

# *Flux tubes in QCD*

## *with (2+1) HISQ fermions*

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*in collaboration with:* Paolo Cea, Francesca Cuteri, Alessandro Papa

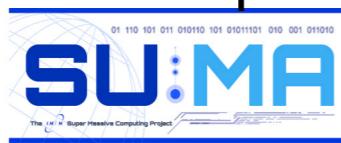


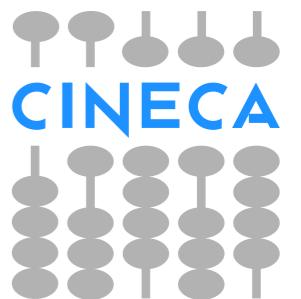
*Lattice 2016*  
*July 24-30, 2016, University of Southampton*

# Outline

- **Introduction**
- **Lattice setup and numerical simulations**
- **Results**
- **Conclusions**

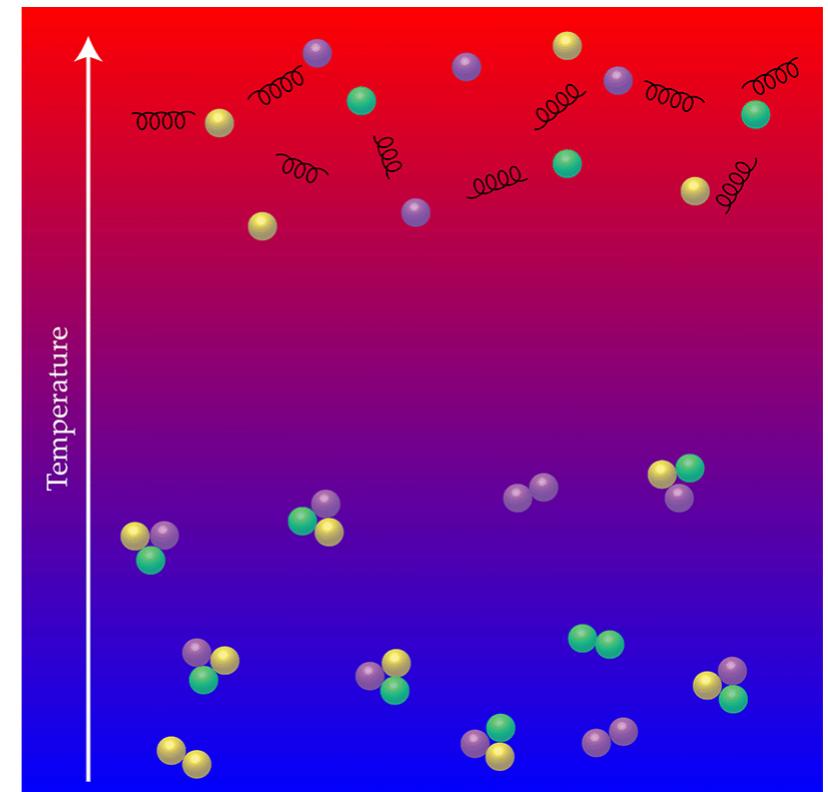
## Acknowledgements

- ★ This work has been partially supported by the INFN SUMA (SUper MAssive Computing) Project The logo for the INFN SUMA project features the word "SUMA" in large blue letters with a colon between the "S" and "U". Below the letters is a stylized globe icon composed of binary digits (0s and 1s). At the bottom, it says "The INFN Super Massive Computing Project".
- ★ Simulations have been performed at CINECA (on IBM BlueGene/Q “FERMI”, IBM NeXtScale “GALILEO”, Lenovo NeXtScale “MARCONI”, under CINECA-INFN agreement).
- ★ This work was in part based on the MILC collaboration’s public lattice gauge theory code. See <http://physics.utah.edu/~detar/milc.html>



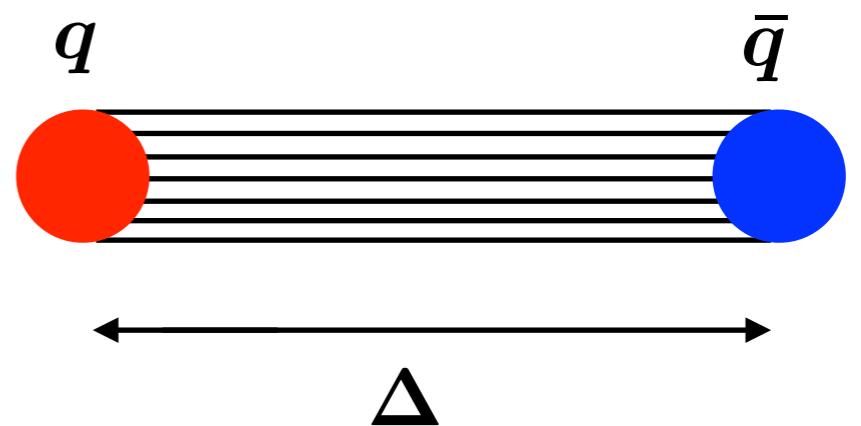
# Introduction

- Reaching a detailed understanding of color confinement is one of the central goals of nonperturbative studies of QCD.



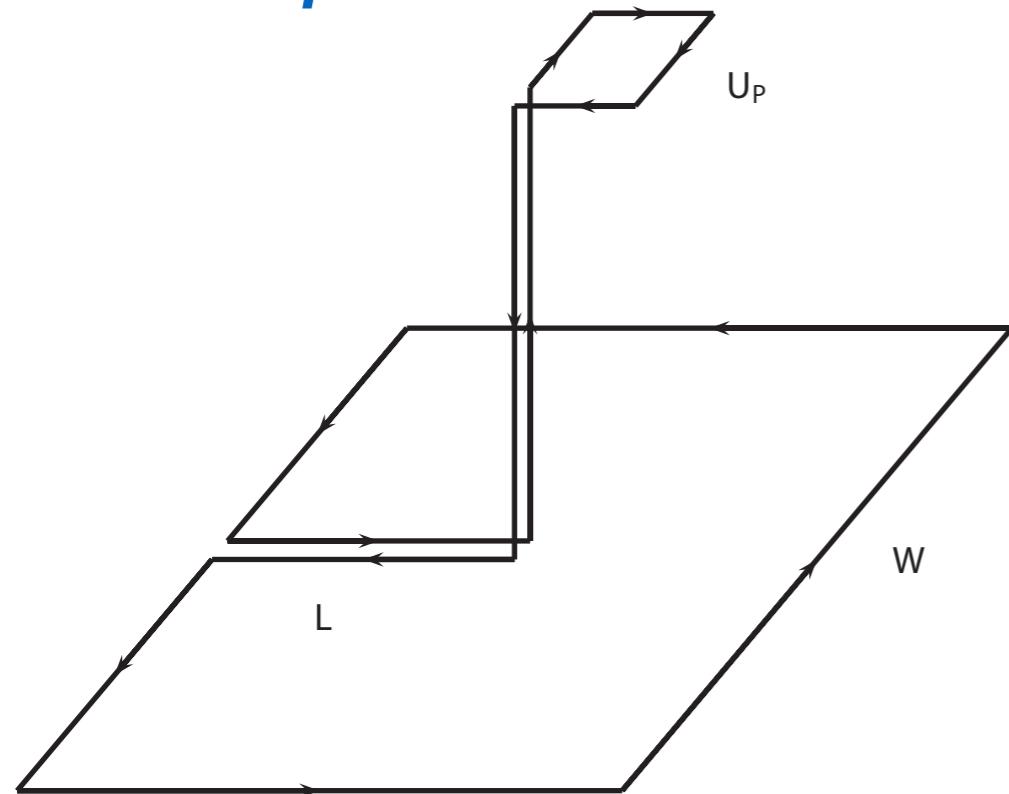
[Credit: APS/[Joan Tycko](#)]

- It is known since long that, in lattice numerical simulations, tubelike structures emerge by analyzing the chromoelectric fields between static quarks. Such tubelike structures naturally lead to a linear potential between static color charges and, consequently, to a direct numerical evidence of color confinement.

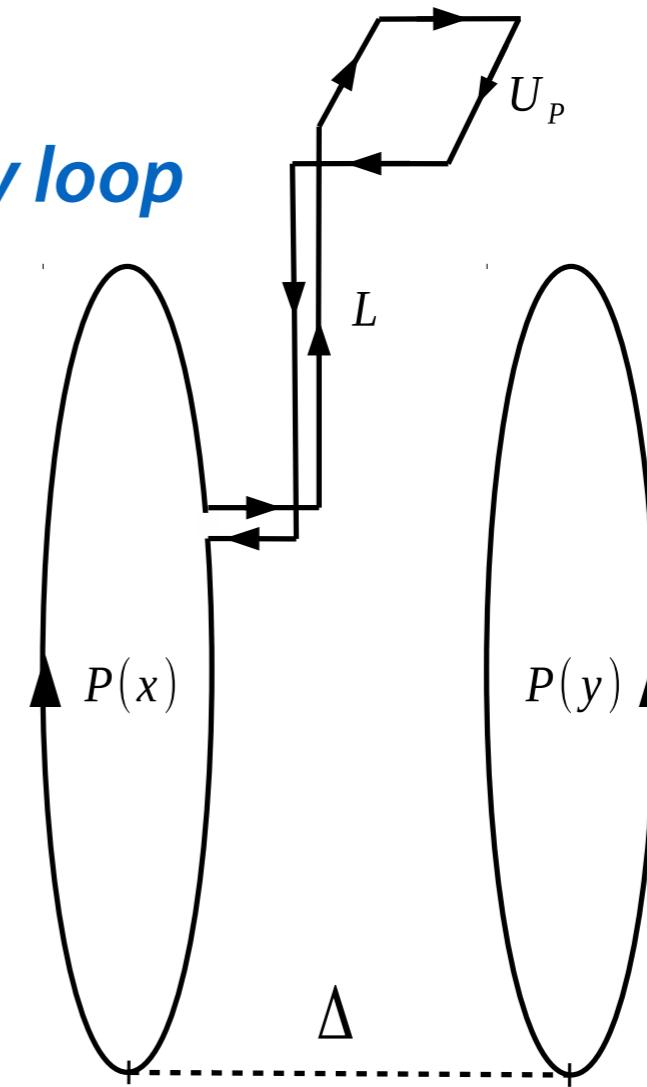


# Measuring the chromoelectric field on the lattice

*Wilson loop*



*Polyakov loop*



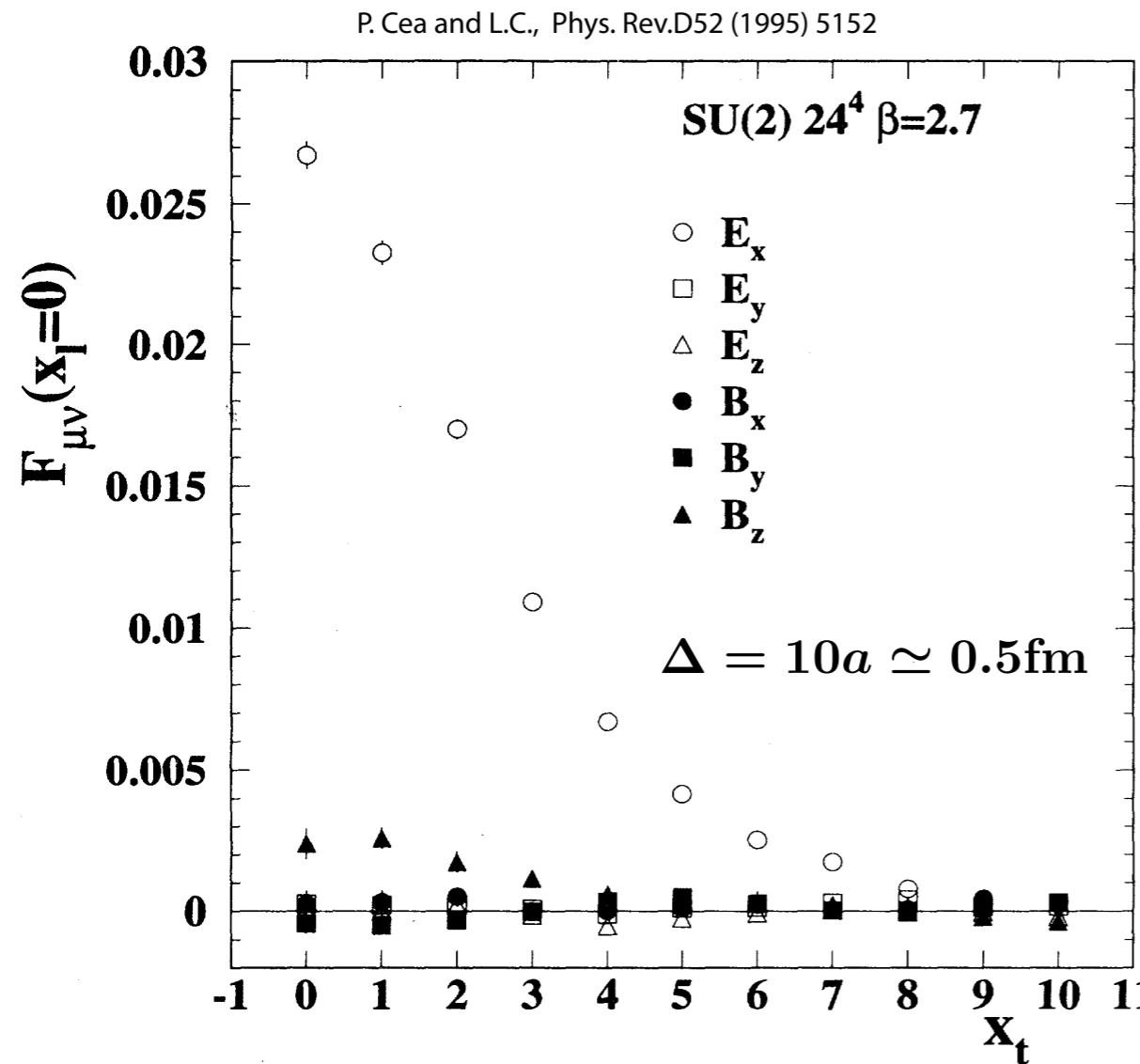
$$\rho_W^{\text{conn}} = \frac{\langle \text{tr}(W L U_P L^\dagger) \rangle}{\langle \text{tr}(W) \rangle} - \frac{1}{N} \frac{\langle \text{tr}(U_P) \text{tr}(W) \rangle}{\langle \text{tr}(W) \rangle}$$

$$\rho_P^{\text{conn}} =$$

$$- \frac{\langle \text{tr}(P(x) L U_P L^\dagger) \text{tr} P^\dagger(y) \rangle}{\langle \text{tr}(P(x)) \text{tr}(P^\dagger(y)) \rangle} \\ \frac{1}{N} \frac{\langle \text{tr}(P(x)) \text{tr}(P^\dagger(y)) \text{tr}(U_P) \rangle}{\langle \text{tr}(P(x)) \text{tr}(P^\dagger(y)) \rangle}$$

$$F_{\mu\nu}(x) = \sqrt{\frac{1}{g^2}} \rho_W^{\text{conn}}(x)$$

# Chromoelectric longitudinal field



$$F_{\mu\nu}(x) = \sqrt{\frac{1}{g^2}} \rho_W^{\text{conn}}(x)$$

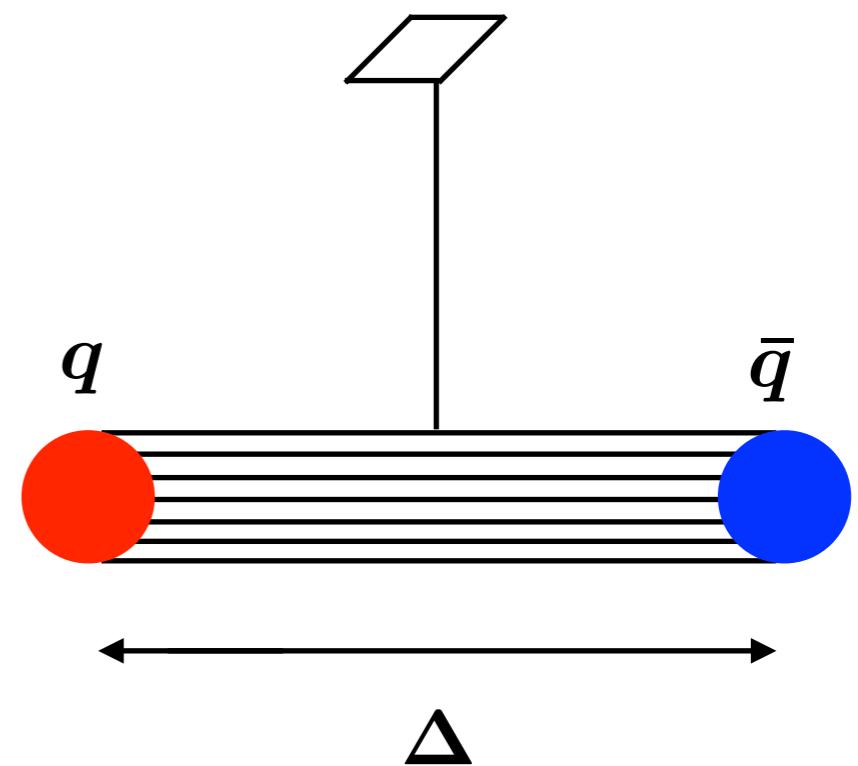


FIG. 2. The field strength tensor  $F_{\mu\nu}(x_l, x_t)$  evaluated at  $x_l = 0$  on a  $24^4$  lattice at  $\beta = 2.7$ , using Wilson loops of size  $10 \times 10$  in Eq. (2.1).

The flux tube is almost completely formed by the longitudinal chromoelectric field, which is constant along the flux tube (not too close to the sources) and decreases rapidly in the transverse direction.

# Dual superconductivity picture of the QCD vacuum

- The vacuum of QCD could be modeled as a coherent state of color magnetic monopoles: '[tHooft \(1976\)](#), [Mandelstam\(1974\)](#) —> DUAL SUPERCONDUCTOR
- In the dual superconductor model of the QCD vacuum the condensation of color magnetic monopoles is analogous to the formation of Cooper pairs in the BCS theory of superconductivity.
- There are several numerical evidences for the color magnetic condensation in QCD vacuum [ [Shiba-Suzuki, 1994](#); [Arasaki-Ejiri-Kitahara-Matsubara-Suzuki, 1996](#); [Cea-Cosmai, 2000](#); [DiGiacomo-Lucini-Montesi-Paffuti, 1999](#); [19991999fa](#),[DiGiacomo:1999fb](#),[Carmona-D'Elia-DiGiacomo-Lucini-Paffuti, 2001](#); [Cea-Cosmai, 2004](#); [D'Alessandro-D'Elia-Shuryak, 2010](#); [Kato-Kondo-Shibata \(2014\)](#) ]
- However, it should be recognised ('[tHooft, 2004](#)) that *the color magnetic monopole condensation in the confinement mode of QCD could be a consequence rather than the origin of the mechanism of color confinement, that could actually arise from additional dynamical causes.*
- Notwithstanding, the dual superconductivity picture of the QCD vacuum remains at least a very useful phenomenological frame to interpret the vacuum dynamics.

# Dual superconductivity and the color flux tube

$$E_l(x_t) = \frac{\phi}{2\pi} \frac{\mu^2}{\alpha} \frac{K_0((\mu^2 x_t^2 + \alpha^2)^{1/2})}{K_1(\alpha)}$$

J.R. Clem, J. Low Temp. Phys. 18 (1975) 427  
 P. Cea, L.C., A.Papa, Phys. Rev. D86 (2012)054501

$$\mu = \frac{1}{\lambda}, \quad \frac{1}{\alpha} = \frac{\lambda}{\xi_v}$$

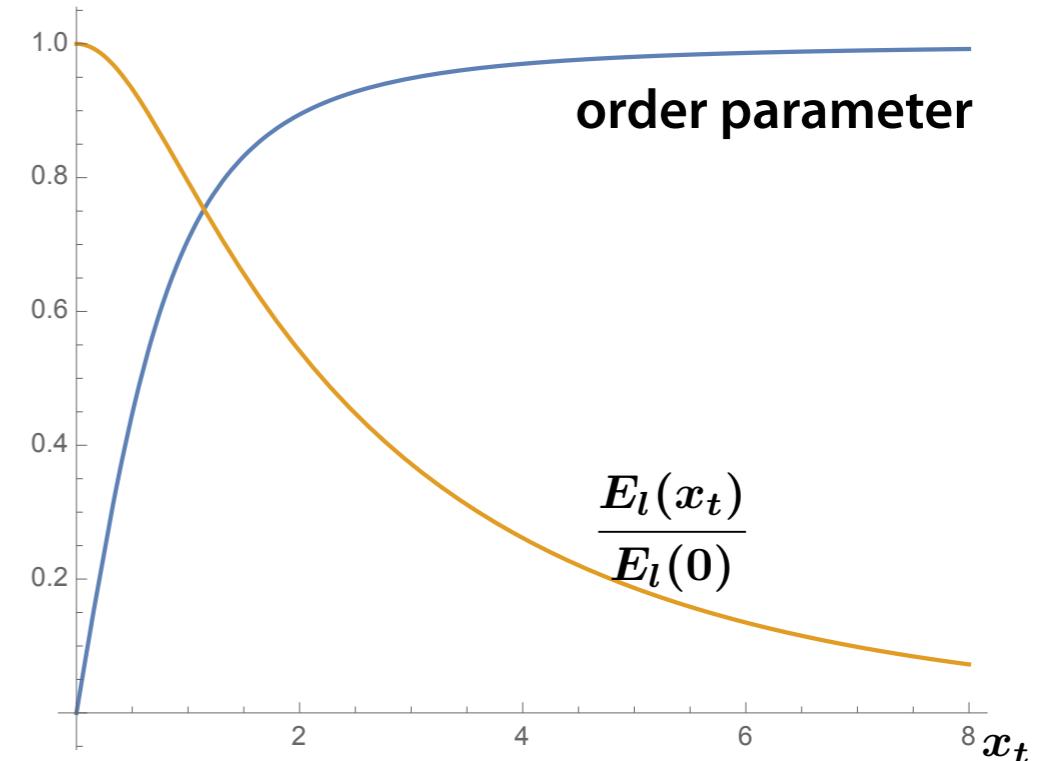
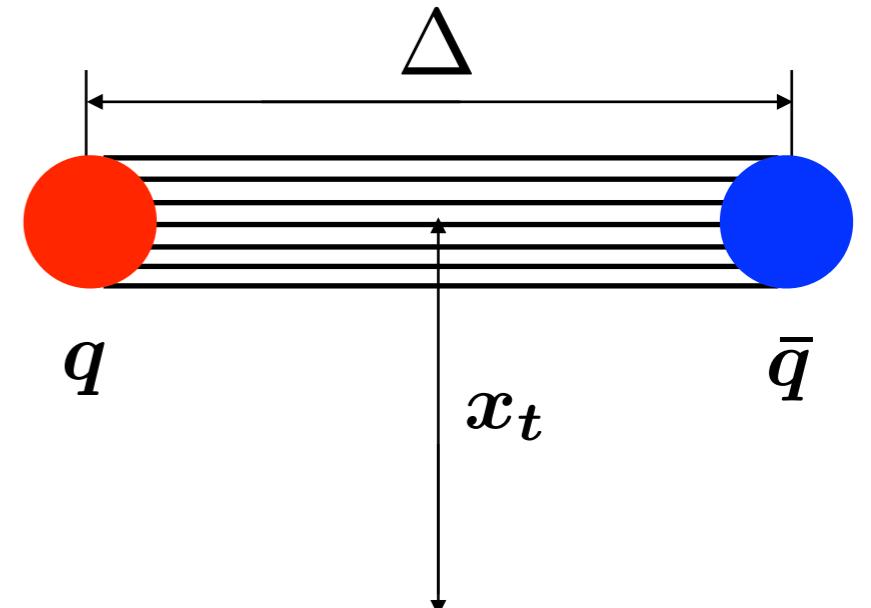
$\lambda$  London penetration length

$\xi_v$  variational core radius

$\xi$  coherence length

$\kappa$  Ginzburg – Landau parameter

$$\kappa = \frac{\lambda}{\xi} = \frac{\sqrt{2}}{\alpha} [1 - K_0^2(\alpha)/K_1^2(\alpha)]^{1/2}$$



# New preliminary results at T=0

- SU(3) pure gauge source distances up to 1.14 fm
- QCD (2+1) flavours HISQ fermions  
0.76 fm, 1.14 fm

