Impact of dynamical charm quarks

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Motivation

- An enormous effort is made to generate $N_f = 2 + 1$ configurations
  - BMW, CLS, JLQCD, RBC/UKQCD, ...
- Advantages
  - Cheaper
    odd numbers of flavors are expensive, e.g inclusion of $s \sim$ doubling of costs
  - Easier to define (and keep!) a chiral trajectory
  - The charm quark has a very small impact on low energy observables (decoupling)
  - No lattice artifacts $O([am_c]^{\alpha})$ which may be difficult to control
  - With Wilson fermions: one may get away with b_X from PT
- Disadvantage
  - Unknown systematical errors in high energy observables
    (where decoupling does not apply)

Goal

Estimate the difference between $N_f = 2 + 1 + 1$ and $N_f = 2 + 1$ for high energy observables
e.g. charmonia masses or static force at small distances

Decoupling

The effect of heavy sea quarks at low energies

- Effective theory $\mathcal{L}_{\text{eff}} = \mathcal{L}^{N_\text{l}} + \frac{1}{M^2}\mathcal{L}_6 + \ldots$

  [S.Weinberg (1979)]

  - N_q quarks in total
  - N_l light quarks
  - Effective theory contains only the light quarks. Leading order describes full theory up to power-corrections $O((\Lambda_\text{q}/M)^2)$

- Detailed study

  [M.Bruno, J.Finkenrath, F.Knechtli, B.Leder, R.Sommer (2014)]

  [A.Athenodorou, M.Bruno, J.Finkenrath, F.Knechtli, B.Leder, M.Marinkovic, R.Sommer (2014)]

  - Factorization formula

$$\frac{m_q^{\text{had}}(M)}{m_q^{\text{had}}(0)} = \underbrace{Q_{\text{l,q}}^{\text{had}}}_{\textit{massless}} \times \underbrace{P_{\text{l,q}}(M/\Lambda_\text{q})}_{\textit{perturbative}} \quad + \quad O((\Lambda_\text{q}/M)^2)$$

  - Particularly simple: $\frac{m_q^{\text{had1}}(M)}{m_q^{\text{had2}}(M)} = r + O(M^{-2})$
  - Numerical study with $N_\text{q} = 2$, $N_\text{l} = 0$:
    $r_0/\sqrt{t_0}$ has a 0.1(6)% effect at $M \sim M_c/2$

Strategy

We compare QCD with $N_\mathrm{f} = 2$ heavy ($M \sim M_c$) quarks to quenched QCD

- We want to
  - Further confirm decoupling
    - ★ Full theory $=$ QCD$^{N_\mathrm{f}=2}$
    - ★ Effective theory $=$ QCD$^{N_\mathrm{f}=0}$
  - Investigate "high" energy observables for which decoupling does not apply

Strategy

- Simulate QCD$^{N_\mathrm{f}=2}$ at $M = M_c$, several lattice spacings
- Compute: $r_0/a,\ \ t_0/a^2,\ \ a\,m_\mathrm{P},\ \ a\,m_\mathrm{V}$
- Continuum extrapolate dimensionless ratios, e.g.
  $r_0/\sqrt{t_0},\ \ \sqrt{t_0}m_\mathrm{P},\ \ \sqrt{t_0}m_\mathrm{P},\ \ m_\mathrm{V}/m_\mathrm{P}$
- Simulate QCD$^{N_\mathrm{f}=0}$. Matching:

$$
\begin{aligned}
[t_0/a]^{N_\mathrm{f}=0} &\approx [t_0/a]^{N_\mathrm{f}=2} && \Rightarrow \beta \text{ for similar lattice spacings} \\
\left[\sqrt{t_0}m_P\right]^{N_\mathrm{f}=0} &\overset{!}{=} \left[\sqrt{t_0}m_P\right]^{N_\mathrm{f}=2}_{\mathrm{cont}} && \Rightarrow \mu
\end{aligned}
$$

Simulations

Dynamical quarks

- Gauge action: plaquette, $\beta \in \{5.7, 6.0, 6.2\}$
- Fermion action: doublet of twisted mass fermions $\psi = \begin{pmatrix} c \\ c' \end{pmatrix}$
  - $c_{\rm sw}$ [K.Jansen, R.Sommer (1997)]
  - κ_c interpolation of [P.Fritzsch et al (2012)], [P.Fritzsch, N.Garron, J.Heitger (2015)]
  - $a\,\mu = Z_P \times \frac{M_c}{\Lambda^{(2)}} \times \Lambda^{(2)}\, L_1 \times \frac{\overline{m}}{M} \times \frac{a}{L_1}$
    [P.Fritzsch, F.Knechtli, B.Leder, M.Marinkovic, S.Schaefer, R.Sommer, F.Virotta (2012)]

