Massachusetts Institute of Technology

First Lattice QCD Study of Gluonic Transversity
The 'Exotic Glue' Structure Function

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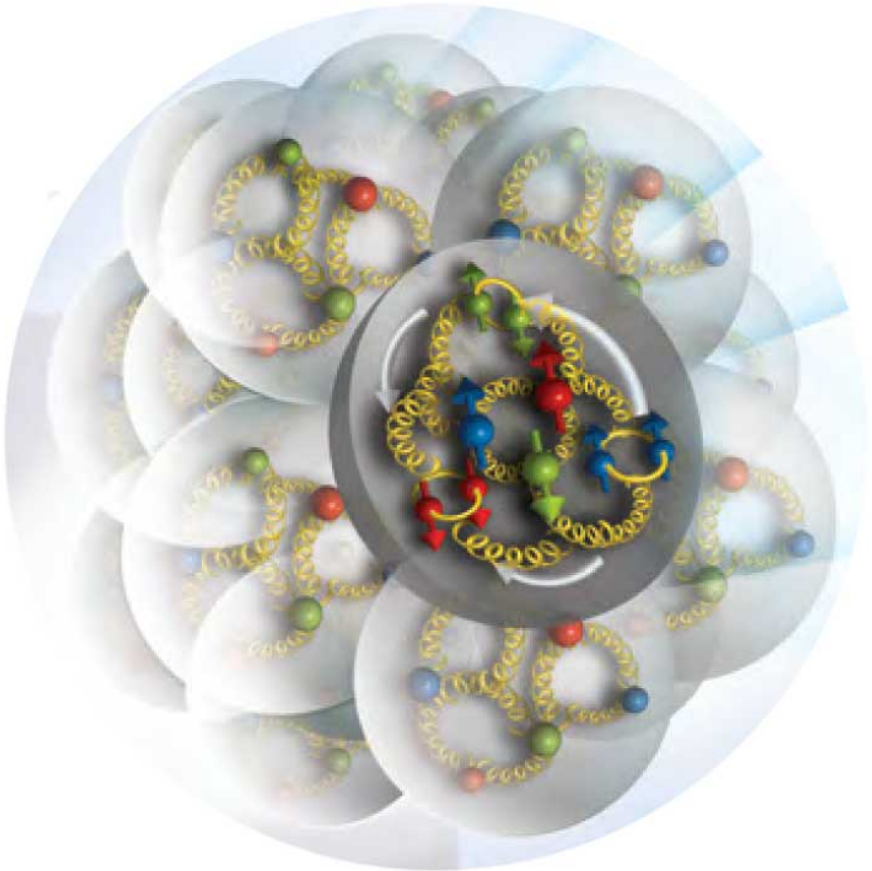
Outline

Motivation


Geometric Scaling
BK/JIMWLK
BFKL
DGLAP
Electron Ion Collider:
The Next QCD Frontier
Understanding the glue
that binds us all

‘Exotic’ Glue in the Nucleus



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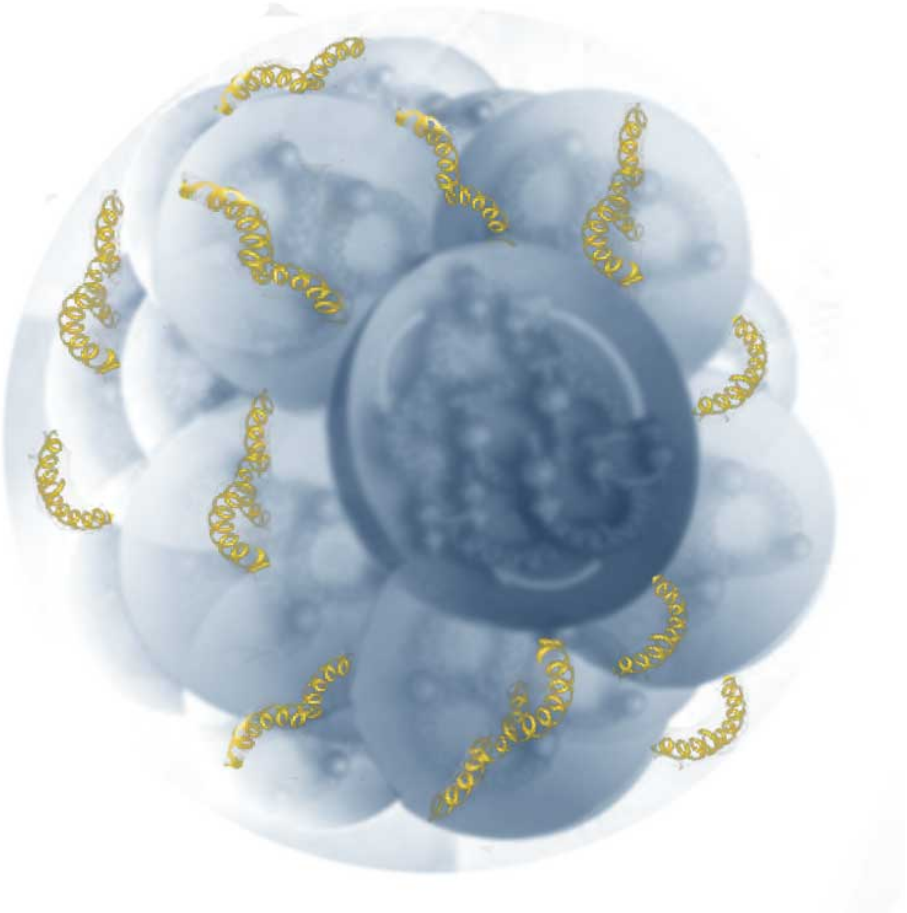


‘Exotic’ Glue

Contributions to gluon observables that are not from nucleon degrees of freedom.

Exotic glue operator:

operator in nucleon $= 0$

operator in nucleus $\neq 0$

Outline

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Jaffe and Manohar (1989)

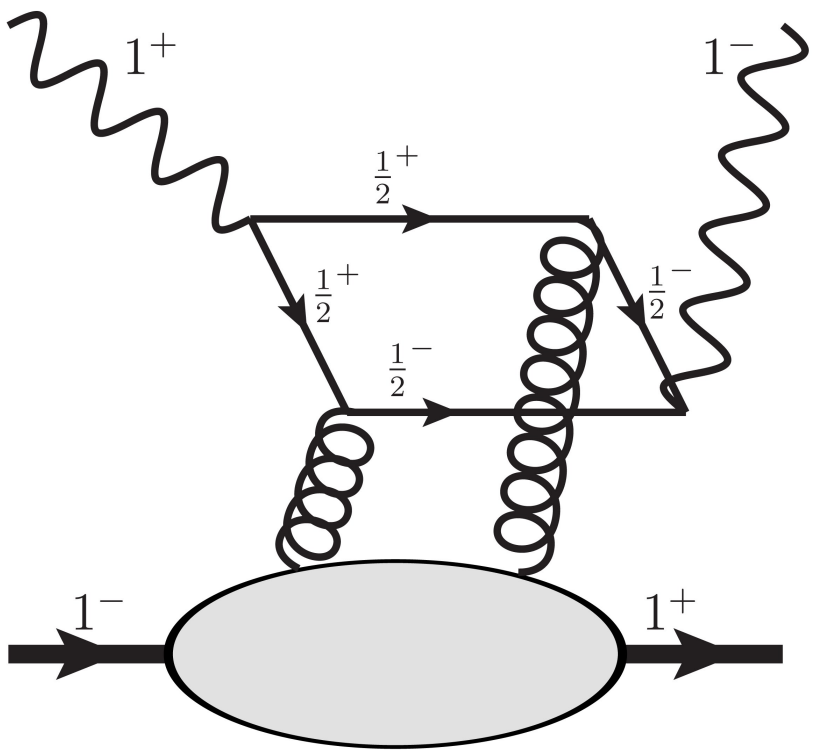
Leading-twist, double-helicity-flipping structure function $\Delta(x, Q^2)$

- Clear signature for exotic glue in nuclei with spin ≥ 1:
  NO analogous twist-2 quark PDF $\rightarrow$ unambiguous
- In single hadrons: gluon transversity structure function
- Experimentally measurable (JLab LOI 2016)
- Moments are calculable on the lattice

First Lattice Study: arXiv:1606.04505 (PRD)

- First moment of $\Delta(x, Q^2)$ in spin-1 ϕ meson

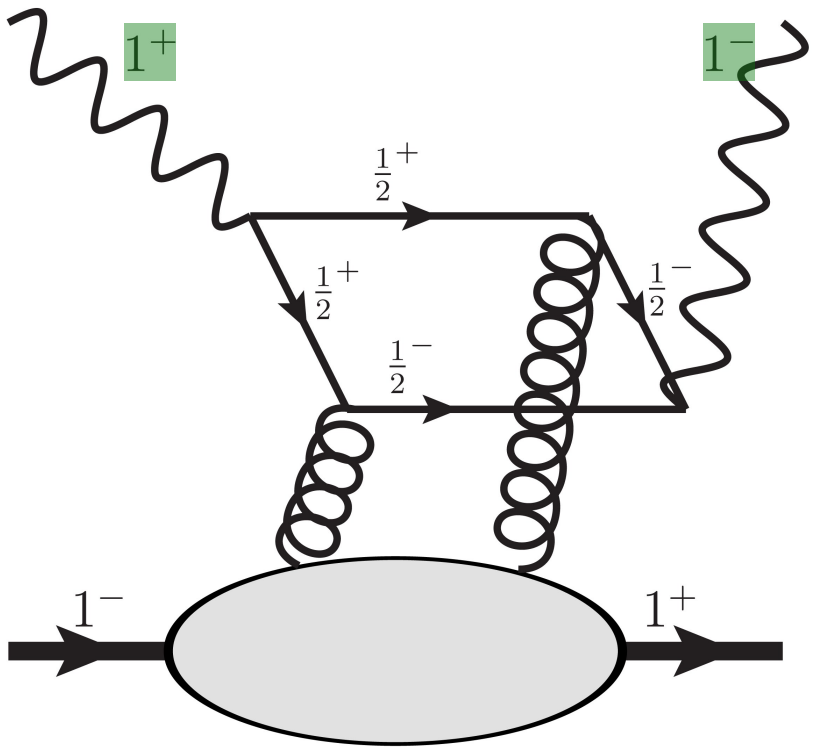
Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$



Double helicity flip amplitude:

$$\Delta(x, Q^2) = A_{+-,-+} = A_{-+,+-}$$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$



