On the width of the confining flux tube in the 3D U(1) lattice gauge theory

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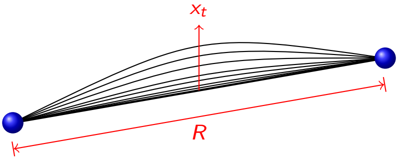
Outline

- Motivation
- Theoretical expectation for the flux tube width
- Results for the $\mathrm{U}(1)$ LGT in $3D$
- Conclusions

Motivation - 1

The shape of the flux tube and the confining mechanism



Relevant observables

- The chromoelectric field parallel to the charges axis $E_l(x_t)$, measured in their symmetry plane.
- The flux tube squared width

$$w^2(R) = \int \mathrm{d}x_t x_t^2 E_l(x_t)$$

(Bali et al., 1995),(Cardaci et al., 2011)
(Takahashi et al.2001),(Amado et al., 2013)

Two pictures

- Effective String Theory.
- The Dual superconductor model.

Motivation - 2

The 3D U(1) LGT and the rigid string

Rigid Strings

In the 3D U(1) lattice gauge theory, the static potential can be given an effective description in terms a NambuGoto (NG) + Rigid string (Polyakov (1996), Caselle et al.(2014))

$$S_{\text{eff}} = \underbrace{\sigma \int d^2\xi \sqrt{g}}_{\text{NG}} + \underbrace{\alpha \int d^2\xi \sqrt{g} K^2,}_{\text{Rigidity}}$$

ξ worldsheet coordinates, g induced metric, K extrinsic curvature. (Gliozzi and Meineri, 2012)

Consequences for the flux tube

- Its transverse profile should be exponentially decaying, in contrast to NG (gaussian).
- Its squared width should be constant with R, logarithmic with NG.

Effective String Theory - 1

The flux tube as a fluctuating string

$$Z_R = \int [\mathcal{D}X]\, e^{-S_{\text{eff}}[X]_{\text{RxL}}}$$

R charge separation,
L time extent of the system,
X_i transverse displacements.

- Long string approximation (valid down to $R \sim 1/\sqrt{\sigma}$), σ string tension,

$$S_{\text{eff}} = \sigma RL + \frac{\sigma}{2}\int \mathrm{d}^2\xi \underbrace{(\partial_\alpha X \cdot \partial^\alpha X)}_{\text{L. O.}} + c_2 \int \mathrm{d}^2\xi\, (\partial_\alpha X \cdot \partial^\alpha X)^2 + \cdots + \alpha \int \mathrm{d}^2\xi K^2$$

- S_{eff} strongly constrained by Poincaré symmetry: few free parameters.

(Gliozzi, 2011), (Aharony and Komargodski, 2009)

Predictions at L.O.

$$V(R) = \sigma R - \frac{(D-2)\pi}{24R}, \qquad w^2(R) = \frac{(D-2)}{2\pi\sigma}\log R/R_c$$

(Luscher et al., 1981)

Perfect agreement with data both for abelian, non abelian models, and at zero and finite T, and beyond the LO, so far...

(Athenodorou et al., 2011, Caselle, 2010)
(Brandt, 2011, Gliozzi et al., 2011)

The dual superconductor model

A phenomenological model of color confinement

The Condensation of (dynamically formed) chromomagnetic monopoles causes Confinement of chromoelectric charges via the (dual) Meissner effect.

(Mandelstam, 1976, Nambu, 1979, 't Hooft, 1979)

The Abelian case

$$\mathcal{L} = -\frac{1}{4}\bar{F}^{\mu\nu}\bar{F}_{\mu\nu} + \frac{1}{2}(D_\mu\psi)^\star D^\mu\psi - \frac{1}{2}b(\psi^\star\psi - v^2)^2$$

- $1/\xi = m_H = 2v\sqrt{b}$ mass of the higgs field (ξ coherence length of the condensate).
- $1/\mu = m_v = gv$ mass of the gauge field (μ London penetration depth of the field).

For SU(3) see: (Cea et al., 2014)

Tubelike solutions to the EOM in D=3

$$E_l(x_t) = \Phi\, m_v^2\, e^{-m_v|x_t|}, \quad \text{for } |x_t| \gg \mu,\ \Phi \text{ chromoelectric flux.}$$

Summarizing...

