Glueball Spectrum from Nf=2 QCD on Anisotropic Lattices

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Outline

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II. Numerical details

III. Discussions on the isoscalar pseudoscalar channel

IV. Summary and outlook

I. Introduction

- QCD predicts the existence of glueballs
- Quenched LQCD predicts that
  lowest-lying glueballs have masses in the range 1~3GeV

J^{PC}	$r_0 M_G$	M_G (MeV)
0^{++}	4.16(11)(4)	1710(50)(80)
2^{++}	5.83(5)(6)	2390(30)(120)
0^{-+}	6.25(6)(6)	2560(35)(120)
1^{+-}	7.27(4)(7)	2980(30)(140)
2^{-+}	7.42(7)(7)	3040(40)(150)
3^{+-}	8.79(3)(9)	3600(40)(170)
3^{++}	8.94(6)(9)	3670(50)(180)
1^{--}	9.34(4)(9)	3830(40)(190)
2^{--}	9.77(4)(10)	4010(45)(200)
3^{--}	10.25(4))(10)	4200(45)(200)
2^{+-}	10.32(7)(10)	4230(50)(200)
0^{+-}	11.66(7)(12)	4780(60)(230)

Y. Chen et al,
Phys. Rev. D 73, 014516 (2006)

- Quenched LQCD predicts that
  scalar and tensor glueballs have large production rate in J/psi radiative decays.

$$\Gamma(J/\psi \to \gamma G_{0^+})/\Gamma_{tot} = 3.8(9)\times 10^{-3}$$
$$\Gamma(J/\psi \to \gamma G_{2^+})/\Gamma_{tot} = 1.1(2)\times 10^{-2}$$

(CLQCD, PRL110(2013)021601, PRL111(2013)091601)

- Experimental candidates for glueballs,
  scalar: f0(1370), f0(1500), f0(1710),
  tensor: f_J(2220)(???), f2(2340)
  pseudoscalar:
  eta(1405),
  X(2120), X(2370), X(2500)

- f0(1710) has large production rate, f2(2340) has large branching fractions in J/psi radiative decays.

BESIII new results for $J/\psi \to \gamma\eta\eta$

(M. Ablikim et al. (BES Collaboration),
Phys. Rev. D 87, 092009 (2013) (arXiv:1301.0053)

Resonance	Mass(MeV/c^2)	Width(MeV/c^2)	$\mathcal{B}(J/\psi \to \gamma X \to \gamma\eta\eta)$	Significance
$f_0(1500)$	1468^{+14+23}_{-15-74}	$136^{+41+28}_{-26-100}$	$(1.65^{+0.26+0.51}_{-0.31-1.40}) \times 10^{-5}$	$8.2\ \sigma$
$f_0(1710)$	$1759\pm6^{+14}_{-25}$	$172\pm10^{+32}_{-16}$	$(2.35^{+0.13+1.24}_{-0.11-0.74}) \times 10^{-4}$	$25.0\ \sigma$
$f_0(2100)$	$2081\pm13^{+24}_{-36}$	273^{+27+70}_{-24-23}	$(1.13^{+0.09+0.64}_{-0.10-0.28}) \times 10^{-4}$	$13.9\ \sigma$
$f_2'(1525)$	$1513\pm5^{+4}_{-10}$	75^{+12+16}_{-10-8}	$(3.42^{+0.43+1.37}_{-0.51-1.30}) \times 10^{-5}$	$11.0\ \sigma$
$f_2(1810)$	1822^{+29+66}_{-24-57}	$229^{+52+88}_{-42-155}$	$(5.40^{+0.60+3.42}_{-0.67-2.35}) \times 10^{-5}$	$6.4\ \sigma$
$f_2(2340)$	$2362^{+31+140}_{-30-63}$	$334^{+62+165}_{-54-100}$	$(5.60^{+0.62+2.37}_{-0.65-2.07}) \times 10^{-5}$	$7.6\ \sigma$

In this analysis, the best fit favors the presence of f2(2340) with a mass of 2362(30)MeV and a width of 334(60) MeV. No evident narrow peak around 2.2GeV over the broad bump is observed in the eta-eta mass specturm.

BESIII new results for $J/\psi \to \gamma\phi\phi$

(M. Ablikim et al. (BES Collaboration),
Phys. Rev. D 93, 112011 (2016) (arXiv:1602.01523)

TABLE I. Mass, width, $\mathcal{B}(J/\psi \to \gamma X \to \gamma\phi\phi)$ (B.F.) and significance (Sig.) of each component in the baseline solution. The first errors are statistical and the second ones are systematic.

