Charm physics by $N_f = 2 + 1$ Iwasaki gauge and the six stout smeared $O(a)$-improved Wilson quark actions on a 96^4 lattice

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for PACS collaboration

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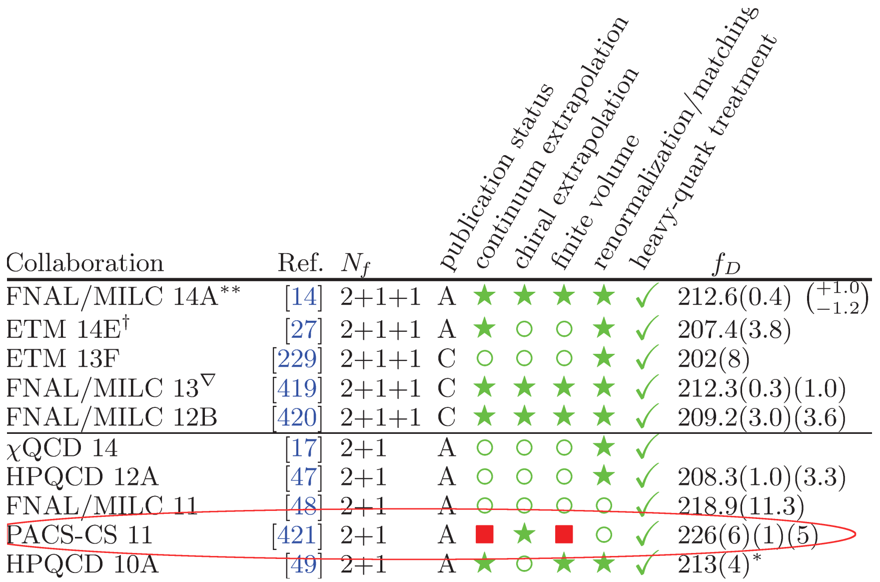
1 Introduction

We have performed simulations of charm physics PACS-CS(2011,2013), featuring

- On the physical point, $m_\pi = 135$ [MeV]
- Small volume, $L = 2.9$ [fm] ($m_\pi L = 2.0$)
- Finite lattice spacing, $a^{-1} = 2.2$ [GeV]
- ($N_f = 2+1$, not $N_f = 2+1+1$)



Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization/matching	heavy-quark treatment	f_D
FNAL/MILC 14A**	[14]	2+1+1	A	★	★	★	★	✓	212.6(0.4) $\binom{+1.0}{-1.2}$
ETM 14E†	[27]	2+1+1	A	★	○	○	★	✓	207.4(3.8)
ETM 13F	[229]	2+1+1	C	○	○	○	★	✓	202(8)
FNAL/MILC 13$^\nabla$	[419]	2+1+1	C	★	★	★	★	✓	212.3(0.3)(1.0)
FNAL/MILC 12B	[420]	2+1+1	C	★	★	★	★	✓	209.2(3.0)(3.6)
χQCD 14	[17]	2+1	A	○	○	○	★	✓	
HPQCD 12A	[47]	2+1	A	○	○	○	★	✓	208.3(1.0)(3.3)
FNAL/MILC 11	[48]	2+1	A	○	○	○	○	✓	218.9(11.3)
PACS-CS 11	[421]	2+1	A	■	★	■	○	✓	226(6)(1)(5)
HPQCD 10A	[49]	2+1	A	★	○	★	★	✓	213(4)*

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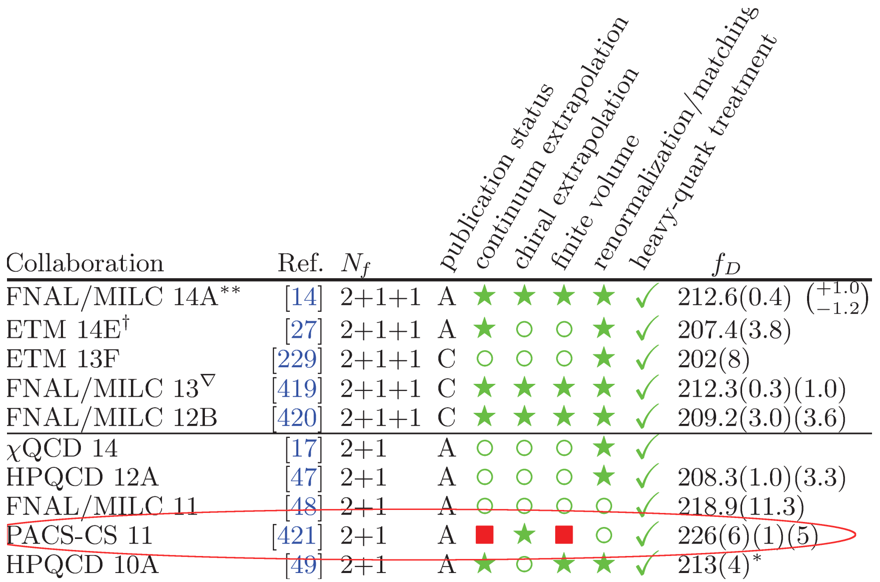
- Finite lattice spacing, $a^{-1} = 2.2$ [GeV]
- $(N_f = 2 + 1$, not $N_f = 2 + 1 + 1)$



