with tree-level corrections

Interesting

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May explain partial success of rainbow-ladder truncation

Lattice results for a few form factors

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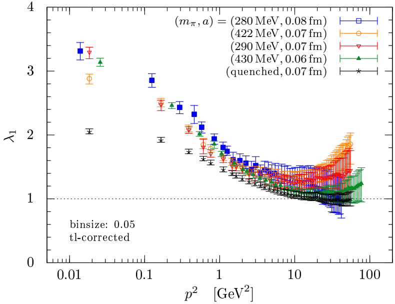

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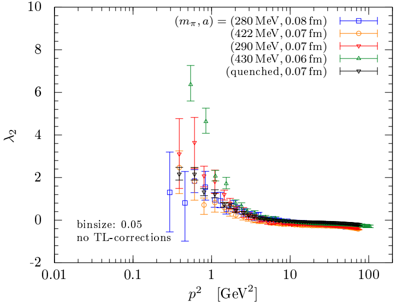

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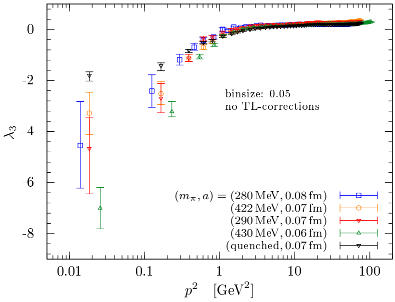

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Work in progress: Other kinematics $\rightarrow \tau_i(p,q)$ and 64^4 lattices.

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New lattice data for triple-gluon and quark-gluon vertex in Landau gauge

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Thank you for coming Friday evening!