Resonance	M(MeV/c^2)	Γ(MeV/c^2)	B.F.(×10^{-4})	Sig.
$\eta(2225)$	$2216^{+4}_{-5}{}^{+21}_{-11}$	$185^{+12}_{-14}{}^{+43}_{-17}$	$(2.40 \pm 0.10^{+2.47}_{-0.18})$	$28\ \sigma$
$\eta(2100)$	$2050^{+30}_{-24}{}^{+75}_{-26}$	$250^{+36}_{-30}{}^{+181}_{-164}$	$(3.30 \pm 0.09^{+0.18}_{-3.04})$	$22\ \sigma$
$X(2500)$	$2470^{+15}_{-19}{}^{+101}_{-23}$	$230^{+64}_{-35}{}^{+56}_{-33}$	$(0.17 \pm 0.02^{+0.02}_{-0.08})$	$8.8\ \sigma$
$f_0(2100)$	2101	224	$(0.43 \pm 0.04^{+0.24}_{-0.03})$	$24\ \sigma$
$f_2(2010)$	2011	202	$(0.35 \pm 0.05^{+0.28}_{-0.15})$	$9.5\ \sigma$
$f_2(2300)$	2297	149	$(0.44 \pm 0.07^{+0.09}_{-0.15})$	$6.4\ \sigma$
$f_2(2340)$	2339	319	$(1.91 \pm 0.14^{+0.72}_{-0.73})$	$11\ \sigma$
0^{-+} PHSP			$(2.74 \pm 0.15^{+0.16}_{-1.48})$	$6.8\ \sigma$

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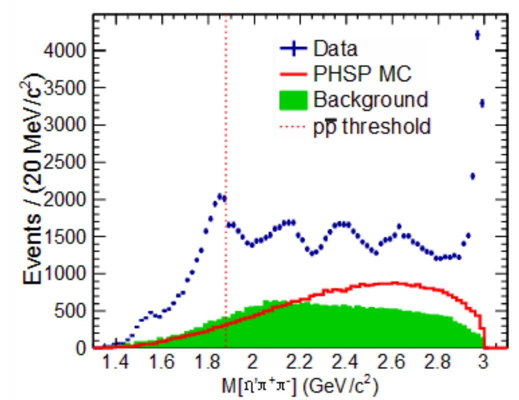
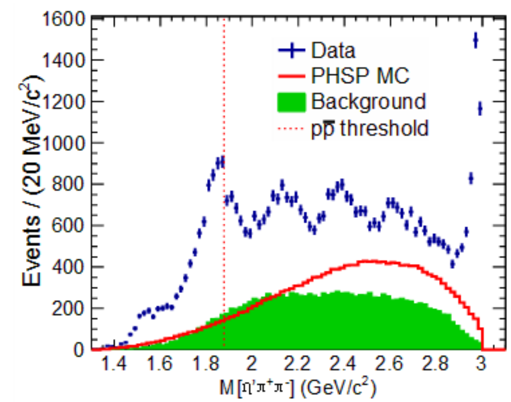
BESIII, PRD93(2016)112011

$J/\psi \to \gamma\eta'\pi^+\pi^-$



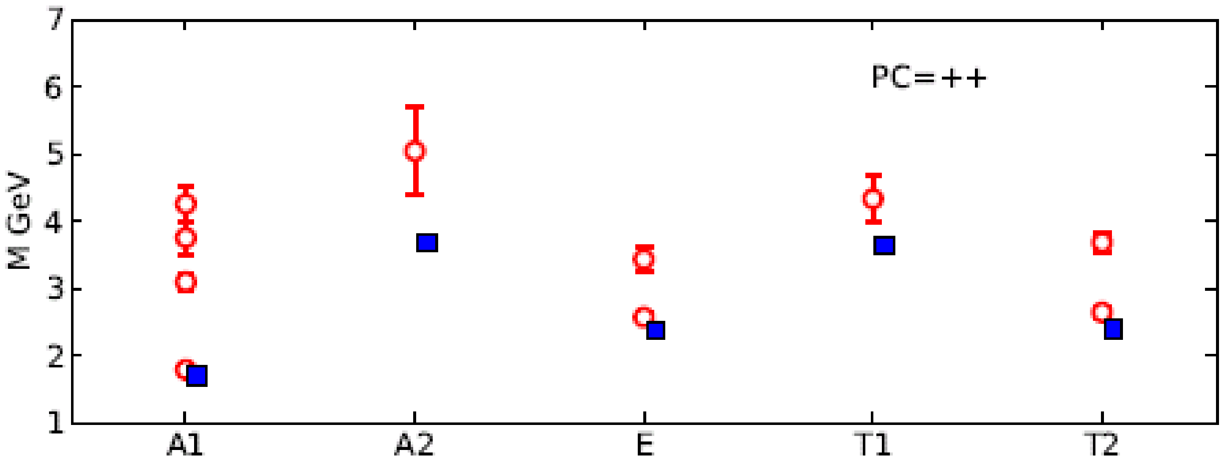
$\eta' \to \gamma\pi^+\pi^-$



$\eta' \to \eta(\to \gamma\gamma)\pi^+\pi^-$

BESIII, arXiv:1603.09653

- Preliminary glueball spectrum from 2+1 flavor dynamical lattice QCD study, which confirms the prediction of the quenched lattice QCD. [E.Gregory et al, JHEP 10 (2012) 170, arXiv:1208.1858(hep-lat)]