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[Development of computers]

- Thanks to a new supercomputer, called K-computer, simulations with a large spatial volume can be performed.

Year	Machine	Speed [TFlops]	m_π [MeV]	$m_\pi L$
1996-2005	CP-PACS	0.6	700	7.1
2006-2011	PACS-CS	14	160	2.3
2008-2014	T2K	235	135	2.0
2012-	K-computer	10510	135	5.6
	Experiment		135	

2 Simulation setup

- Action :
  Iwasaki gauge + $N_{\mathrm{stout}} = 6$, $O(a)$ improved Wilson fermion for sea quarks PACS(2015) + relativistic heavy fermion for valence charm quark
  cf. pioneering work, Symanzik gauge + $N_{\mathrm{stout}} = 6$, $C_{\mathrm{SW}} = 1.0$ Wilson fermion BMW(2009)
- Lattice size :
  $96^3 \times 96$ ($L = 8.1$ fm, $a^{-1} = 2.3$ GeV)
- Sea quark masses :
  almost on the physical point ($m_\pi = 145$ MeV, shortly extrapolated to 135 MeV using reweighted data in $m_\pi = 144 - 156$ MeV)
- Inputs : m_π,m_K,m_Ω for m_{ud},m_s,a; $m(1S)$ for m_{charm}
- Statistics : $N_{\mathrm{config}} = 40$ (2000 MD time), not full statistics, yet

◇ We show our preliminary results for charm physics, focusing on stout smearing and finite size influence.

[Improved action for the charm quark]

We employ the relativistic heavy quark action(Tsukuba-type) S.Aoki et al.(2003)

- Since the charm quark is not too heavy, relativistic approach is needed.
- This action is designed to control heavy quark mass corrections.

$\leftarrow$ $O(m_Q a)$ and $O((m_Q a)(a\Lambda_{QCD}))$ terms are removed, once all of the parameters in the heavy quark action are determined nonperturbatively.

◇ We employ perturbative values for the heavy quark action, except for a parameter ν → Next page

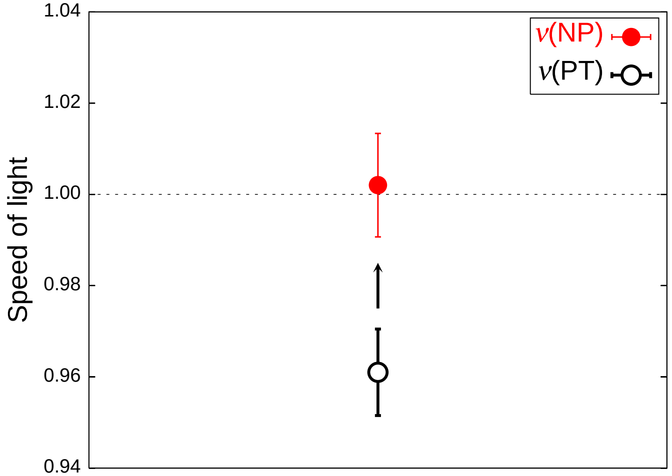
$$\begin{aligned} S_{RHQ} &= \sum_{x,y} \bar{q}(x) D(x,y) q(y), \\ D(x,y) &\equiv \delta_{x,y} - \kappa_{\text{heavy}} \Big\{ (1-\gamma_4) U_4(x) \delta_{x+4,y} + (1+\gamma_4) U_4^\dagger(x) \delta_{x,y+4} \\ &\quad + \sum_i \big((r_s - \nu\gamma_i) U_i(x) \delta_{x+i,y} + (r_s + \nu\gamma_i) U_i^\dagger(x) \delta_{x,y+i} \big) \Big\} \\ &\quad - \delta_{x,y} \kappa_{\text{heavy}} \Big\{ C_{SW}^s \sum_{i<j} \sigma_{ij} F_{ij} + C_{SW}^t \sum_i \sigma_{4i} F_{4i} \Big\}. \end{aligned}$$