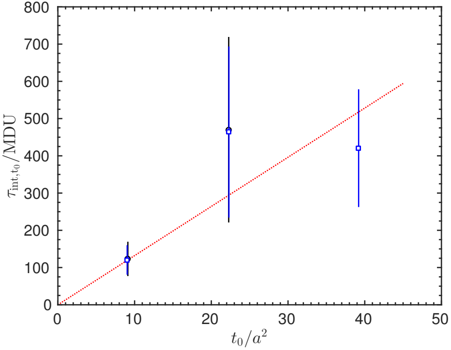
Quenched quarks

- Gauge action: plaquette, $\beta \in \{6.34, 6.672, 6.9\}$
  Estimated from $\frac{r_0}{a}(\beta)$ [S.Necco, R.Sommer (2001)] and t_0/r_0^2 [M.Bruno]
- Valence quarks: doublet of twisted mass fermions
  - $c_{\rm sw}$ [M.Lüscher, S.Sint, R.Sommer, P.Weisz, U.Wolff(1996)]
  - κ_c interpolation of [M.Lüscher, S.Sint, R.Sommer, P.Weisz, U.Wolff (1996)]
  - $a\,\mu$: For each β, 3 values $\rightarrow$ interpolation to matching point

Ensembles

- Open boundaries in time, periodic in space
  - Milder critical slowing down than on torus
  - `openQCD-1.2` [M.Lüscher, S.Schaefer (2013)]
  - $c_G = c_F = 1$

β	$\frac{L}{a} \times \frac{T}{a}$	a/fm	$a\,\mu$	MDUs
5.700	32×120	0.051	0.113200	17k (17k)
6.000	48×192	0.033	0.072557	22k (11k)
6.340	32×120	0.051	-	20k (20k)
6.672	48×192	0.033	-	74k (21k)
6.900	64×192	0.025	-	100k (65k)



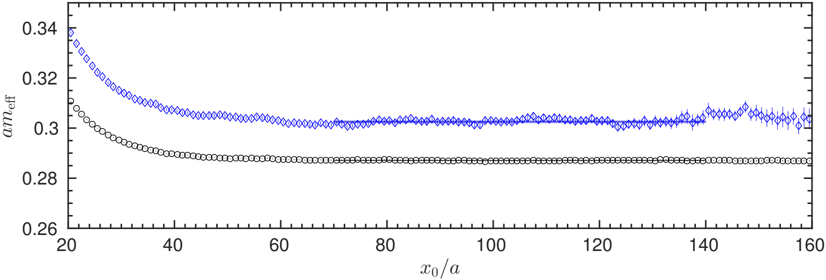
critical slowing down compatible with $z \sim 2$

Measurements

- r_0/a, Sommer scale
  Computed as in [M.Donnellan, F.Knechtli, B.Leder, R.Sommer (2011)]
  - HYP-smeared Wilson loops, 3-4 levels
  - GEVP for static potential $aV(r)$
  - $r^2 V'(r)\big|_{r=r_0} = 1.65$
- t_0/a^2, scale from gradient flow
  [M.Lüscher (2010)]
  - Wilson discretization of the gradient flow
  - Clover discretization of the action density $E = \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$
  - $t^2 \langle E(t) \rangle\big|_{t=t_0} = 0.3$
- Meson masses
  - From zero momentum correlation functions
    $f(x_0, y_0) = \sum_{\boldsymbol{x},\boldsymbol{y}} \langle J(x) J^\dagger(y) \rangle$
  - Stochastic propagators
  - Local interpolating fields
    $J \in \{\underbrace{\bar{c}\gamma_5 c'}_{\to m_P}, \underbrace{\bar{c}\gamma_k c'}_{\to m_V} \ldots\}$

Effective masses

- Use T symmetry: $g(x_0) = \frac{f(a,x_0)+f(T-a,T-x_0)}{2}$
- Effective mass: $a\,m_{\text{eff}}(x_0) = \log\left(\frac{g(x_0)}{g(x_0+a)}\right)$
- Black$= m_P$, blue$= m_V$, finest lattice