Open circles are full-QCD results, and the filled squares are from quenched lattice QCD studies

- We would like to extend our previous studied in the quenched approximation to the full-QCD regime. As the first step, we study the spectrum using only the gluonic operators.

II. Numerical details

- Recently, we generated gauge ensembles with Nf=2 clover Wison fermions on anisotropic lattices

[Y. Chen et al (CLQCD), in preparation]

Table 1. Parameters of configurations

m_π	β	$L^3 \times T$	ξ	a_s	N_{conf}
$\sim 655 Mev$	2.5	$12^3 \times 128$	5	$0.113 fm$	4800
$\sim 1000 Mev$	2.5	$12^3 \times 128$	5	$0.118 fm$	10400

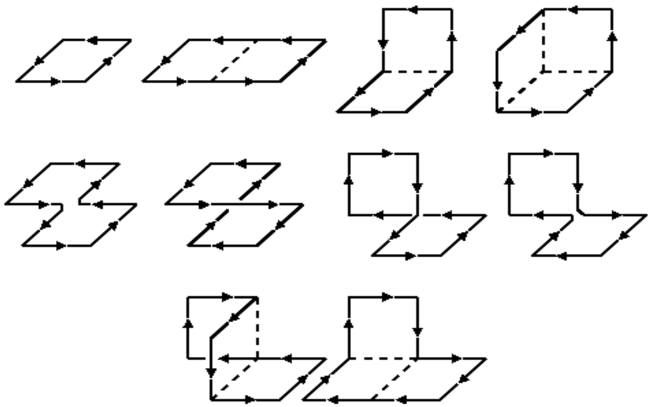
Gauge action: Tadpole improved Symanzik's action
Fermion action: Wilson clover action

The pion masses are still very large.
The physical volumes are very small.
The statistics is relatively large.

The lattice spacings have explicit quark mass dependences.

- Gluonic operators in the scalar, tensor, and pseudoscalar channels.

	Continuum limit	Finite lattice
Symmetry Group	$SO(3) \otimes P \otimes T$	$O \otimes P \otimes T$
Irreducible Representation (R)	J^{PC}, $J = 0,1,2,\ldots$	R^{PC}, $R = A_1, A_2, E, T_1, T_2$



C.Morningstar and M. Peardon, Phys. Rev. D 60, 034509, 1999

- Use these Wilson loops as prototypes
- They can be contructed through smeared gauge links
- Different irreps can be realized through proper linear conbinations of the different spatial orientation of these prototypes.
- Finally, one can build a set of operators for a specific quantum number R^{PC} .

Solving the generalized eigenvalue problem (GEVP)

The essence of the VM is to find a set of combinational coefficients

$$\{v_\alpha, \alpha = 1,2,\ldots 24\}$$

such that the operator

$$\Phi = \sum_\alpha v_\alpha \phi_\alpha$$

couples mostly to a specific state.

$$\tilde{C}(t_D)\mathbf{v}^{(R)} = e^{-t_D \tilde{m}(t_D)} \tilde{C}(0)\mathbf{v}^{(R)}$$

$$\tilde{C}_{\alpha\beta}(t) = \sum_\tau \langle 0|\phi_\alpha(t+\tau)\phi_\beta(\tau)|0\rangle$$

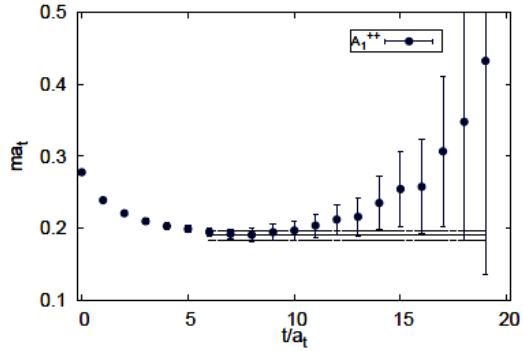
$$\tilde{m}(t_D) = -\frac{1}{t_D} \ln \frac{\sum_{\alpha\beta} v_\alpha v_\beta \tilde{C}_{\alpha\beta}(t_D)}{\sum_{\alpha\beta} v_\alpha v_\beta \tilde{C}_{\alpha\beta}(0)}$$

These techniques have been successfully implemeted previous quenched studies.