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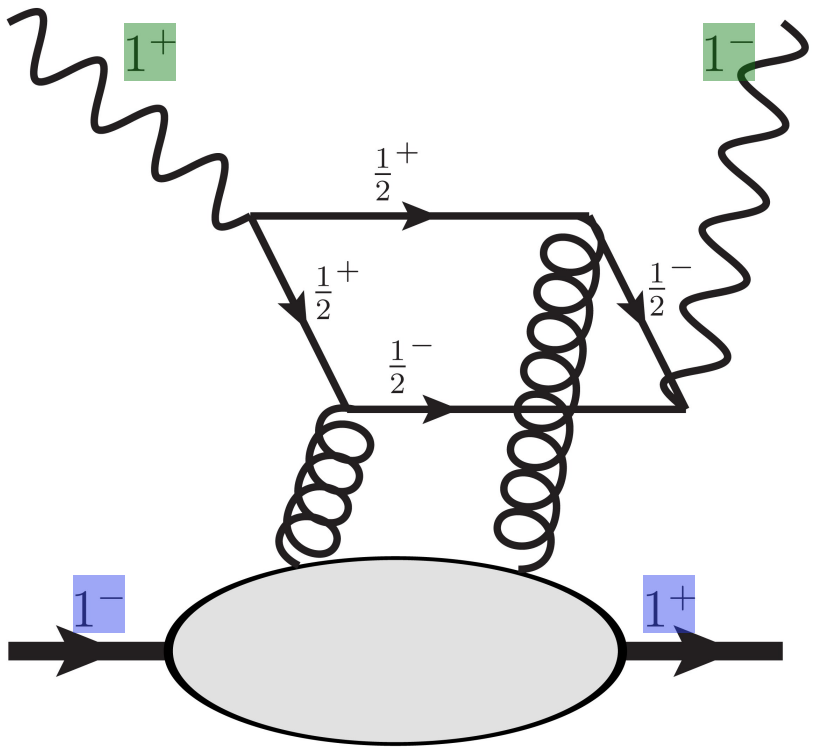

Double helicity flip amplitude:

Photon helicity
Target helicity

$$\Delta(x, Q^2) = A_{+-,-+} = A_{-+,+-}$$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Operator Product Expansion to relate to matrix elements of operator

$$\langle pE'|\mathcal{O}_{\mu\nu\{\mu_1\ldots\mu_n\}} - \text{Tr}|pE\rangle$$

Symmetrize and trace subtract in $\mu_1, \ldots, \mu_n$

$$= (-2i)^{n-2} \underline{S} \left[(p_\mu E'^*_{\mu_1} - p_{\mu_1} E'^*_\mu)(p_\nu E_{\mu_2} - p_{\mu_2} E_\nu) + (\mu \leftrightarrow \nu) \right] p_{\mu_3} \ldots p_{\mu_n} \boxed{A_n(Q^2)} \ldots,$$

Reduced Matrix Element

where

Symmetrize and trace subtract in $\mu_1, \ldots, \mu_n$

$$\boxed{\mathcal{O}_{\mu\nu\mu_1\ldots\mu_n}} = \underline{S} \left[G_{\mu\mu_1} \overleftrightarrow{D}_{\mu_3} \ldots \overleftrightarrow{D}_{\mu_n} G_{\nu\mu_2} \right]$$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

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Optical theorem, dispersion relation for hadronic forward scatt. amplitude, analytic continuation give **moments**:

$$\int_0^1 dx x^{n-1}\Delta(x,Q^2) = \frac{\alpha_s(Q^2)}{3\pi}\frac{A_n(Q^2)}{n+2}, \qquad n = 2, 4, 6\ldots,$$

Unpolarized scattering: symmetric piece of hadronic tensor $W_{\mu\nu}$, $\rightarrow$ even n

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Outline

Lattice Operators

First moment of $\Delta(x, Q^2)$

Matrix elt. of $\mathcal{O}_{\mu\nu\mu_1\mu_2} = S[G_{\mu\mu_1}G_{\nu\mu_2}]$

- Relate $\mathcal{O}_{\mu\nu\mu_1\mu_2}$ to Euclidean operator
- Find linear combs. of Euclidean operator (with different indices) that
  1. Transform irreducibly under appropriate representations of $H(4)$
  2. Don't mix with same or lower-dimensional quark or gluon operators
     - 3 irreps. of dimension 2, 6, 2, i.e., 10 basis vectors
- Lattice simulation of matrix element in ϕ meson (spin-1)

Lattice Details

Luscher-Weisz gauge action with a clover-improved quark action

L/a	T/a	β	am_l	am_s
24	64	6.1	-0.2800	-0.2450
a (fm)	L (fm)	T (fm)	m_π (MeV)	m_K (MeV)
0.1167(16)	2.801(29)	7.469(77)	450(5)	596(6)
m_ϕ (MeV)	$m_\pi L$	$m_\pi T$	N_{cfg}	N_{src}
1040(3)	6.390	17.04	1042	10^5

- All ϕ polarization states ($\{1,2,3\}$ or $\{+,-,0\}$)
  - on-diagonal
  - off-diagonal
- Momenta up to (1,1,1) in lattice units (1 unit $\sim$ 0.4GeV)
- Gradient Flow (100 steps to t=1) or HYP smearing of gauge fields in operator (2-5 steps)

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m_ϕ (MeV)				N_{src}
1040(3)	64	6.590	1042	10^5

SYSTEMATICS IGNORED

- Quark mass effects
- Volume effects
- Discretization effects
- Renormalization (for now)

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  - on-diagonal
  - off-diagonal
- Momenta up to (1,1,1) in lattice units (1 unit $\sim$ 0.4GeV)
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Extraction of A_2

Moment of Structure Function

Reduced Matrix Element

$$\int_0^1 dx x^{n-1} \Delta(x, Q^2) = \frac{\alpha_s(Q^2)}{3\pi} \frac{A_n(Q^2)}{n+2}, \qquad n = 2, 4, 6 \ldots,$$

We calculate on the lattice:

factors of m and p

$$\left[\frac{C^{EE'}_{\text{3pt}}}{C^{EE'}_{\text{2pt}}} \right] (t_{\text{sink}}, \tau) \propto A_2, \qquad 0 \ll \tau \ll t_{\text{sink}}$$

Extraction of A_2: 3pt/2pt ratio

Some choice of irrep. and basis vector

ϕ momentum $(0,0,0)$

$$\begin{array}{c} \\ \rho_0 \\ \rho_+ \\ \rho_- \end{array} \begin{array}{c} \begin{array}{ccc} \rho_0 & \rho_+ & \rho_- \end{array} \\ \begin{pmatrix} \frac{2m^2 A_2}{\sqrt{3}} & 0 & 0 \\ 0 & -\frac{m^2 A_2}{\sqrt{3}} & 0 \\ 0 & 0 & -\frac{m^2 A_2}{\sqrt{3}} \end{pmatrix} \end{array}$$

Extraction of A_2: 3pt/2pt ratio

Some choice of irrep. and basis vector

ϕ momentum (p, p, p)

$$
\begin{array}{c|ccc}
 & \rho_0 & \rho_+ & \rho_- \\
\rho_0 & \frac{2\left(m^3+\sqrt{m^2+3p^2}m^2+4p^2m+2p^2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} & \frac{(1-i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{(1+i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} \\
\rho_+ & \frac{(1+i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{\left(m^3+\sqrt{m^2+3p^2}m^2+4p^2m+2p^2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{2ip^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} \\
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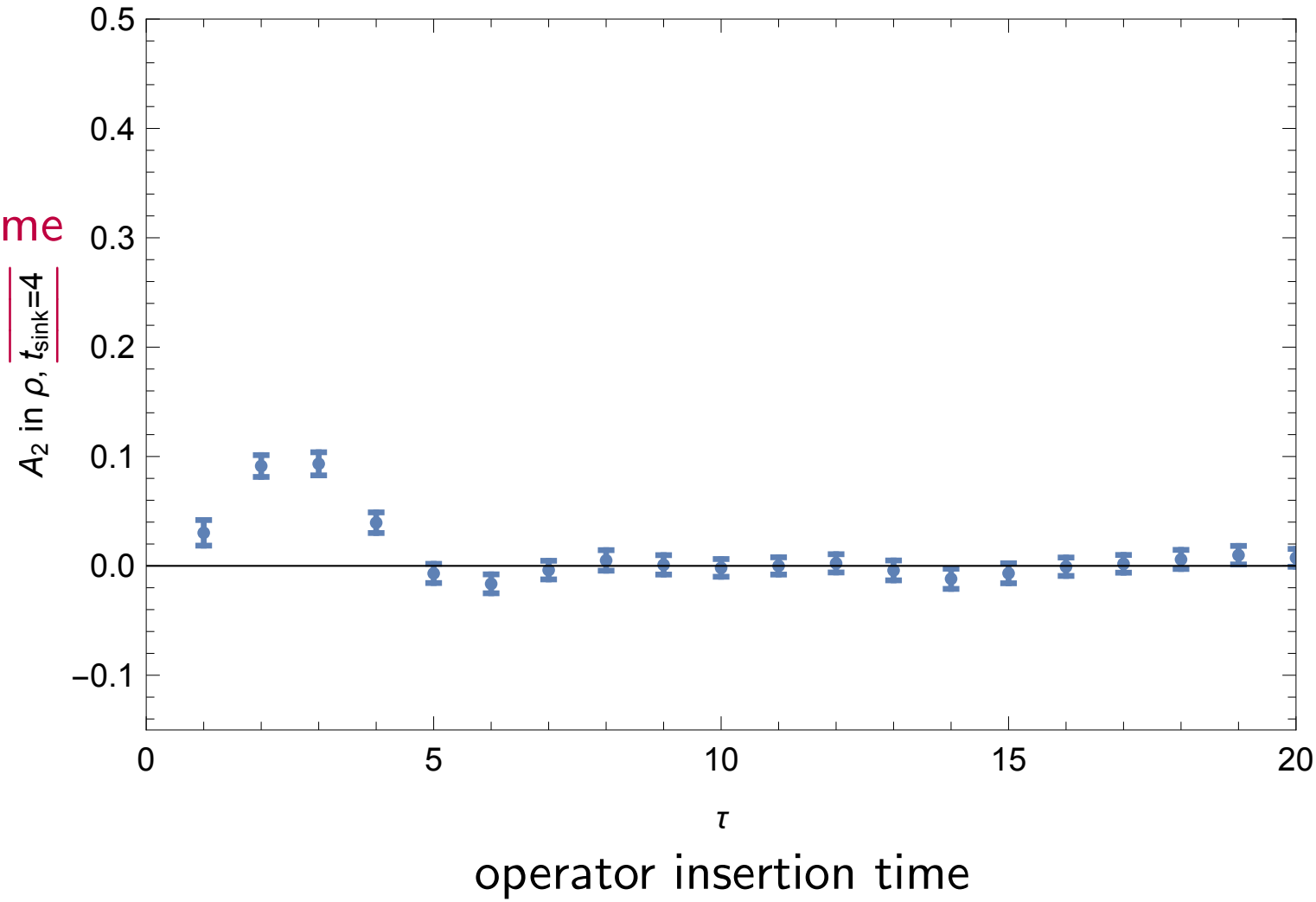
\end{array}
$$