# SU(3) pure gauge - LATTICE SETUP

- 32<sup>4</sup> lattice - MILC code (suitably modified to measure the chromoelectric field)
- distance between sources up to 12 in lattice spacing (1.14 fm in physical units)
- smoothing of the gauge configurations: several APE smearings for spatial links, one HYP smearing for temporal links
- scale setting: (Edwards, Heller, Klassen, Nucl. Phys. B517 (1998) 377)

$$(a \sqrt{\sigma})(g) = f_{\text{SU}(3)}(g^2) \{ 1 + 0.2731 \hat{a}^2(g) - 0.01545 \hat{a}^4(g) + 0.01975 \hat{a}^6(g) \} / 0.01364$$

$$\hat{a}(g) = \frac{f_{\text{SU}(3)}(g^2)}{f_{\text{SU}(3)}(g^2(\beta = 6))}$$
$$\beta = \frac{6}{g^2}, \quad 5.6 \leq \beta \leq 6.5$$

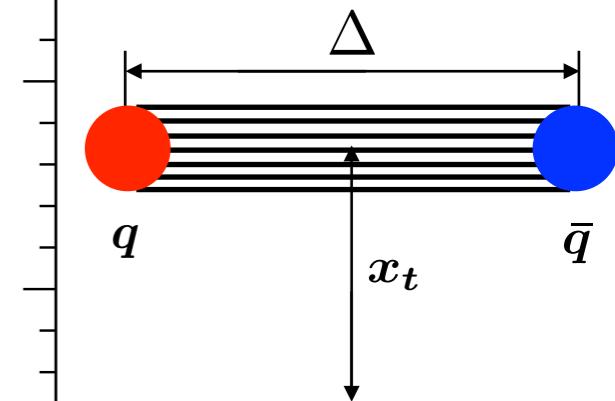
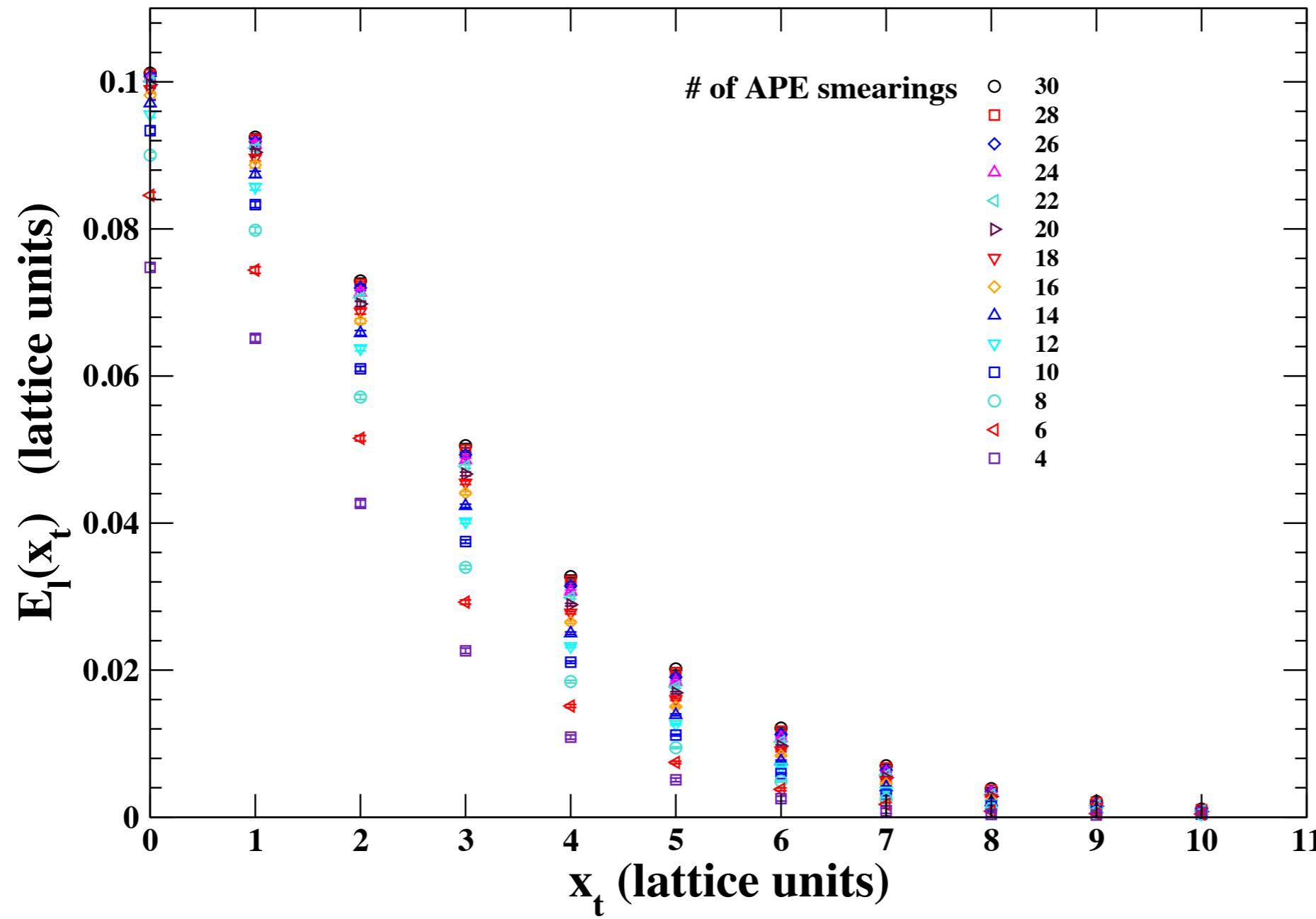
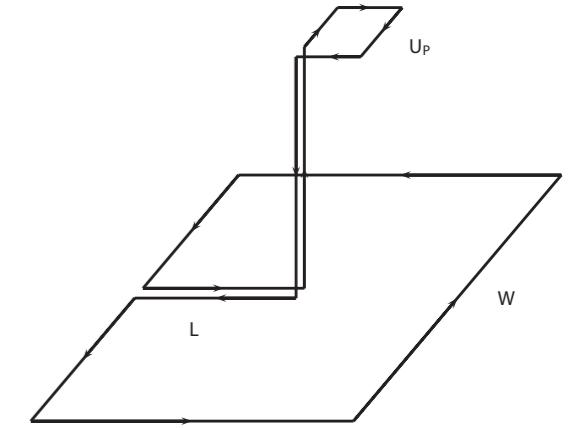
$$\sqrt{\sigma} = 420 \text{ MeV}$$

$$f_{\text{SU}(3)}(g^2) = (b_0 g^2)^{-b_1/2b_0^2} \exp\left(-\frac{1}{2b_0 g^2}\right) \quad b_0 = \frac{11}{(4\pi)^2}, \quad b_1 = \frac{102}{(4\pi)^4}$$

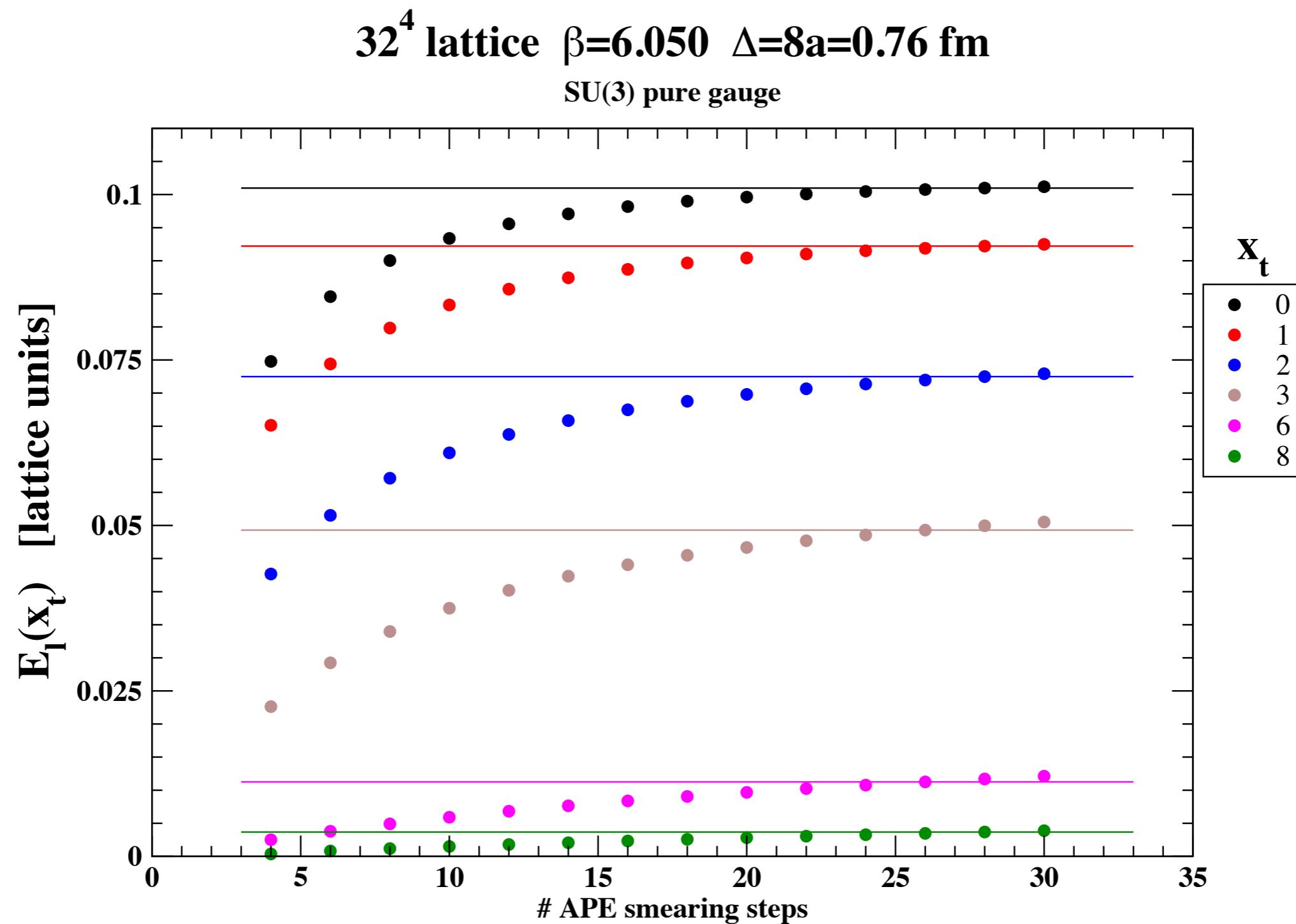
- # of measurements for each value of the gauge coupling: 500 to 2000.

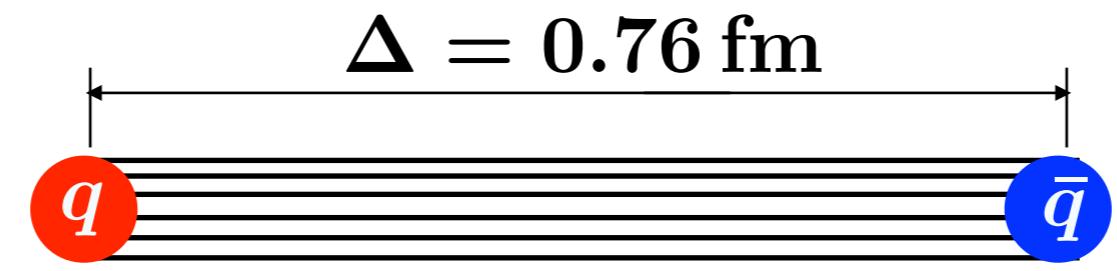
# smoothing of the gauge configurations

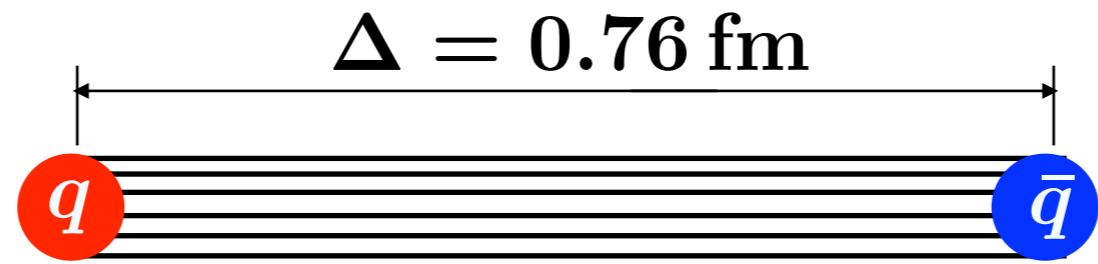
$32^4$   $\beta=6.050$   $\Delta=8a=0.76$  fm  
SU(3) pure gauge