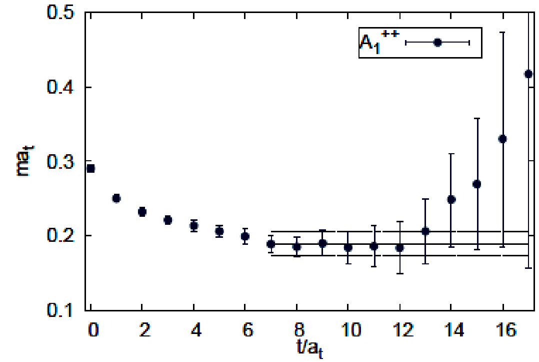
C.Morningstar and M. Peardon, Phys. Rev. D 60, 034509, 1999

Y. Chen et al, Phys. Rev. D 73, 014516, 2006

A_1^{++}



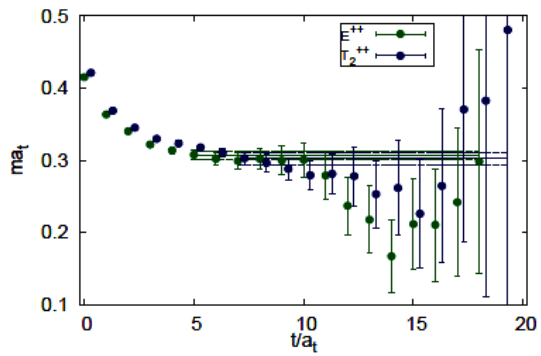
(a) $m_\pi \sim 920Mev$



(b) $m_\pi \sim 580Mev$

0^{++}

E^{++} T_2^{++}



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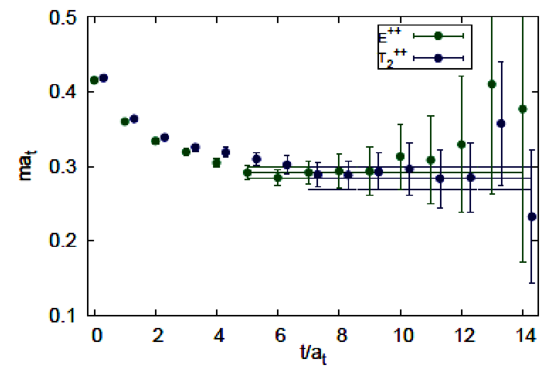
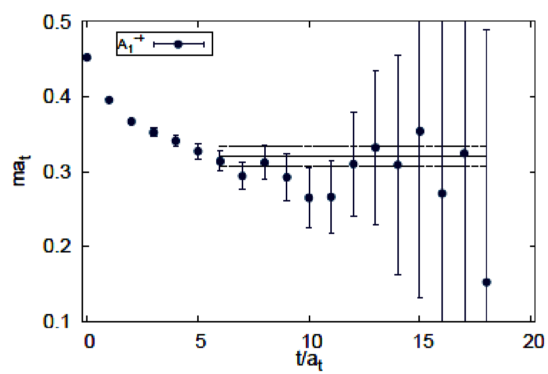


(b) $m_\pi \sim 580Mev$

2^{++}

A_1^{-+}



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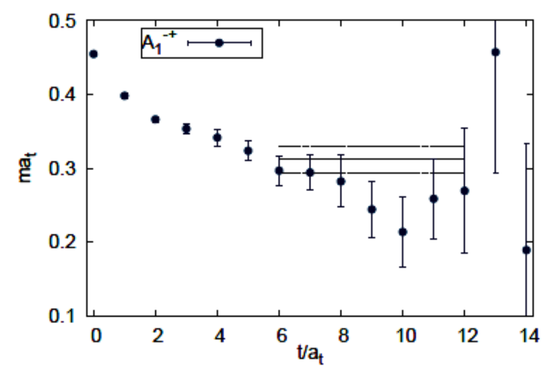


(b) $m_\pi \sim 580Mev$

0^{-+}

- Our results (very preliminary)

m_π	R^{PC}	fit range	ma_t	$m_G(Mev)$	χ^2/dof
$\sim 920Mev$	A_1^{++}	6-19	0.1900(65)	1589(54)	0.75
	A_1^{-+}	6-18	0.3204(132)	2680(110)	0.87
	E^{++}	5-18	0.3068(63)	2566(53)	0.71
	T_2^{++}	7-18	0.3023(78)	2528(65)	0.66
$\sim 580Mev$	A_1^{++}	7-17	0.1860(161)	1624(141)	1.64
	A_1^{-+}	6-19	0.3135(175)	2738(153)	1.71
	E^{++}	5-14	0.2921(76)	2551(66)	0.84
	T_2^{++}	7-14	0.2841(144)	2481(126)	0.60