Extraction of A_2: 3pt/2pt ratio

sink time



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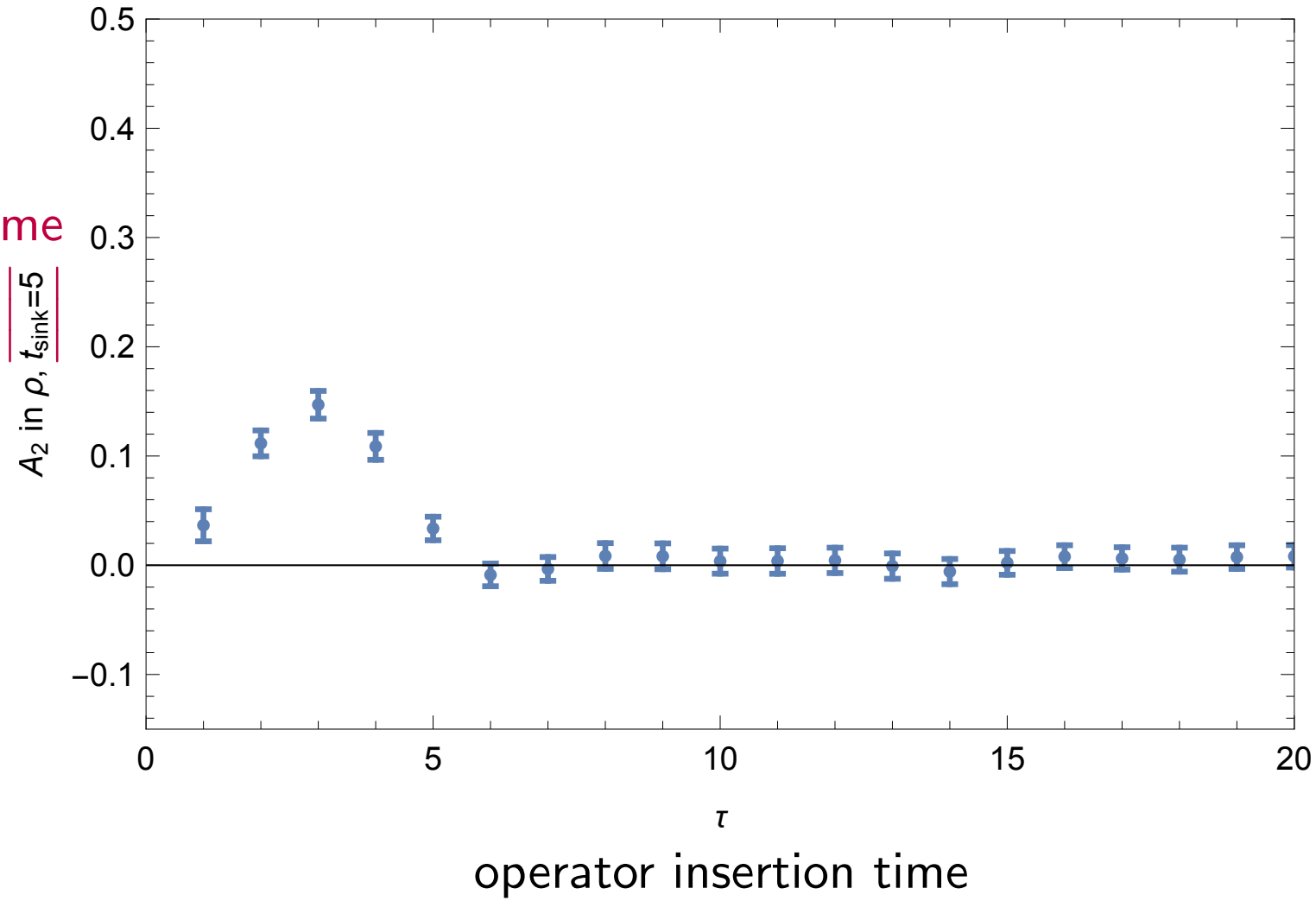

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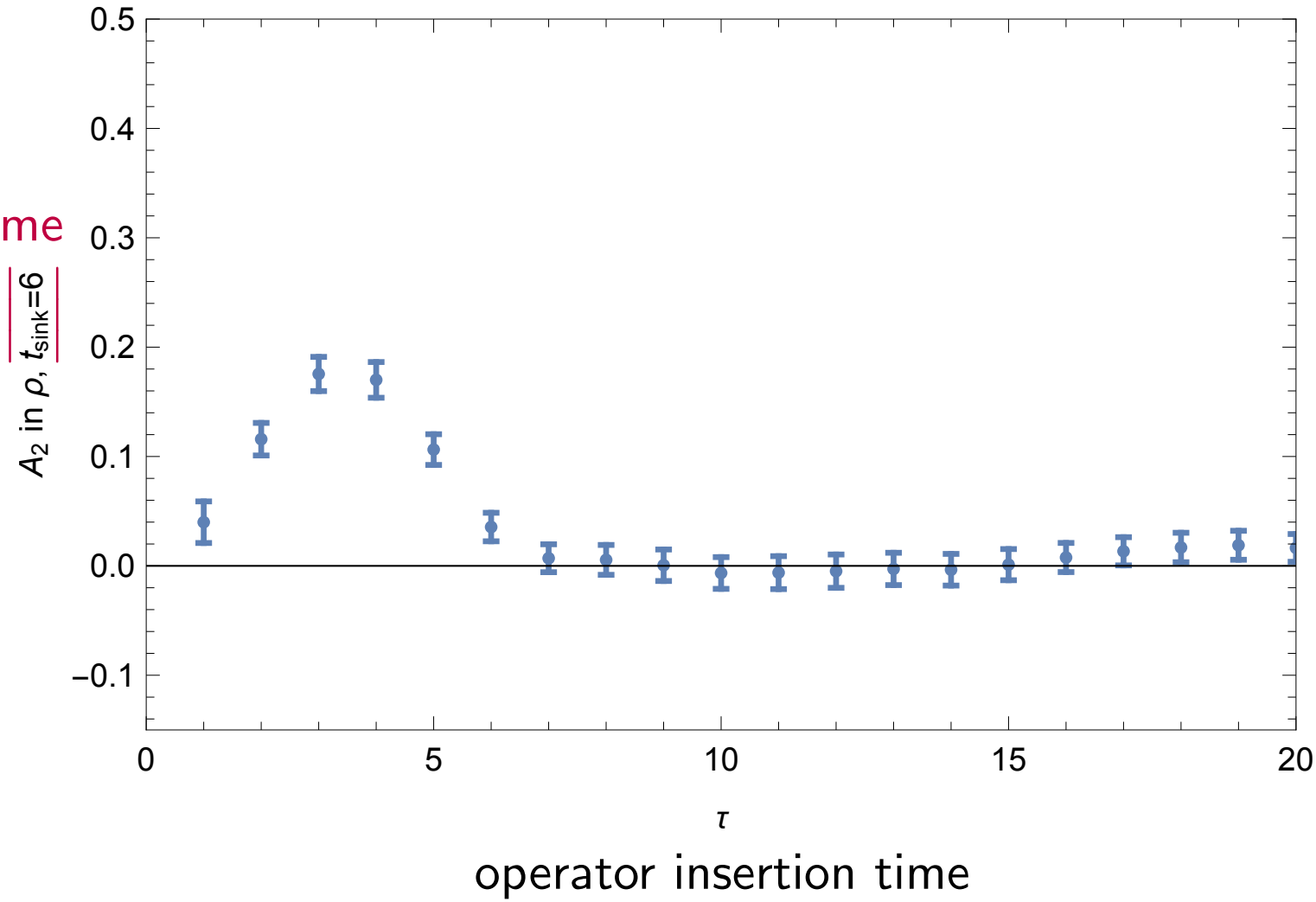

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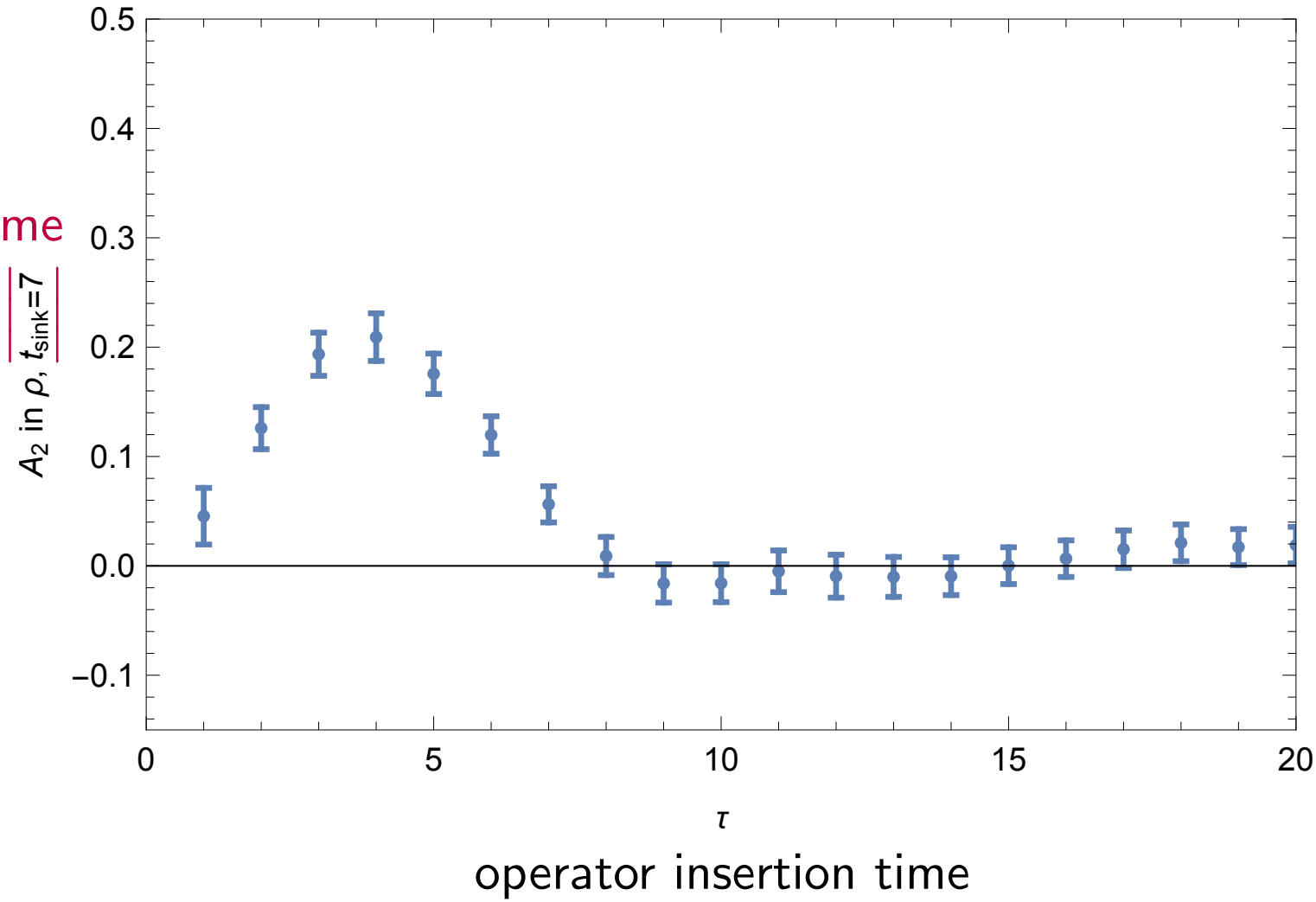