# smoothing of the gauge configurations (*continued*)

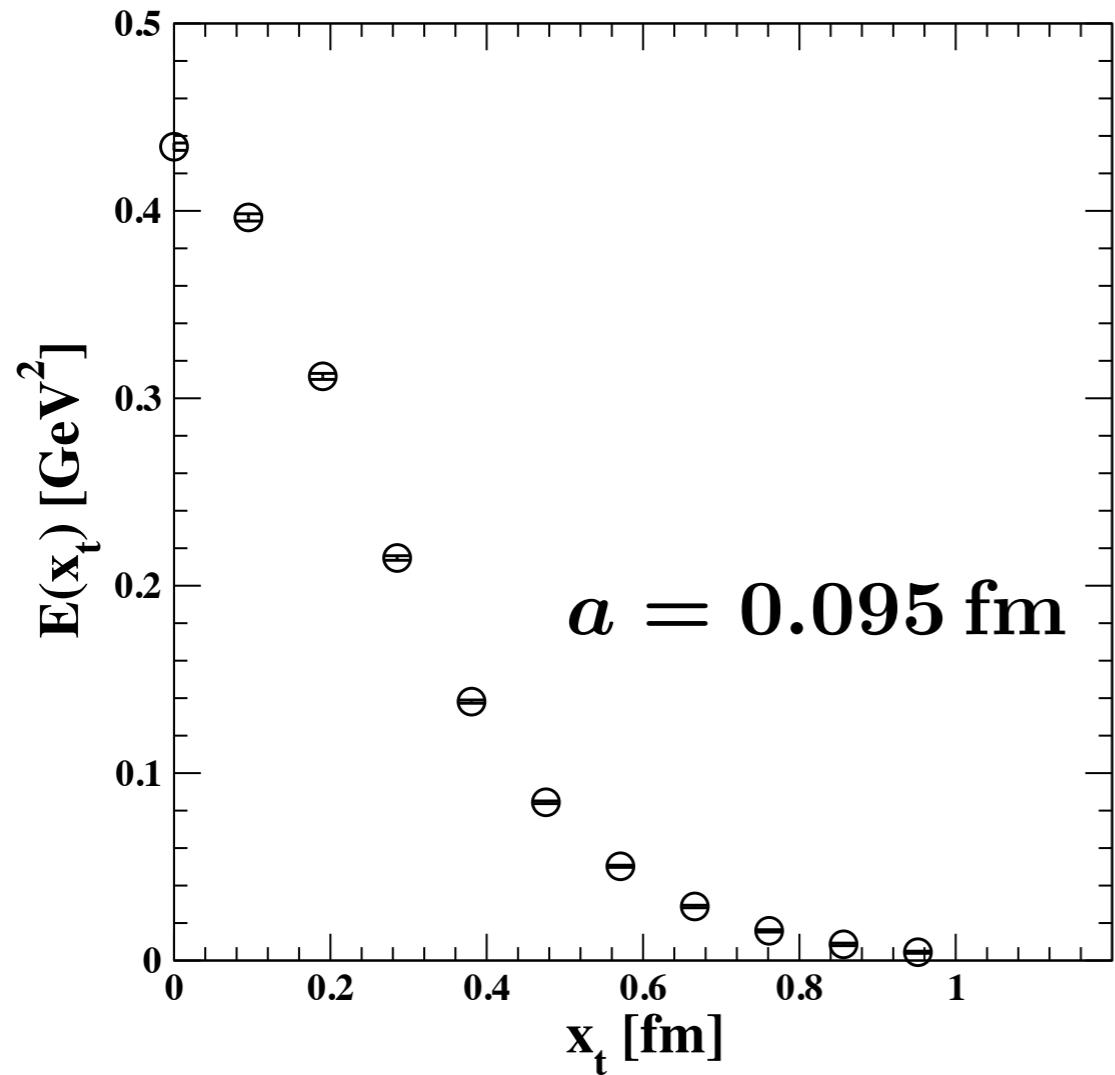


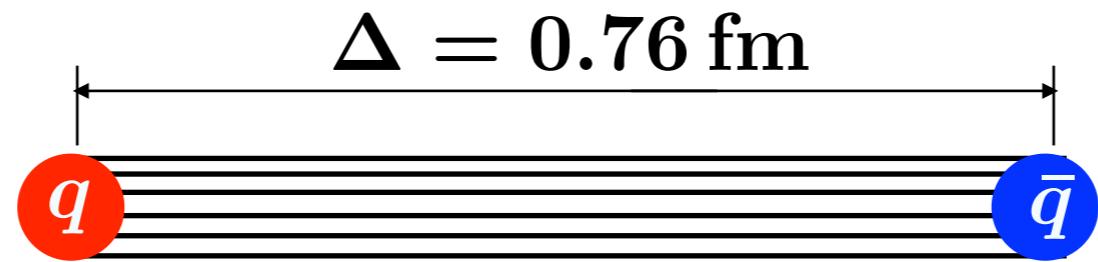




$\beta=6.050 \Delta=8a=0.76 \text{ fm}$

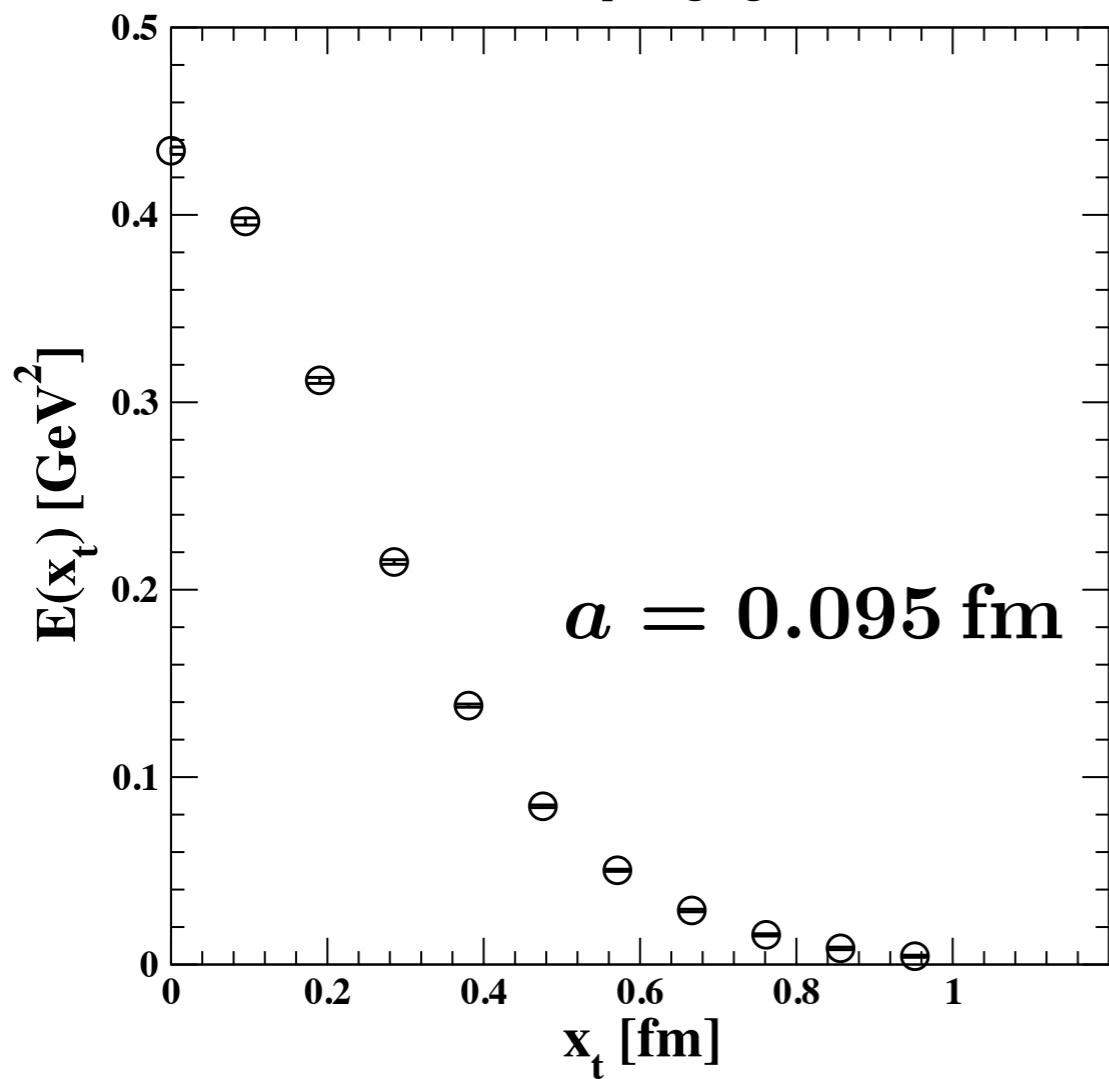
SU(3) pure gauge





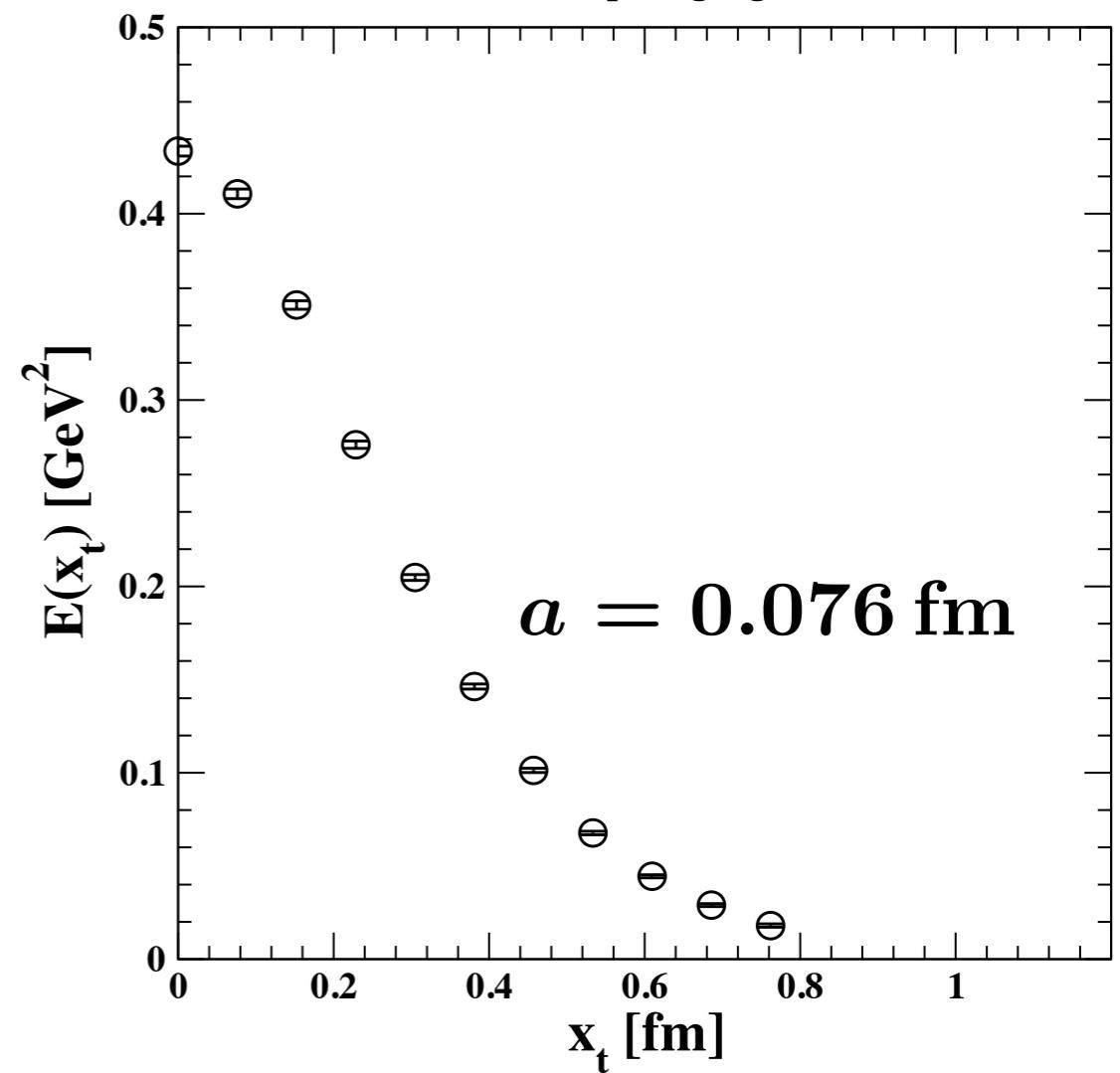
$\beta=6.050 \Delta=8a=0.76 \text{ fm}$

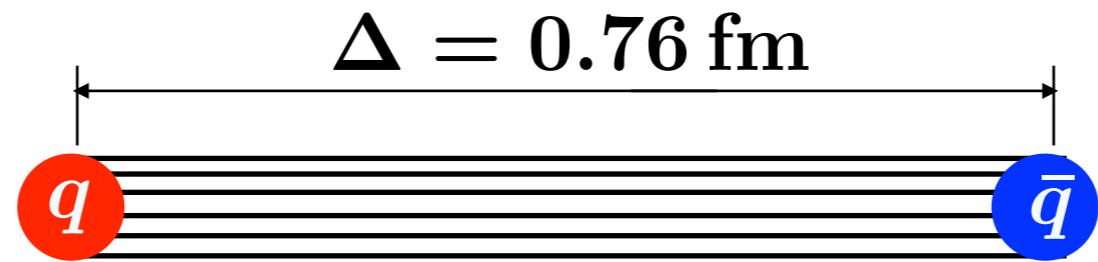
SU(3) pure gauge



$\beta=6.195 \Delta=10a=0.76 \text{ fm}$

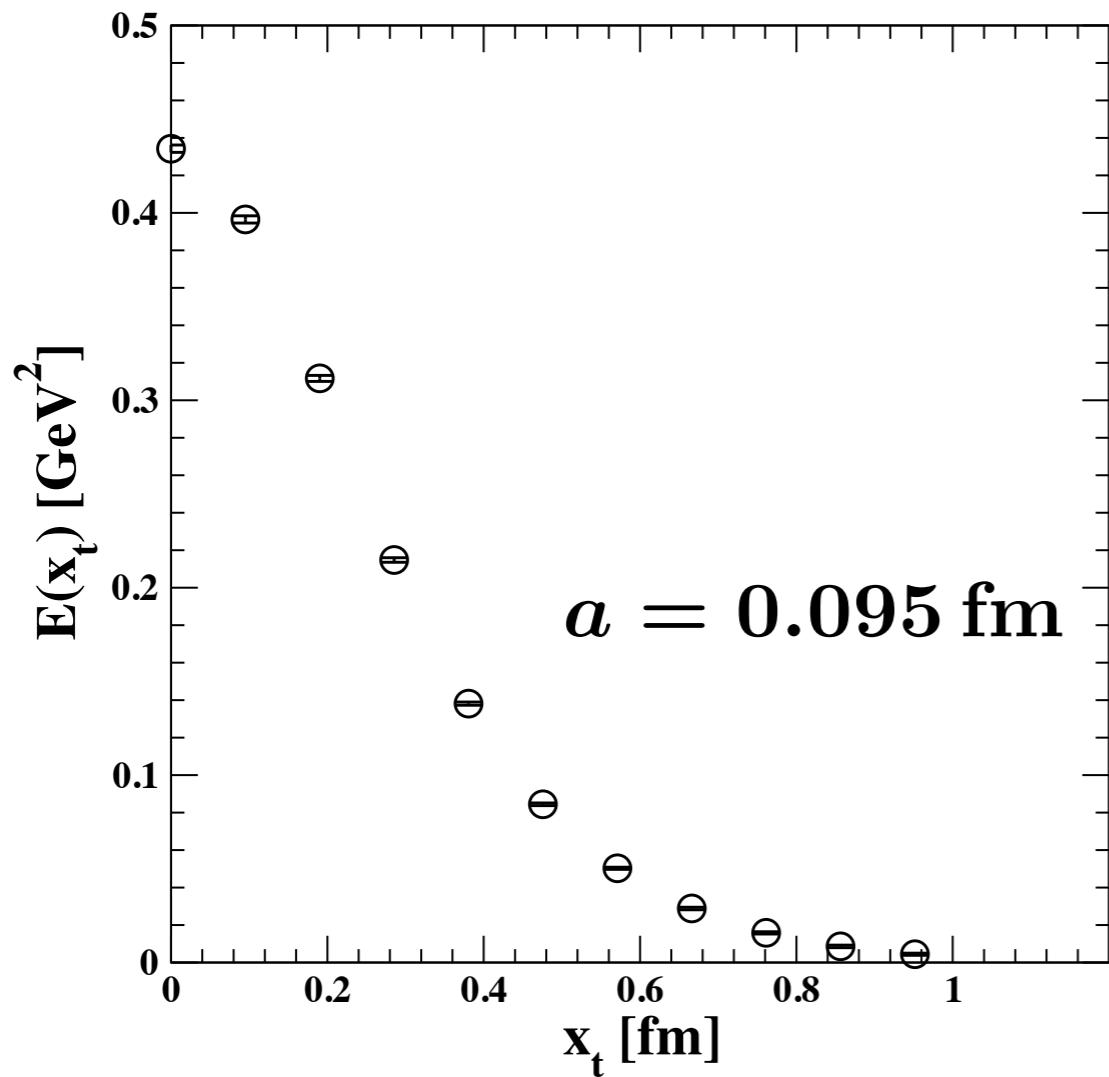
SU(3) pure gauge





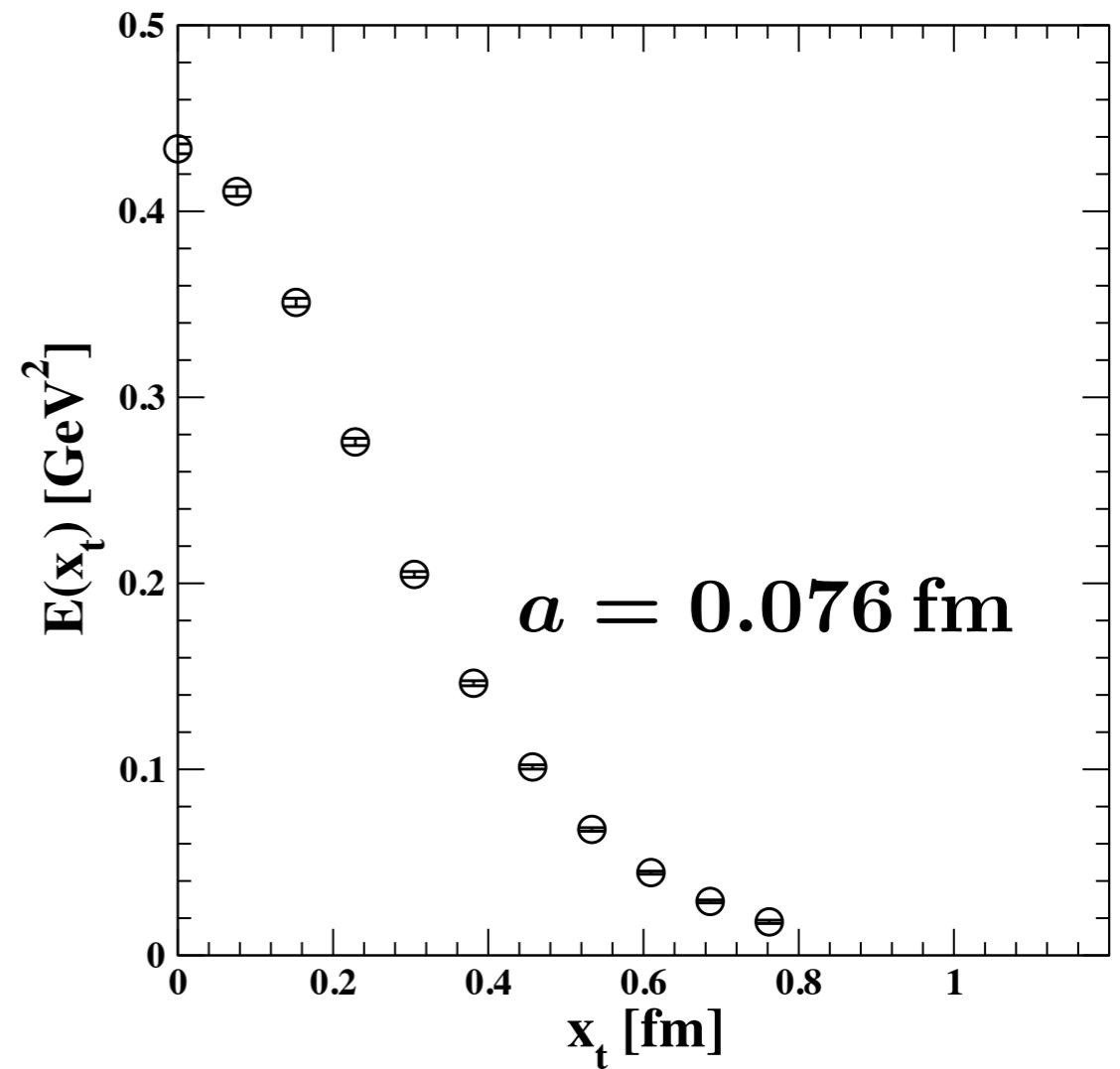
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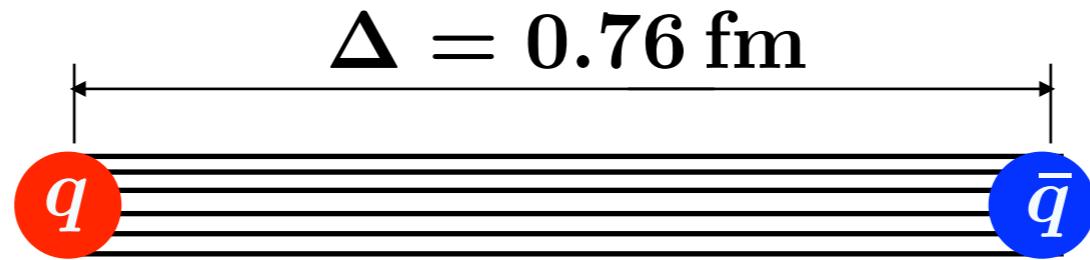
SU(3) pure gauge



$$E_l(x_t) = \frac{\phi}{2\pi} \frac{\mu^2}{\alpha} \frac{K_0((\mu^2 x_t^2 + \alpha^2)^{1/2})}{K_1(\alpha)}$$

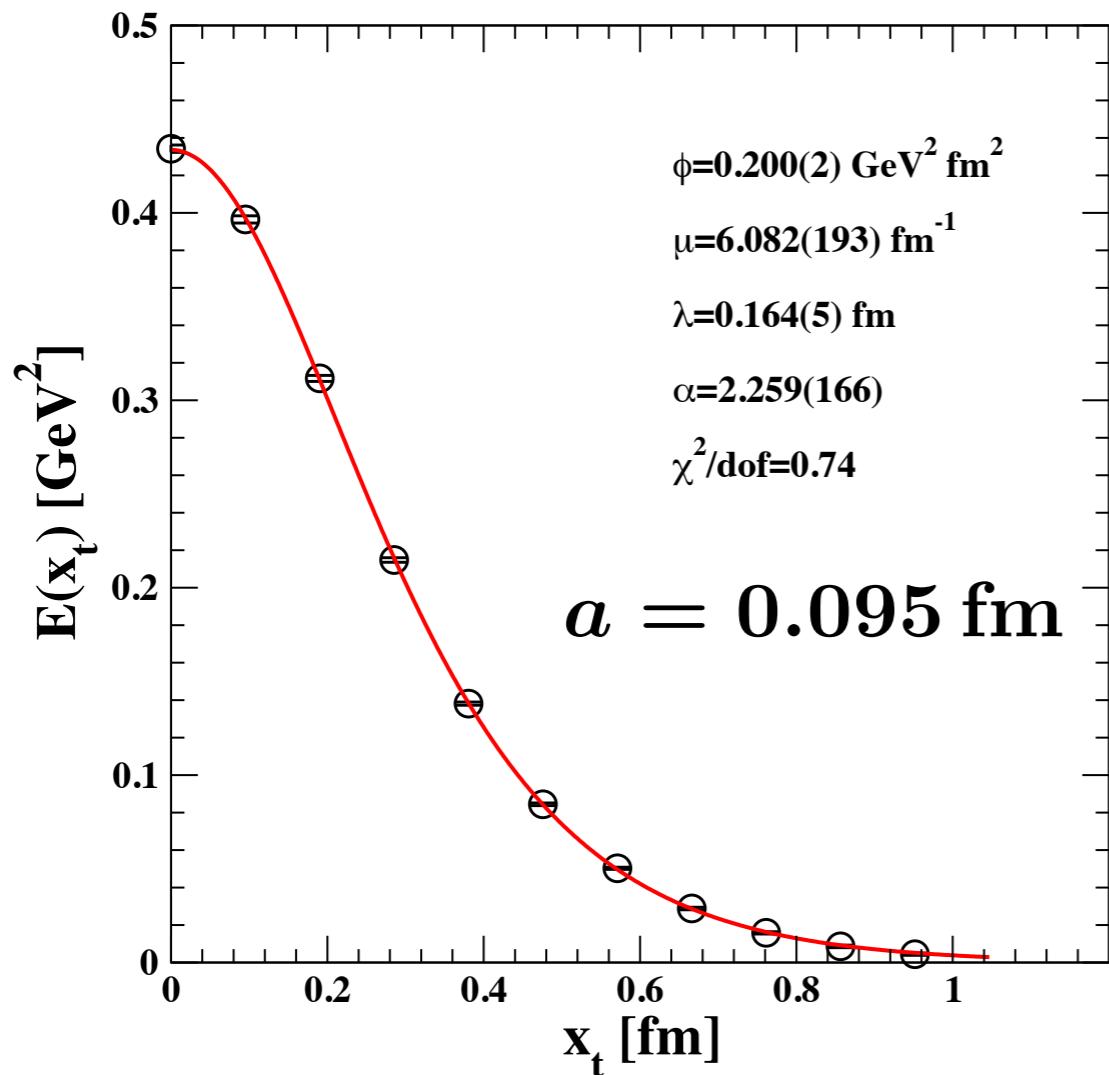
$$\kappa = \frac{\lambda}{\xi} = \frac{\sqrt{2}}{\alpha} \left[ 1 - K_0^2(\alpha)/K_1^2(\alpha) \right]^{1/2}$$

$$\mu = \frac{1}{\lambda}, \quad \frac{1}{\alpha} = \frac{\lambda}{\xi_v}$$



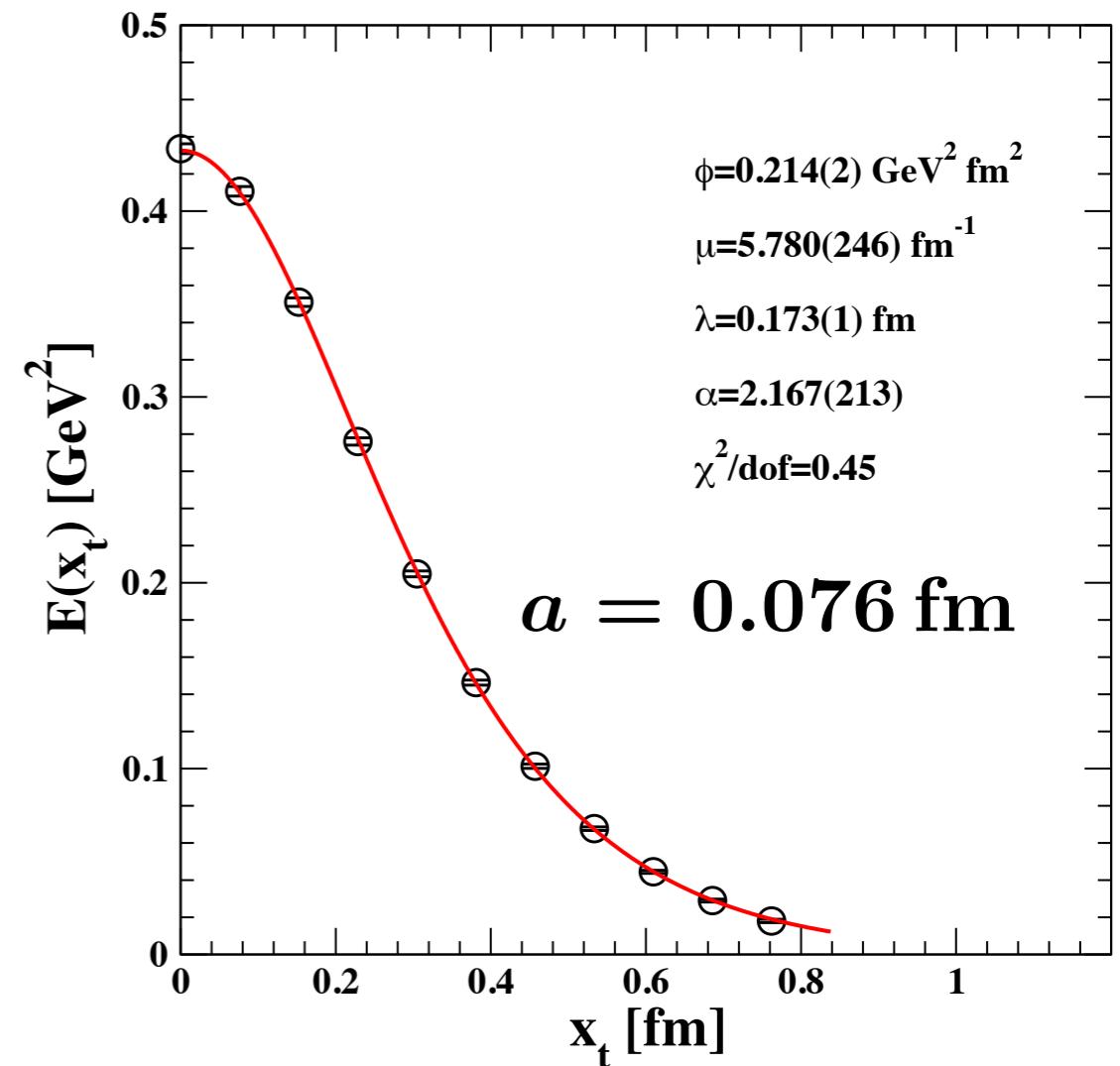
$\beta=6.050 \Delta=8a=0.76 \text{ fm}$

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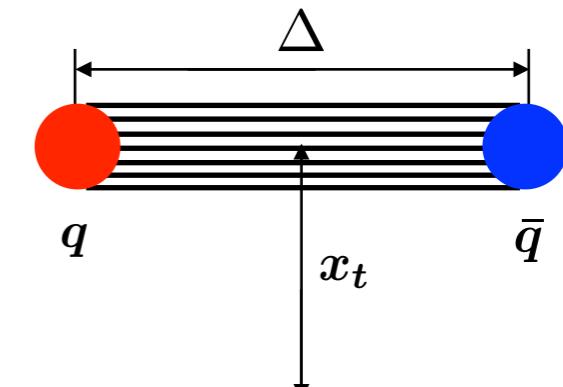
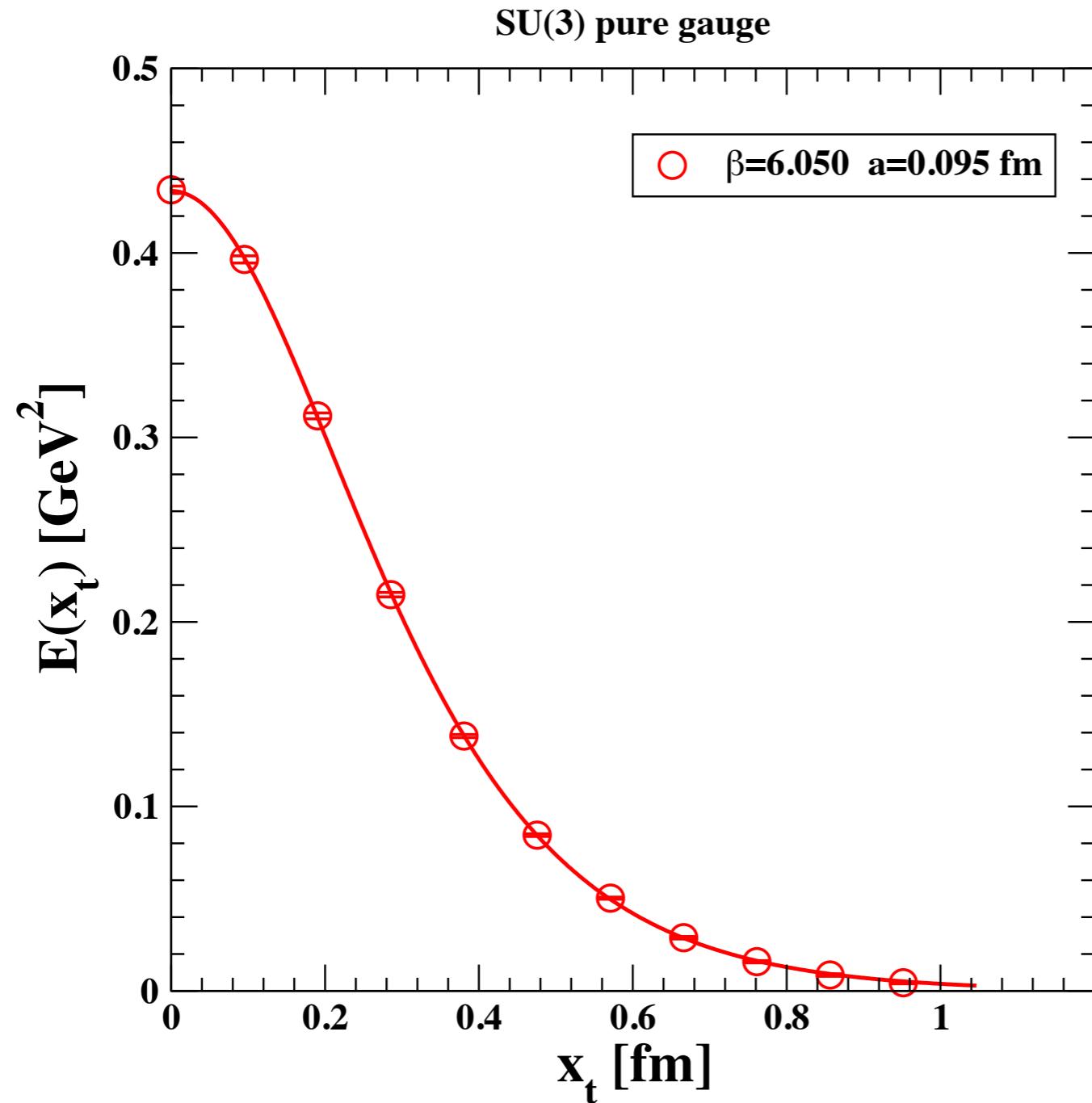
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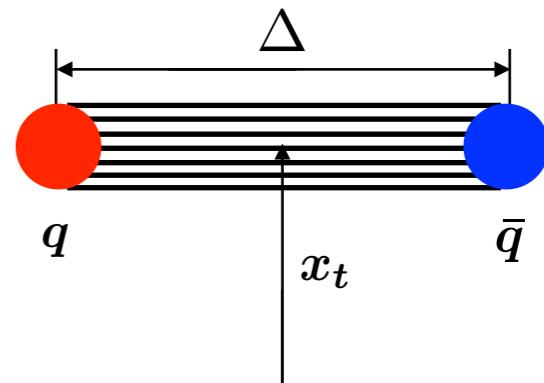
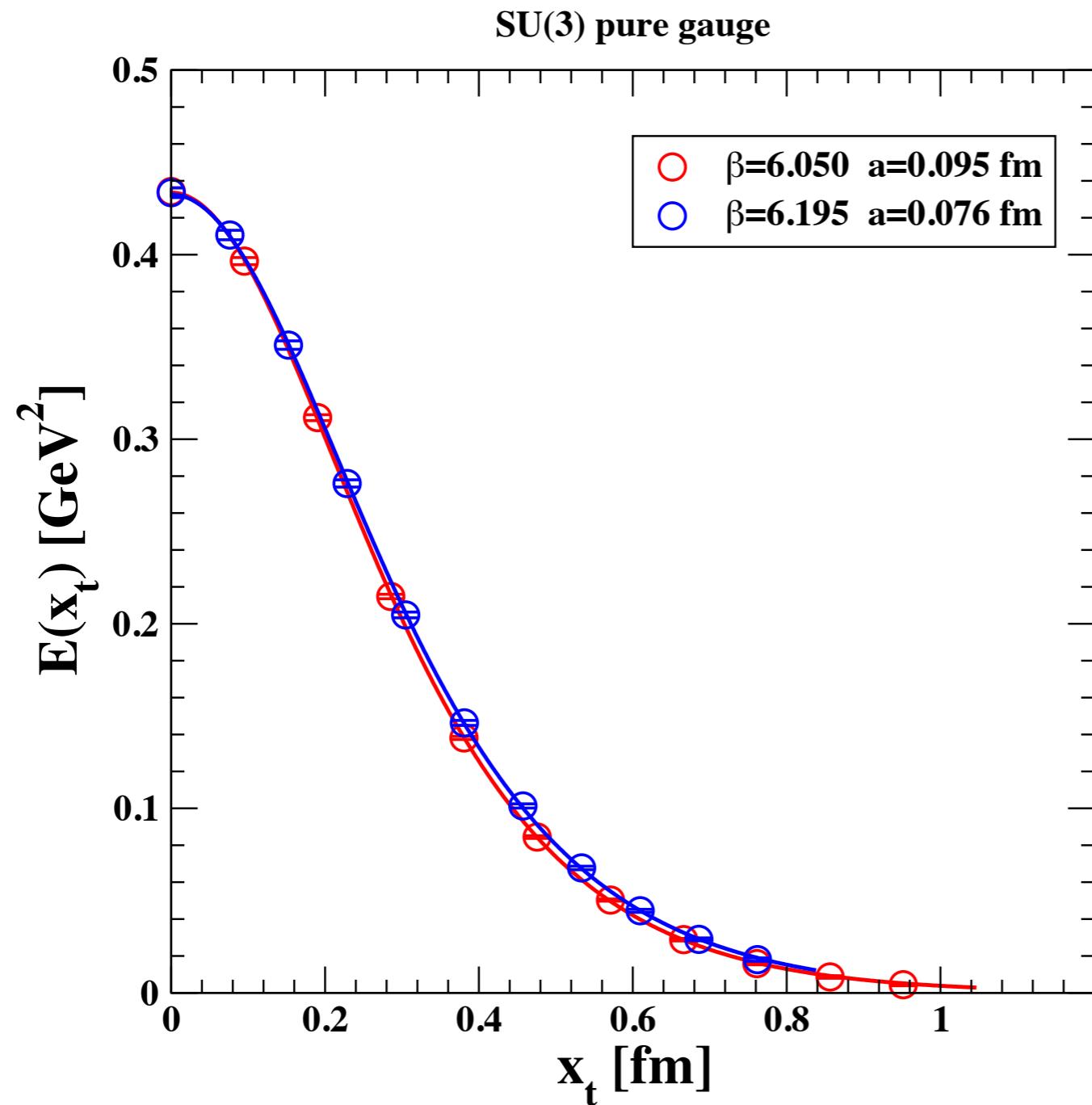