- Comparison with previous results

J^{PC} / Researches		ground states spectrum(Mev)		
		0^{++}	0^{-+}	2^{++}
this work	$m_\pi \sim 920Mev$	1589(54)	2680(110)	2546(58)
	$m_\pi \sim 580Mev$	1624(141)	2738(153)	2516(95)
M&P		1730(50)(80)	2590(40)(130)	2400(25)(120)
Chen *et.al.*		1710(50)(80)	2560(35)(120)	2390(30)(120)
Gregory *et.al.*		1795(60)		2620(50)

No substantial differences from the quenched Results !

- Comments: A more sophisticated data analysis is undergoing.

III. Discussions on the isoscalar pseudoscalar channel

- For the flavor singlet pseudoscalar channel,

$U_A(1)$ Anomaly $$\partial_\mu A^\mu(x) = 2mP(x) - \frac{N_f}{16\pi^2}\varepsilon^{\mu\nu\rho\sigma} TrF_{\mu\nu}F_{\rho\sigma}$$

Topological charge density $$q(x) = -\frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma} TrF^{\mu\nu}(x)F^{\rho\sigma}(x)$$

It is expected that the gluonic operator q(x) has a large overlap to the flavor singlet pseudoscalar meson.

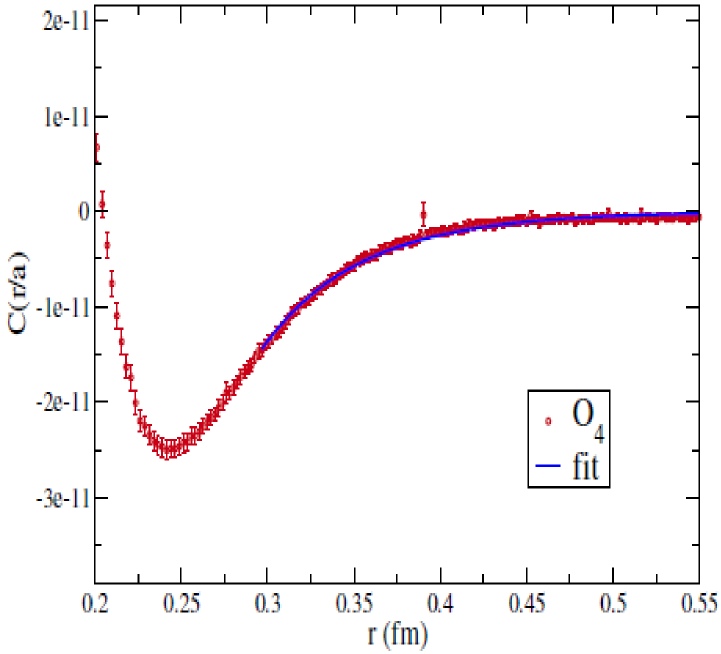
It was proposed to investigate the lowest flavor singlet pseudo scalar mesons uing the correlator of the topological charge density operator q(x)

$$C(r) = \langle q(x)q(y)\rangle, r = |x-y|.$$

$$C(r) = N\frac{m}{4\pi^2 r}K_1(m_{\eta'}r)$$

In the quenched approximation

(A. Chowdhury et al., Phys. Rev. D 92 (2015)111501, [arXiv:1509.00944])

