Near threshold states D_{s0}^{*} (2317) and D_{s1} (2460)

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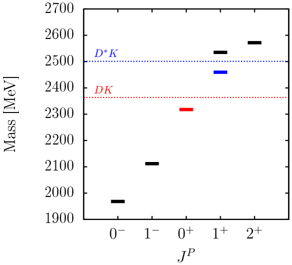
Experimental D_s spectrum

J^P	State	Mass [MeV]	Width [MeV]
2^+	$D_{s2}^*(2573)$	2571.9 ± 0.8	17 ± 4
1^+	$D_{s1}(2536)$	2535.11 ± 0.06	0.92 ± 0.05
1^+	$D_{s1}(2460)$	2459.5 ± 0.6	< 3.5
0^+	$D_{s0}^*(2317)$	2317.7 ± 0.6	< 3.8
1^-	D_s^*	2112.1 ± 0.4	< 1.9
0^-	D_s	1968.30 ± 0.10	$\tau = 5.00\,(7) \times 10^{-13}$ s



- Unexpectedly, the $D_{s0}^*(2317)$ lies below the DK threshold.
- Possible interpretations: tetraquark, molecule state etc..

Including the scattering state

- Close-by energy levels: standard $\overline{q}q$ interpolators alone do not distinguish the ground state from the scattering state.
- Need to include the scattering state in the analysis.
- This is done by computing the correlator matrix:

$$C_{ij}(t) = \begin{pmatrix} \langle O_{D_s}(t)\, O^{\dagger}_{D_s} \rangle & \langle O_{D_s}(t)\, O^{\dagger}_{DK} \rangle \\ \langle O_{DK}(t)\, O^{\dagger}_{D_s} \rangle & \langle O_{DK}(t)\, O^{\dagger}_{DK} \rangle \end{pmatrix} = \sum_n z^{\dagger}_{in} z_{nj} e^{-E_n t}$$

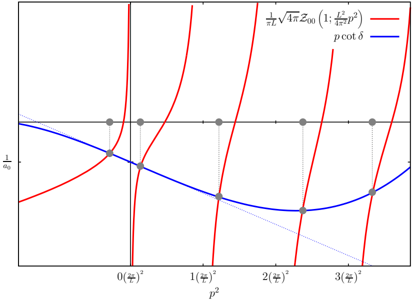
- The energy levels are obtained from the exponential decay of the eigenvalues.

$$C(t)\, v_i = \lambda C(t_0)\, v_i \quad \Longrightarrow \quad \begin{array}{l} \lambda_1(t) \sim e^{-m_{D_s} t} \\ \lambda_2(t) \sim e^{-E_{DK} t} \end{array}$$

- The eigenvectors are related to the overlaps of the operators used with the physical states, $v^{-1}_{ni} \propto z_{ni} = \langle n | O^{\dagger}_i | 0 \rangle$.
- The inclusion of four-quark operators significantly increases the computational cost.

Finite volume dependence

$$
\begin{aligned}
E_n &= \sqrt{m_A^2 + k_n^2} + \sqrt{m_B^2 + k_n^2} + \Delta E_n \qquad & \boldsymbol{k_n} = \frac{2\pi}{L}\boldsymbol{n} \\
&= \sqrt{m_A^2 + p_n^2} + \sqrt{m_B^2 + p_n^2} \qquad & \boldsymbol{p_n} = \frac{2\pi}{L}\boldsymbol{q_n}
\end{aligned}
$$

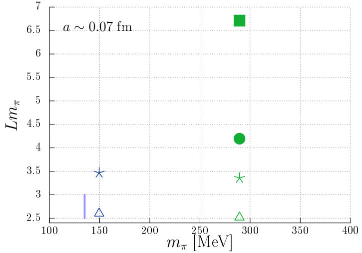


- Lüscher's relation:
  $$p\cot\delta(p) = \frac{1}{\pi L}\sqrt{4\pi}\mathcal{Z}_{00}\left(1;\frac{L^2}{4\pi^2}p^2\right)$$
- Effective range approx.:
  $$p\cot\delta(p) \approx \frac{1}{a_0} + \frac{1}{2}r_0 p^2$$
- ∞ volume bound state pole condition:
  $$p\cot\delta(p) = ip$$
- Volume dependence:
  $$
  \begin{aligned}
  \Delta E_{n=0}(L) &= -\frac{2\pi a_0}{\mu L^3}\left(1 + c_1\frac{a_0}{L} + c_2\frac{a_0^2}{L^2}\right) + \ldots \\
  \Delta E_{BS}(L) &= \Delta E_{BS}(\infty)\left(1 + \frac{12}{|p_\infty|L}\frac{1}{1-2A|p_\infty|}e^{-|p_\infty|L} + \ldots\right)
  \end{aligned}
  $$