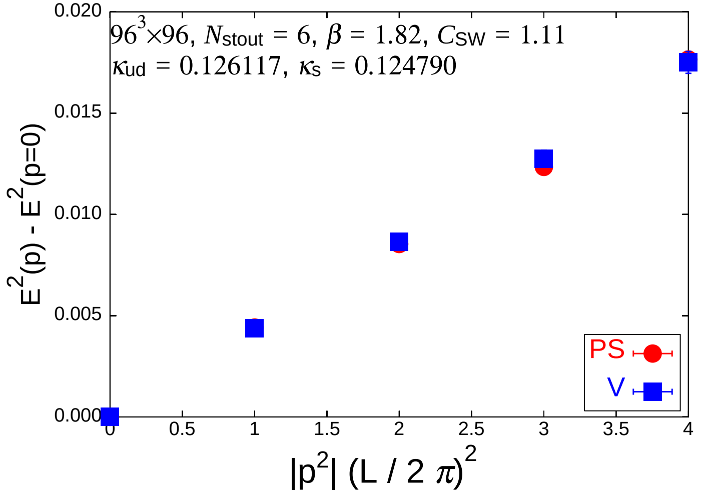
[Non-perturbative tuning of ν on a larger spatial volume]

- A perturbative choice of the parameter ν in the relativistic heavy quark action is not bad. The effective speed of light is $c_{\mathrm{eff}} = 0.96(1)$.
- Non-perturbative tuning of ν is performed to reproduce the relativistic dispersion relation, $c_{\mathrm{eff}} = 1.00(1)$.

◇ Non-perturbative tuning of the relativistic heavy quark action is easier, due to finer resolution in momentum, thanks to larger volume.

$\leftarrow p = 2\pi/L = 0.43$ GeV PACS-CS(2011,2013) → 0.15 GeV in this work



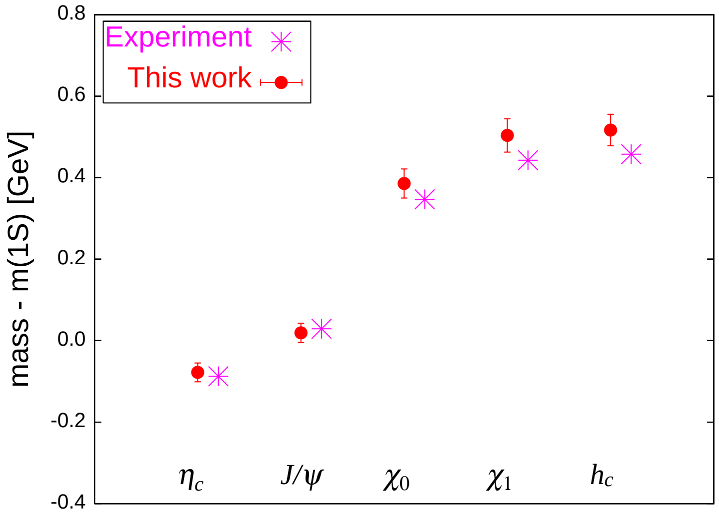


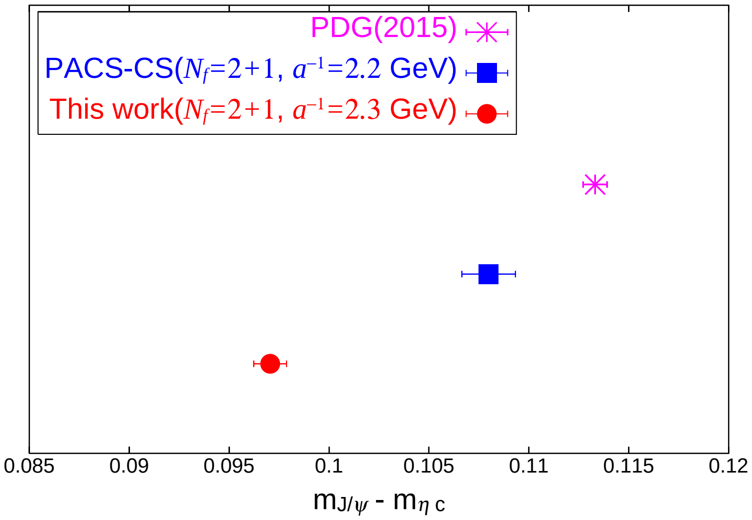
3 Results

[Mass spectrum of charmonium]

- Our results agree with experiments, except for the hyperfine splitting. More detailed analysis including continuum extrapolation is needed.
  - ◇ Smearing may not be advantageous to the hyperfine splitting.