## scaling test at distance 0.76 fm between sources



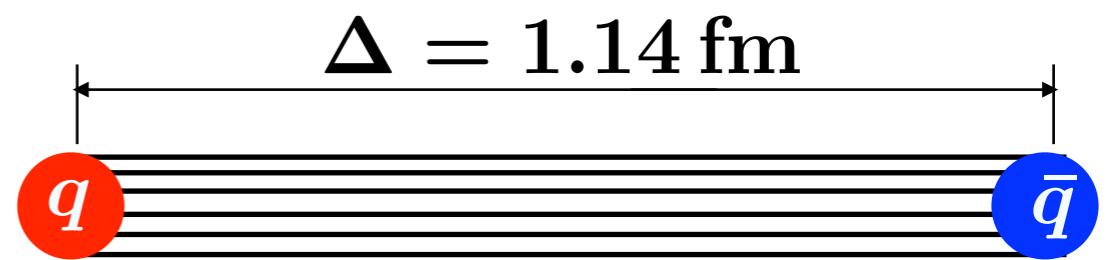
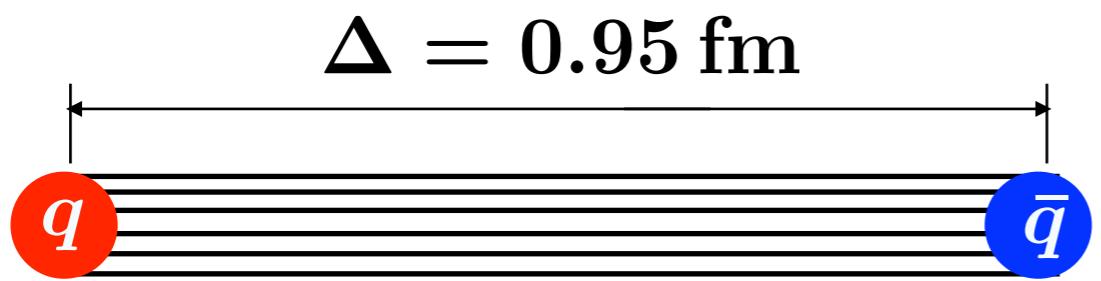
$$\beta = 6.050 \quad \Delta = 8a = 0.76 \text{ fm}$$

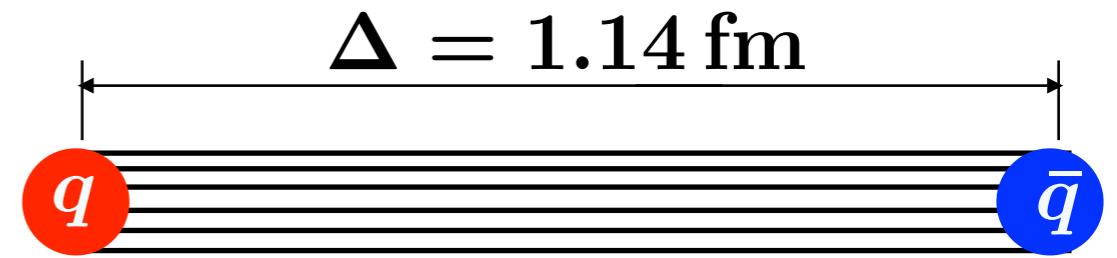
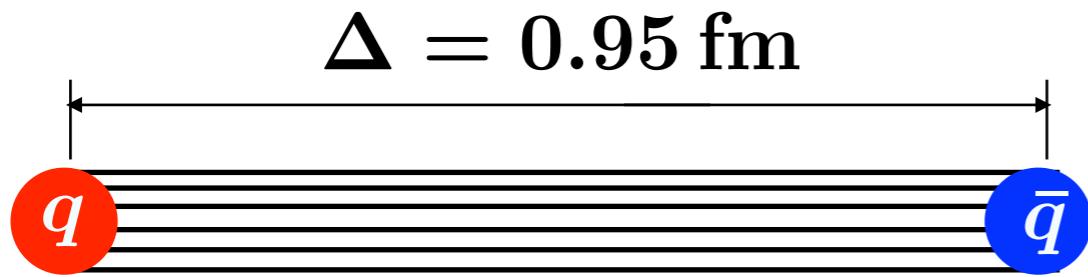
## scaling test at distance 0.76 fm between sources



$$\beta = 6.050 \quad \Delta = 8a = 0.76 \text{ fm}$$

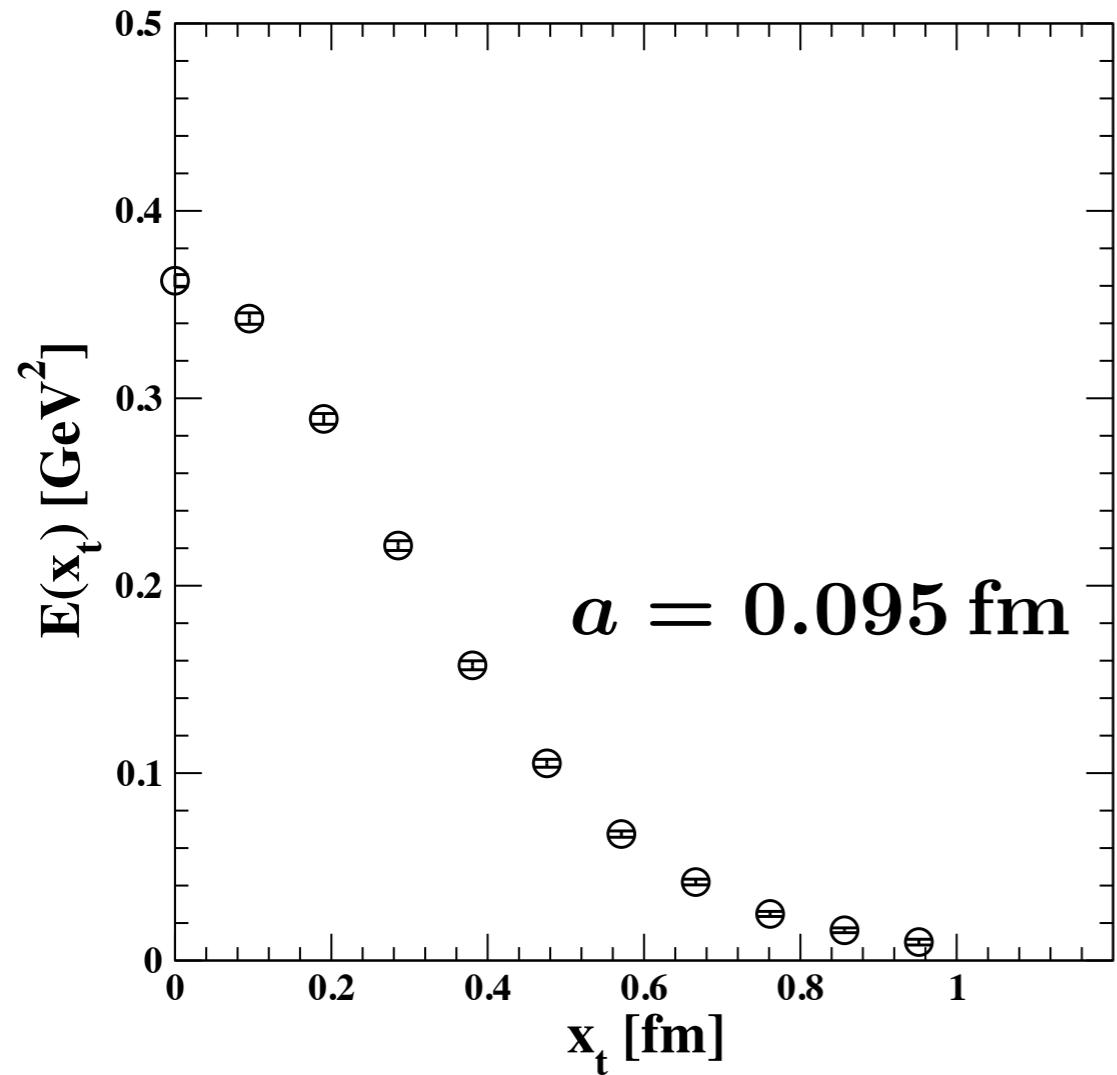
$$\beta = 6.195 \quad \Delta = 10a = 0.76 \text{ fm}$$

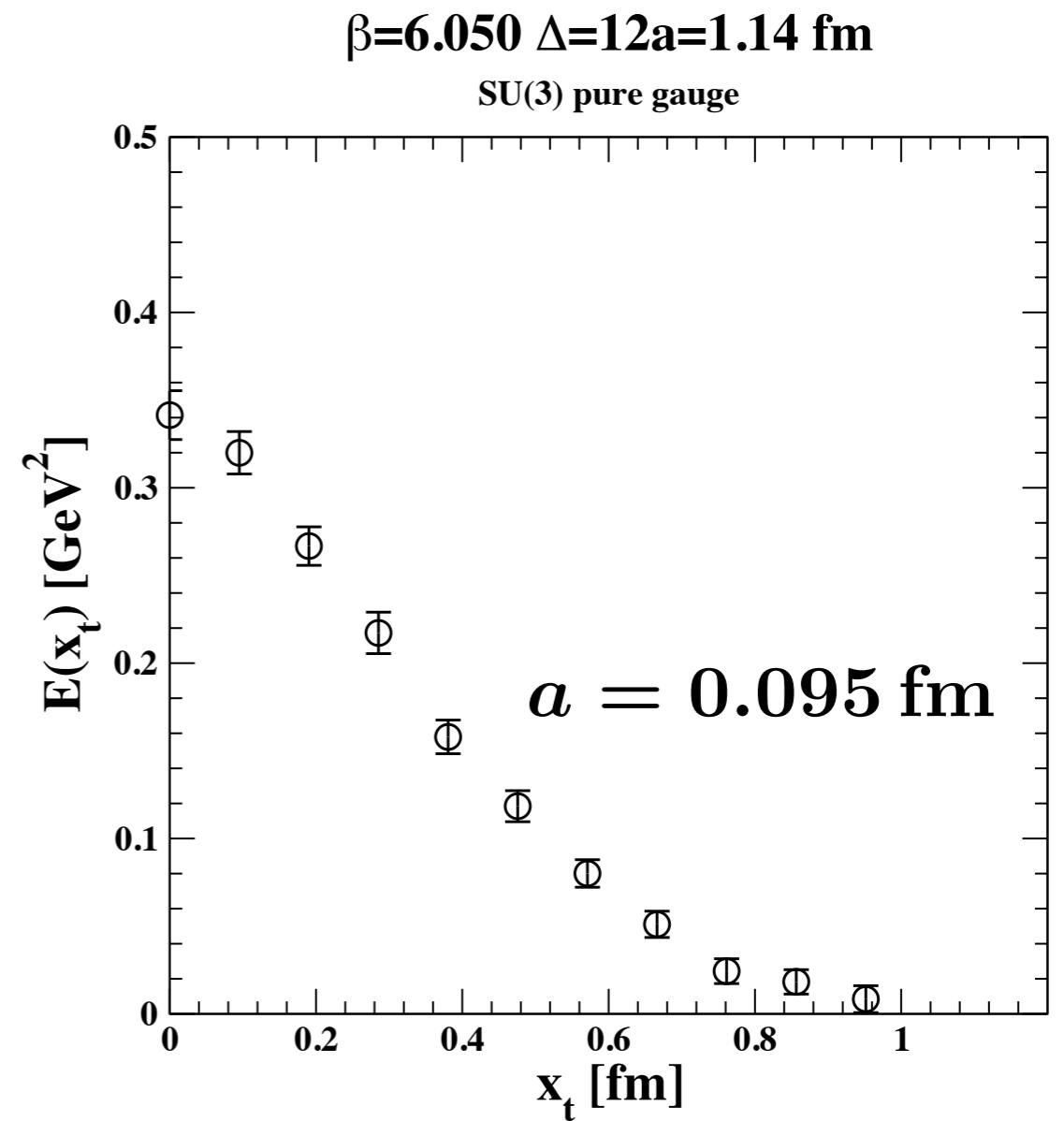
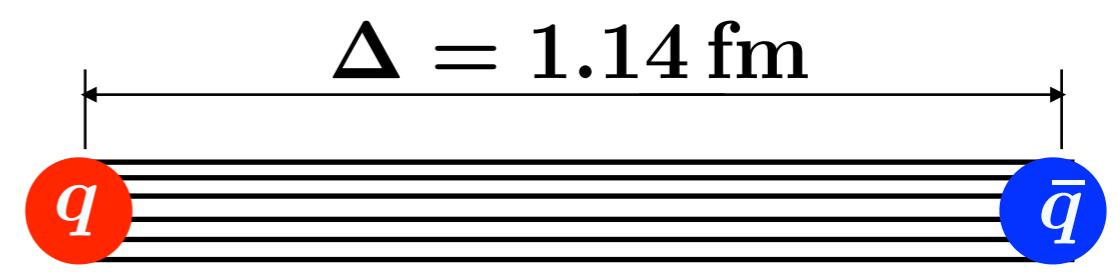
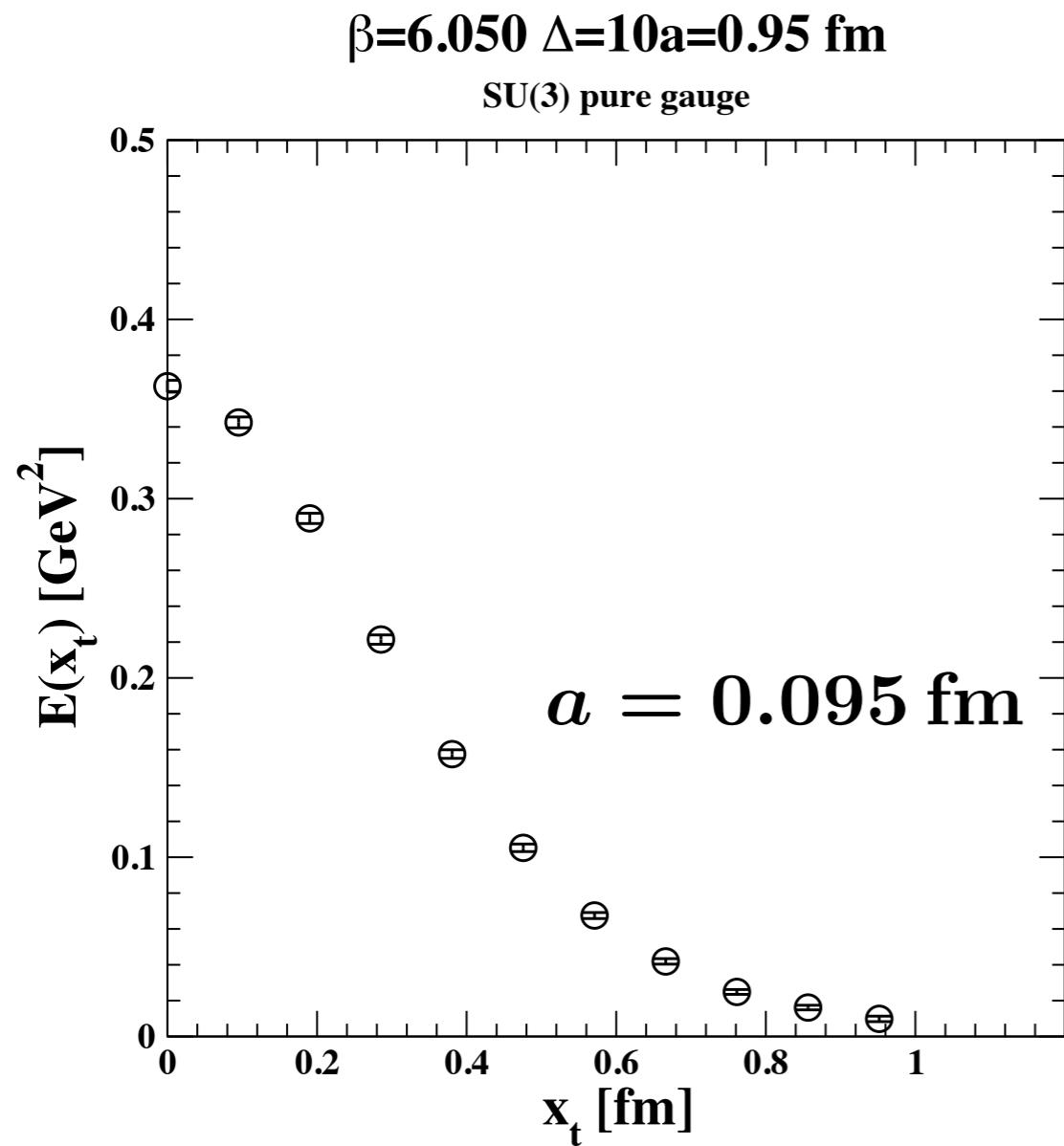
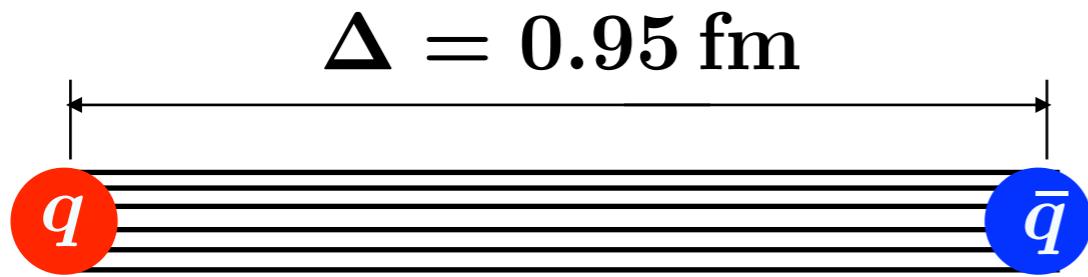


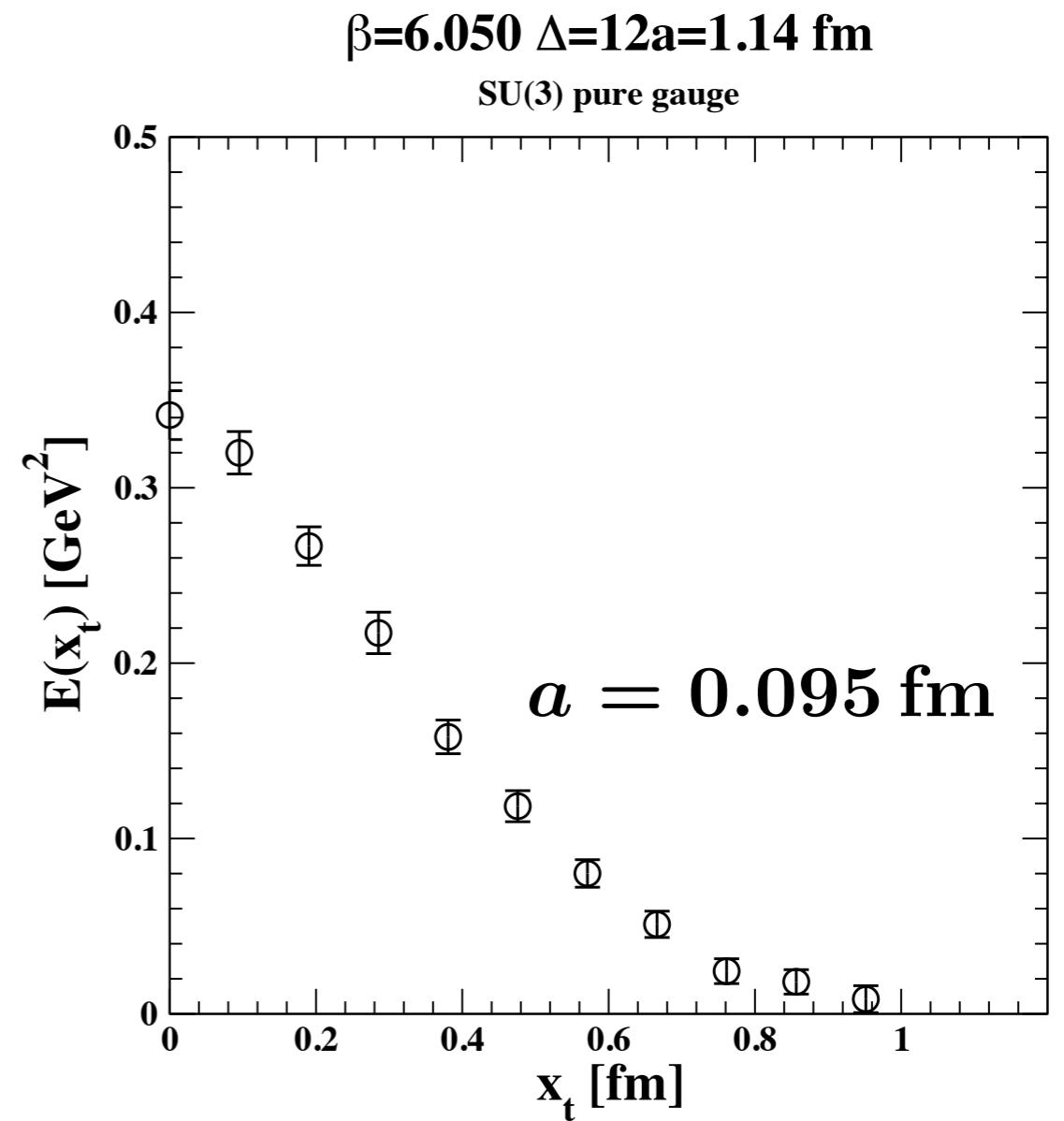
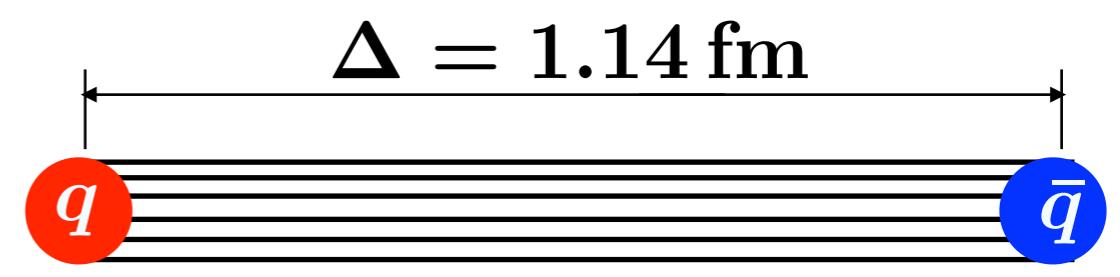
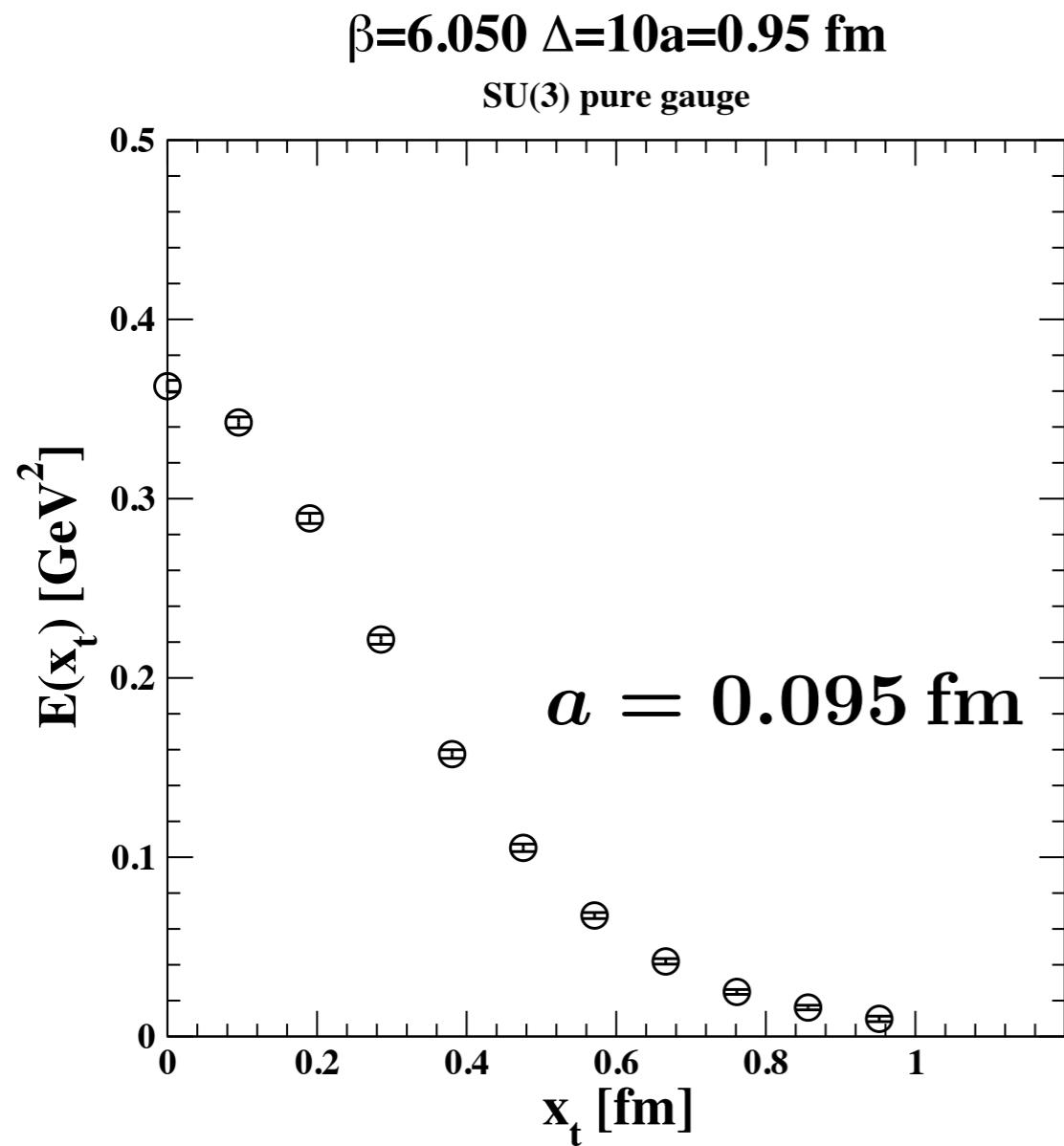
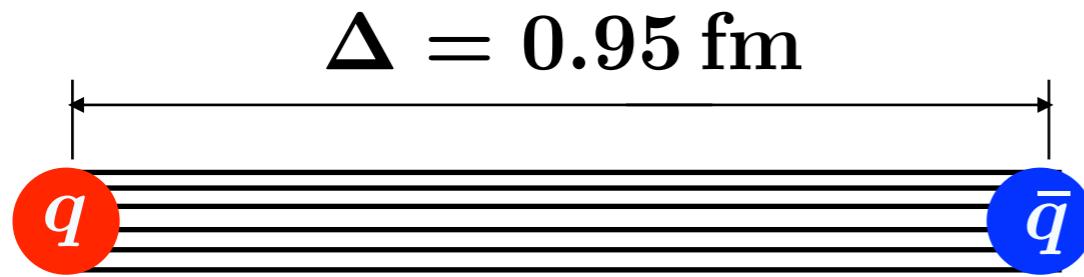