Lattice set up

Multiple volume ensembles are employed, splitted into two sets according to m_π:

κ_l	a [fm]	V	am_π	m_π [MeV]	Lm_π	m_K [MeV]	m_D [MeV]	m_{D^*} [MeV]	N_{conf}
0.13632	0.071	$24^3 \times 48$	0.1112(9)	306.9(2.5)	2.52	540(2)	1907(3)	2038(5)	2222
	0.071	$32^3 \times 64$	0.10675(52)	294.6(1.4)	3.42	528(1)	1902(3)	2030(5)	1453
	0.071	$40^3 \times 64$	0.10465(38)	288.8(1.1)	4.19	527(1)	1901(2)	2030(4)	2000
	0.071	$64^3 \times 64$	0.10487(24)	289.5(0.7)	6.70	526(1)	1898(1)	2030(2)	1463
0.13640	0.071	$48^3 \times 64$	0.05786(55)	159.7(1.5)	2.77	500(1)	1880(2)	2007(3)	2501
	0.071	$64^3 \times 64$	0.05425(49)	149.7(1.4)	3.49	497(1)	1877(1)	1996(3)	1591



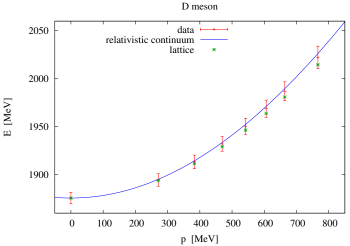


Figure: Dispersion relation for the D.

Diagrams

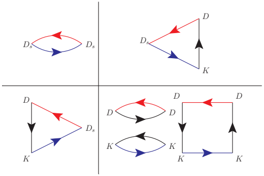
- Two main channels are studied, the 0^+ and 1^+.
- Operators with the corresponding quantum numbers are required.
- Both D_s-like operators and DK-like operators are present.

J^P	Physical states	Two-quark op.	Four-quark op.
0^+	$D_{s0}^*(2317)\,,\ DK$	$O_{D_s} = \bar{s}\mathbb{1}c, \quad O_{D_s'} = \bar{s}\gamma_t c$	$O_{DK} = (\bar{u}\gamma_5 c)(\bar{s}\gamma_5 u) + (\bar{d}\gamma_5 c)(\bar{s}\gamma_5 d)$
1^+	$D_{s1}(2460)\,,\ D_{s1}(2536)\,,\ D^*K$	$O_{D_s} = \bar{s}\gamma_i\gamma_5 c, \quad O_{D_s'} = \bar{s}\gamma_t\gamma_i\gamma_5 c$	$O_{DK} = (\bar{u}\gamma_i c)(\bar{s}\gamma_5 u) + (\bar{d}\gamma_i c)(\bar{s}\gamma_5 d)$

- Five smearing levels are performed in total for the D_s and one for the DK.

The Wick contractions: 2-point, triangular and box diagrams.

Cost increase: light quark inversions and sequential propagators.



- Sequential propagators and the one-end-trick

$$\frac{1}{N}\sum_{r=1}^{N}\eta^{(r)}(\mathbf{x},t_s)\,\eta^{(r)}(\mathbf{y},t_s) = \delta(\mathbf{x}-\mathbf{y}) + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$$

$$M_{ij}^{f}(y) = \sum_{x} G_{ij}^{f}(y;x)\,\eta(x) \qquad f = l,s,c$$

$$S_{ij}^{f'f}(y;t_I) = \sum_{z,k} G_{ik}^{f'}(y,z)\left(\Gamma Q^{f}\right)_{kj}(z) \qquad f'f = ls, cl$$

- Number of inversions per config. per spin-colour:

	M	S	Total
Charm	6	8	14
Strange	3	0	3
Light	3	N_T	$N_T + 3$

Energies I

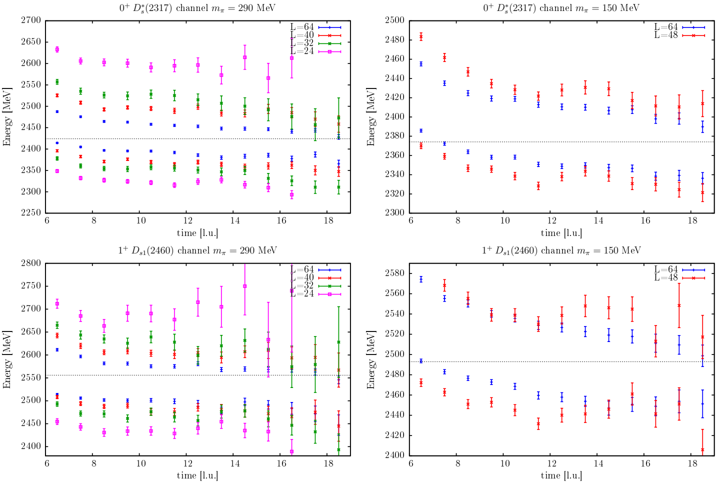
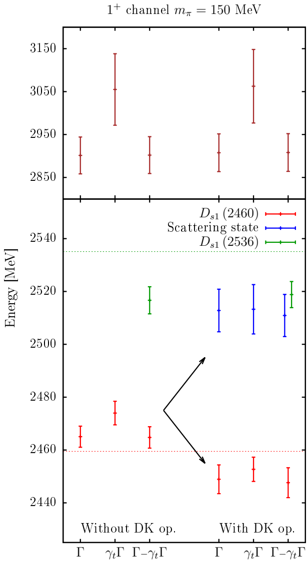


Figure: The effective masses of the eigenvalues $E_n \approx \log \frac{\lambda_n(t,t_0)}{\lambda_n(t+1,t_0)}$

Energies II

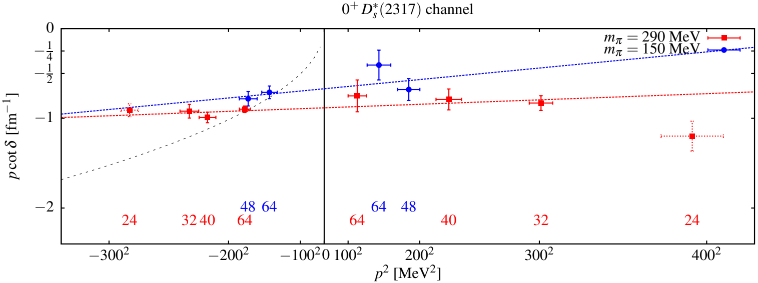


- One exp. fits vs two exp. fits performed.
- Splitting of the lowest eigenvalue when including DK operator
- Independence of D_{s1} (2536) from the scattering operator.

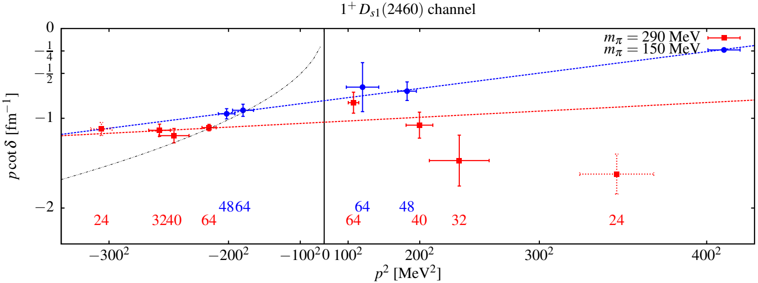
	Scalar		Vector	
	BS	*SS*	*BS*	*SS*
24	2318(5)	2594(13)	2435(6)	2691(16)
32	2352(5)	2529(5)	2469(6)	2621(14)
40	2362(4)	2485(6)	2477(8)	2602(6)
64	2382(3)	2440(5)	2496(4)	2570(3)
48	2332(5)	2417(6)	2440(4)	2535(4)
64	2344(4)	2402(6)	2449(5)	2513(8)

Table: Fitted energy levels in MeV.