Effective string theory

- The profile should be a pure Gaussian, at LO.
- The squared width should grow logarithmically with quark separation.

Dual superconductor model

- The profile should be described by a exponential decay.
- The squared width should be constant and close to μ^2.

The $U(1)$ Lattice gauge theory in 3D

Definition

On a 3D euclidean spacetime lattice Λ (spacing a),

$$S = \beta \sum_{x\in\Lambda} \sum_{1\leq\mu<\nu\leq 3} \left[1 - \cos\vartheta_{\mu\nu}(x)\right], \quad \beta = \frac{1}{ae^2}, \quad \vartheta_\mu(x) \in (-\pi, \pi],$$

The semiclassical approximation ($\beta \gg 1$)

$$m_0 a = c_0(\simeq 1)\sqrt{8\pi^2\beta}\, e^{-\pi^2\beta v(0)}, \quad \sigma a^2 \geq \frac{c_\sigma(\simeq 8)}{\sqrt{2\pi^2\beta}} e^{-\pi^2\beta v(0)}, \quad v(0) = 0.2527$$

(Göpfert and Mack, 1981, Polyakov, 1977)

- The model is always in the confined phase in $3D$
- The ratio

$$\frac{m_0}{\sqrt{\sigma}} = \frac{2\pi c_0}{\sqrt{c_\sigma}}(2\pi\beta)^{3/4} e^{-\pi^2 v(0)\beta/2},$$

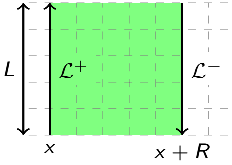
  can be tuned at will by an appropriate choice of β, in contrast to the general Yang-Mills case.

The $U(1)$ Lattice gauge theory in 3D
The dual model

We obtain a globally $\mathbb{Z}$ symmetric spin model

$$Z_{\mathrm{U}(1)} = e^{-\beta N_l} \sum_{\{^\star l=-\infty\}}^{\{\infty\}} \prod_{^\star c_1} \mathrm{I}_{|\mathrm{d}^\star l|}(\beta), \quad {}^\star c_1 \text{ dual links.}$$

Addition of static charges



$$Z^R_{\mathrm{U}(1)} = e^{-\beta N_l} \sum_{\{^\star l=-\infty\}}^{\{\infty\}} \prod_{^\star c_1} \mathrm{I}_{|\mathrm{d}^\star l+^\star n|}(\beta)$$

where $^\star n$ is an integer valued dual 1-chain which is nonvanishing only on the links dual to the green surface.

$$\langle F(x)\rangle_{q\bar{q}} = \frac{\langle \mathrm{d}^\star l +^\star n\rangle}{\sqrt{\beta}}$$

(Zach et al., 1998)

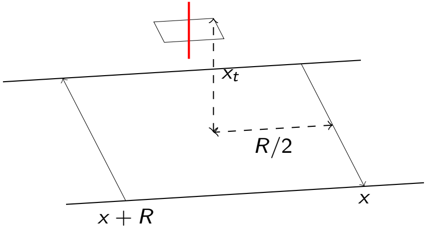
Measurement of the Electric field

Simulation setup

The measured quantity

No Schwinger line!

$$E_l(x_t)a^2 = \frac{\langle \mathrm{d}^\star l +^\star n \rangle}{\sqrt{\beta}}$$



Update algorithm

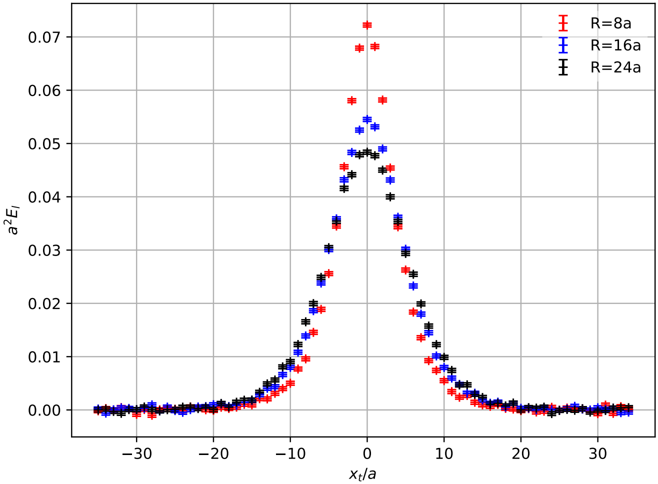
- Local Metropolis update of the dual model.
- Hierarchical update around the local measured quantity.

