Form factors in the $B_s \to K\ell\nu$ decays using HQET and the lattice

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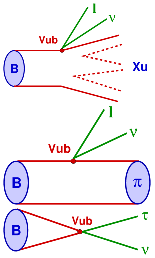
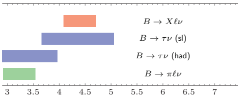
Motivation

- CP-violation within the S(tandard) M(odel) $+$ new physics needs a good understanding of flavor physics, CKM matrix elements.
- Precise (non-perturbative, first principles) determination of $|V_{ub}|$, currently the least well determined.
- $\sim 2.5 - 3\sigma$ discrepancy [PDG] :
  - Inclusive $B \to X_u \ell \nu$:
    $V_{ub} = (4.41 \pm 0.15^{+0.15}_{-0.17}) \times 10^{-3}$
  - Exclusive $B \to \pi \ell \nu$:
    $V_{ub} = (3.28 \pm 0.29) \times 10^{-3}$
  - leptonic $B \to \tau \nu$ via f_B :
    $V_{ub} = (4.22 \pm 0.42) \times 10^{-3}$
- theoretical and experimental input needed
- This talk: form factors for $B_s \to K \ell \nu$ decay
- No experimental data *yet* - predictions.
- Easier on the lattice (valence $m_K = m_K^{\mathrm{phys}}$)





Form Factors

- Use Heavy Quark Effective Theory.
- *Ground state* matrix elements $\langle \mathrm{K}| V^{\mu}(0) |\mathrm{B_s}\rangle$.
- Renormalize the currents in EFT and relate to QCD ("matching").
- Take their continuum limit.
- Extrapolate to physical quark masses in Nature.
- Map out the q^2 dependence.

The QCD matrix element

$$\langle \mathrm{K}(p_{\mathrm{K}})| V^{\mu}(0) |\mathrm{B_s}(p_{\mathrm{B_s}})\rangle = \sqrt{2m_{\mathrm{B_s}}}\left[v^{\mu} \cdot h_{\parallel}(p_{\mathrm{K}} \cdot v) + p_{\perp}^{\mu} \cdot h_{\perp}(p_{\mathrm{K}} \cdot v)\right]$$

with $v^{\mu} = p^{\mu}_{\mathrm{B_s}}/m_{\mathrm{B_s}}$ and $p^{\mu}_{\perp} = p^{\mu}_{\mathrm{K}} - (v \cdot p_{\mathrm{K}})\, v^{\mu}$ defines $h_{\parallel}$ and $h_{\perp}$.
In rest frame ($\mathbf{p}_{\mathrm{B_s}} = 0$), we get

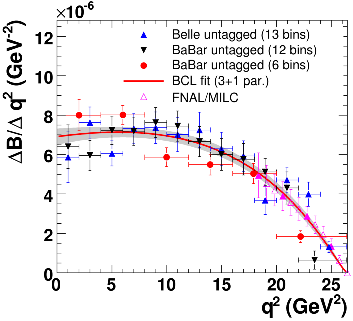
$$\begin{aligned} (2m_{\mathrm{B_s}})^{-1/2}\langle \mathrm{K}(p_{\mathrm{K}})| V^{0}(0) |\mathrm{B_s}\rangle &= h_{\parallel}(E_{\mathrm{K}}) \\ (2m_{\mathrm{B_s}})^{-1/2}\langle \mathrm{K}(p_{\mathrm{K}})| V^{k}(0) |\mathrm{B_s}\rangle &= p^{k}_{\mathrm{K}} h_{\perp}(E_{\mathrm{K}}). \end{aligned}$$

$V^{\mu}(x) \equiv \bar{\psi}_{\mathrm{u}}(x)\gamma^{\mu}\psi_{\mathrm{b}}(x)$ has effective (mass-independent) heavy quark fields, such that $h_{\parallel}, h_{\perp}$ only weakly depend on $m_{\mathrm{B_s}}$.