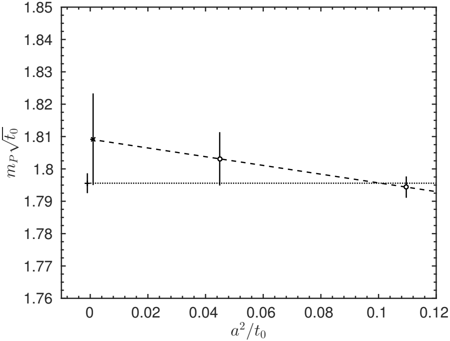


Systematical errors

- $a\,\mu = Z_P \times \frac{M_c}{\Lambda^{(2)}} \times \Lambda^{(2)} L_1 \times \frac{\overline{m}}{M} \times \frac{a}{L_1}$
  - $\frac{M_c}{\Lambda^{(2)}} = 4.87$
  - Largest errors: $\Lambda^{(2)} L_1$, $\frac{\bar{m}}{M}$ common to all points
- κ_c mistuning
  - Maximal twist: $m_{\mathrm{PCAC}} = 0$
  - $\bar{m} = \frac{1}{Z_P}\sqrt{\mu^2 + Z_A^2 m_{\mathrm{PCAC}}^2}$
  - We have on all ensembles $\frac{\bar{m}-\mu/Z_P}{\bar{m}} < 0.3\%$ (2%)
- Finite volume effects: negligible $\frac{L}{\sqrt{t_0}} > 10$
- Lattice artifacts: $O(a^2)$

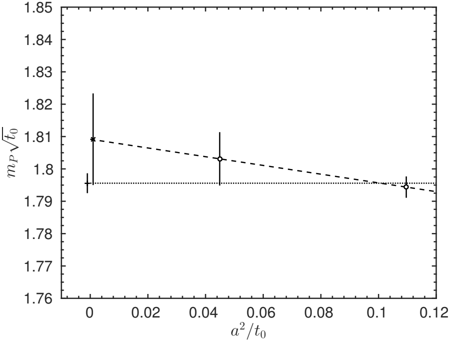
Matching zero and two flavor QCD

$N_\mathrm{f} = 2$

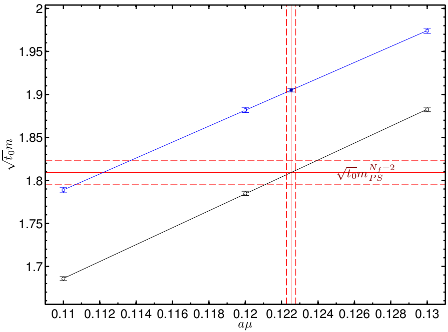


- Linear & constant fits
  $\rightarrow$ compatible continuum values
- We work with value from linear fit

$N_f = 2$



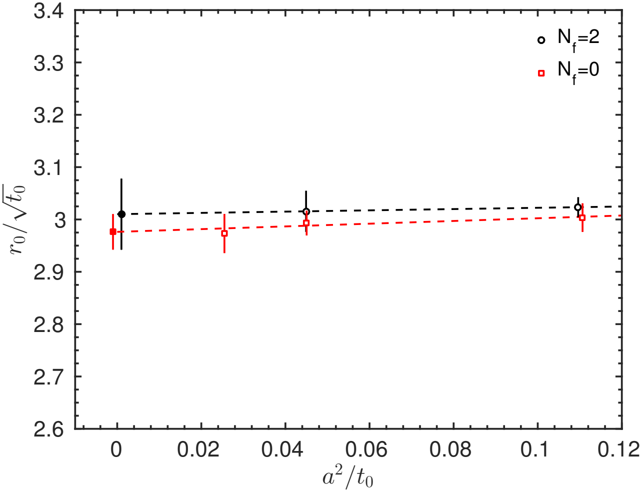
$N_f = 0$



- Linear & constant fits
  $\rightarrow$ compatible continuum values
- We work with value from linear fit

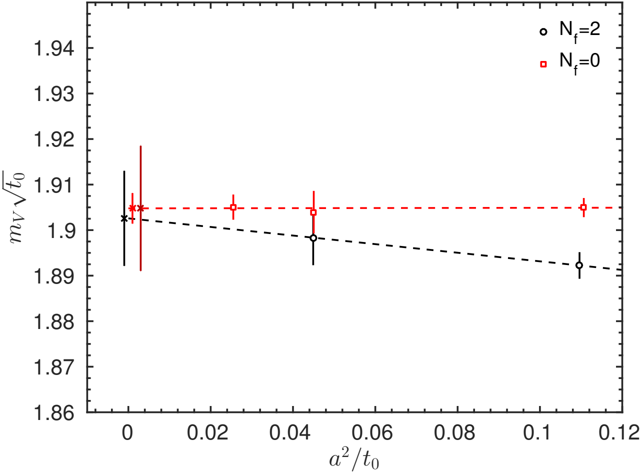
- Black: $\sqrt{t_0}\, m_P$
  Blue: $\sqrt{t_0}\, m_V$
- m_P linear in μ (like HQET)