$\beta=6.050 \Delta=10a=0.95 \text{ fm}$

SU(3) pure gauge



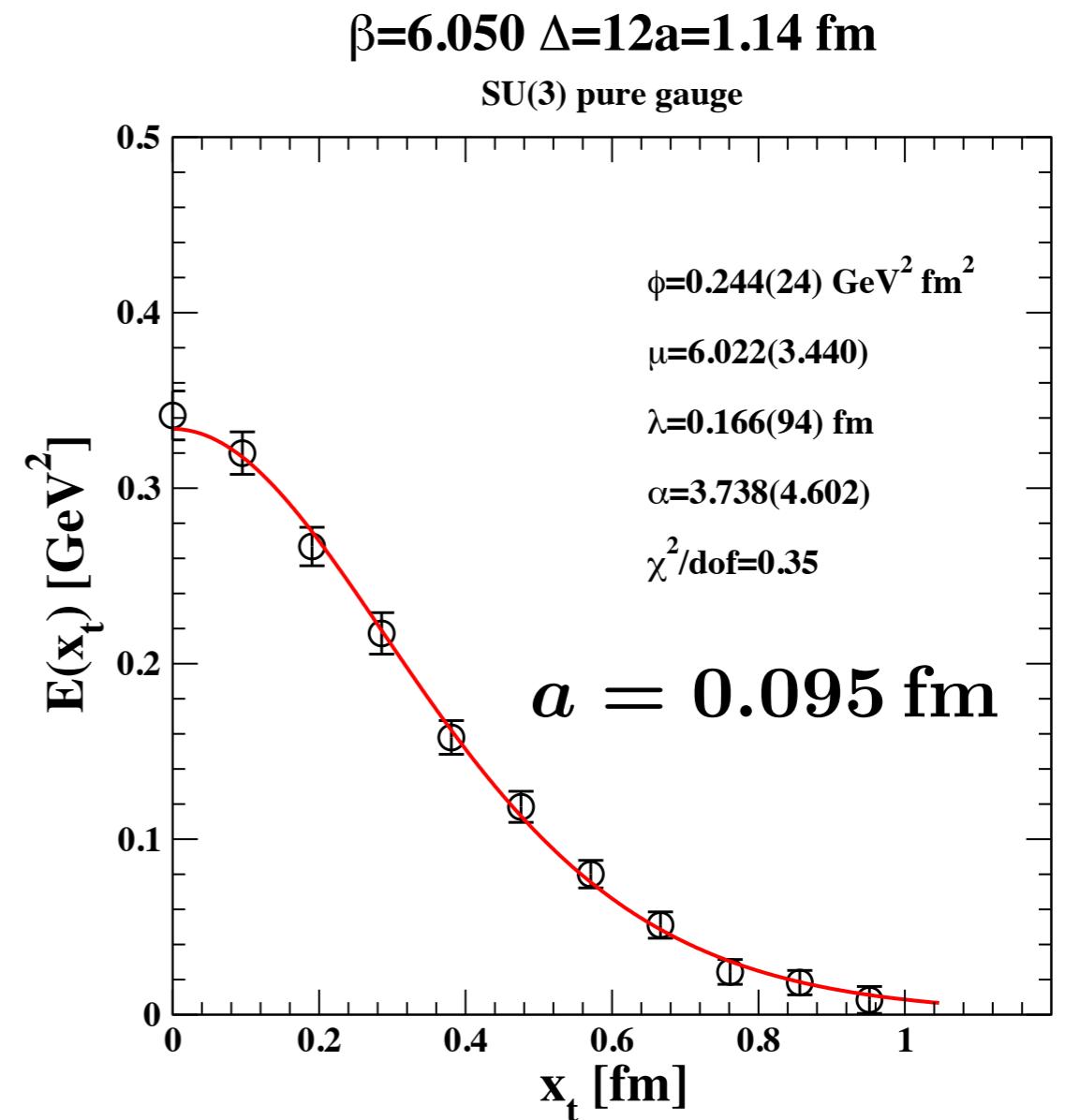
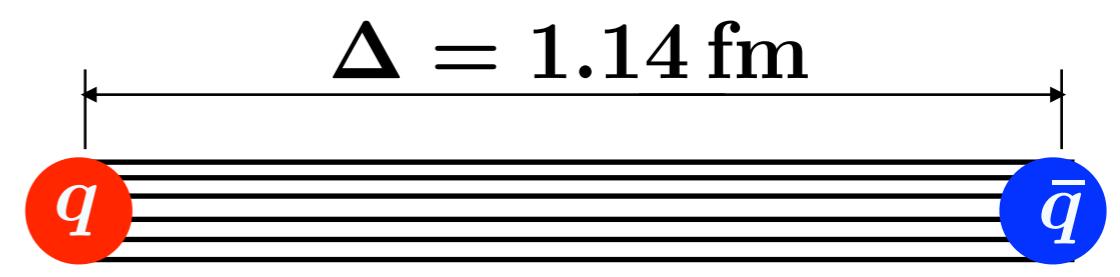
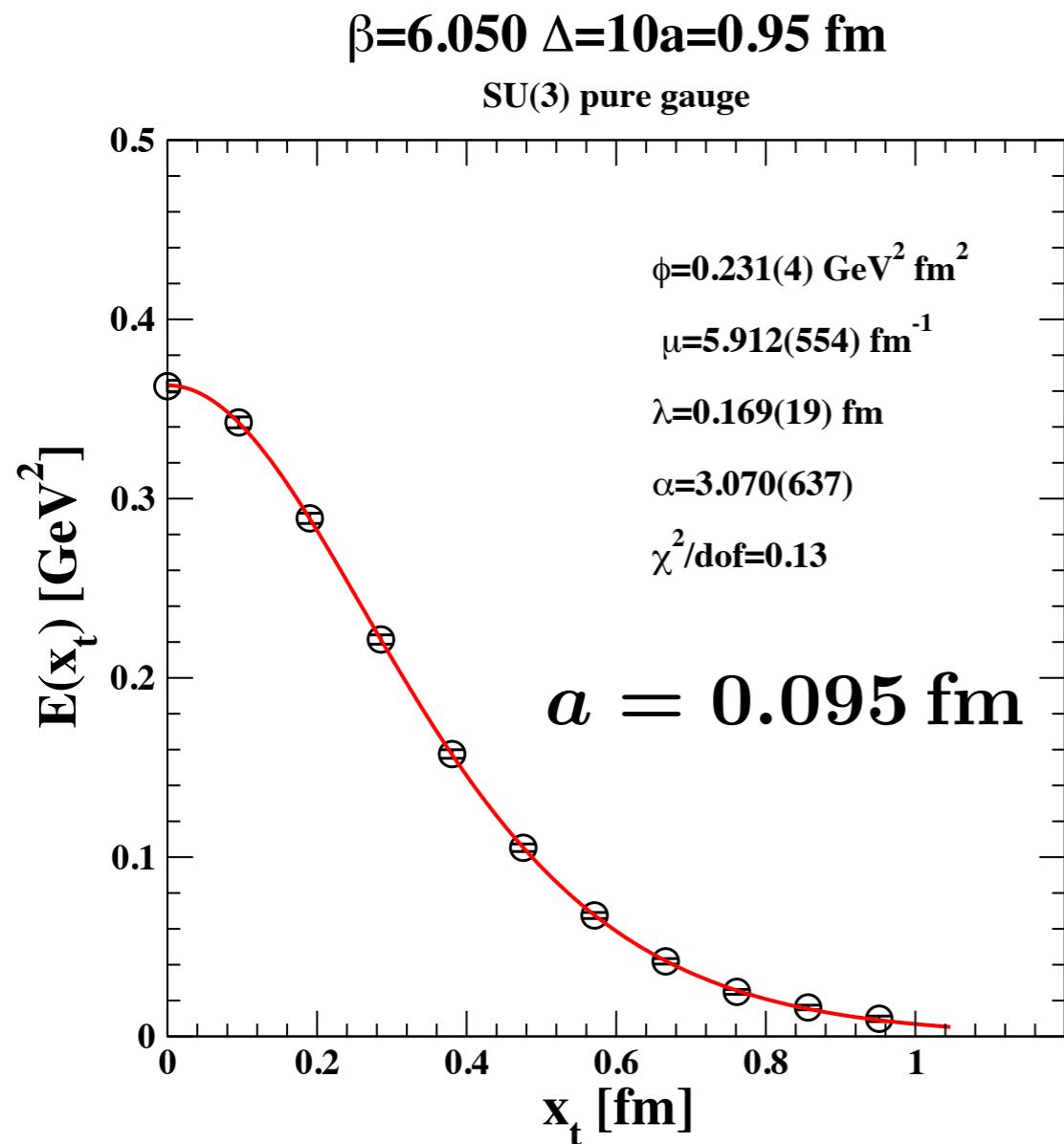
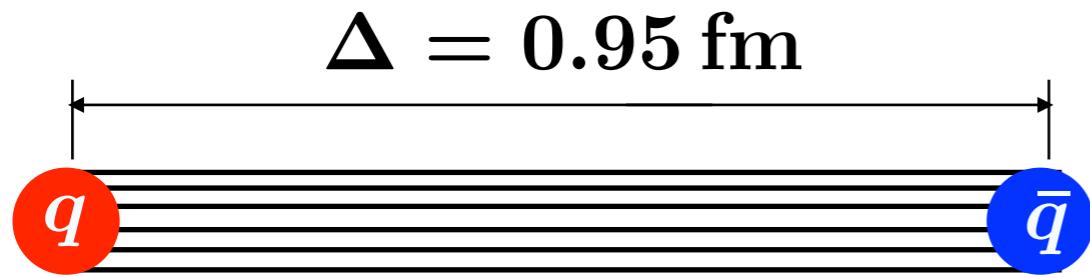




$$E_l(x_t) = \frac{\phi}{2\pi} \frac{\mu^2}{\alpha} \frac{K_0((\mu^2 x_t^2 + \alpha^2)^{1/2})}{K_1(\alpha)}$$

$$\kappa = \frac{\lambda}{\xi} = \frac{\sqrt{2}}{\alpha} \left[ 1 - K_0^2(\alpha)/K_1^2(\alpha) \right]^{1/2}$$

$$\mu = \frac{1}{\lambda}, \quad \frac{1}{\alpha} = \frac{\lambda}{\xi_v}$$



$$E_l(x_t) = \frac{\phi}{2\pi} \frac{\mu^2}{\alpha} \frac{K_0((\mu^2 x_t^2 + \alpha^2)^{1/2})}{K_1(\alpha)}$$

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$$\mu = \frac{1}{\lambda}, \quad \frac{1}{\alpha} = \frac{\lambda}{\xi_v}$$

width of the flux tube:

$$\sqrt{w^2} = \sqrt{\frac{\int d^2x_t x_t^2 E_l(x_t)}{\int d^2x_t E_l(x_t)}} = \sqrt{\frac{2\alpha}{\mu^2} \frac{K_2(\alpha)}{K_1(\alpha)}}$$

energy in the flux tube per unit length:

$$\varepsilon = \int d^2x_t \frac{E_l(x_t)^2}{2} = \frac{\phi^2}{8\pi} \mu^2 \left(1 - \left(\frac{K_0(\alpha)}{K_1(\alpha)}\right)^2\right)$$

$$\frac{\sqrt{\varepsilon}}{\phi} = \sqrt{\frac{\mu^2}{8\pi} \left(1 - \left(\frac{K_0(\alpha)}{K_1(\alpha)}\right)^2\right)}$$

**SU(3) pure gauge summary:**

$\beta$	$\Delta$ [fm]	$\phi$	$\lambda$ [fm]	$\kappa = \lambda/\xi$	$\xi$ [fm]	$\sqrt{w^2}$ [fm]	$\sqrt{\varepsilon}/\phi$ [GeV]
6.050	0.76	5.143(39)	0.164(5)	0.348(208)	0.472(283)	0.458(17)	0.133(5)
6.195	0.76	5.485(56)	0.173(7)	0.369(229)	0.469(293)	0.476(24)	0.128(7)
6.050	0.95	5.932(114)	0.169(16)	0.229(103)	0.738(339)	0.517(59)	0.116(14)
6.050	1.14	6.254(617)	0.166(95)	0.174(65)	0.953(651)	0.542(386)	0.109(82)

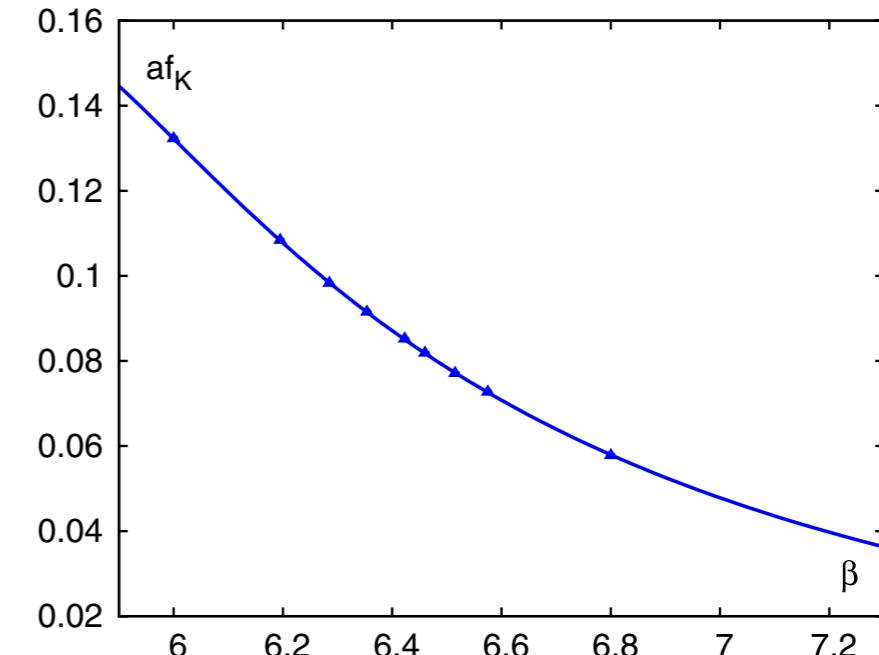
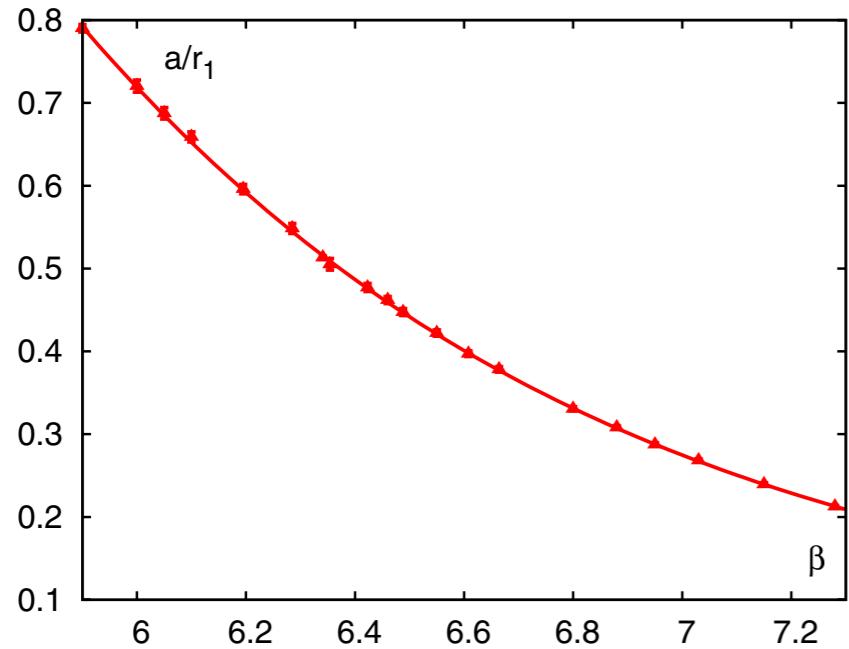
- penetration depth almost stable within errors
- hint of a slow increasing for  $\xi$  and  $\sqrt{w^2}$  even though with large errors

# QCD (2+1) flavors - LATTICE SETUP

- *Highly improved staggered quark action* with *tree level improved Symanzik gauge action* (**HISQ/tree**) with 2+1 flavors.
- We work on a *line of constant physics (LCP)* determined (\*) by fixing the *strange quark mass* to its physical value  $m_s$  at each value of the gauge coupling  $\beta$ . The *light-quark mass* has been fixed at  $m_l = m_s/20$ . ( $M_\pi = 160 \text{ MeV}$ )  
(\*) as determined in A. Bazavov et al (HotQCD Collaboration), PRD 85, 054503 (2012)
- To perform numerical simulations we used the **MILC** code suitably modified in order to measure the chromoelectric field
- All simulations make use of the rational hybrid Monte Carlo (RHMC) algorithm.  
(The length of each RHMC trajectory has been set to 1.0 in molecular dynamics time units.)
- smoothing of the gauge configurations: several APE smearings for spatial links,  
one HYP smearing for temporal links
- scale setting —> (HotQCD Collaboration), PRD 85, 054503 (2012)

# Setting the lattice scale (QCD (2+1) flavors)

The lattice spacing can be determined using the slope of the static quark-antiquark potential on zero-temperature lattices or the value of the decay constant  $f_K$  (we use results of HotQCD collaboration (\*)).