Experimental decay rates

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_{B_s}^3} \lambda^{3/2}(q^2) |f_+(q^2)|^2$$

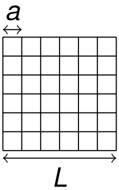
$$\lambda(q^2) = (m_{B_s}^2 + m_K^2 - q^2)^2 - 4 m_{B_s}^2 m_K^2$$



- experimentally measured decay rate
- form factor $f_+(q^2)$ computed in LQCD
- $\Rightarrow$ determine V_{ub}
- The so-called BCL (Bourelly, Caprini, Lellouch) parametrization can be used to obtain results for a whole range of q^2.
  - Upto $1/m_h$ terms, the $h_\perp$ we calculate is directly related to f_+ as $f_+ = \sqrt{m_{B_s}/2}\, C_{V_k} h_\perp^{\mathrm{stat,RGI}}$.
  - For f_+, the $h_\parallel$ contribution is $1/m_h$ suppressed.

Heavy Quark, HQET expansion of $\langle \mathrm{K}|V^\mu|\mathrm{B}\rangle$

Problem: $L^{-1} \ll m_\pi \approx 140\,\mathrm{MeV}, \ldots, m_\mathrm{B} \approx 5\,\mathrm{GeV} \ll a^{-1}$



Solution: Heavy Quark Effective Theory (HQET) [ALPHA collab. '01-'13]

- Effective theory: expansion in $1/m_\mathrm{h}$
- *Non-perturbatively* renormalizable (order by order in $1/m_\mathrm{h}$)
- well-defined continuum limit
- valid for kaon momenta $p_\mathrm{K} \ll m_\mathrm{b}$
- in practice $p_\mathrm{K} \lesssim 1\,\mathrm{GeV} \Rightarrow q^2$ close to q^2_max

$$\langle \mathcal{O} \rangle = \langle \mathcal{O} \rangle_\mathrm{stat} + \omega_\mathrm{kin} a^4 \sum_x \langle \mathcal{O}\mathcal{O}_\mathrm{kin}(x) \rangle_\mathrm{stat} + \omega_\mathrm{spin} a^4 \sum_x \langle \mathcal{O}\mathcal{O}_\mathrm{spin}(x) \rangle_\mathrm{stat}$$

$$\omega_{\{\mathrm{kin,spin}\}} \sim 1/m_\mathrm{h}$$

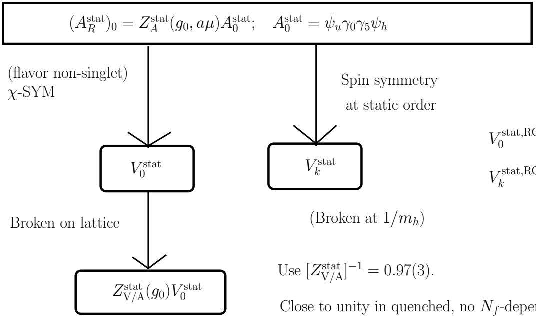
$$\mathcal{O}_\mathrm{kin}(x) = \overline{\psi}_\mathrm{h}(x) \vec{D}^2 \psi_\mathrm{h}(x), \quad \mathcal{O}_\mathrm{spin}(x) = \overline{\psi}_\mathrm{h}(x) \vec{\sigma} \cdot \vec{B} \psi_\mathrm{h}(x)$$

HQET (current) renormalization

$$V_0^{\rm stat} = \bar{\psi}_u \gamma_0 \psi_h + a c_{V_0}(g_0) \bar{\psi}_l \sum_l \overleftarrow{\nabla}_l^S \gamma_l \psi_h$$

$$V_k^{\rm stat} = \bar{\psi}_u \gamma_k \psi_h - a c_{V_k}(g_0) \bar{\psi}_l \sum_l \overleftarrow{\nabla}_l^S \gamma_l \gamma_k \psi_h$$

- At static order, heavy quark fields $\longrightarrow \psi_h$, HYP1 and HYP2 action.
- Improvement coefficients c_{V_0}, c_{V_k} known to 1-loop order.
- Use symmetries to relate renormalization of staic axial current $A_0^{\rm stat}$.



$$V_0^{\rm stat,RGI} = Z_{\rm A,RGI}^{\rm stat}(g_0) Z_{\rm V/A}^{\rm stat} V_0^{\rm stat}$$

$$V_k^{\rm stat,RGI} = Z_{\rm A,RGI}^{\rm stat}(g_0) V_k^{\rm stat}$$

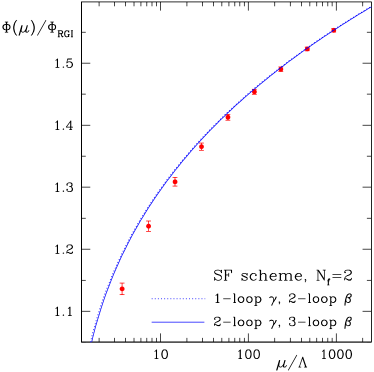
Use $[Z_{\rm V/A}^{\rm stat}]^{-1} = 0.97(3)$.

