Charm physics by $N_f = 2 + 1$ Iwasaki gauge and the six stout smeared $O(a)$-improved Wilson quark actions on a $96^4$ lattice

Yusuke Namekawa (Univ of Tsukuba) for PACS collaboration

1 Introduction

We have performed simulations of charm physics PACS-CS(2011,2013), featuring

- On the physical point, \( m_\pi = 135 \) [MeV]
- Small volume, \( L = 2.9 \) [fm] (\( m_\pi L = 2.0 \))
- Finite lattice spacing, \( a^{-1} = 2.2 \) [GeV]
- \( (N_f = 2 + 1, \text{ not } N_f = 2 + 1 + 1) \)

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FLAG(2016)
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- Small volume, $L = 2.9$ [fm] ($m_{\pi}L = 2.0$) $\rightarrow L = 8.1$ [fm] ($m_{\pi}L = 5.6$)
- Finite lattice spacing, $a^{-1} = 2.2$ [GeV]
- ($N_f = 2 + 1$, not $N_f = 2 + 1 + 1$)

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FLAG(2016)
Thanks to a new supercomputer, called K-computer, simulations with a large spatial volume can be performed.

<table>
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2 Simulation setup

- Action:
  Iwasaki gauge + $N_{\text{stout}} = 6$, $O(a)$ improved Wilson fermion for sea quarks \textit{PACS(2015)} + relativistic heavy fermion for valence charm quark
  cf. pioneering work, Symanzik gauge + $N_{\text{stout}} = 6$, $C_{SW} = 1.0$ Wilson fermion \textit{BMW(2009)}

- Lattice size:
  $96^3 \times 96$ ($L = 8.1$ fm, $a^{-1} = 2.3$ GeV)

- Sea quark masses:
  almost on the physical point ($m_\pi = 145$ MeV, shortly extrapolated to 135 MeV using reweighted data in $m_\pi = 144 - 156$ MeV)

- Inputs:
  $m_\pi, m_K, m_\Omega$ for $m_{ud}, m_s, a$; $m(1S)$ for $m_{charm}$

- Statistics:
  $N_{\text{config}} = 40$ (2000 MD time), not full statistics, yet

  ◆ We show our preliminary results for charm physics, focusing on stout smearing and finite size influence.
[Improved action for the charm quark]
We employ the relativistic heavy quark action (Tsukuba-type) \textsuperscript{S.Aoki et al.(2003)}

- Since the charm quark is not too heavy, relativistic approach is needed.
- This action is designed to control heavy quark mass corrections.

\[ \mathcal{O}(m_{Qa}) \text{ and } \mathcal{O}((m_{Qa})(a\Lambda_{QCD})) \text{ terms are removed, once all of the parameters in the heavy quark action are determined nonperturbatively.} \]

\[ \diamond \text{ We employ perturbative values for the heavy quark action, except for a parameter } \nu \rightarrow \text{ Next page} \]

\[
S_{RHQ} = \sum_{x,y} \bar{q}(x) D(x, y) q(y),
\]

\[
D(x, y) \equiv \delta_{x,y} - \kappa_{\text{heavy}} \left\{ (1 - \gamma_4)U_4(x)\delta_{x+4,y} + (1 + \gamma_4)U_4^\dagger(x)\delta_{x,y+4}
\right.
\]

\[
+ \sum_i \left( (r_s - \nu \gamma_i)U_i(x)\delta_{x+i,y} + (r_s + \nu \gamma_i)U_i^\dagger(x)\delta_{x,y+i} \right) \right\}
\]

\[-\delta_{x,y} \kappa_{\text{heavy}} \left\{ C^s_{SW} \sum_{i<j} \sigma_{ij} F_{ij} + C^t_{SW} \sum_i \sigma_{4i} F_{4i} \right\}. \]
[Non-perturbative tuning of $\nu$ on a larger spatial volume]

- A perturbative choice of the parameter $\nu$ in the relativistic heavy quark action is not bad. The effective speed of light is $c_{\text{eff}} = 0.96(1)$.
- Non-perturbative tuning of $\nu$ is performed to reproduce the relativistic dispersion relation, $c_{\text{eff}} = 1.00(1)$.

◊ Non-perturbative tuning of the relativistic heavy quark action is easier, due to finer resolution in momentum, thanks to larger volume.