$$a(\beta)|_{m_l=0.05m_s} = r_1 \frac{c_0 f(\beta) + c_2 (10/\beta) f^3(\beta)}{1 + d_2 (10/\beta) f^2(\beta)}$$

$$r_1 = 0.3106 \text{ fm}$$

$$c_0 = 44.06$$

$$c_2 = 272102$$

$$d_2 = 4281$$

$$af_K(\beta)|_{m_l=0.05m_s} = \frac{c_0^K f(\beta) + c_2^K (10/\beta) f^3(\beta)}{1 + d_2^K (10/\beta) f^2(\beta)}$$

$$r_1 f_K = 0.1738$$

$$c_0^K = 7.66$$

$$c_2^K = 32911$$

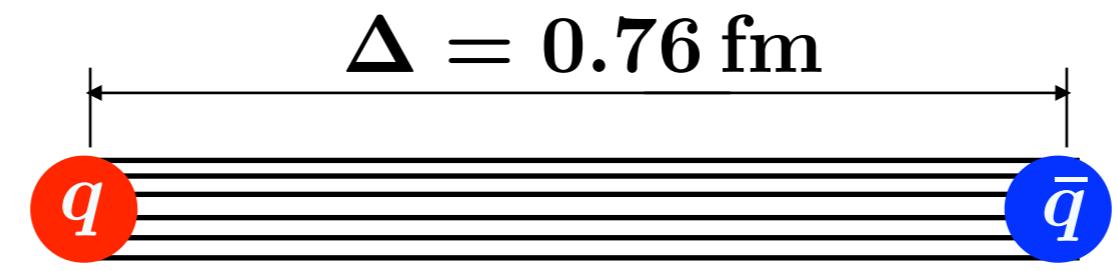
$$d_2^K = 2388$$

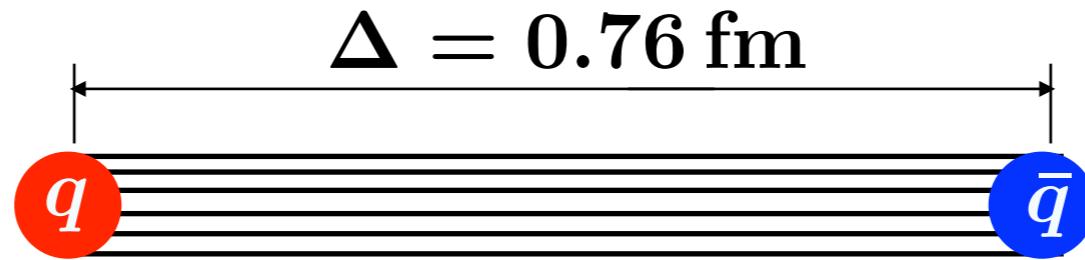
$$f(\beta) = (b_0(10/\beta))^{-b_1/(2b_0^2)} \exp(-\beta/(20b_0))$$

$$b_0, b_1$$

coefficients of the  
universal two-loop  
beta function

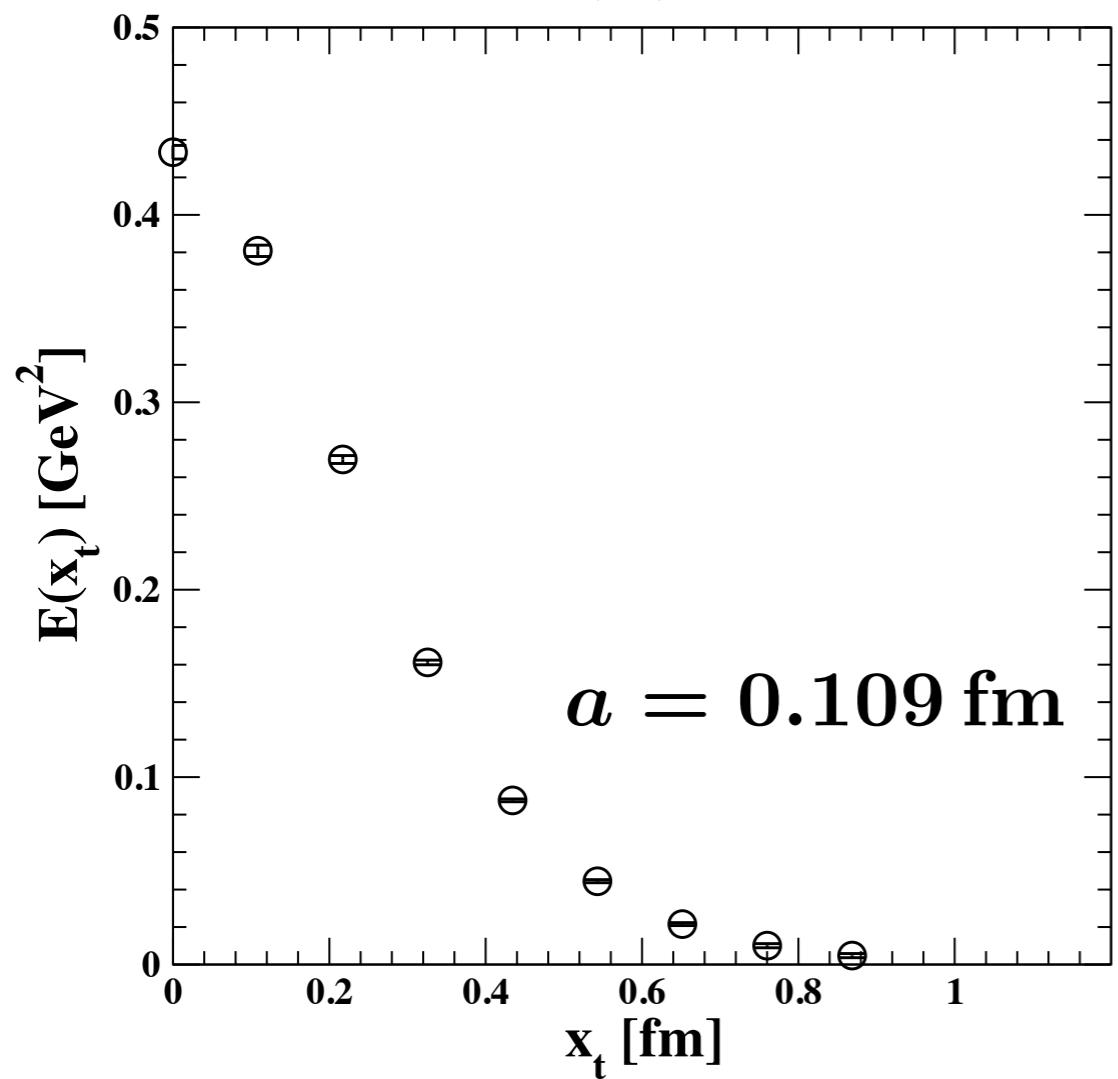
(\*) as discussed in Appendix B of A. Bazavov et al (HotQCD Collaboration), PRD 85, 054503 (2012)

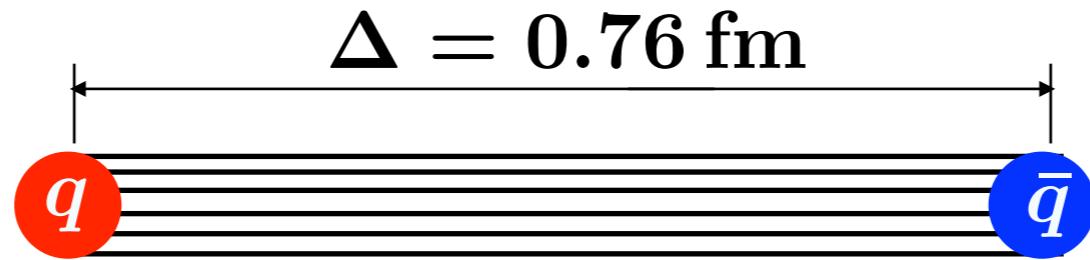




$\beta=6.743 \Delta=7a=0.76 \text{ fm}$

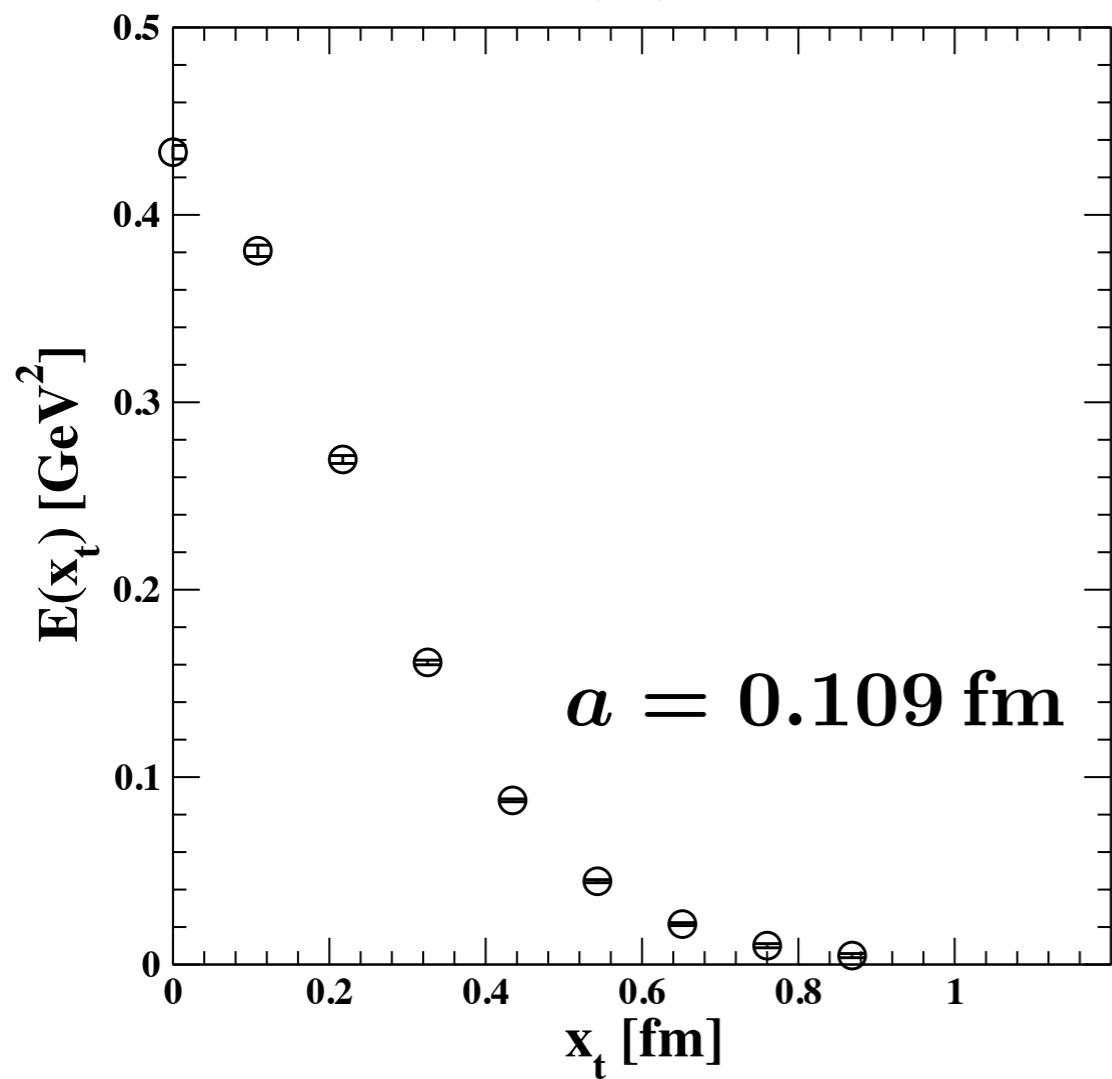
QCD (2+1) flavors





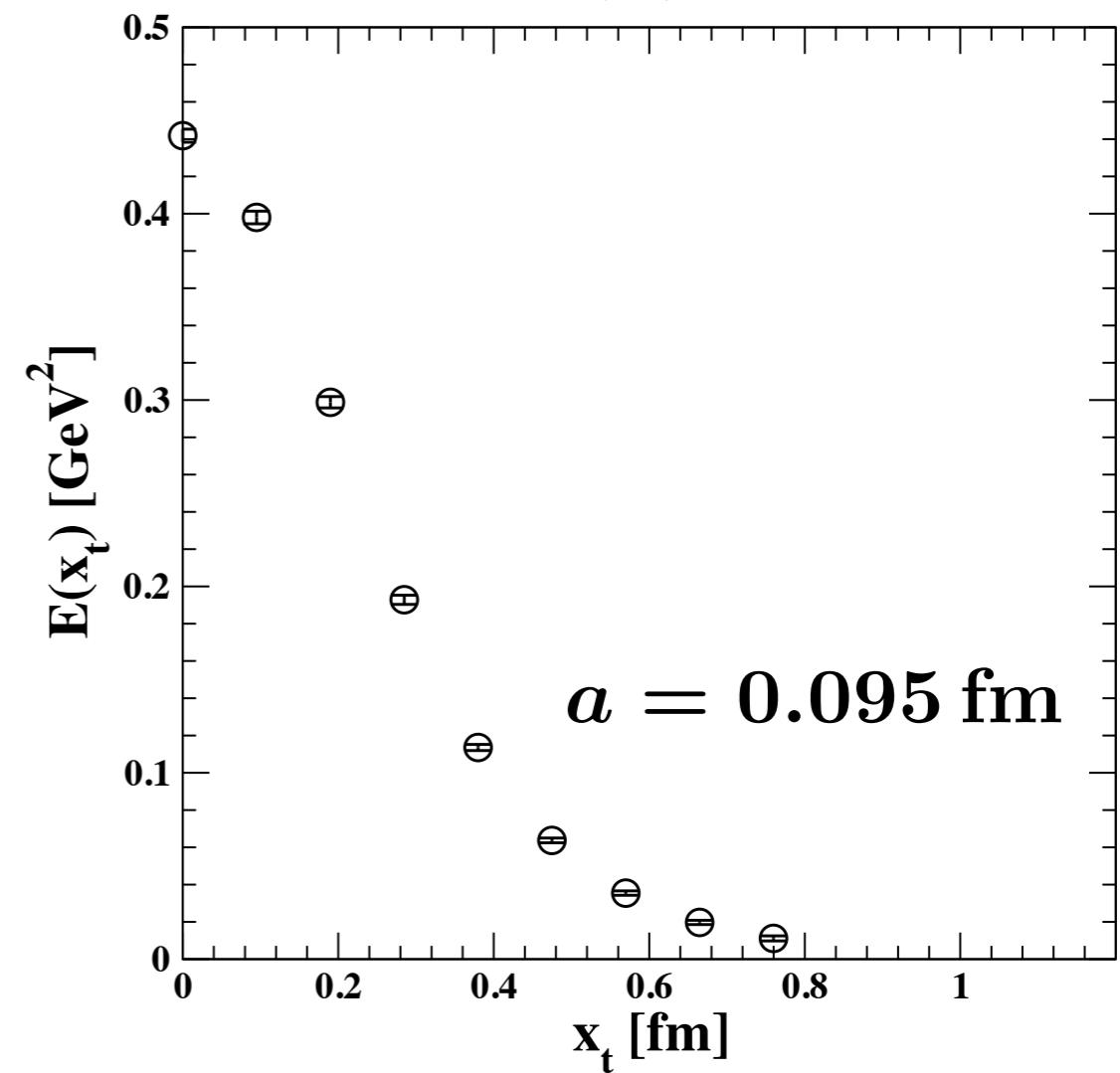
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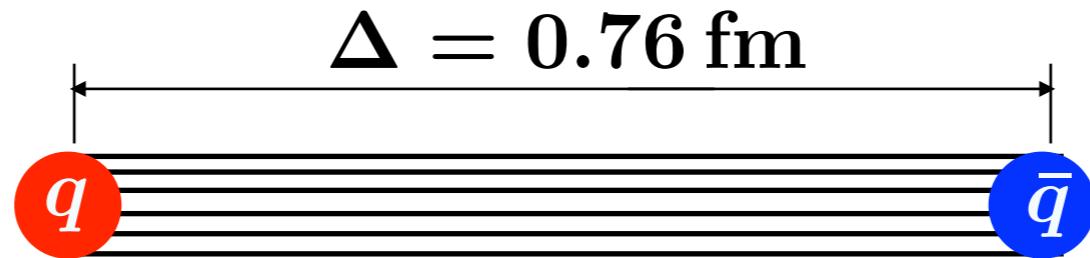
QCD (2+1) flavors



$\beta=6.885 \Delta=8a=0.76 \text{ fm}$

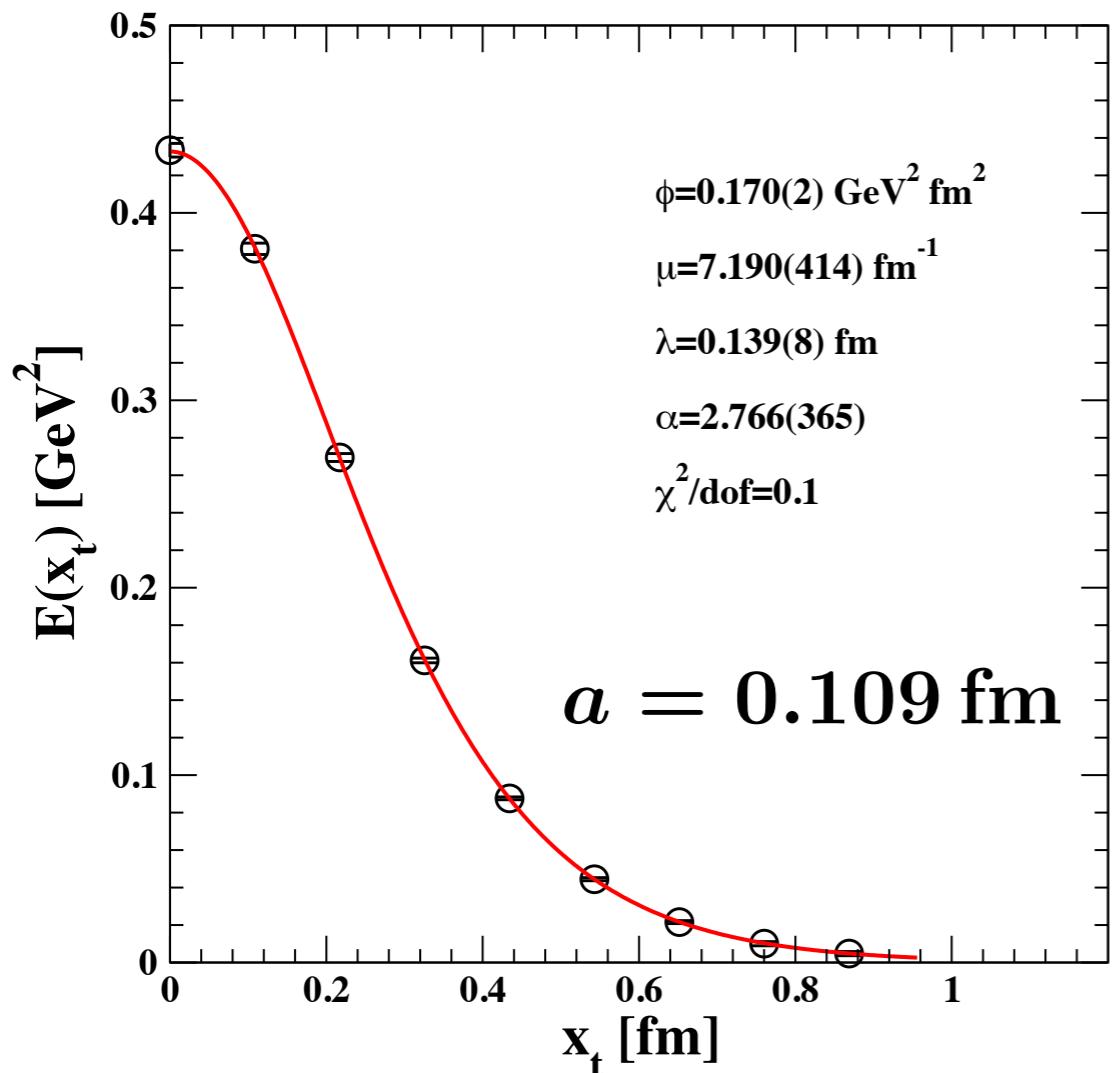
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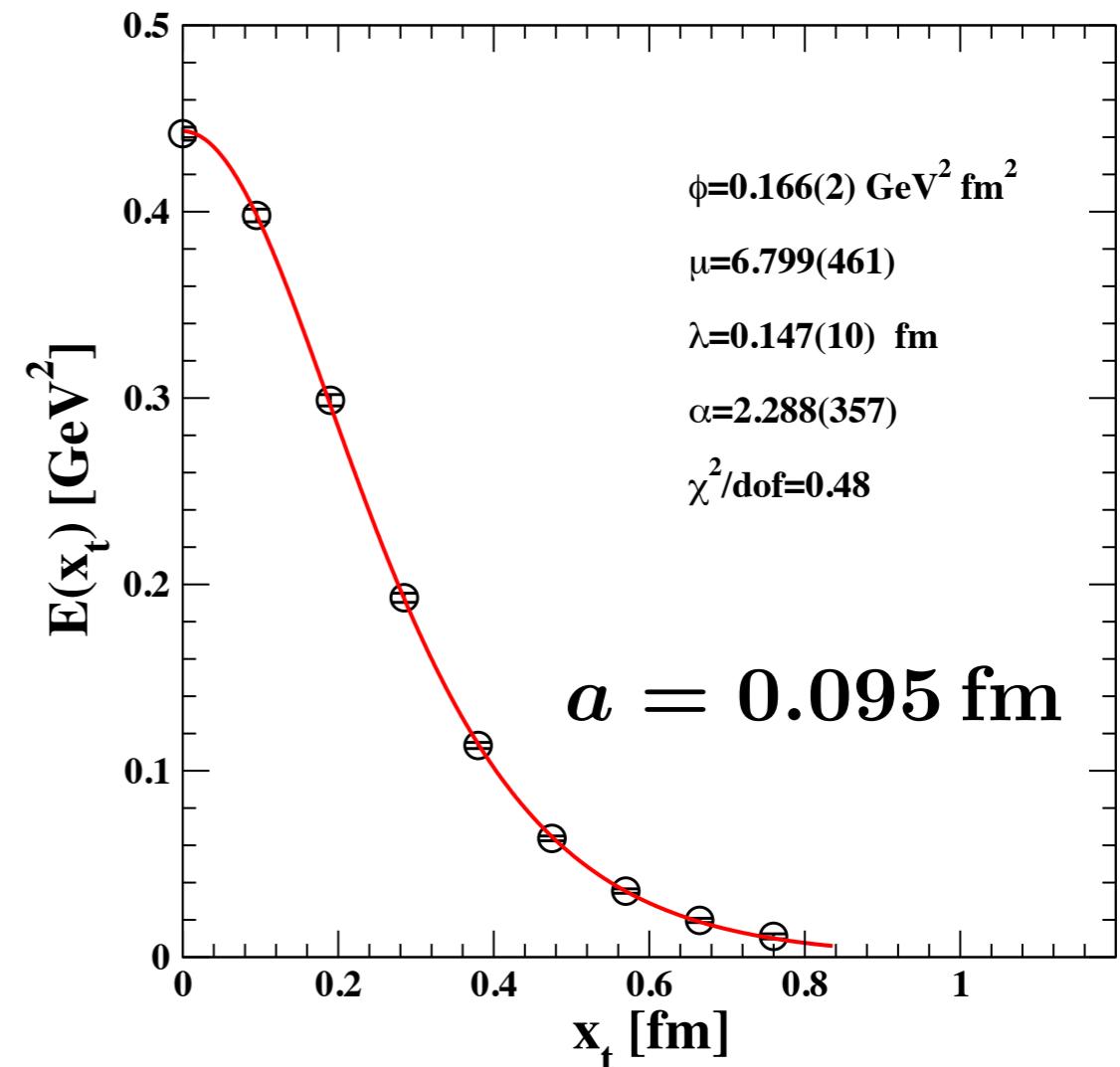
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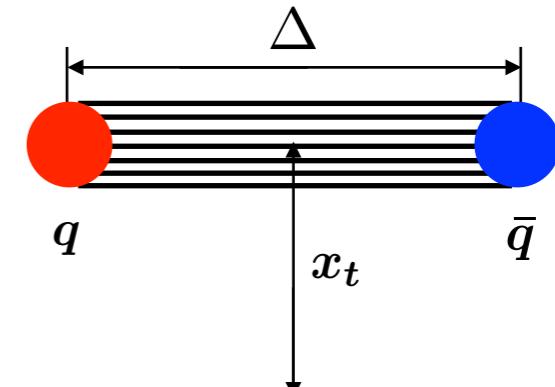
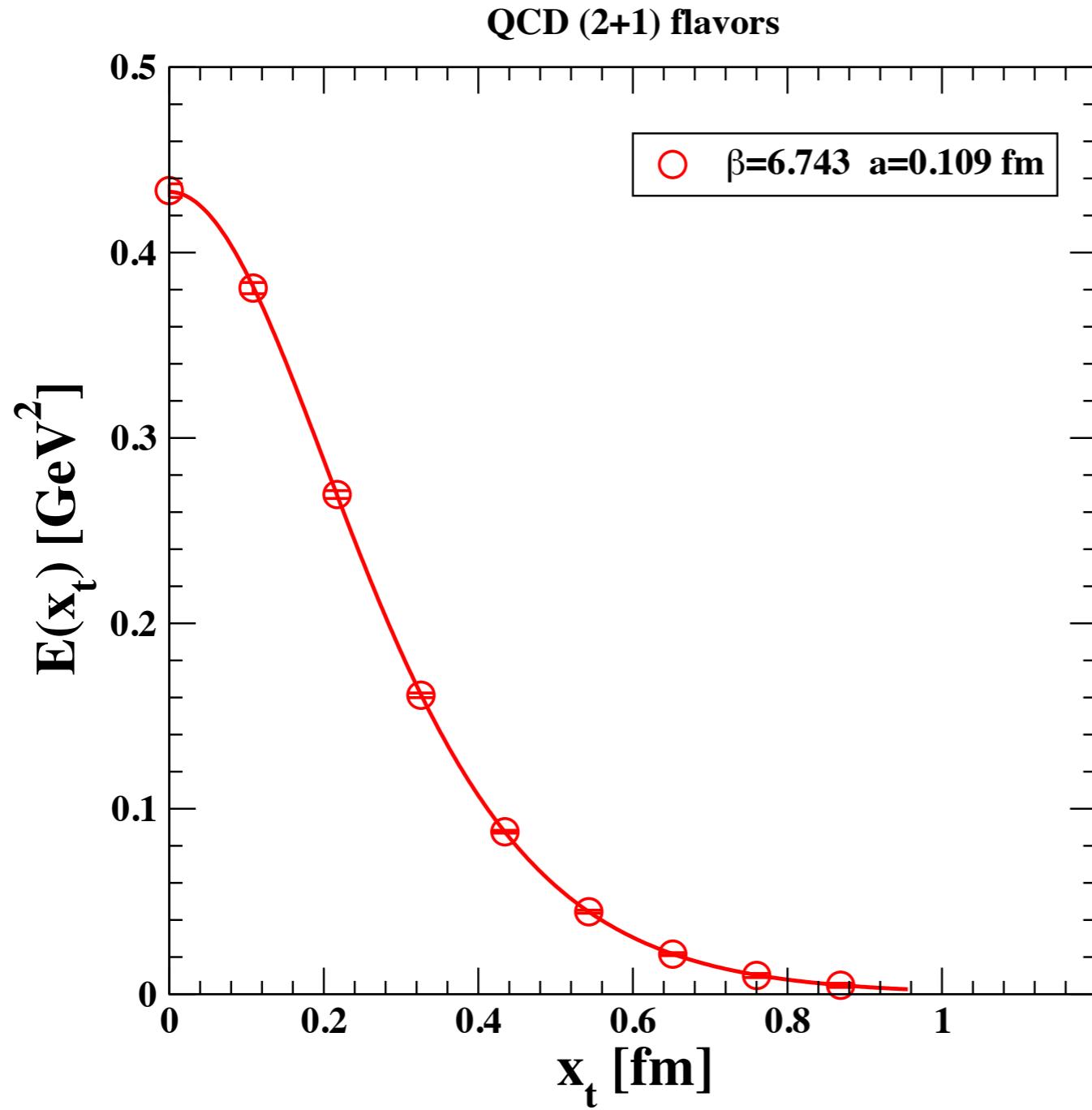
$$E_l(x_t) = \frac{\phi}{2\pi} \frac{\mu^2}{\alpha} \frac{K_0((\mu^2 x_t^2 + \alpha^2)^{1/2})}{K_1(\alpha)}$$