    ← The reason may be tadpole contribution(tadpole improvement is employed in the previous work, while not in this work, due to plaq(smear) = 0.97), finite size effects, …

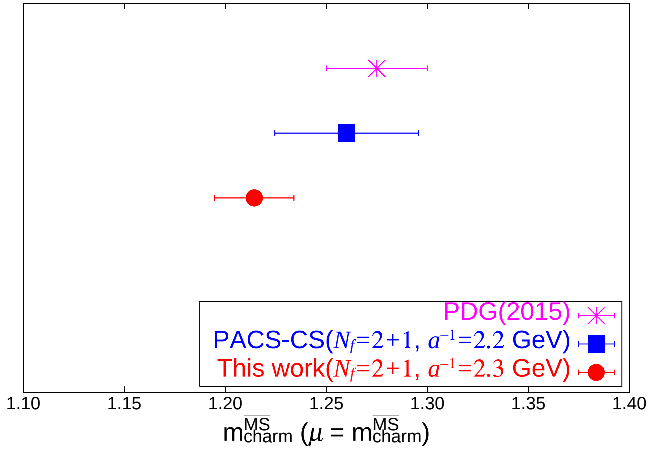




[Result of charm quark mass]

- Charm quark mass is obtained by the axial Ward-Takahashi identity.
- Our result is more accurate thanks to smearing, which reduces systematic errors from renormalization factors.
    - ◇ Smearing is valuable to charm quark mass calculation.
    - ◇ No clear finite size effects are observed.

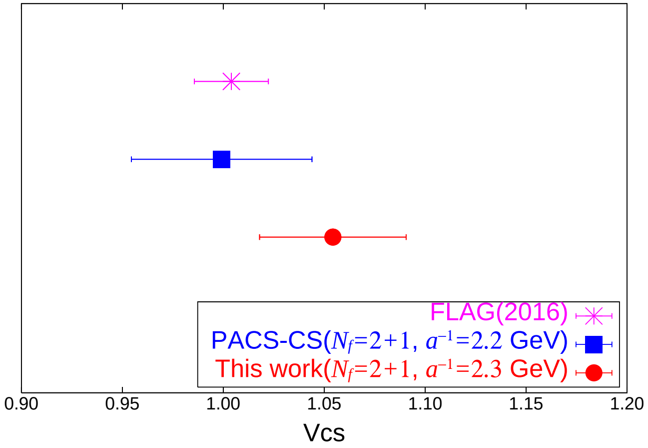
$$\begin{aligned} m^{\overline{\rm MS}}_{\rm charm}(\mu = m^{\overline{\rm MS}}_{\rm charm}) &= Z_m(\mu, m^{\rm AWI}_{\rm charm})\, m^{\rm AWI}_{\rm charm}, \quad m^{\rm AWI}_{\rm charm} = m_{PS}\frac{\left\langle 0|A_4^{\rm imp}|PS\right\rangle}{\langle 0|PS|PS\rangle}, \\ Z_m(\mu, m^{\rm AWI}_{\rm charm}) &= Z_m^{\rm NP}(\mu, m=0) + \left(Z_m(\mu, m^{\rm AWI}_{\rm charm}) - Z_m(\mu, m=0)\right)^{\rm PT} \end{aligned}$$



[Cabbibo-Kobayashi-Maskawa matrix element]

- CKM matrix elements are extracted from our mass and pseudoscalar decay constant combined with experiment for the leptonic decay width.
- Our result of CKM matrix is not improved much by smearing, due to precision limitation of the experimental data.
  - ◇ Smearing is not advantageous to CKM matrix elements, waiting for experimental update, such as Belle II starting in 2016.
  - ◇ No clear finite size effects are observed.

$$\Gamma(D_s \to l\nu) = \frac{G_F^2}{8\pi} m_l^2 m_{D_s} f_{D_s}^2 \left(1 - \frac{m_l^2}{m_{D_s}^2}\right)^2 |V_{cs}|^2$$



[as explained in Introduction]