Close to unity in quenched, no N_f-dependence at 1-loop order.
Affects only at $1/m_h$. With non-perturbative matching, these issues can be eliminated.

(non-pert) Determination of $Z^{\mathrm{stat}}_{\mathrm{A,RGI}}$

The strategy is to obtain the so-called Renormalization Group Invariant quantities, Φ^{RGI} (scale and scheme independent).
In PT, for example, the RGI corresponding to the renormalized static heavy-light current at a scale μ is given by

$$(A^{\mathrm{RGI}})_0 = \lim_{\mu\to\infty}\left[2b_0\bar{g}^2(\mu)\right]^{-\gamma_0/2b_0}(A^{\mathrm{stat}}_{\mathrm{R}})_0(\mu).$$



Della Morte, Fritzsch, Heitger (2006)

The corresponding non-perturbative analogue is

$$Z^{\mathrm{stat}}_{\mathrm{A,RGI}}(g_0) = \frac{\Phi^{\mathrm{RGI}}}{\Phi(\mu)} \times Z^{\mathrm{stat}}_{\mathrm{A}}(g_0, a\mu)\Big|_{\mu=\frac{1}{2L_{\mathrm{max}}}}$$

The first universal factor relates the renormalization of A^{stat}_0 at scale $\mu_0 = 1/L_{\mathrm{max}}$ calculated in the SF scheme to the RGI operator.

Matching to QCD

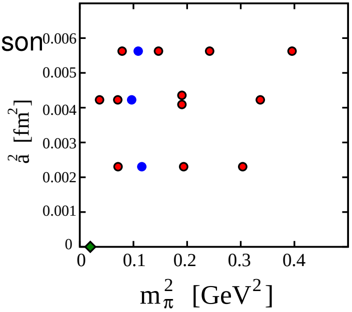
The matching to QCD is done as

$$h_{\parallel}(E_K) = C_{V_0}(M_b/\Lambda_{\overline{MS}}) h_{\parallel}^{\mathrm{stat,RGI}}(E_K) \cdot [1 + O(1/m_b)],$$
$$h_{\perp}(E_K) = C_{V_k}(M_b/\Lambda_{\overline{MS}}) h_{\perp}^{\mathrm{stat,RGI}}(E_K) \cdot [1 + O(1/m_b)]$$

- Because the RGI quantities are used, expressions from continuum PT can be used for the C_x factors, upto $O(\alpha^3)$ uncertainty. S. Bekavac et. al. (2010)
- For $N_f = 2$ QCD, these numbers are:
  $C_{V_0}(M_b/\Lambda_{\overline{MS}}) = 1.214(6)(13)$ and
  $C_{V_k}(M_b/\Lambda_{\overline{MS}}) = 1.134(7)(47)$ and
  $M_b/\Lambda_{\overline{MS}} = 21.2(1.2)$.
- No extra m_b dependent factors appear in $h_x^{\mathrm{stat,RGI}}$.
- Non-perturbative matching of HQET with QCD non-perturbatively, also for the vector currents (Heitger, (Wed)).