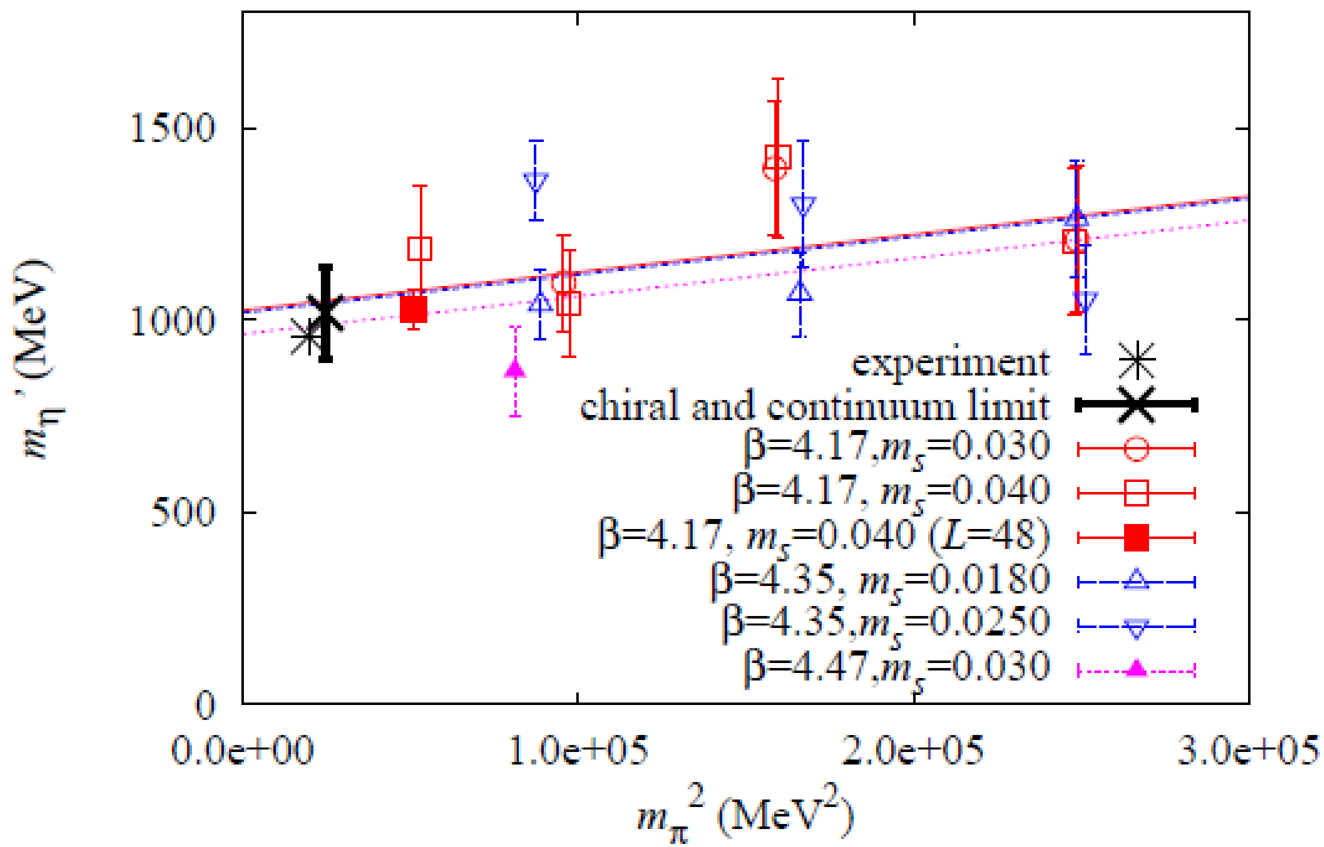


Lattice	$r_{\min}$ (fm)	am	m (MeV)
O_1	0.31	0.887(39)	2624(114)
P_1	0.30	0.831(36)	2459(108)
O_2	0.29	0.648(18)	2590(78)
P_2	0.33	0.648(25)	2560(100)
O_3	0.28	0.535(29)	2625(140)
P_3	0.27	0.524(17)	2573(81)
O_4	0.31	0.445(11)	2545(63)

The mass of the lowest pseudoscalar is in good agreement with the pseudoscalar glueball mass.

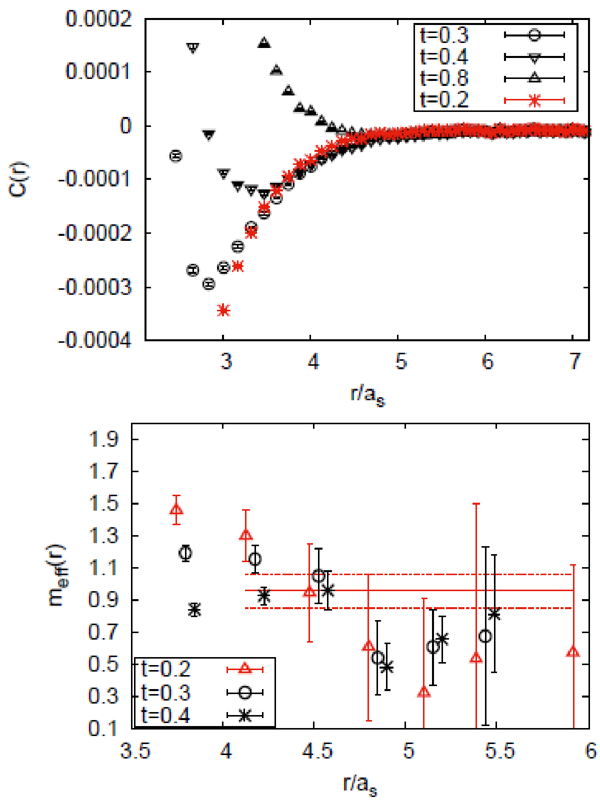
Nf=2+1 full QCD calculation

(H. Fukaya et al, Phy. Rev. D92 (R), 111501 (2015), arXiv: 1509.00944)