Results: scales



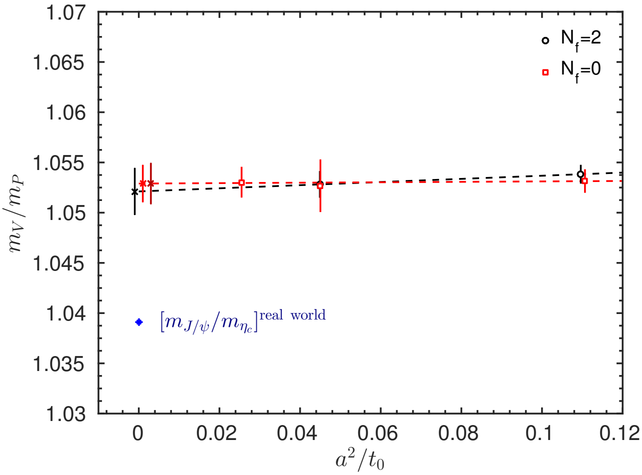
- Expected effect (decoupling): below 0.3%
- No disagreement found at a precision of $\sim$ 2%

Results: masses



- Decoupling not applicable
- No effect resolvable at a precision of 0.7%
- Error dominated by $\Delta_{[\sqrt{t_0}m_P]_{\mathrm{cont}}^{N_f=2}}$

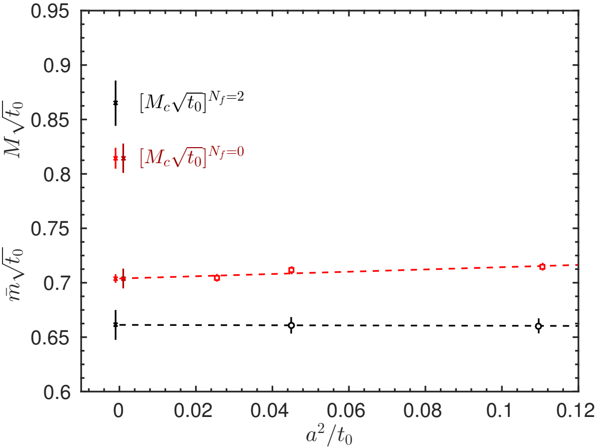
Results: masses



- Errors in μ cancel to a large extent
- No effect resolvable at a precision of 0.2%

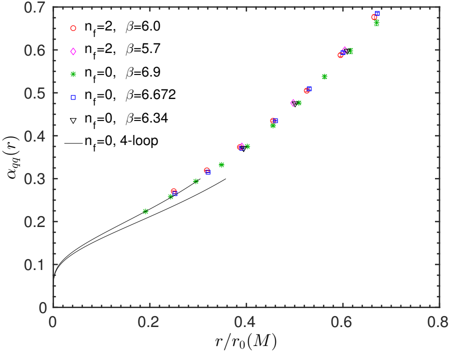
Results: RGI mass

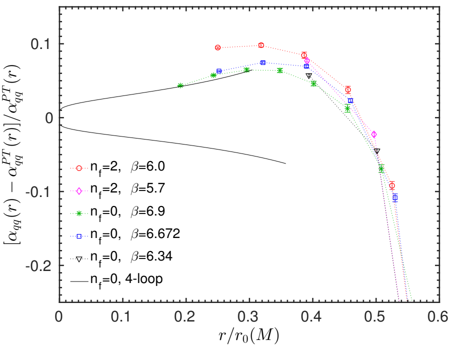
- $\bar{m} = \frac{1}{Z_P}\sqrt{\mu^2 + Z_A^2 m_{PCAC}^2}$
  - $Z_P^{N_f=2}$, $M/\bar{m}$ [P.Fritzsch et al (2012)]
  - $Z_A^{N_f=2}$ [M.Della Morte et al (2005)]
  - $Z_P^{N_f=0}$, $M/\bar{m}$ [A.Jüttner (2004)]
  - $Z_A^{N_f=0}$ [M.Lüscher et al (1997)]
- $\bar{m}$ values at different scales but M values comparable



- $\sim 5\%$ effect, but large errors

Results: strong coupling





- Strong coupling from the static force: $\alpha_{qq}(\mu) = \frac{1}{C_F} r^2 V'(r)$
- Significant effect at $\mu = 1/r \sim 1.6$ GeV and above
- Not a lattice artifact

Conclusions

Conclusions

- Effects of dynamical charm quarks
  - tiny in charmonium masses
  - significant in α_{qq} at large energies
  - quite sizable in the RGI mass
- Lattice artifacts appear to be $O(a^2)$ below $a = 0.05$ fm

Outlook

- Higher statistics
- Third lattice spacing with $N_f = 2 \Rightarrow$ smaller errors everywhere
- More observables
  - Charmonium spectrum
  - Matrix elements
  - Quenched strange quark $\rightarrow f_{D_s}$