Lattice setting

- Lattices $(64a)^3$ and $(96a)^3$ respectively at $\beta = 1.7, 2.0$ and $\beta = 2.2, 2.4$
- $T \sim 0$
- Charges separation $R = 4a \div 24a$.

Measurement of the Electric field

Example at $\beta = 2.0$ for various intercharge distances.



The Squared width

Computation - 1

Two ways of computing w^2

- Fit the profile with some arbitrarily complicated analytically integrable function, then analitically compute w^2: its error will depend on the estimated parameters.
- Use a discretized version of the continuum formula

$$w^2 a^{-2} = \frac{\sum_{x_t} x_t^2 E_l(x_t)}{\sum_{x_t} E_l(x_t)}.$$

For large x_t, $E_l(x_t) \simeq 0$ and the corresponding points only contribute to the (systematical) error of $w^2 a^{-2}$.

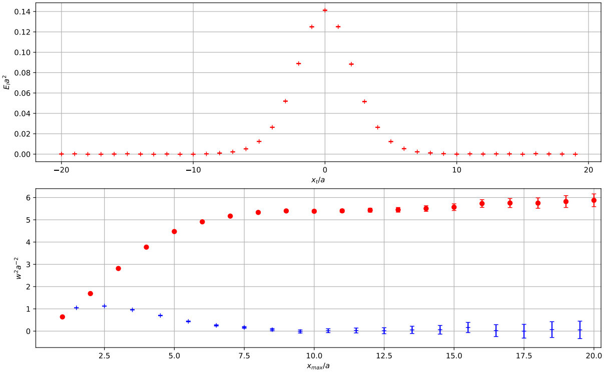
The Squared width

Computation - 2

We used the second way: introducing a cutoff $x_{\max}$ in the sum...

$$w^2 a^{-2} = \frac{\sum^{x_{\max}} x_t^2 E_l(x_t)}{\sum^{x_{\max}} E_l(x_t)}.$$

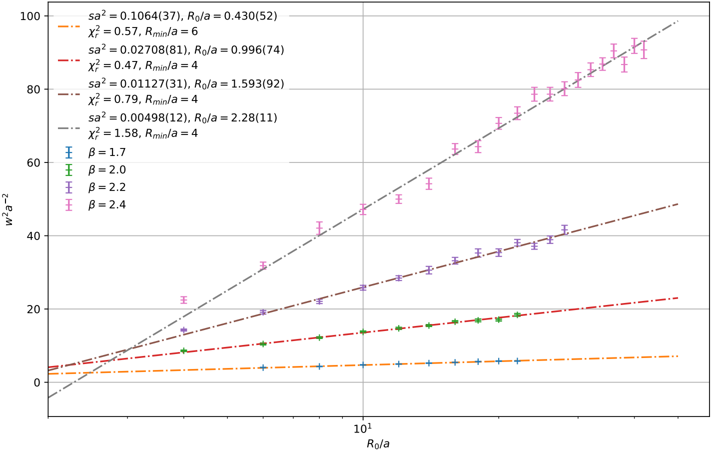
For $\beta = 1.7$ and $R = 16a$:



The Squared width

Fit of the prediction of gaussian EST

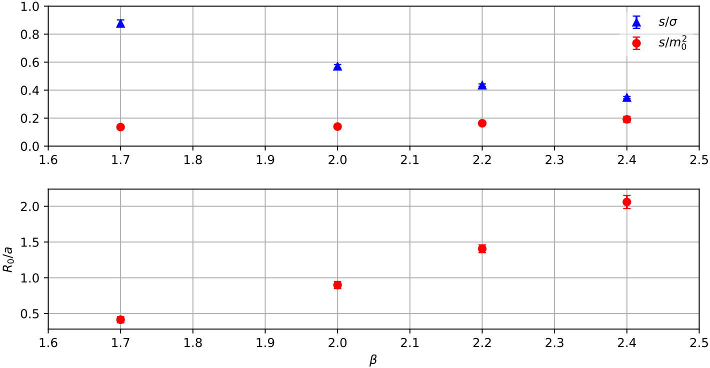
$$w^2(R) = \frac{1}{2\pi s} \log R/R_c$$