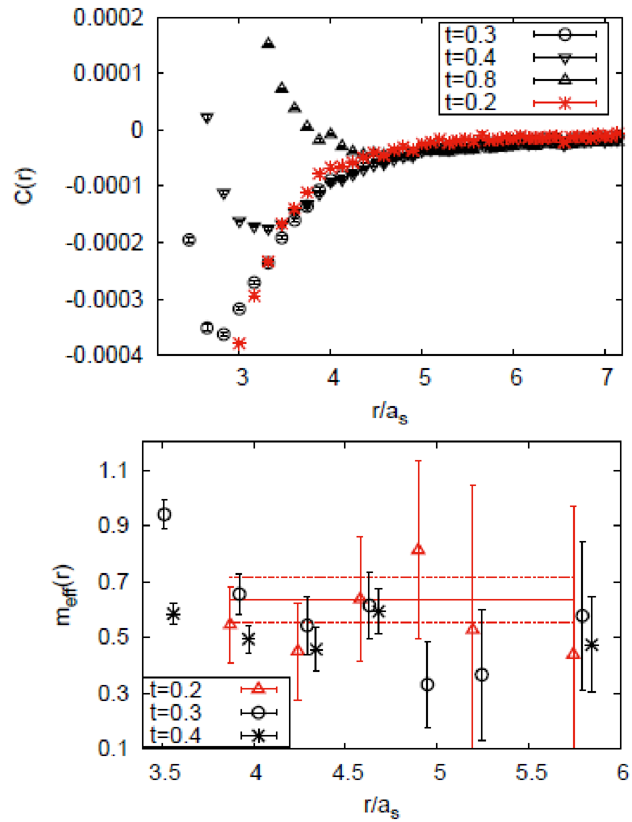


$$m_{\eta'} = 1019(119)(^{+97}_{-86}) \quad \text{MeV}$$

A similar study based on our two ensembles:



(a) $m_\pi \sim 920Mev$



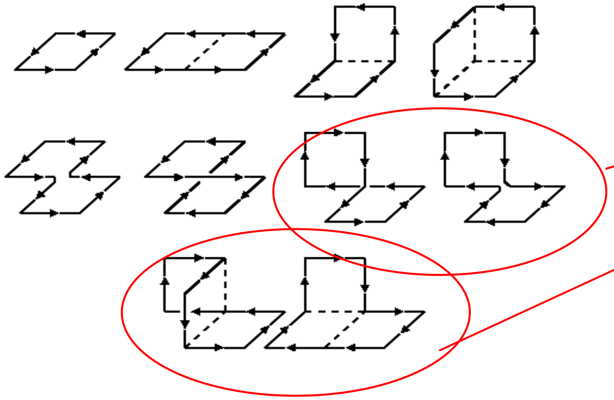
(b) $m_\pi \sim 580Mev$

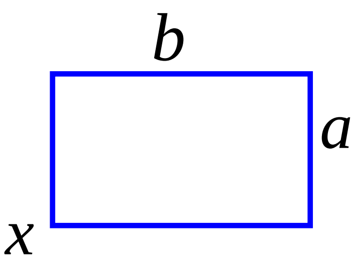
m_π	fit range$[a_s]$	ma_s	$m_{\eta'}(Mev)$	χ^2/dof
$\sim 920Mev$	4.1-5.9	0.9522(1075)	1468(166)	0.48
$\sim 580Mev$	3.8-5.7	0.6336(807)	977(124)	0.41

We don't take the values seriously, but only get the impression they are much lower than the mass of pseudosclar glueball.

In the calculation of the glueball spectrum, the gluonic operator for the pseudoscalar is defined as

$$\phi_{\alpha}^{A_1^{-+}}(\mathbf{x},t) = \sum_{R\in O} c_{A_1} ReTr\left[R \circ W_{\alpha}(\mathbf{x},t) - \mathcal{P}R \circ W_{\alpha}(\mathbf{x},t)\mathcal{P}^{-1}\right]$$





$$P_{\mu\nu}^{a\times b}(x) = 1 + ab(F_{\mu\nu}(x) + \frac{1}{2}(aD_{\mu} + bD_{\nu})F_{\mu\nu}(x) + \ldots)$$