Ensembles and simulation

- non-perturbatively $O(a)$ improved Wilson fermions
- $N_\mathrm{f} = 2$ CLS ensembles
- scale setting via f_K [Fritzsch et al. '12]
- $m_\pi L \gtrsim 4$
- Error estimates taking into account auto-correlations [Schaefer et al. '12]

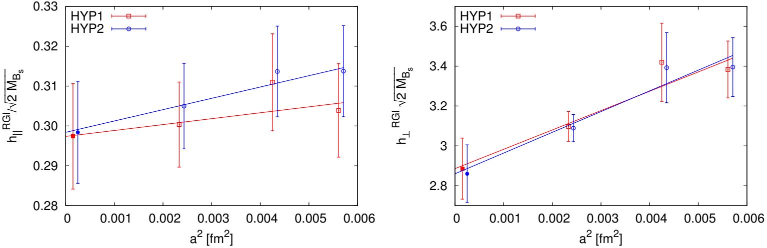


id	$T \times L^3$	a [fm]	m_π [MeV]	$m_\pi L$	# meas.
A5	64×32^3	0.0749(8)	330	4.0	1000
F6	96×48^3	0.0652(6)	310	5.0	300
N6	96×48^3	0.0483(4)	340	4.0	300

- for now: one value of q^2 only, $q^2 = 21.23\,\mathrm{GeV}^2$.
- Fixed value of q^2 is realized by the use of twisted boundary conditions in the spatial direction: $\psi(x + L\hat{k}) = e^{i\theta_k}\,\psi(x)$, and the momenta $\vec{p}_K^{\,\theta} = (2\pi\vec{n} + \vec{\theta})/L$, keeping $\mathrm{B_s}$ at rest.

Towards the continuum limit

Details of obtaining the bare estimates of $h_{\parallel}^{\rm stat,bare}$ and $h_{\perp}^{\rm stat,bare}$ will be discussed by M. Koren in the next talk.



Combining the different discretizations, in the continuum limit, we have $h_{\parallel}^{\rm stat,RGI} = 0.976(41){\rm GeV}^{1/2}$ and $h_{\perp}^{\rm stat,RGI} = 0.876(43){\rm GeV}^{-1/2}$.

Form factor $f_{+}(21.22{\rm GeV}^2) = \sqrt{m_{\rm B_s}/2}\, C_{\rm V_k}\, h_{\perp}^{\rm stat,RGI}(E_{\rm K}) = 1.63(8)(6) \pm 0.24$ allowing for a $\sim$ 15% ambiguities for the $1/m_{\rm b}$ terms.
The latter will get reduced to $1-2\%$ with all the $1/m_{\rm b}$ terms included.

Conclusions and Outlook

Our results: $f_+(21.22\mathrm{GeV}^2) = 1.63(8)(6) \pm 0.24$

Conclusions

- $f_+(q^2)$ for $\mathrm{B_s} \to \mathrm{K}$ in HQET.
- *Fully non-pertubative* renormalisation setup (at LO, soon at NLO in $1/m_\mathrm{h}$).
- *Small discretisation errors*.
- Agreement with other results $\longrightarrow$ V_ub puzzle remains.

Outlook

- Inclusion of $O(1/m_\mathrm{h})$ effects in analysis (in progress).
- Measure at one or two more q^2.
- $N_\mathrm{f} = 2+1$, open BC, wrappers gone.
- Chiral extrapolation: $m_\pi \to m_\pi^\mathrm{phys}$.
- $\mathrm{B} \to \pi$.

Parameterisation of $f(q^2) \times V_{\text{ub}}$

Our ultimate plan:
BCL-Parameterisation [Bourrely, Caprini, Lellouch '09] :

$$f_+(q^2) = \frac{1}{1 - q^2/m^2_{B_s^*}} \sum_{k=0}^{K-1} b_k \left[z^k(q^2) - (-1)^{k-K} \frac{k}{K} z^K(q^2) \right]$$

- Correlated, combined fit of our data and experimental data
- Minimize $\chi^2 = \chi^2_{\text{th}} + \chi^2_{\text{exp}}$
- fit parameters b_k, V_{ub}

Error budget – rough estimates

- extraction of FF through fits / ratios ($\approx 2\%$)
- lattice spacing (scale setting): determination of q^2 ($\approx 1\%$)
- continuum extrapolations ($2 \ldots 5\%$)
- chiral extrapolations (seems flat: small)
- BCL parameterisation, experimental data (none yet, for $B \to \pi$ $\approx 10\%$)
- $N_f = 2$ (*"To date, no significant differences between results with different values of N_f have been observed."* [FLAG '13])
- HQET truncation (static: $\sim 10\%$, at $O(1/m_h)$: $\sim 1\%$; [$< 1\%$ for f_{B_s} [Bernardoni et al. '14]])