

Temperature dependence of shear viscosity in $SU(3)$ -gluodynamics

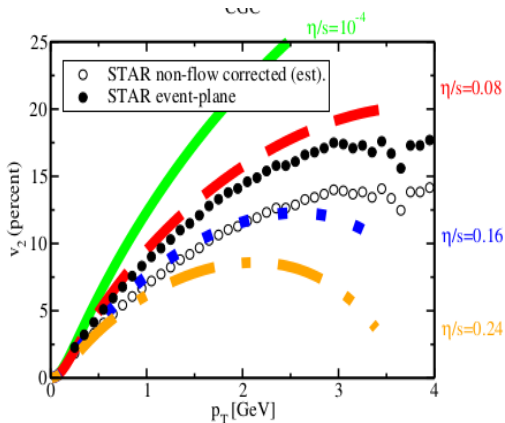
N.Yu. Astrakhantsev, V.V. Braguta, A.Yu. Kotov

ITEP

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Outline:

- Introduction
- Details of the calculation
- Fitting of the data
- Backus-Gilbert method
- Conclusion

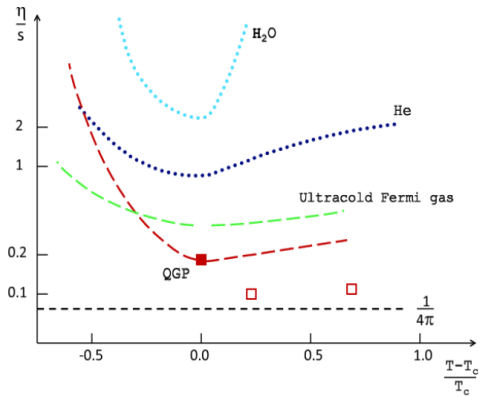


Elliptic flow from STAR experiment (Nucl. Phys. A 757, 102 (2005))

$$\frac{dN}{d\phi} \sim (1 + 2v_1 \cos(\phi) + 2v_2 \cos^2(\phi)), \phi\text{-scattering angle}$$

Quark-gluon plasma is close to ideal liquid ($\frac{\eta}{s} = (1 - 3) \frac{1}{4\pi}$)

M. Luzum and P. Romatschke, Phys. Rev. C 78, 034915 (2008)



S.Cremonini, U.Gursoy, P.Szepietowski, JHEP 1208 (2012) 167

Comparison of different liquids

Quark-gluon plasma is the most ideal liquid

Other works (SU(3) gluodynamics):

- Karsch, F. et al. Phys.Rev. D35 (1987)
- A. Nakamura, S. Sakai Phys. Rev. Lett. 94, 072305 (2005)
- H. B. Meyer, Phys.Rev. D76 (2007) 101701
- H. B. Meyer, Nucl.Phys. A830 (2009) 641C-648C
- **Next talk: Dr. Attila PASZTOR**

Results:

- $\frac{\eta}{s} = 0.134 \pm 0.033$ ($T/T_c = 1.65, 8 \times 28^3$)
- $\frac{\eta}{s} = 0.102 \pm 0.056$ ($T/T_c = 1.24, 8 \times 28^3$)
- $\frac{\eta}{s} = 0.20 \pm 0.03$ ($T/T_c = 1.58, 16 \times 48^3$)
- $\frac{\eta}{s} = 0.26 \pm 0.03$ ($T/T_c = 2.32, 16 \times 48^3$)

SU(2) gluodynamics:

- $\frac{\eta}{s} = 0.134 \pm 0.057$ ($T/T_c = 1.2, 16 \times 32^3$)

N.Yu. Astrakhantsev, V.V. Braguta, A.Yu. Kotov, JHEP 1509 (2015) 082

Lattice calculation of shear viscosity

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Measurement of the correlation function:

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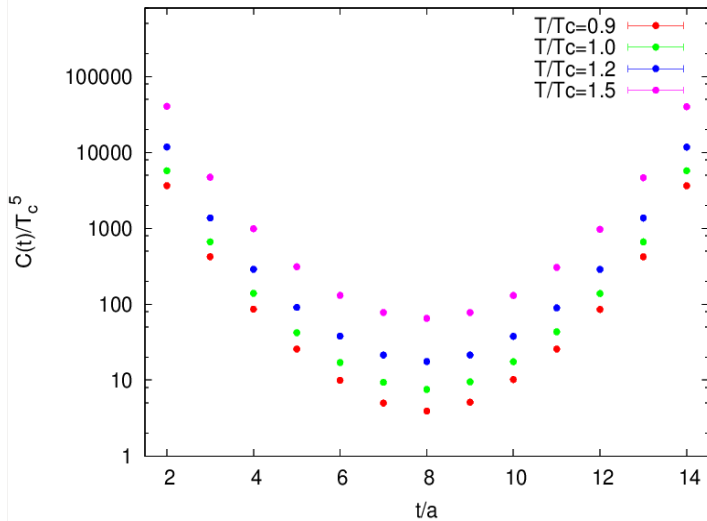
Calculation of the spectral function $\rho(\omega)$:

$$C(t) = \int_0^\infty d\omega \rho(\omega) \frac{\text{ch}\left(\frac{\omega}{2T} - \omega t\right)}{\text{sh}\left(\frac{\omega}{2T}\right)}$$
$$\eta = \pi \lim_{\omega \rightarrow 0} \frac{\rho(\omega)}{\omega}$$

Details of the calculation

- SU(3) gluodynamics
- Two-level algorithm
- Lattice size $32^3 \times 16$
- Temperatures $T/T_c = 0.9, 0.925, 0.95, 1.0, 1.2, 1.35, 1.5$
- Accuracy $\sim 2 - 3\%$ at $t = \frac{1}{2T}$
- $\langle T_{12}(x) T_{12}(y) \rangle \sim (\langle T_{11}(x) T_{11}(y) \rangle - \langle T_{11}(x) T_{22}(y) \rangle)$
- Clover discretization for the $\hat{F}_{\mu\nu}$
- Renormalization of EMT: F. Karsch, Nucl.Phys. B205 (1982) 285-300
- ...