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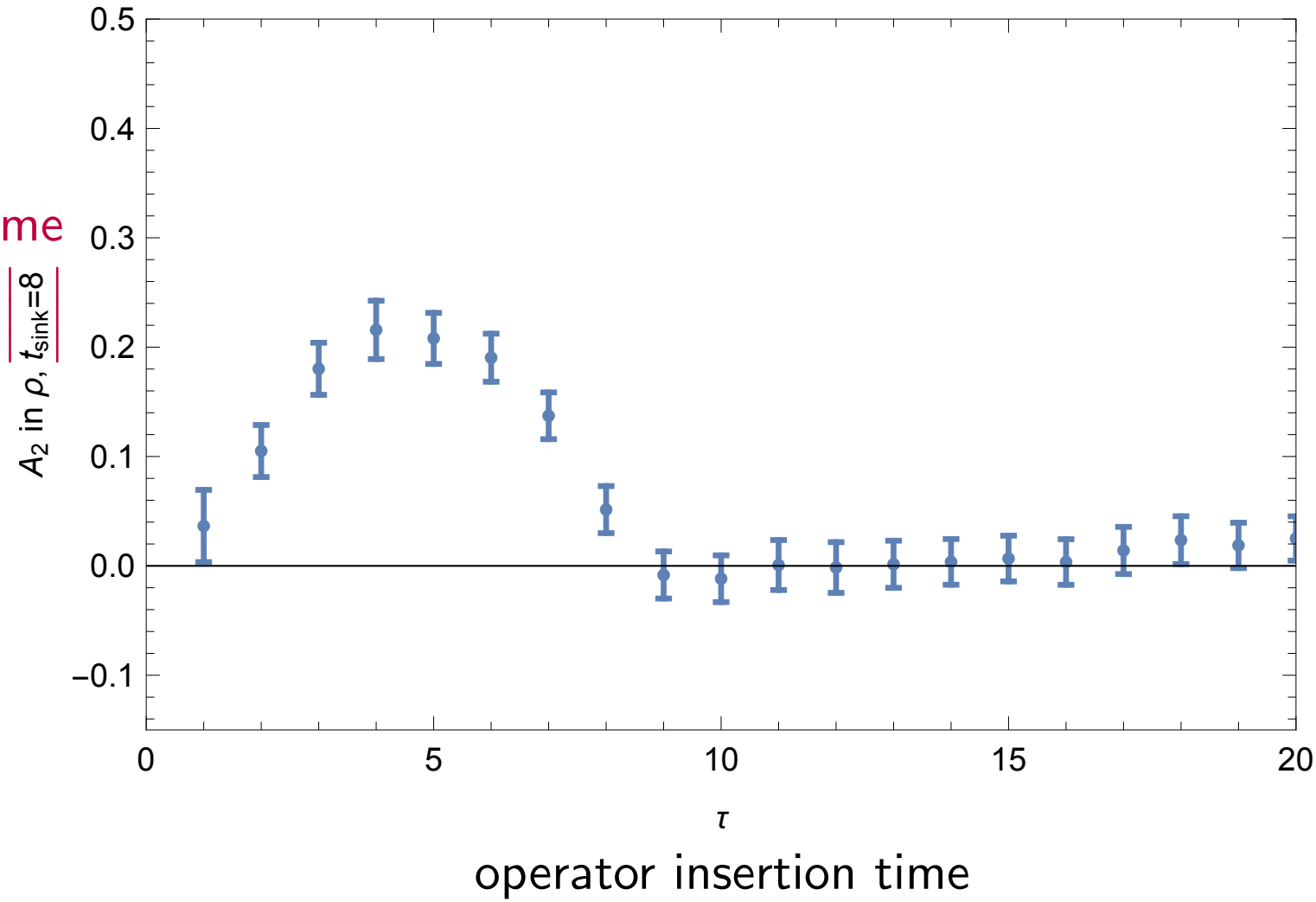

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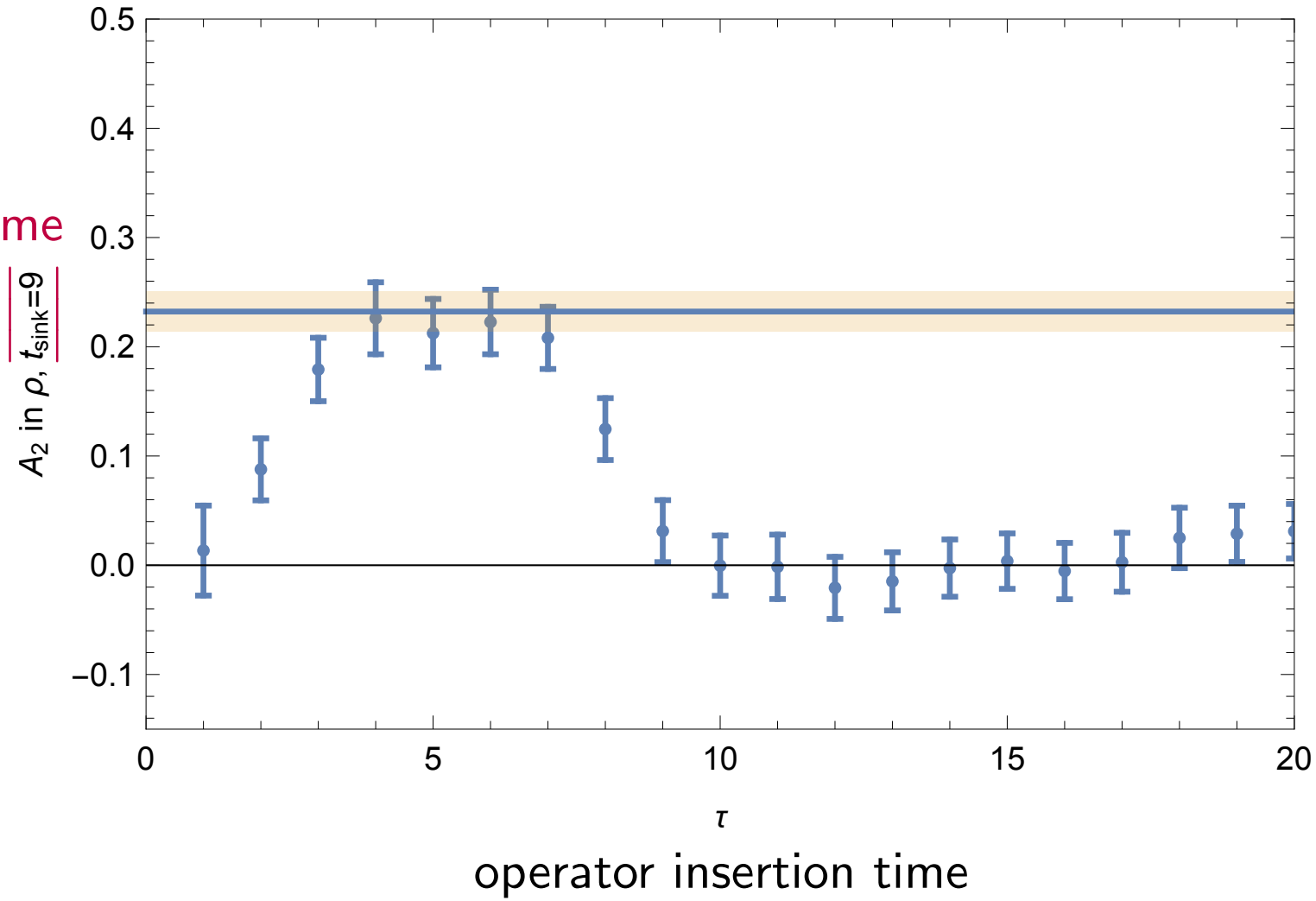