$$\kappa = \frac{\lambda}{\xi} = \frac{\sqrt{2}}{\alpha} \left[ 1 - K_0^2(\alpha)/K_1^2(\alpha) \right]^{1/2}$$

$$\mu = \frac{1}{\lambda}, \quad \frac{1}{\alpha} = \frac{\lambda}{\xi_v}$$

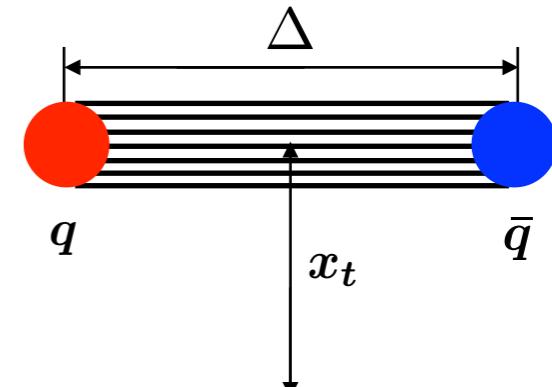
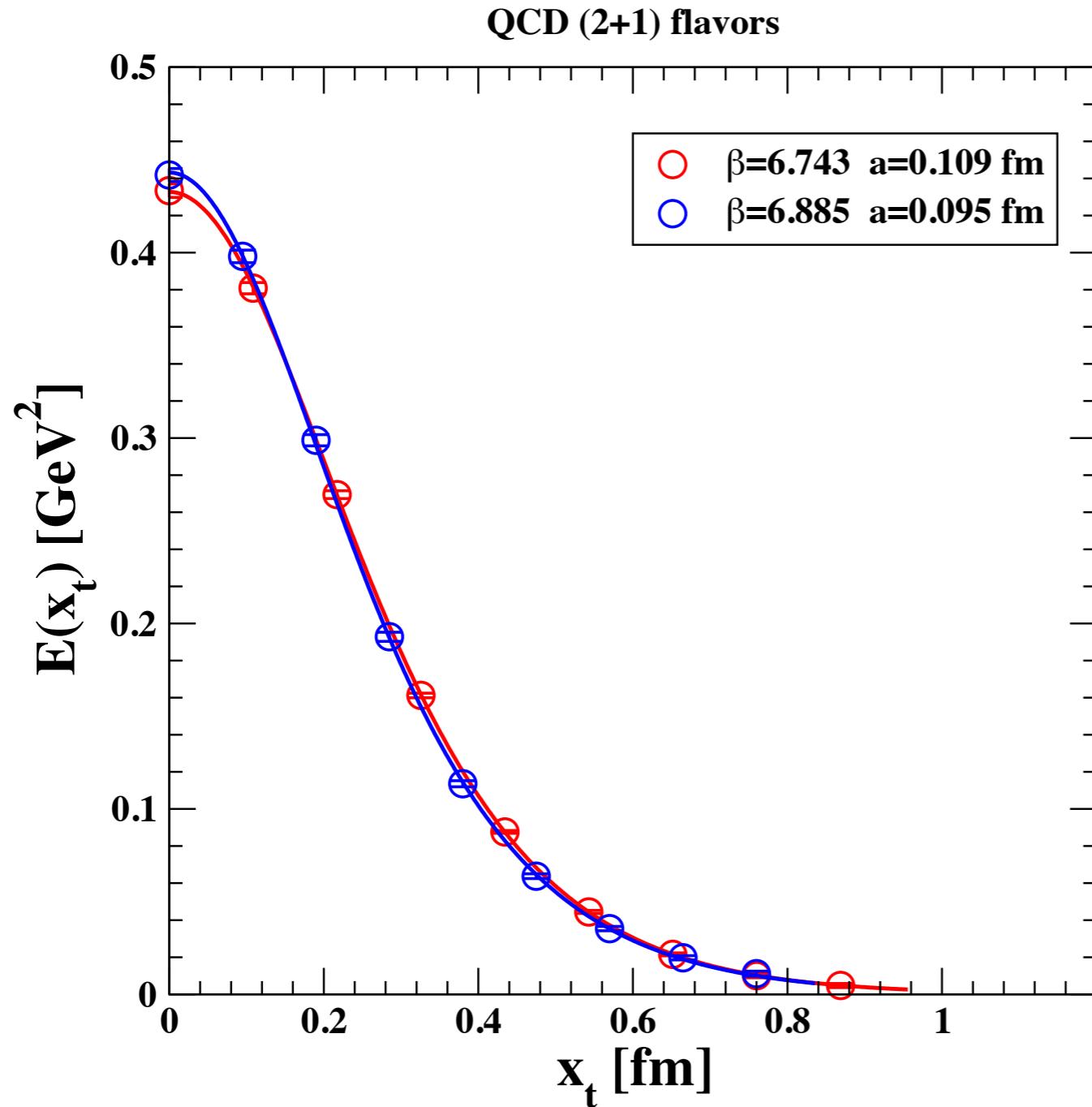


## scaling test at distance 0.76 fm between sources



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width of the flux tube:

$$\sqrt{w^2} = \sqrt{\frac{\int d^2x_t x_t^2 E_l(x_t)}{\int d^2x_t E_l(x_t)}} = \sqrt{\frac{2\alpha}{\mu^2} \frac{K_2(\alpha)}{K_1(\alpha)}}$$

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energy in the flux tube per unit length:

$$\varepsilon = \int d^2x_t \frac{E_l(x_t)^2}{2} = \frac{\phi^2}{8\pi} \mu^2 \left( 1 - \left( \frac{K_0(\alpha)}{K_1(\alpha)} \right)^2 \right)$$

$$\frac{\sqrt{\varepsilon}}{\phi} = \sqrt{\frac{\mu^2}{8\pi} \left( 1 - \left( \frac{K_0(\alpha)}{K_1(\alpha)} \right)^2 \right)}$$

**QCD (2+1) flavours summary:**

$\beta$	$\Delta$ [fm]	$\phi$	$\lambda$ [fm]	$\kappa = \lambda/\xi$	$\xi$ [fm]	$\sqrt{w^2}$ [fm]	$\sqrt{\varepsilon}/\phi$ [GeV]
6.743	0.76	4.366(48)	0.139(8)	0.264(131)	0.526(263)	0.411(29)	0.146(11)
6.885	0.76	4.251(67)	0.147(10)	0.342(203)	0.429(256)	0.411(33)	0.148(13)

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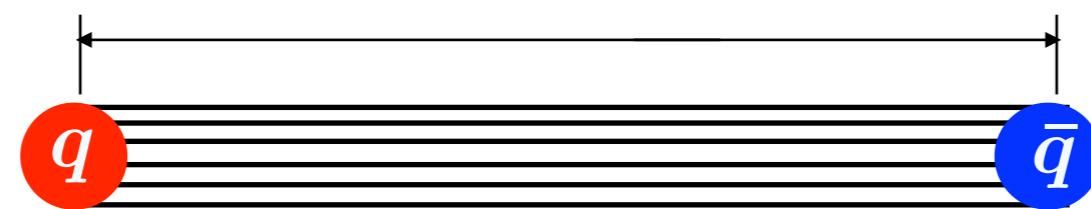
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**SU(3) pure gauge summary:**

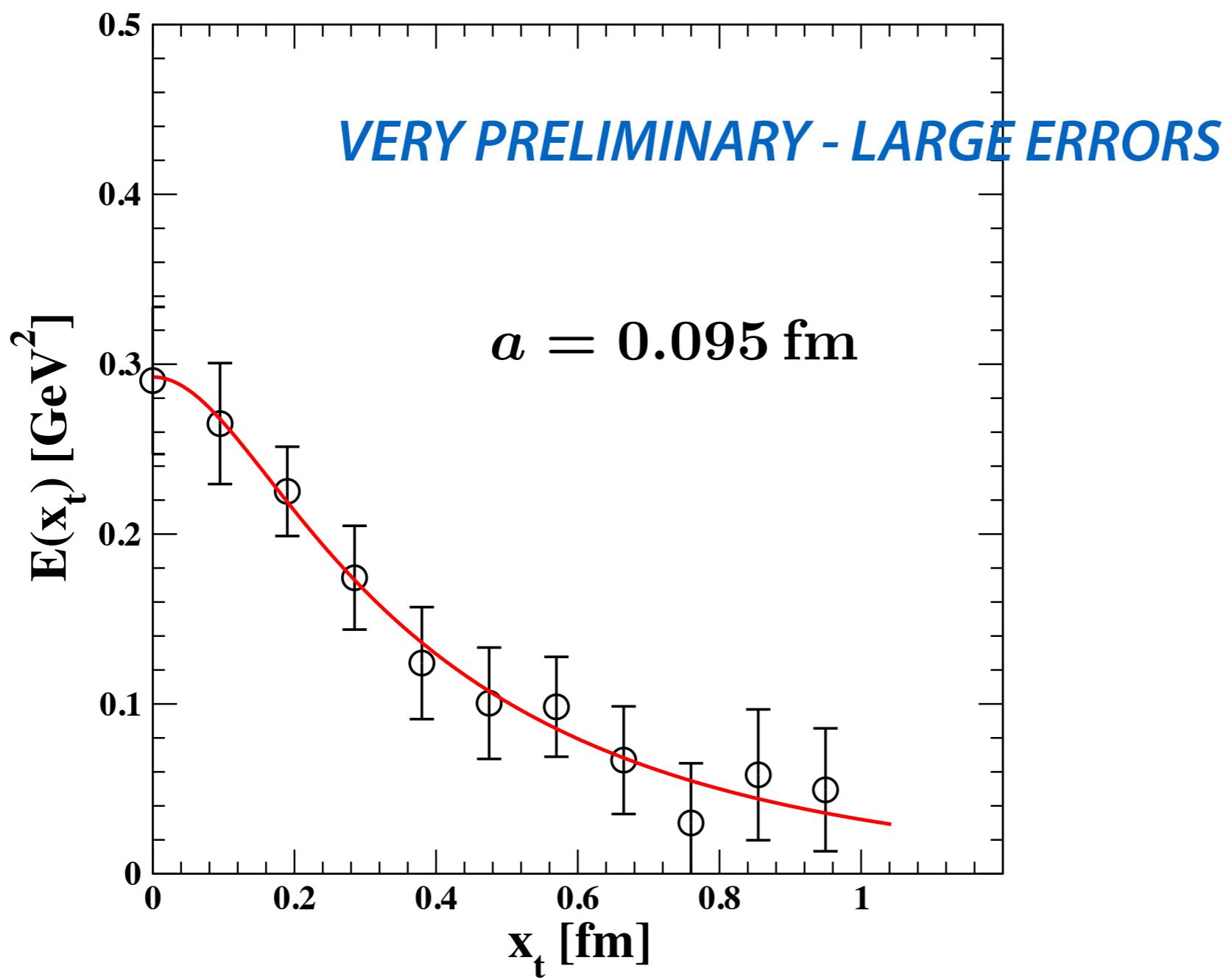
$\beta$	$\Delta$ [fm]	$\phi$	$\lambda$ [fm]	$\kappa = \lambda/\xi$	$\xi$ [fm]	$\sqrt{w^2}$ [fm]	$\sqrt{\varepsilon}/\phi$ [GeV]
6.050	0.76	5.143(39)	0.164(5)	0.348(208)	0.472(283)	0.458(17)	0.133(5)
6.195	0.76	5.485(56)	0.173(7)	0.369(229)	0.469(293)	0.476(24)	0.128(7)
6.050	0.95	5.932(114)	0.169(16)	0.229(103)	0.738(339)	0.517(59)	0.116(14)
6.050	1.14	6.254(617)	0.166(95)	0.174(65)	0.953(651)	0.542(386)	0.109(82)

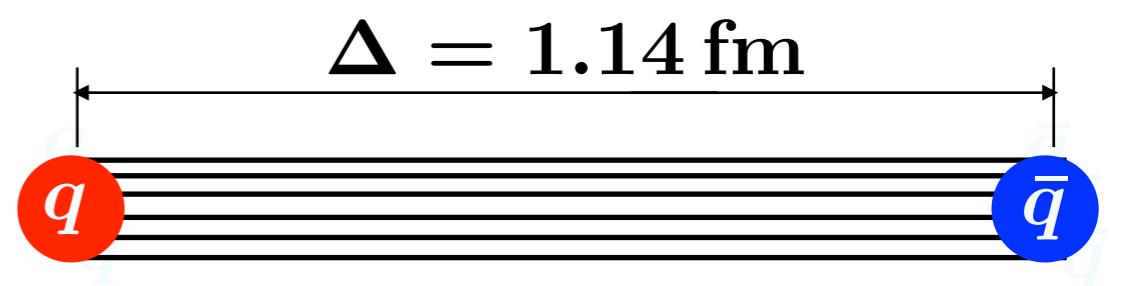
$$\Delta = 0.95 \text{ fm}$$



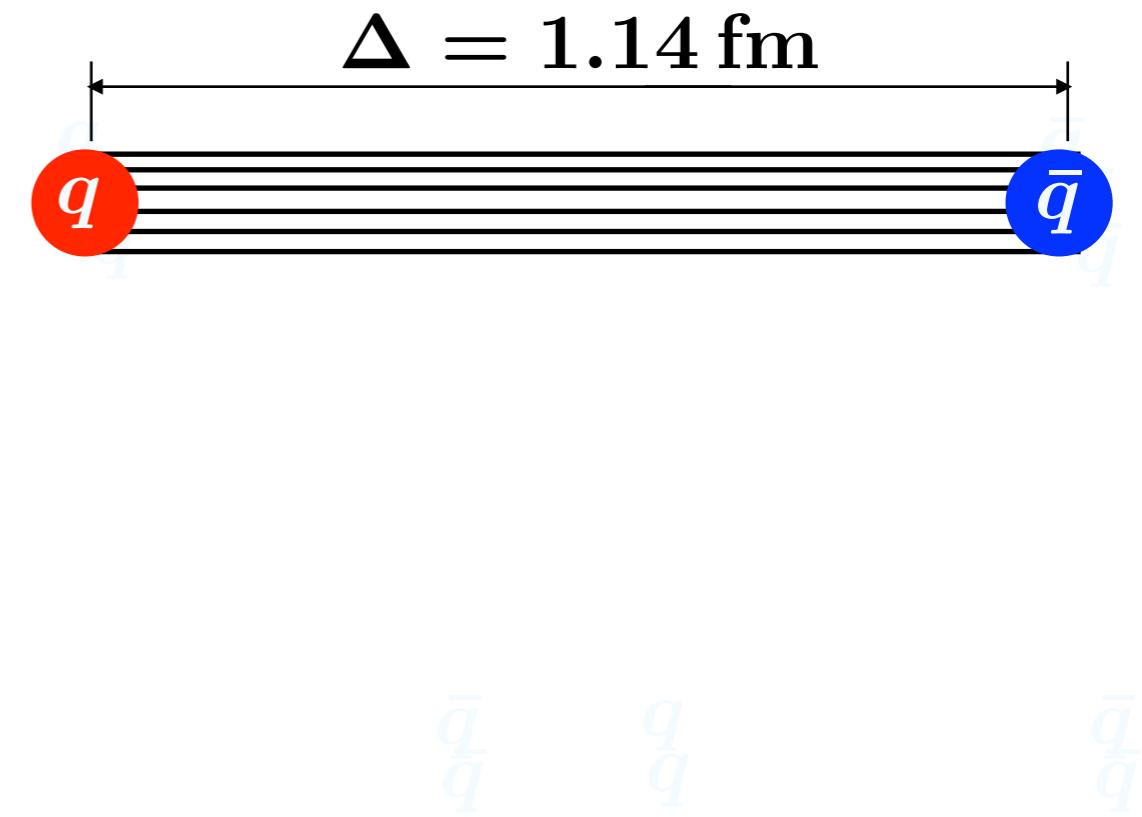
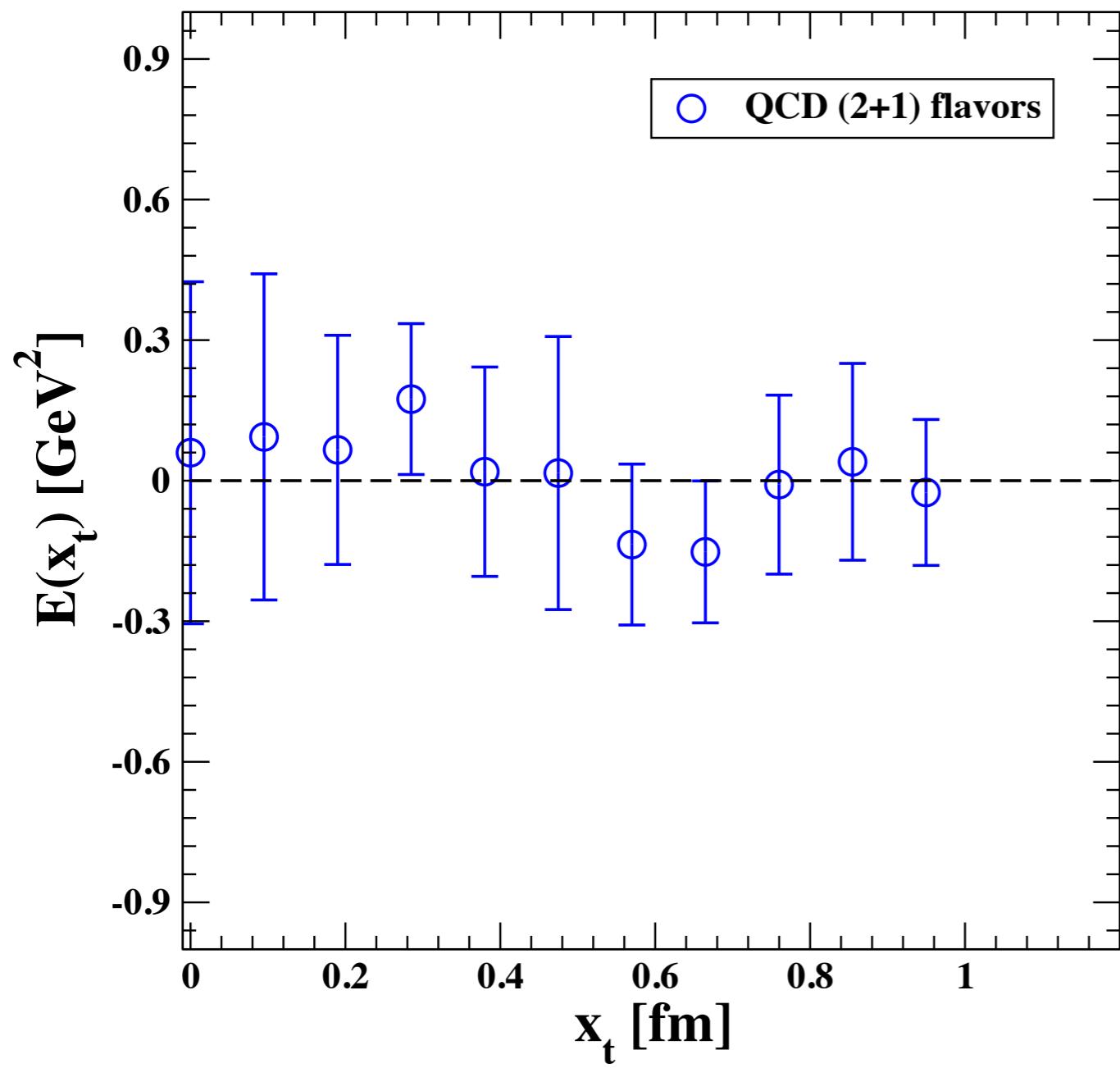
$$\beta=6.885 \Delta=10a=0.95 \text{ fm}$$

QCD (2+1) flavors

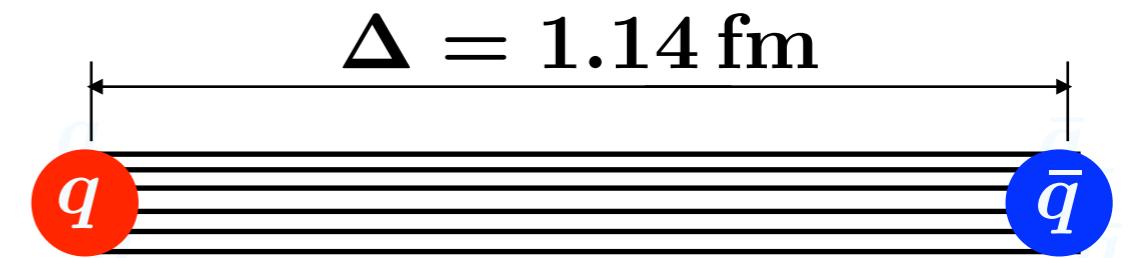
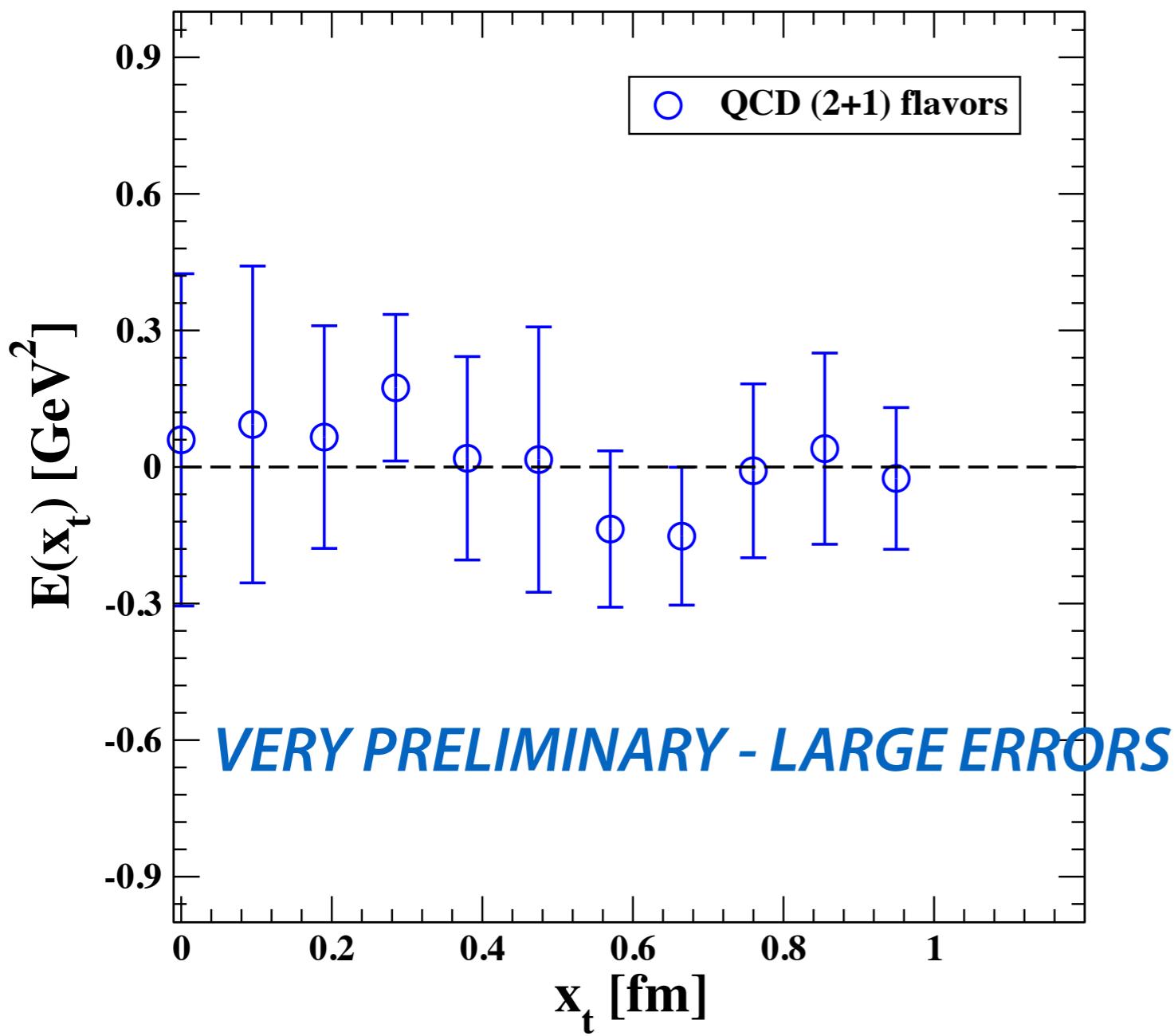