Phase shift

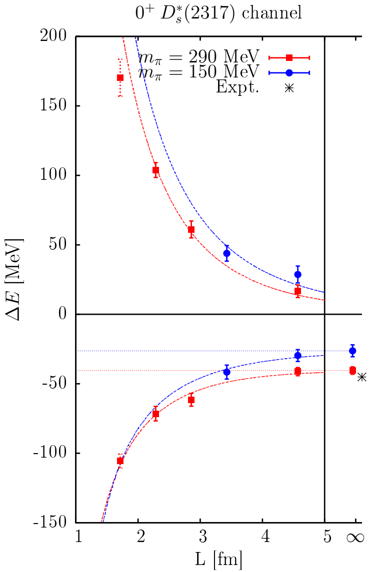


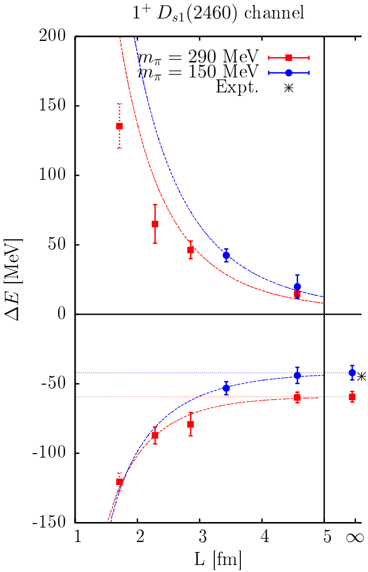
$0^+ D_s^*(2317)$ channel
$m_\pi = 290$ MeV
$m_\pi = 150$ MeV
$p \cot \delta$ [fm^{-1}]
0
$-\frac{1}{4}$
$-\frac{1}{2}$
-1
-2
48 64
24 32 40 64
64 48
64 40 32 24
-300^2
-200^2
-100^2 0 100^2
200^2
300^2
400^2
p^2 [MeV^2]



$1^+ D_{s1}(2460)$ channel
$m_\pi = 290$ MeV
$m_\pi = 150$ MeV
$p \cot \delta$ [fm^{-1}]
0
$-\frac{1}{4}$
$-\frac{1}{2}$
-1
-2
4864
24 3240 64
64 48
64 40 32 24
-300^2
-200^2
-100^2 0 100^2
200^2
300^2
400^2
p^2 [MeV^2]

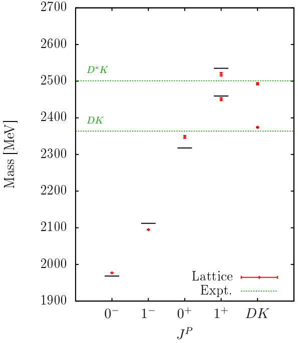
Volume dependence





Infinite volume extrapolations

	0^+ channel			1^+ channel		
	$m_\pi = 290$ [MeV]	$m_\pi = 150$ [MeV]	Expt. [MeV]	$m_\pi = 290$ [MeV]	$m_\pi = 150$ [MeV]	Expt. [MeV]
a_0 [fm]	$-1.13(4)$	$-1.49(13)$		$-0.96(5)$	$-1.24(9)$	
r_0 [fm]	0.077(33)	0.199(87)		0.106(64)	0.265(74)	
$\|p_\infty\|$ [MeV]	180(6)	142(11)		219(7)	180(11)	
Δm [MeV]	40.4(2.7)	26.3(4.3)	45.1	59.3(3.8)	42.0(5.2)	44.7
m_{D_s} [MeV]	2383.5(2.4)	2347.8(3.8)	2317.7	2496.5(3.6)	2450.8(4.0)	2459.5



Conclusions

- Multiple volume simulations were performed in order to establish the finite volume dependence of the ground and first scattering state in the D_s sector.
- We confirm the existence of a bound state below threshold for both 0^+ and 1^+ channels and compute the effective range parameters of DK scattering.
- The use of stochastic sources is a valid method to handle sequential propagators.
- Our work still needs to be refined.
- In particular, the amount of DK in the D_s wave function via Weinberg compositeness condition can be checked.