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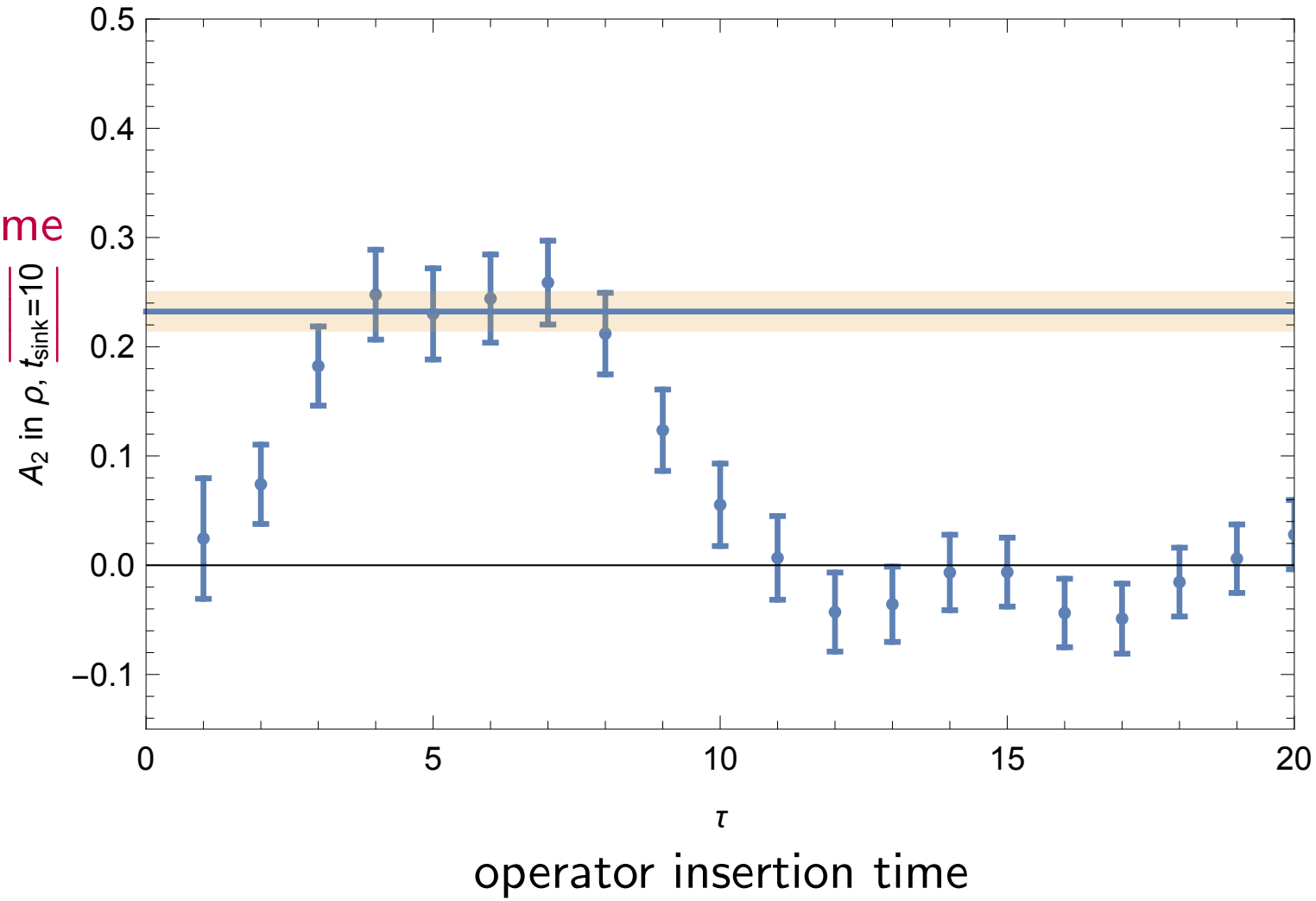

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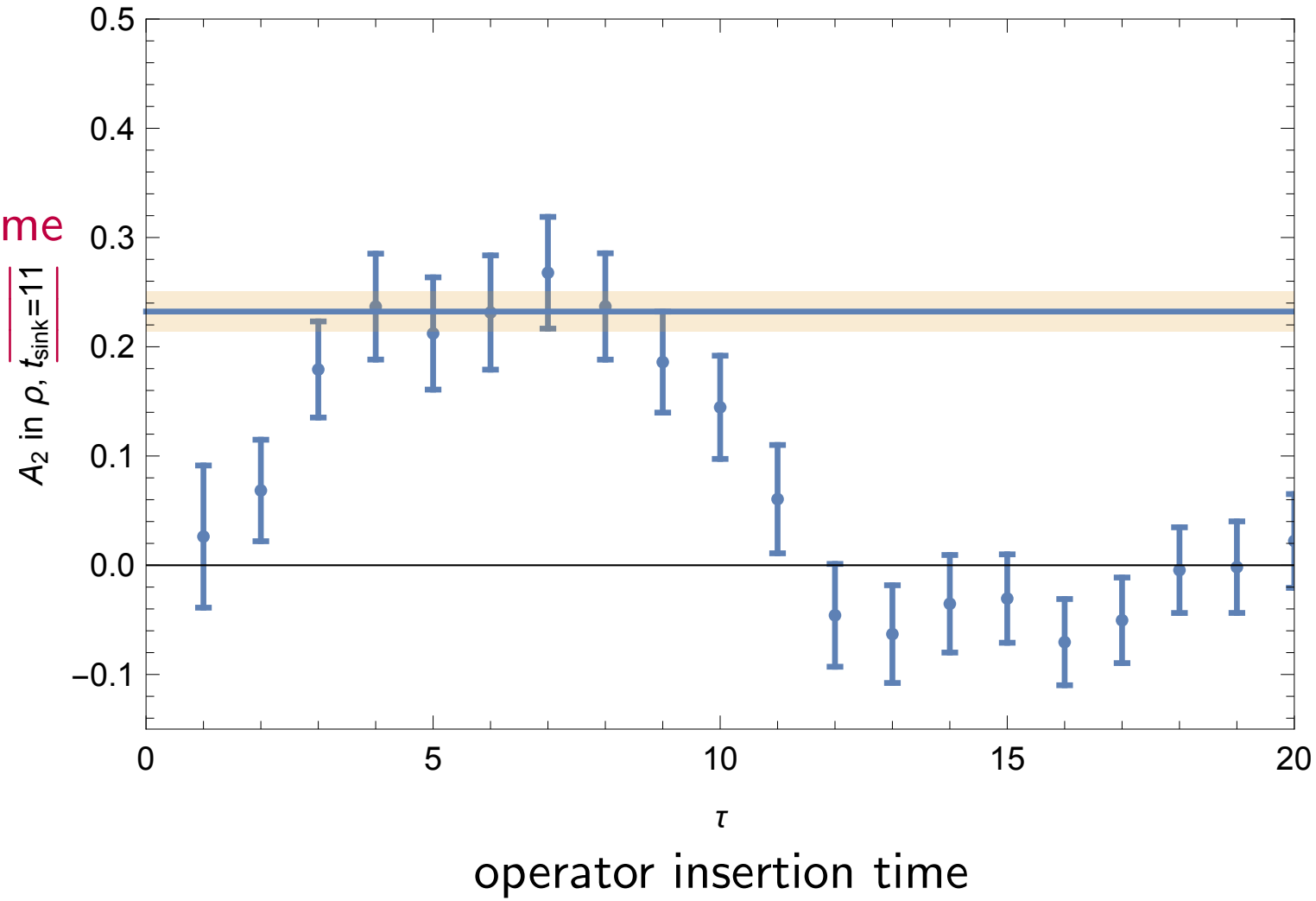

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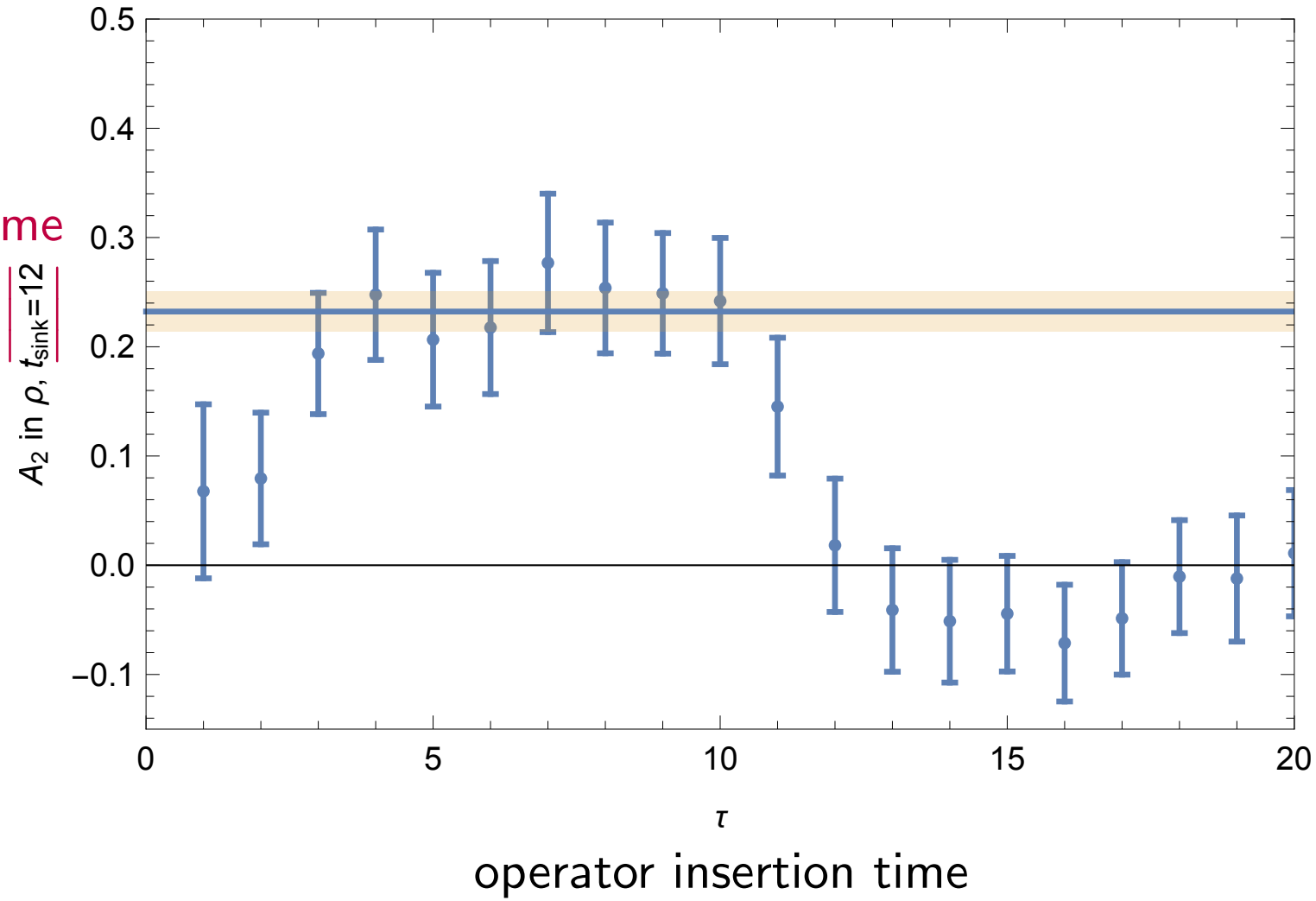