The Squared width

Value and scaling of the parameters fitted with the logarithmic EST prediction

$$w^2(R) = \frac{1}{2\pi s} \log R/R_c$$

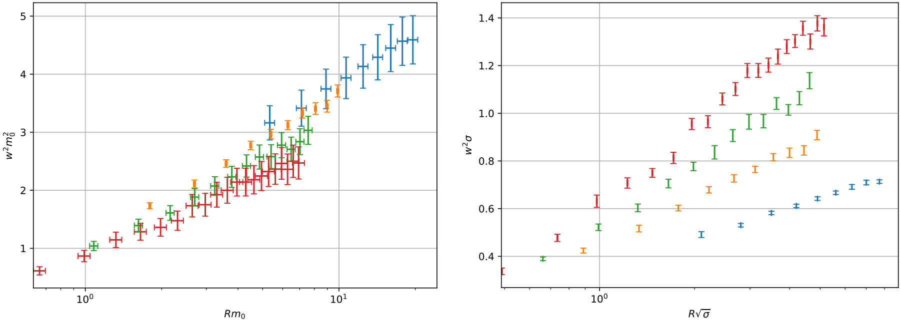


The Squared width

Scaling of the squared width

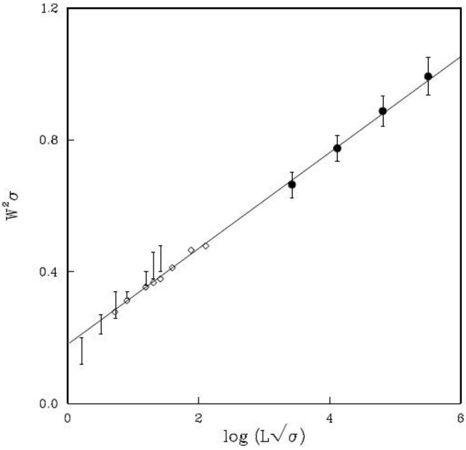
We are led to conclude that

$$w^2 = \frac{A}{m_0^2} \log R/R_c$$



The Squared width

The Ising case at $\beta = 0.7516$ and $\beta = 0.7560$ (σ roughly doubles)



The transverse profile of the flux tube

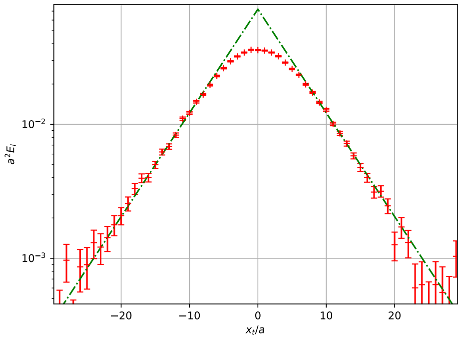
Predictions of the dual superconductor model

No range of the data could be fitted with a gaussian.

$$E_l(x_t) = \Phi m_v^2 e^{-m_v |x_t|}$$

should predict the profile outside of the vortex core: for $x_t > x_{\text{min}}$...

Example of the profile at $R = 20a$, $\beta = 2.2$



Performing the fits for all values of R at each β,

β	m_v/m_0
1.7	0.941(15)
2.0	0.985(27)
2.2	1.02(5)
2.4	1.03(8)

To within the errors m_v is independent of R and

$$m_v = m_0$$

Conclusions

- As a general ansatz,

$$w^2 = \frac{A}{c^2(\beta)\sigma} \log R/R_c$$

with two distinct cases

The 3D U(1) LGT

$$c^2(\beta)\sigma = m_0^2$$

Other LGT's

$$c^2(\beta)\sigma = 2\pi\sigma A$$

- The thickness of the flux tube is the sum of two contributions:
  - the tail contribution, constant with quark separation (and $\sim \mu$).
  - the core contribution, that grows with quark separation and is bell shaped.

Future steps:

- Include the effects of the rigidity term in w^2.
- Work at finite T.

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