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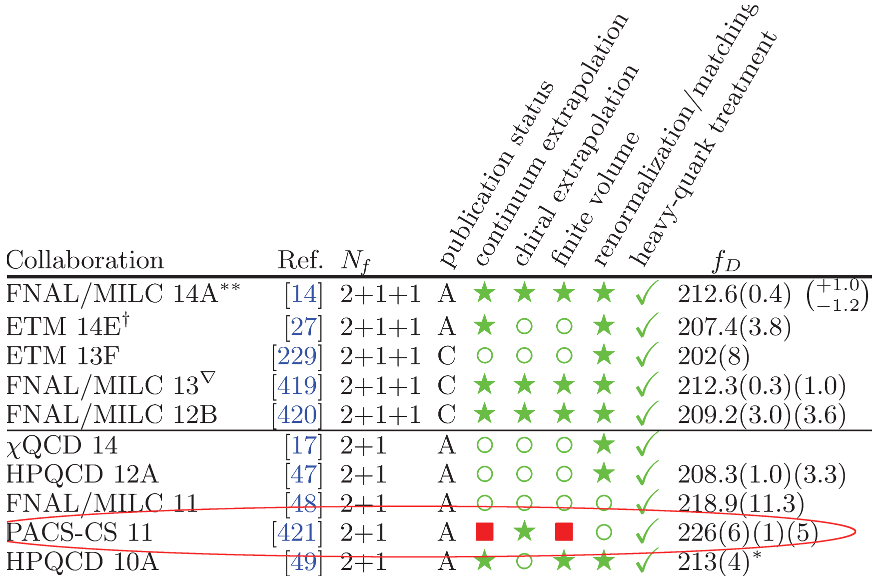
- Finite lattice spacing, $a^{-1} = 2.2$ [GeV] → Calculations at other lattice spacings are ongoing.
- ($N_f = 2+1$, not $N_f = 2+1+1$)



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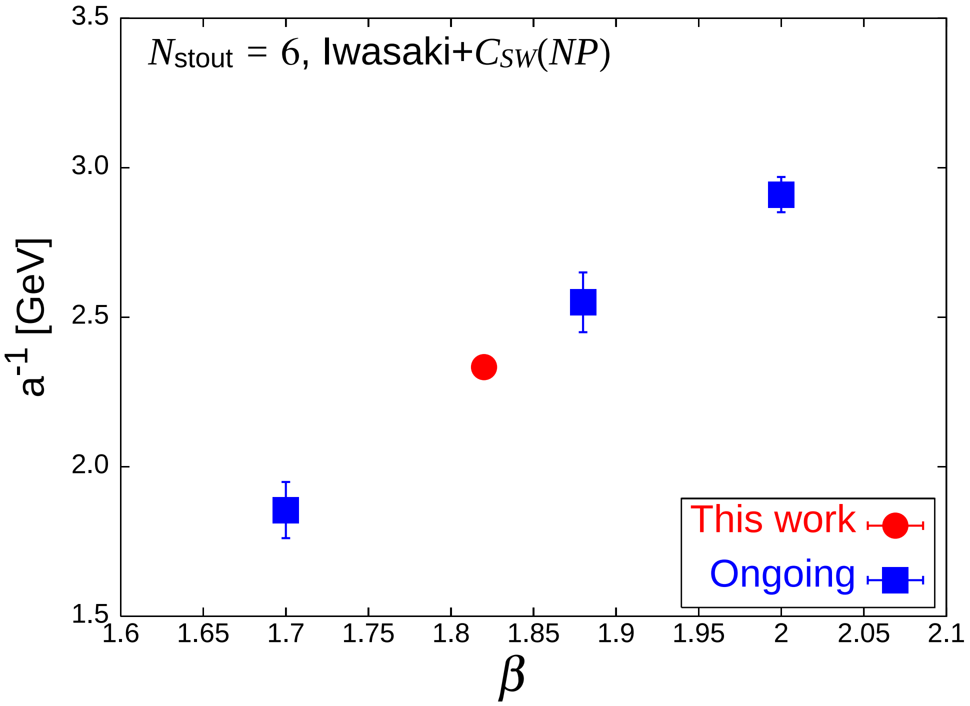
[Simulations toward the continuum limit]

Simulations at other lattice spacings are ongoing to take the continuum limit.



[New supercomputer]

Due to large simulation costs, our lattice spacing is still finite.

→ A new supercomputer will allow us to take the continuum limit.

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2006-2011	PACS-CS	14	160	2.3	yet
2008-2014	T2K	235	135	2.0	yet
2012-	K-computer	10510	135	5.6	yet
2016-	post-T2K†	25000	(135)	(5.6)	(Yes)
	Experiment		135		

† post-T2K will be installed in Dec 2016

4 Summary

Our preliminary results for charm physics on 96^4 are presented.

- Our results for mass spectrum of charmonium reproduce experiments, except for the hyperfine splitting. More detailed analysis is needed.
  - ◇ Smearing may not be advantageous to the hyperfine splitting.
- Our result of charm quark mass is more accurate thanks to smearing, which reduces systematic errors from renormalization factors.
  - ◇ Smearing is valuable to charm quark mass calculation.
  - ◇ No clear finite size effects are observed.
- Our result of CKM matrix is not improved much by smearing, due to precision limitation of the experimental data.
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