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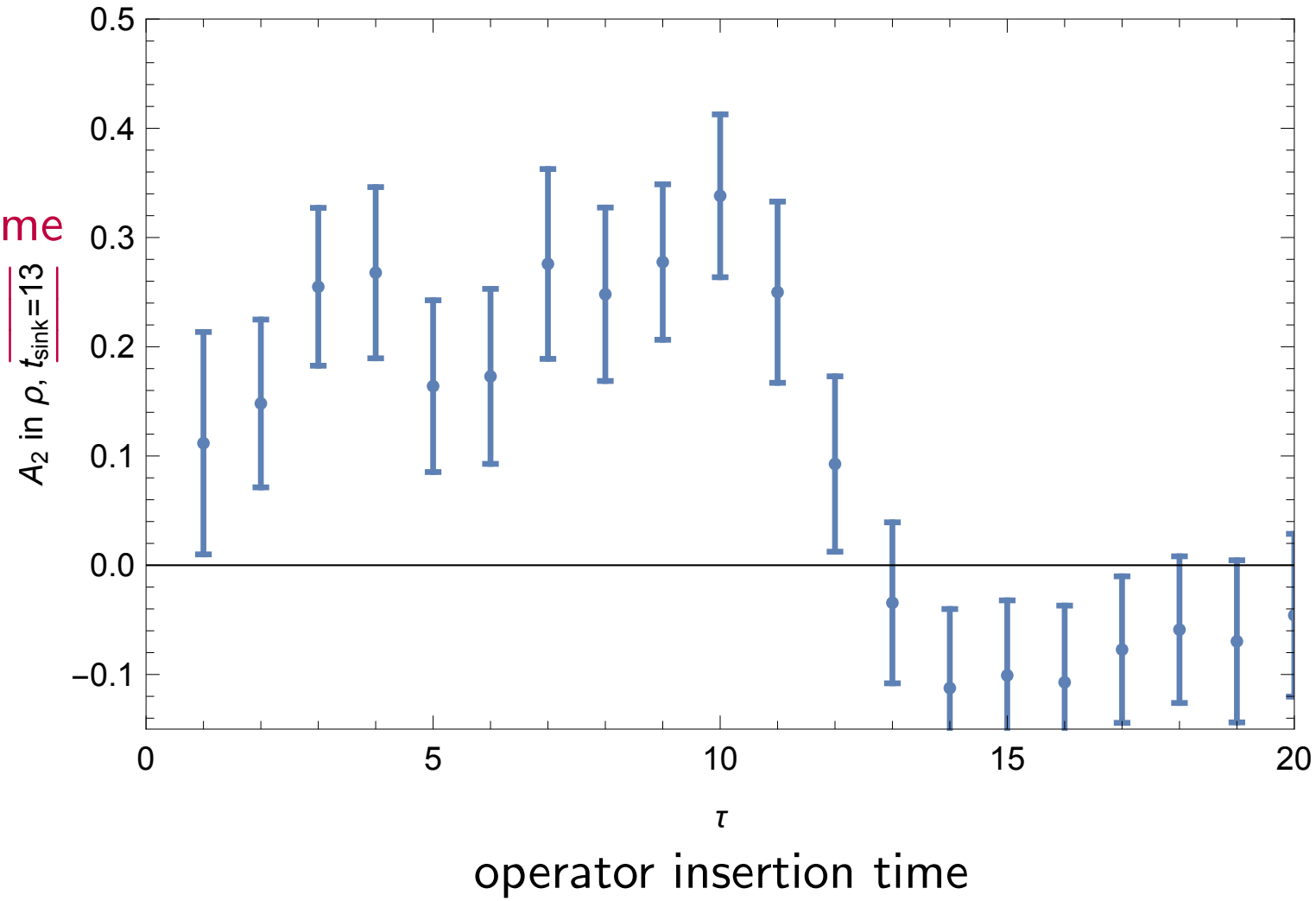

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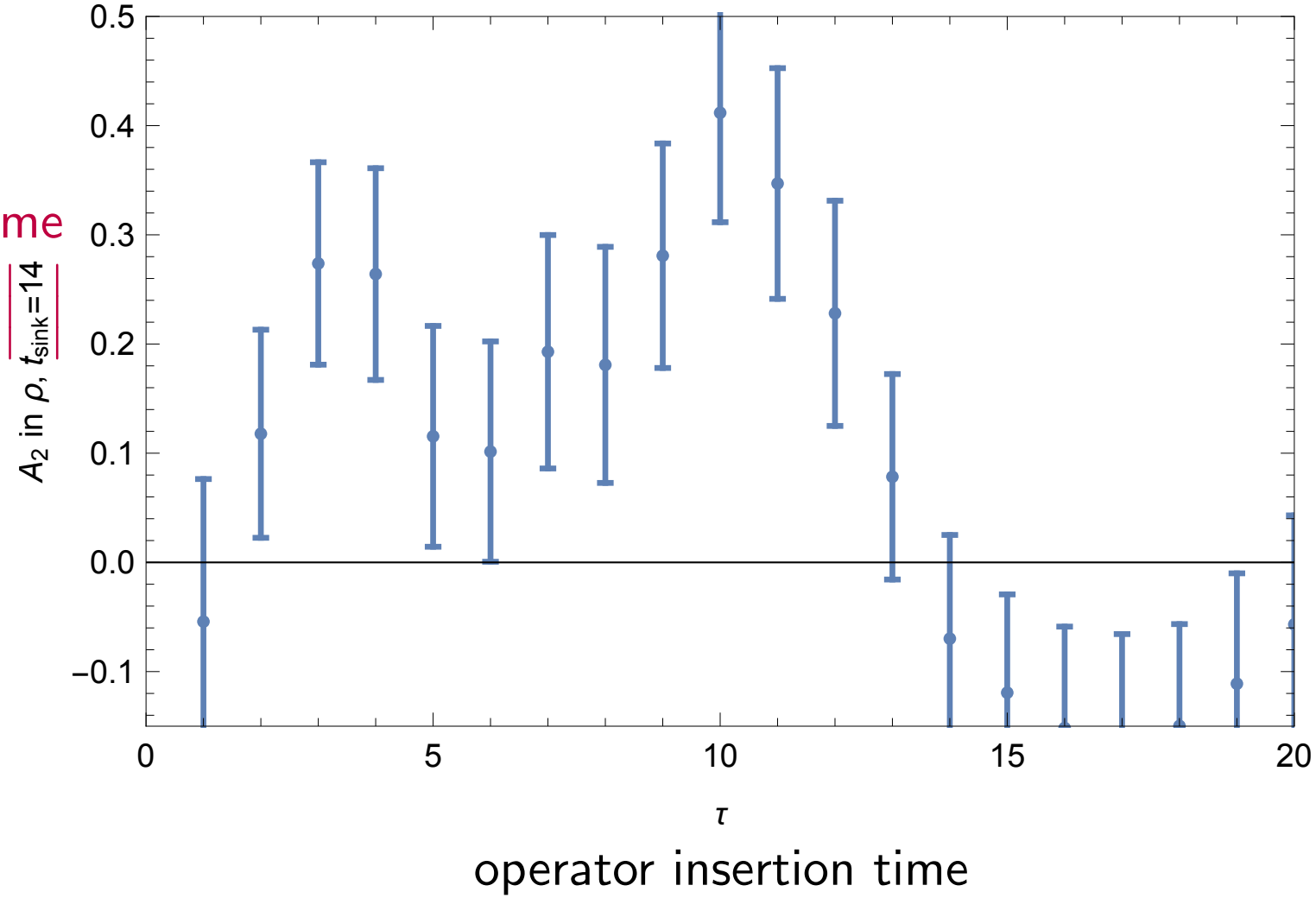
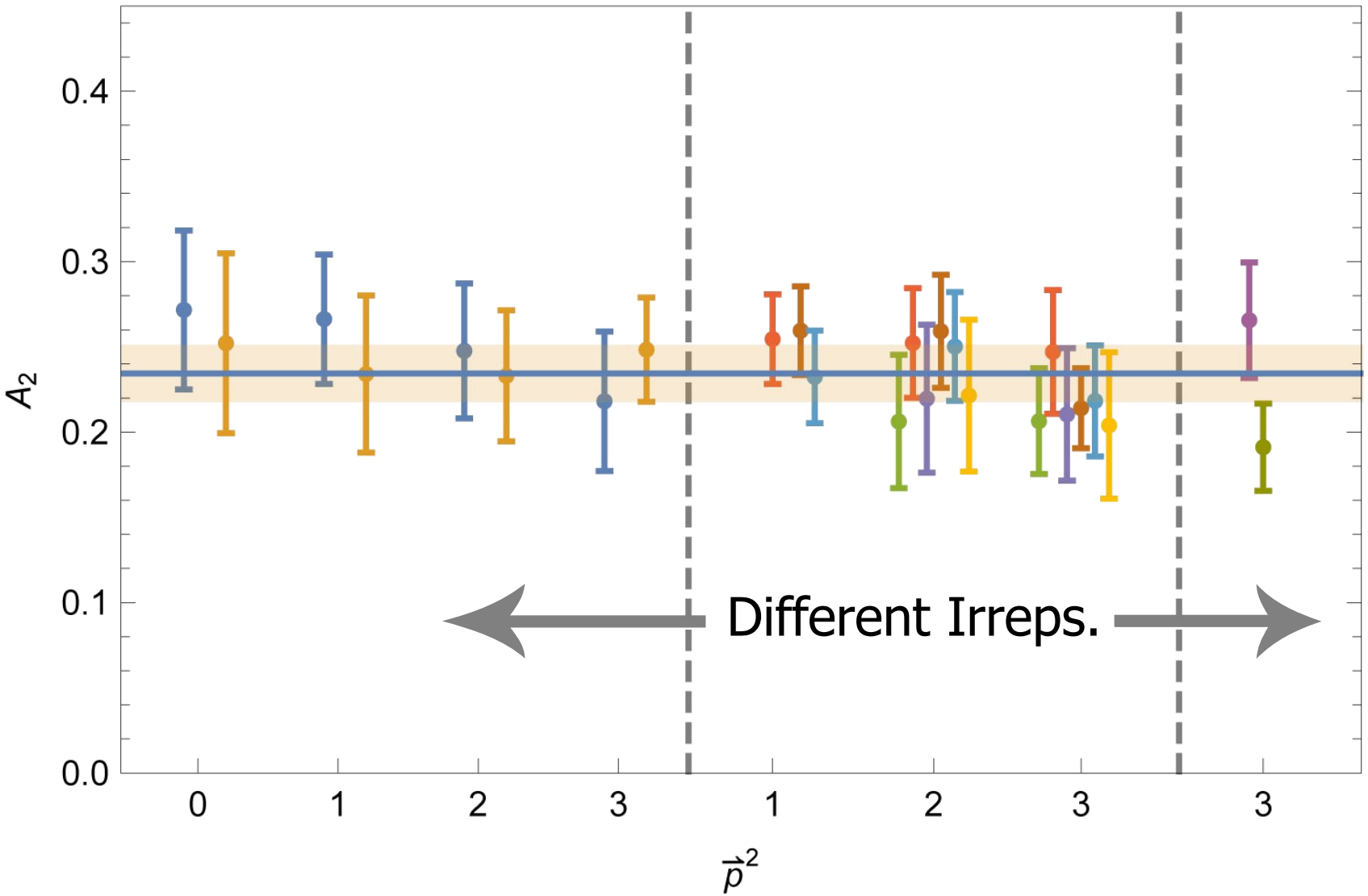