Thus one can easily verify that the continuum form of the operator is

$$\boxed{\phi_{\alpha}^{A_1^{-+}}(\mathbf{x},t) \propto \epsilon_{ijk} TrB_i(\mathbf{x},t)D_jB_k(\mathbf{x},t) + O(a_s^2)}$$

which is in sharp constrast with the topological charge density

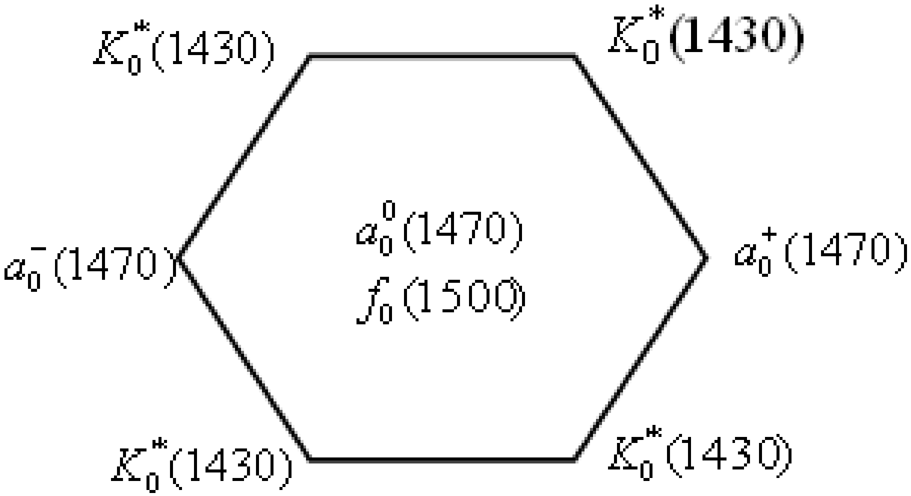
$$q(x) \propto \epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}(x)F^{\rho\sigma}(x) \propto \mathbf{E}(x)\cdot\mathbf{B}(x)$$

IV. Summary and Outlook

- The mass spectrum of the scalar, the tensor, and the pseudoscalar glueballs are calculated in Nf=2 QCD on anisotropic lattices.
- The preliminary results do not show substantial unquenching effects on these glueball masses.
- A relatively heavy pseudoscalar is obtained though the conventional gluonic operator, whose mass is close to the pure guage pseudoscalar glueball.
- The conventional gluonic operator for the pseudoscalar glueball has very different asymptotic from the topological charge density in the continuum limit.
- The mixing between glueball states and qqbar mesons will be investigate in the future.

Thanks!

$q\bar{q}$??



$+ \quad f_0(1370)$

$f_0(1710)$

These ten scalar mesons can be assigned as a q-barq meson nonet plus a possible scalar glueball which can be either one of the three isoscalars or an admixture of them. There are Many mixing models, the details are out of the scope of this talk.

Lattice prediction:

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}|E_1(0)|^2 = 0.35(8) keV$$

$$\Gamma/\Gamma\_tot = 0.33(7)/93.2 = 3.8(9)\times 10^{-3}$$

Experimental results for J/psi radiatively decaying to scalars

C. Amsler et al. (Particle Data Group), Phy. Rev. D 86, 010001 (2012)

$J/\psi \to \gamma f_0(1500) \to \gamma\pi\pi$ $(1.01 \pm 0.32)\times 10^{-4}$

$Br(f_0(1500) \to \pi\pi) = (34.9 \pm 2.3)\%$ $\Rightarrow Br(J/\psi \to \gamma f_0(1500)) = 2.9\times 10^{-4}$

$J/\psi \to \gamma f_0(1710) \to \gamma\eta\eta$	$(2.35^{+1.27}_{-0.77})\times 10^{-4}$
$J/\psi \to \gamma f_0(1500) \to \gamma\eta\eta$	$(1.65^{+0.57}_{-1.50})\times 10^{-4}$

BESIII results (PRD87, 092009)

$J/\psi \to \gamma f_0(1710) \to \gamma K\overline{K}$	$(8.5^{+1.2}_{-0.9})\times 10^{-4}$
$J/\psi \to \gamma f_0(1710) \to \gamma\pi\pi$	$(4.0 \pm 1.0)\times 10^{-4}$
$J/\psi \to \gamma f_0(1710) \to \gamma\omega\omega$	$(3.1 \pm 1.0)\times 10^{-4}$
$J/\psi \to \gamma f_0(1710)$	$> (1.5 \pm 0.3)\times 10^{-3}$

Using Br(f_0(1710)→ KK)=0.36 ⇒ Br(J/ψ→γf_0(1710))= 2.4×10^{-3}

Br(f_0(1710)→ ππ)= 0.15 ⇒ Br(J/ψ→γf_0(1710))= 2.7×10^{-3}

Our result support f0(1710) as the candidate for the scalar glueball