$\leftarrow p = 2\pi/L = 0.43$ GeV $\text{PACS-CS(2011,2013)} \rightarrow 0.15$ GeV in this work

![Graph showing the speed of light versus momentum for non-perturbative (NP) and perturbative (PT) tuning.](attachment:graph.png)

- $\nu^{(\text{NP})}$
- $\nu^{(\text{PT})}$

$|p^2| (L/2\pi)^2$

$E^2(p) - E^2(p=0)$

$96^3 \times 96, N_{\text{stout}} = 6, \beta = 1.82, C_{\text{SW}} = 1.11$

$\kappa_d = 0.126117, \kappa_s = 0.124790$
3 Results

[Mass spectrum of charmonium]

- Our results agree with experiments, except for the hyperfine splitting. More detailed analysis including continuum extrapolation is needed.

◊ Smearing may not be advantageous to the hyperfine splitting.

← The reason may be tadpole contribution (tadpole improvement is employed in the previous work, while not in this work, due to plaq(smear) = 0.97), finite size effects, ...
[Result of charm quark mass]

- Charm quark mass is obtained by the axial Ward-Takahashi identity.
- Our result is more accurate thanks to smearing, which reduces systematic errors from renormalization factors.

◊ Smearing is valuable to charm quark mass calculation.
◊ No clear finite size effects are observed.

\[
\begin{align*}
\overline{m}_{\text{charm}}(\mu = \overline{m}_{\text{charm}}) &= Z_m(\mu, m_{\text{charm}}) m_{\text{charm}}, \quad m_{\text{charm}} = m_P S \frac{\langle 0|A_4^{\text{imp}}|PS\rangle}{\langle 0|PS|PS\rangle}, \\
Z_m(\mu, m_{\text{charm}}) &= Z_N^P(m = 0) + (Z_m(\mu, m_{\text{charm}}) - Z_m(\mu, m = 0))^\text{PT}
\end{align*}
\]

\[\text{PDG(2015)}\]

\[\text{PACS-CS}(N_f=2+1, a^{-1}=2.2 \text{ GeV}) - \text{This work}(N_f=2+1, a^{-1}=2.3 \text{ GeV}) \]
CKM matrix elements are extracted from our mass and pseudoscalar decay constant combined with experiment for the leptonic decay width.

Our result of CKM matrix is not improved much by smearing, due to precision limitation of the experimental data.

◊ Smearing is not advantageous to CKM matrix elements, waiting for experimental update, such as Belle II starting in 2016.

◊ No clear finite size effects are observed.

\[
\Gamma(D_s \to l\nu) = \frac{G_F^2}{8\pi} m_l^2 m_{D_s} f_{D_s}^2 \left(1 - \frac{m_l^2}{m_{D_s}^2}\right)^2 |V_{cs}|^2
\]
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- Small volume, $L = 2.9$ [fm] ($m_\pi L = 2.0$) $\rightarrow$ $L = 8.1$ [fm] ($m_\pi L = 5.6$)
- Finite lattice spacing, $a^{-1} = 2.2$ [GeV] $\rightarrow$ Calculations at other lattice spacings are ongoing.
- $(N_f = 2 + 1$, not $N_f = 2 + 1 + 1)$

![Table of results from various collaborations](image)
Simulations at other lattice spacings are ongoing to take the continuum limit.
Due to large simulation costs, our lattice spacing is still finite. A new supercomputer will allow us to take the continuum limit.

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| Experiment | 135 |

$^\dagger$ post-T2K will be installed in Dec 2016
4 Summary

Our preliminary results for charm physics on $96^4$ are presented.

- Our results for mass spectrum of charmonium reproduce experiments, except for the hyperfine splitting. More detailed analysis is needed.
  - Smearing may not be advantageous to the hyperfine splitting.

- Our result of charm quark mass is more accurate thanks to smearing, which reduces systematic errors from renormalization factors.
  - Smearing is valuable to charm quark mass calculation.
  - No clear finite size effects are observed.

- Our result of CKM matrix is not improved much by smearing, due to precision limitation of the experimental data.
  - Smearing is not advantageous to CKM matrix elements, waiting for experimental update, such as Belle II starting in 2016.
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