Outline

UNRENORMALISED reduced matrix element: ϕ meson



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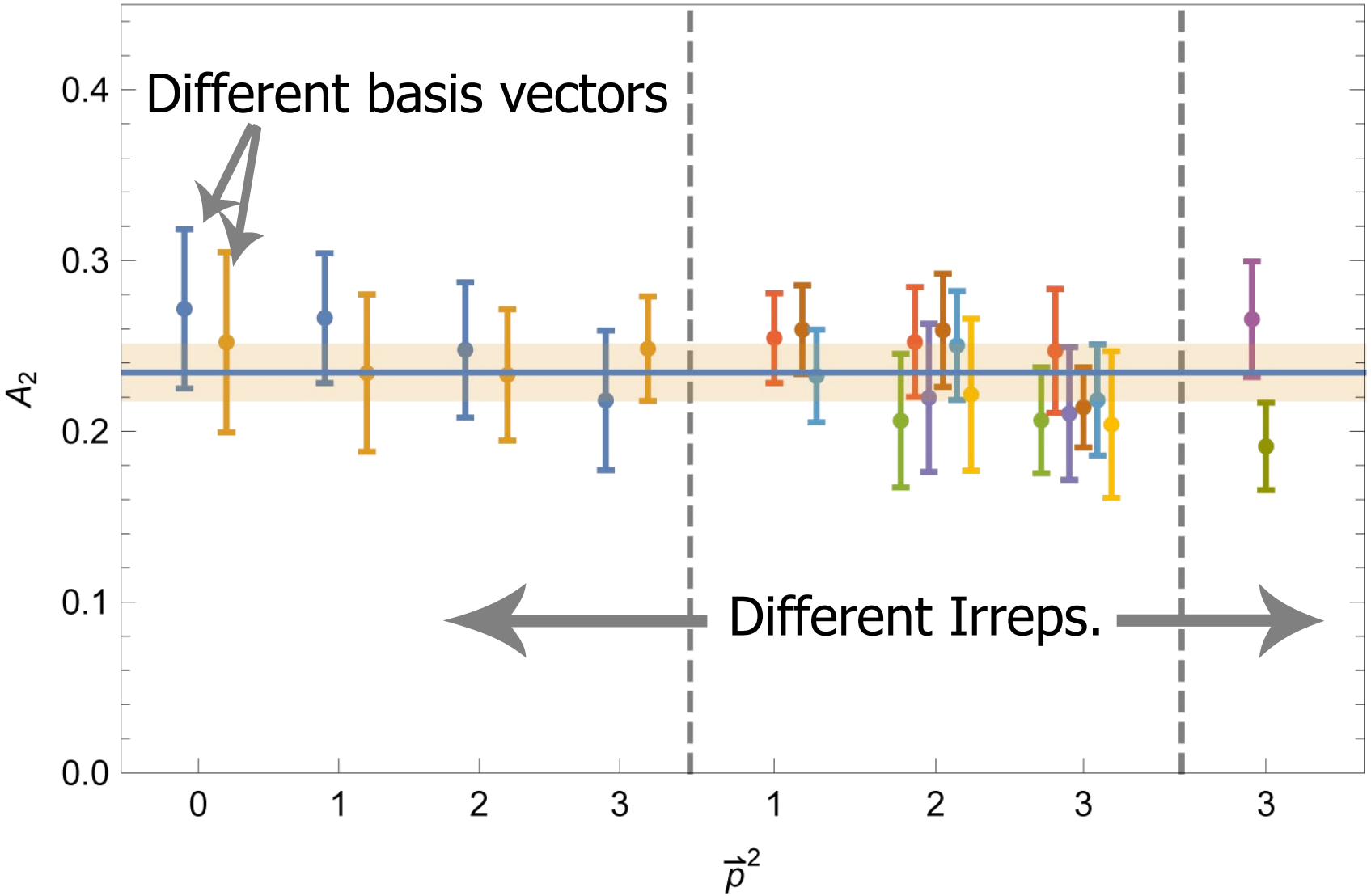

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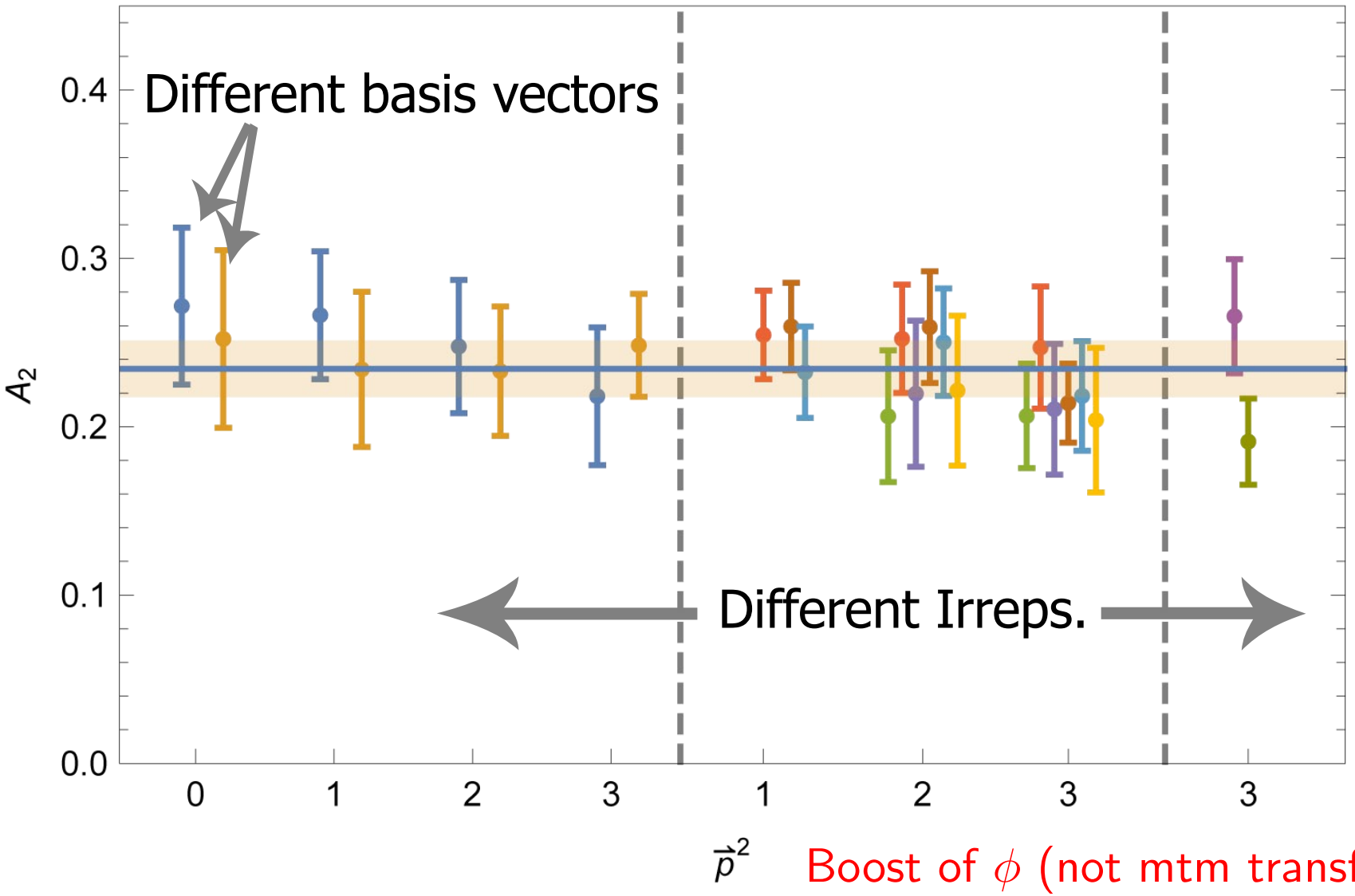

Boost of ϕ (not mtm transfer)

Results: ϕ meson

ROBUST NON-ZERO SIGNAL

Proof of principle: similar signal in a nucleus $\Leftrightarrow$ exotic glue

SYSTEMATICS IGNORED

- Quark mass effects
- Volume effects
- Discretization effects
- Renormalization

Soffer bound analogue

Explore gluon structure of ϕ meson more generally

Soffer bound for transversity quark distributions:

$$|\delta q(x)| \leq \frac{1}{2}\left(q(x) + \Delta q(x)\right)$$

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Transversity

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Spin-independent

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Transversity

Spin-independent

Spin-dependent

Direct analogue for leading moments of gluon distributions:

$$|A_2| \leq \frac{1}{2} B_2$$

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Spin-independent

Spin-dependent

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$G_{\mu\mu_1}G_{\nu\mu_2}$

$G_{\mu_1\alpha}G_{\mu_2}^{\ \ \alpha}$

Soffer bound analogue

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Transversity — Spin-independent — Spin-dependent

Direct analogue for leading moments of gluon distributions:

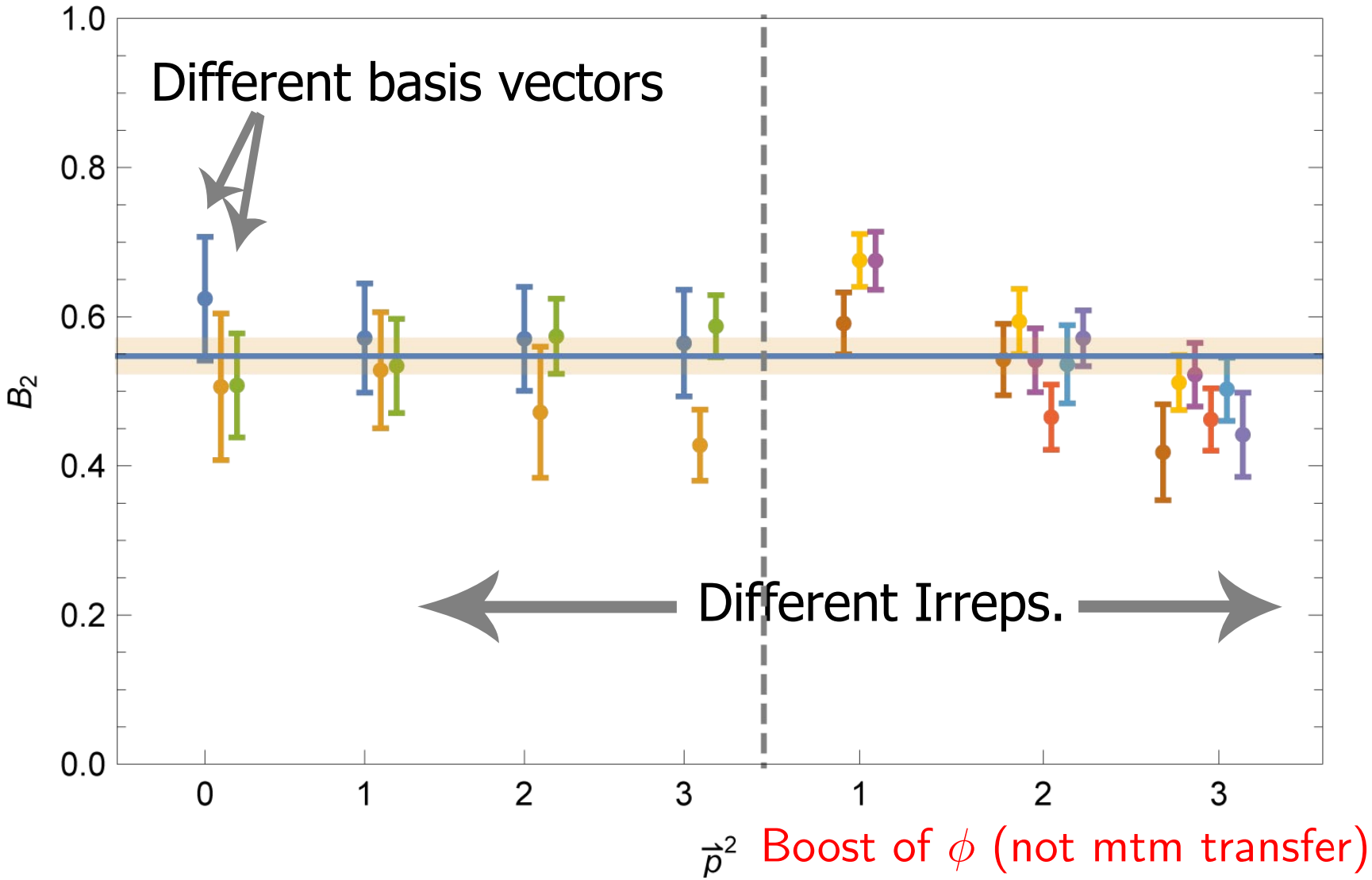
$$|A_2| \leq \frac{1}{2} B_2$$

A_2: $G_{\mu\mu_1} G_{\nu\mu_2}$

B_2: $G_{\mu_1\alpha} G_{\mu_2}^{\ \alpha}$

$$\widetilde{G}_{\mu_1\alpha} G_{\mu_2}^{\ \alpha} \to 0$$

UNRENORMALISED reduced matrix element: ϕ meson



Soffer bound analogue

If we assume approx. the same renormalisation for A_2 and B_2:

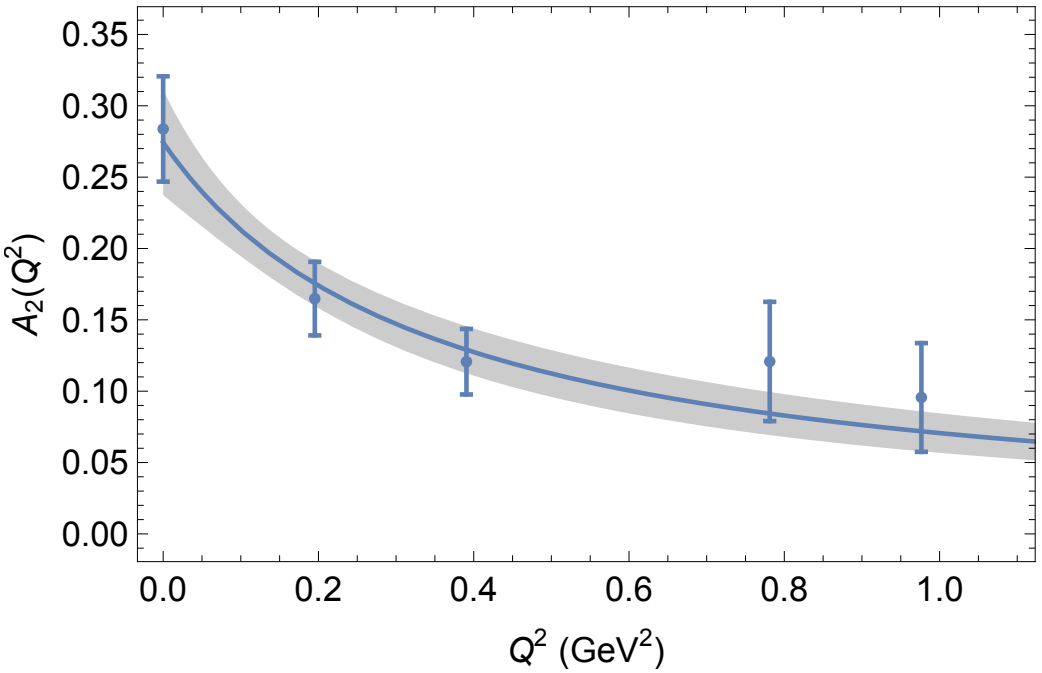
$$G_{\mu\mu_1}G_{\nu\mu_2} \qquad G_{\mu_1\alpha}G_{\mu_2}^{\ \ \alpha}$$

$$\boxed{A_2} \leq \frac{1}{2}\boxed{B_2}$$

$$0.25 \leq \frac{1}{2}\ 0.6$$

First two moments of quark distributions: Soffer bound saturated to 80% (lattice QCD, Diehl *et al.* 2005)

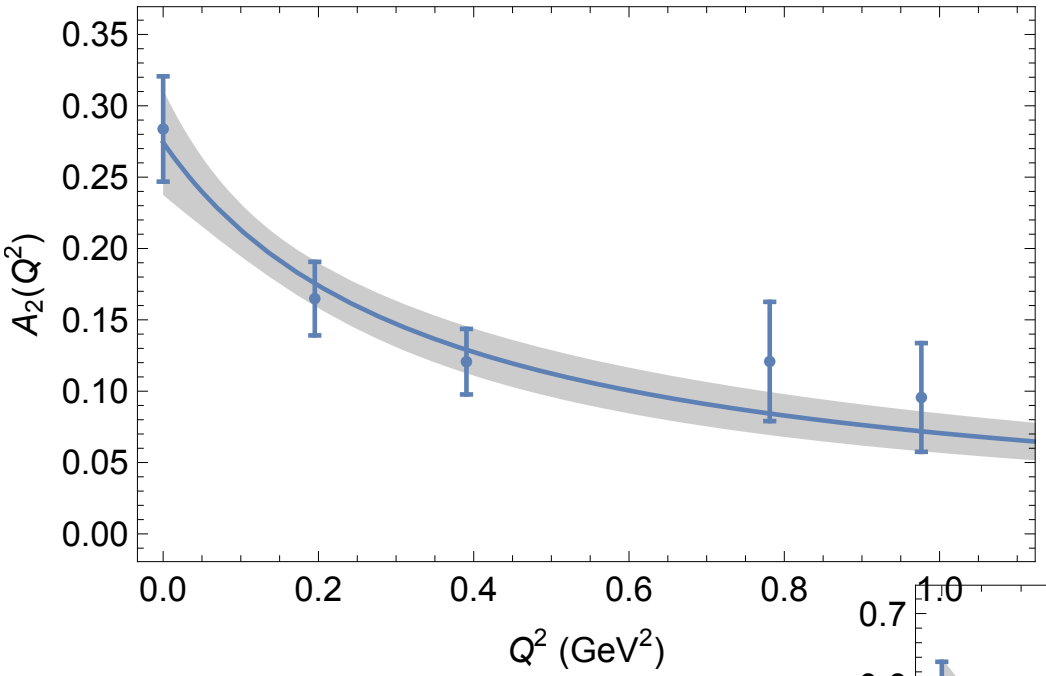
Gluonic Form Factors and Radii: PRELIMINARY



Gluonic Transversity

$\langle r^2 \rangle \approx 0.7$ fm^2

Gluonic Form Factors and Radii: PRELIMINARY

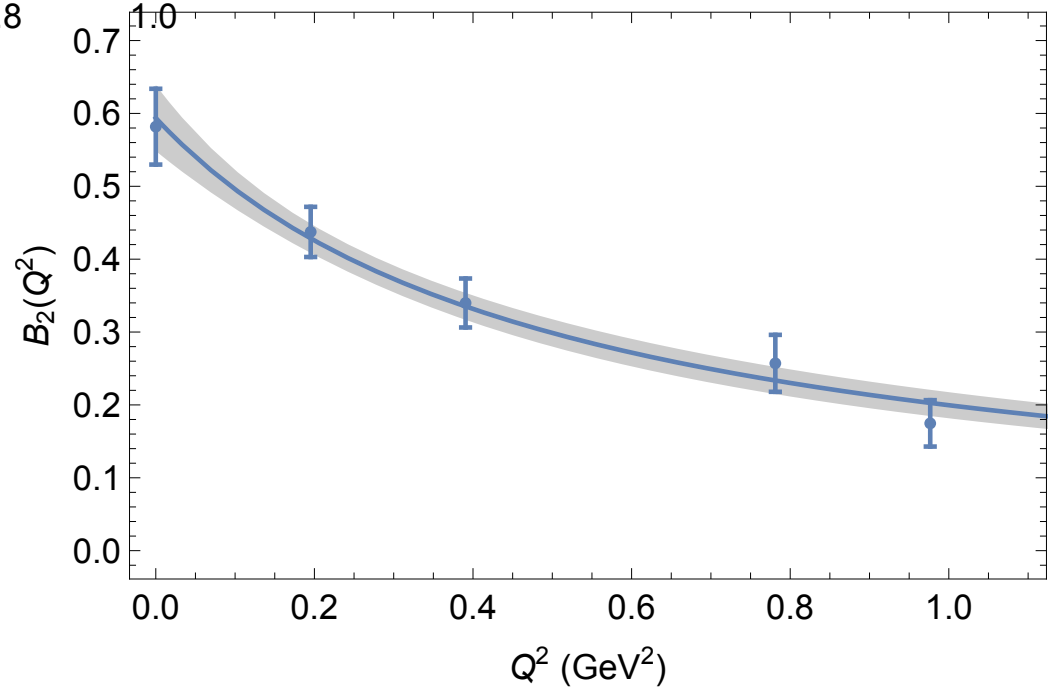


Gluonic Transversity

$\langle r^2 \rangle \approx 0.7$ fm^2

Gluonic Spin-Indep

$\langle r^2 \rangle \approx 0.5$ fm^2



Outline

Summary

ROBUST NON-ZERO signal
for 'exotic glue' operator in the ϕ meson

Proof of principle: similar signal in a nucleus $\Leftrightarrow$ exotic glue

Explore gluon structure of hadrons more generally
e.g., Soffer bound analogue

BUT: SYSTEMATICS IGNORED
$\Rightarrow$ no physically meaningful number (yet)