 $q$  $\bar{q}$  $q$  $\bar{q}$

$\beta=6.885$   $\Delta=12a=1.14$  fm

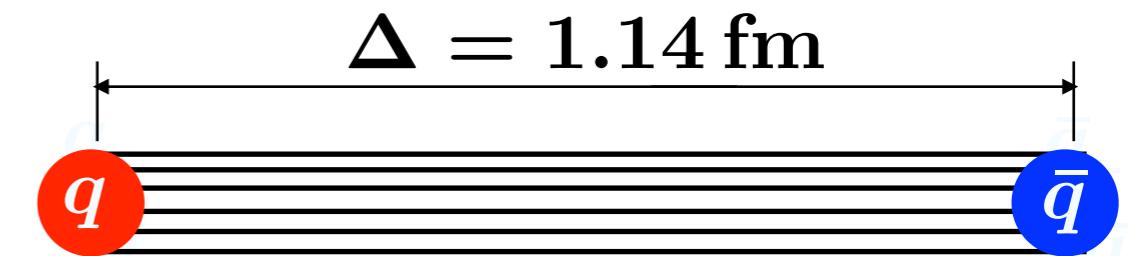
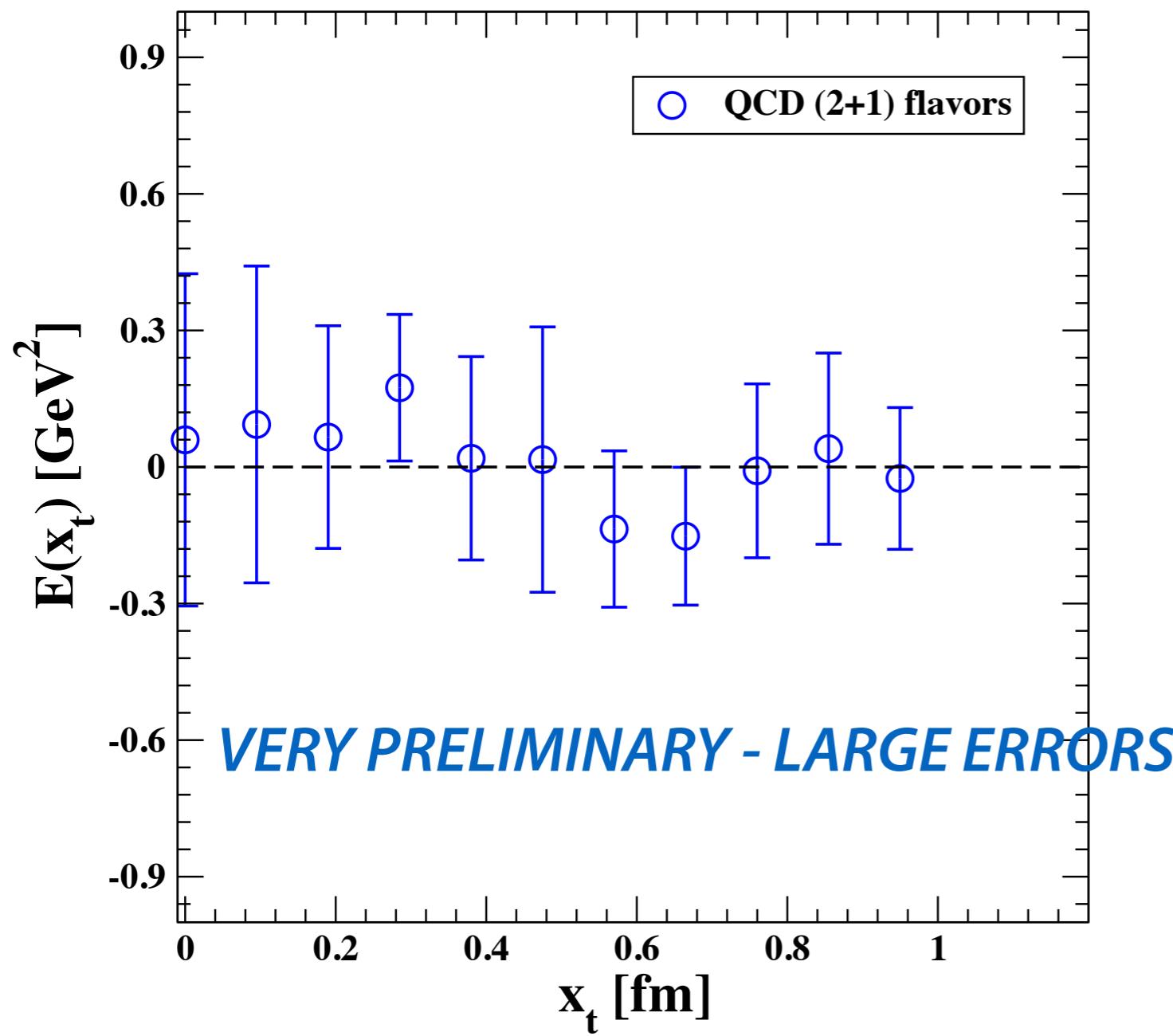


$\beta=6.885$   $\Delta=12a=1.14$  fm



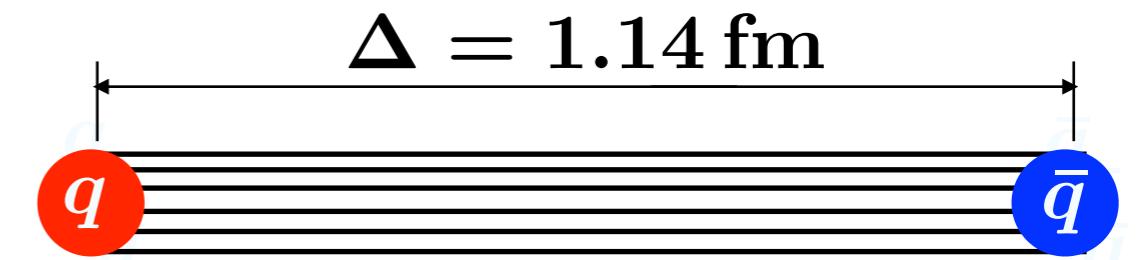
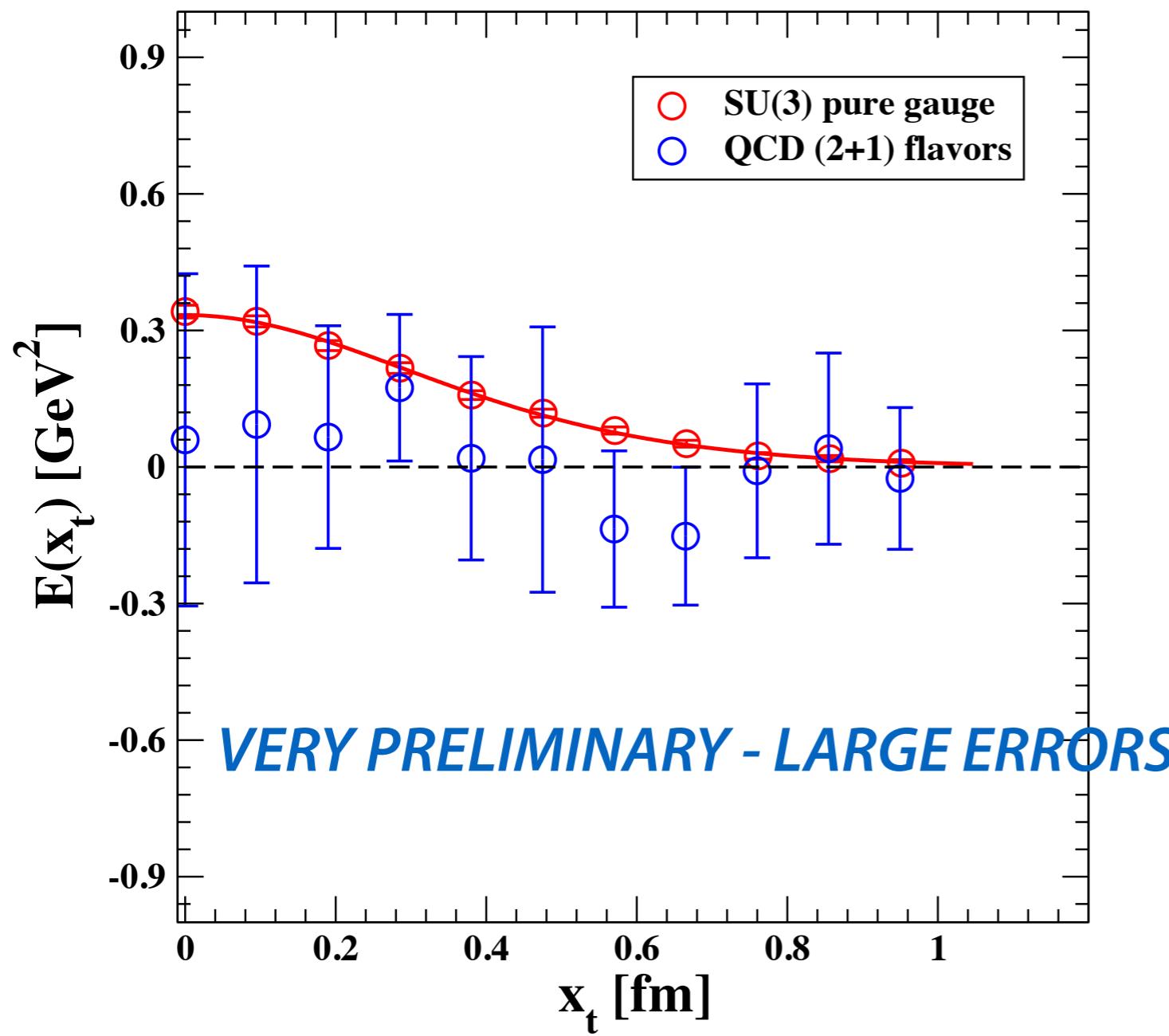
The field signal seems to disappear at distance 1.14 fm between sources:

$$\beta=6.885 \Delta=12a=1.14 \text{ fm}$$



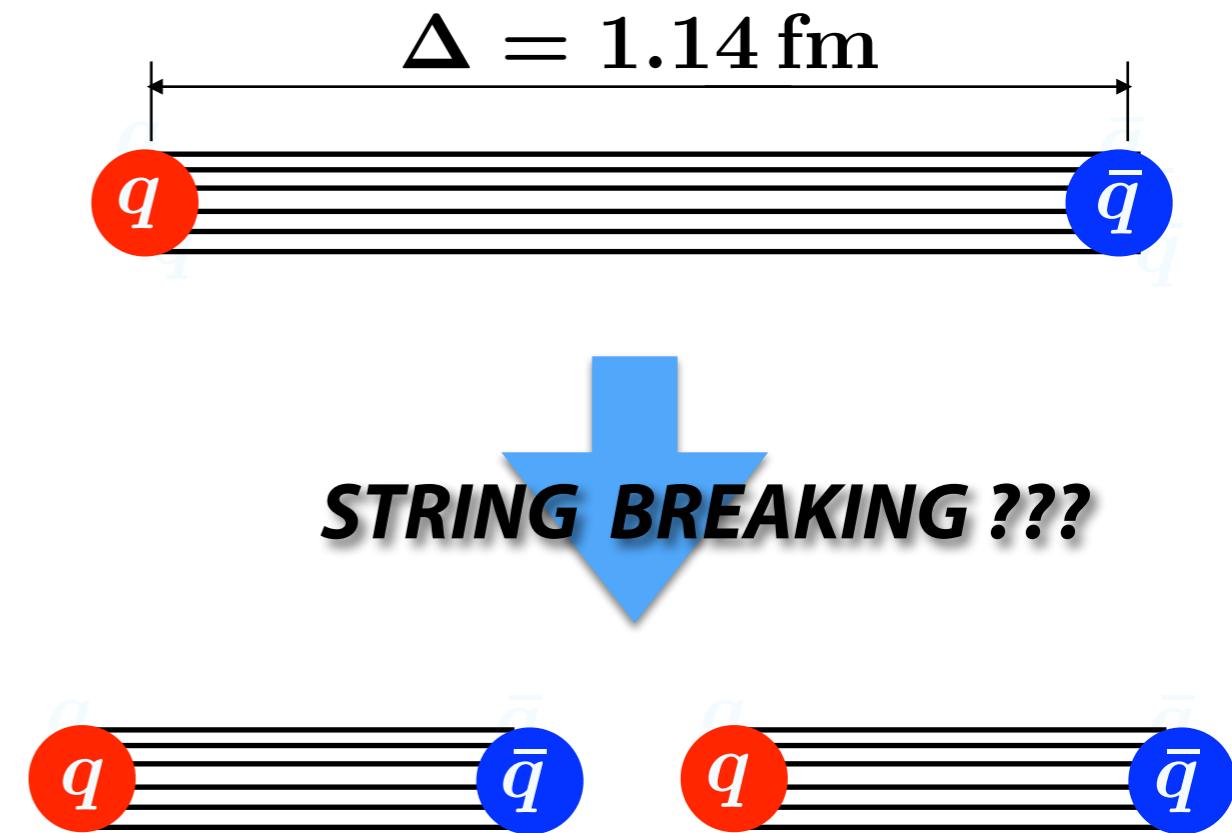
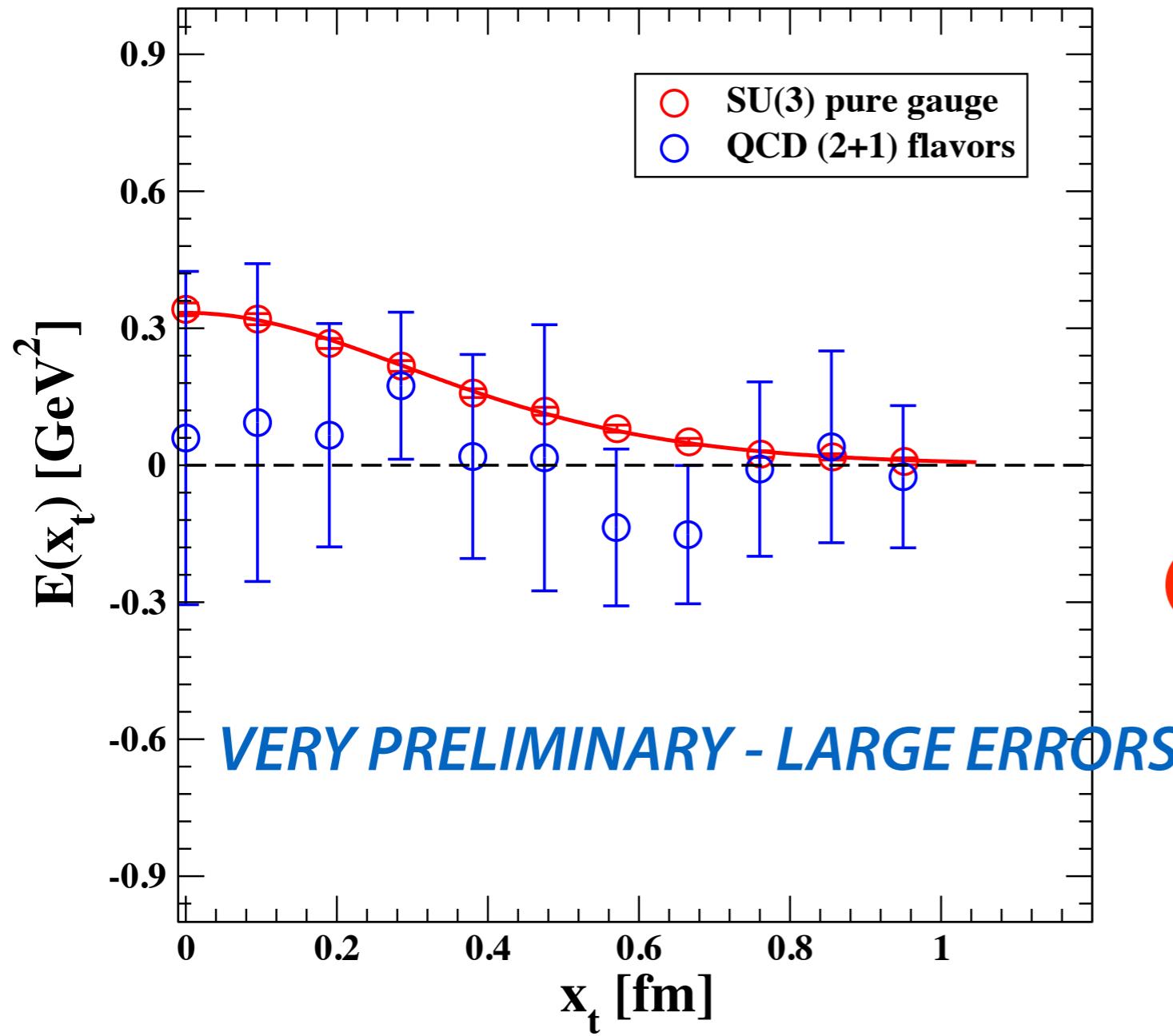
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$$\beta=6.885 \Delta=12a=1.14 \text{ fm}$$



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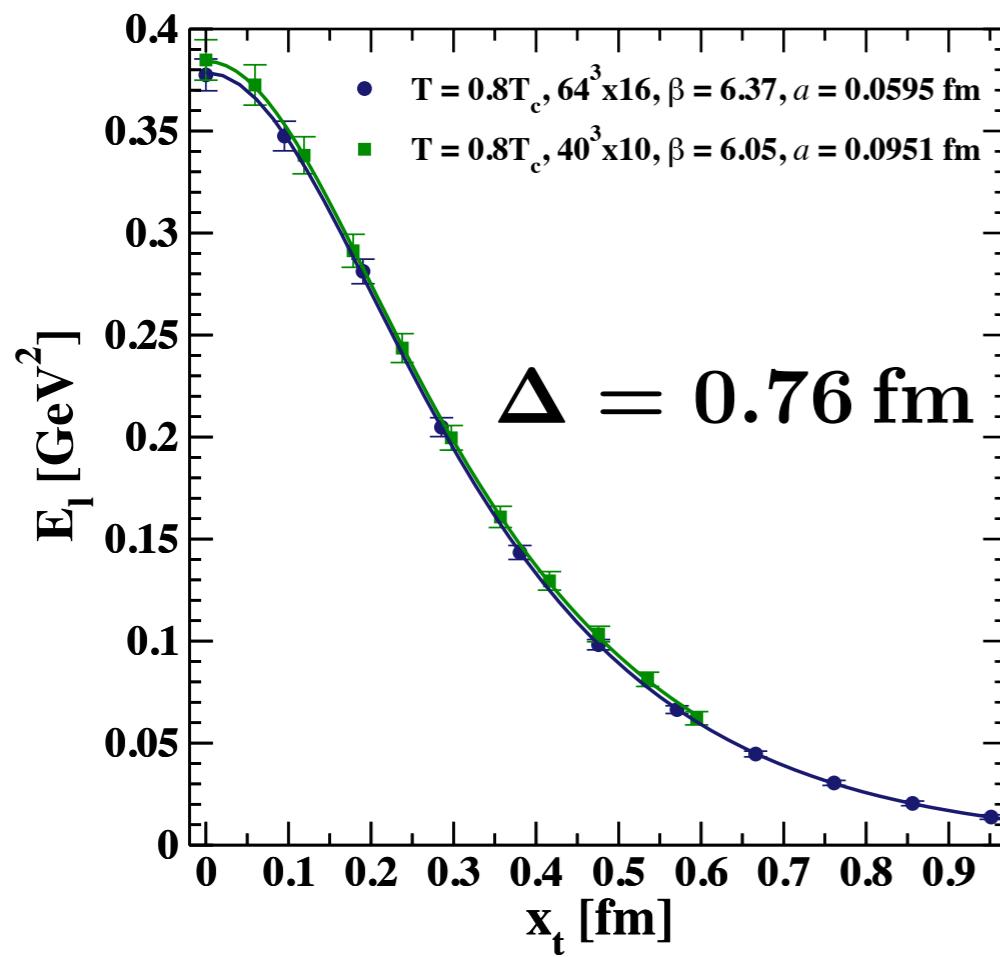
## Summary & Outlook

- We have studied the flux tubes produced by a quark-antiquark pair in the (3+1) dim QCD vacuum, both in the case of SU(3) pure gauge theory and in the case of (2+1) flavor QCD along the line of constant physics with pion mass 160 MeV.
- We have seen that the transverse behaviour of the field inside the flux tube can be well described using a functional form derived from ordinary superconductivity.
- We plan to improve the statistics of our preliminary results and to study systematically the dependence of the flux tube parameters from the distance between the sources.
- We want to check the stability of our results under changes of the smoothing procedure.
- We want to study the fate of the flux tubes at finite temperature in QCD with (2+1) flavours.
- We want to find confirmation of our very preliminary results indicating the possibility of “string breaking” in QCD with (2+1) flavors.

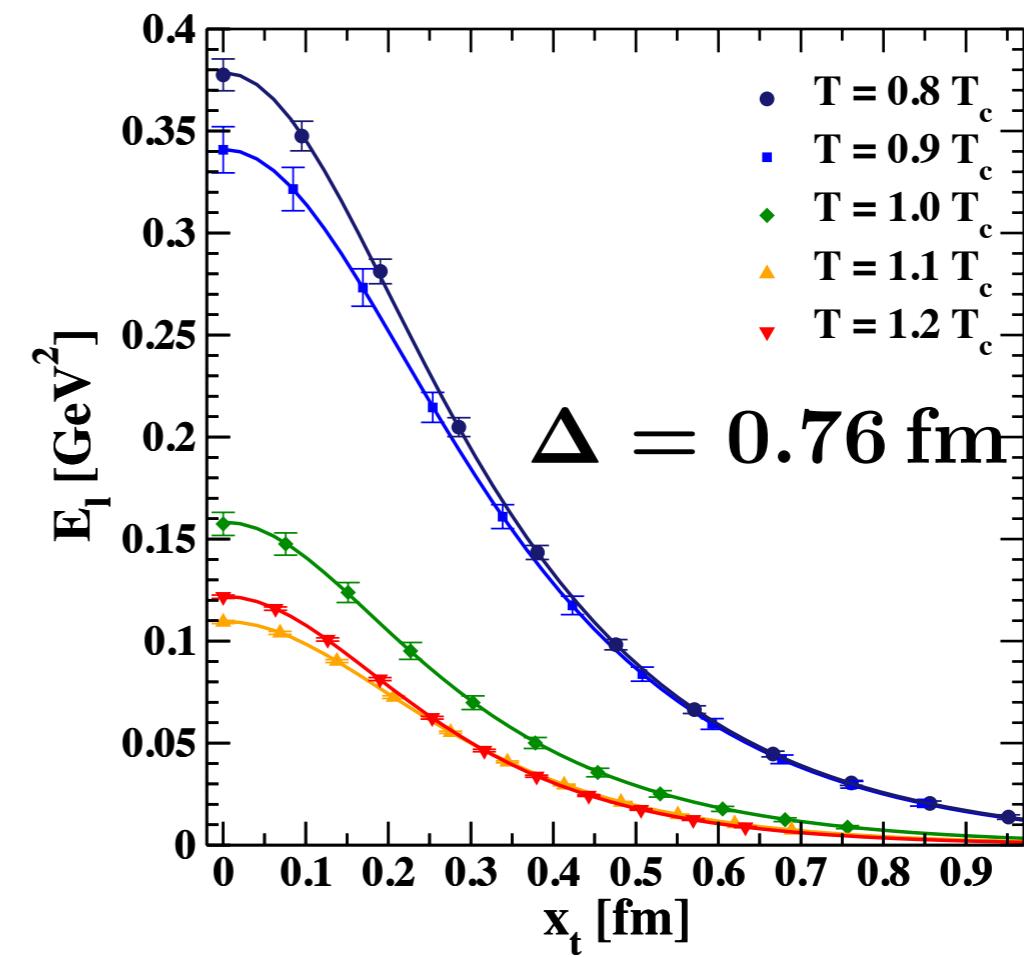
# **BACKUP SLIDES**

# SU(3) pure gauge: flux tubes at finite temperature

scaling test



attenuation of the flux  
across deconfinement (\*)



(\*) could be explained by screening effects.  
Check with larger distances between sources.