Correlation functions



Spectral function

$$C(t) = \int_0^\infty d\omega \rho(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega t\right)}{sh\left(\frac{\omega}{2T}\right)}$$

Properties of the spectral function:

- $\rho(\omega) \geq 0$, $\rho(-\omega) = -\rho(\omega)$
- Asymptotic freedom: $\rho(\omega)|_{\omega \rightarrow \infty}^{NLO} = \frac{1}{10} \frac{d_A}{(4\pi)^2} \omega^4 \left(1 - \frac{5N_c \alpha_s}{9\pi}\right)$
~ 90% of the total contribution $t = 1/2/T$
- Hydrodynamics: $\rho(\omega)|_{\omega \rightarrow 0} = \frac{\eta}{\pi} \omega$

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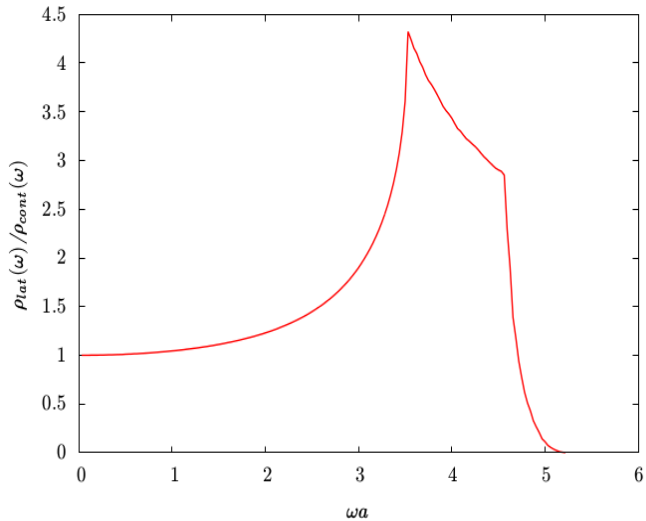
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Ansatz for the spectral function (QCD sum rules motivation)

$$\rho(\omega) = \frac{\eta}{\pi} \omega \theta(\omega_0 - \omega) + A \rho_{lat}(\omega) \theta(\omega - \omega_0)$$

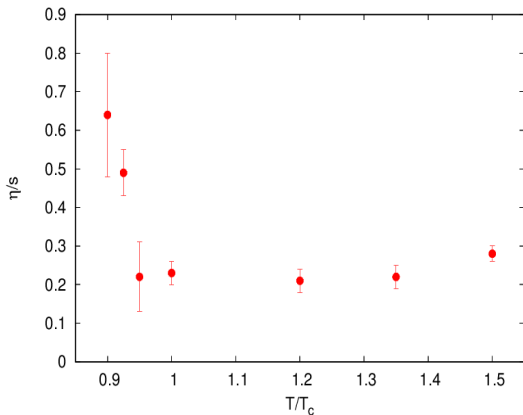
Lattice spectral function ρ_{lat}



Spectral function

$$\rho(\omega) = \frac{\eta}{\pi} \omega \theta(\omega_0 - \omega) + A \rho_{lat}(\omega) \theta(\omega - \omega_0)$$

$$\chi^2/dof \sim 1, \quad A \sim 1, \quad \omega_0/T \sim 7 - 8$$



Properties of the spectral function

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- Hydrodynamical approximation works well up to $\omega < \pi T \sim 1 \text{ GeV}$ (H.B. Meyer, arXiv:0809.5202)
- Asymptotic freedom works well from $\omega > 3 \text{ GeV}$
- Poor knowledge of the spectral function in the region $\omega \in (1, 3) \text{ GeV}$
 \Rightarrow Main source of uncertainty in the fitting procedure

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Conclusion

Accuracy improvement leads to improvement of our knowledge about UV part of spectral function but not about IR one

Backus-Gilbert method for the spectral function

- Problem: find $f(\omega)$ from the integral equation

$$C(x_i) = \int_0^\infty d\omega f(\omega) K(x_i, \omega), \quad K(x_i, \omega) = \frac{\text{ch}\left(\frac{\omega}{2T} - \omega x_i\right)}{\text{sh}\left(\frac{\omega}{2T}\right)}$$

- Define an estimator $\tilde{f}(\bar{\omega})$ ($\delta(\bar{\omega}, \omega)$ - resolution function):

$$\tilde{f}(\bar{\omega}) = \int_0^\infty d\omega \hat{\delta}(\bar{\omega}, \omega) f(\omega)$$

- Let us expand $\delta(\bar{\omega}, \omega)$ as

$$\delta(\bar{\omega}, \omega) = \sum_i b_i(\bar{\omega}) K(x_i, \omega) \quad \tilde{f}(\bar{\omega}) = \sum_i b_i(\bar{\omega}) C(x_i)$$

- Goal: minimize the width of the resolution function

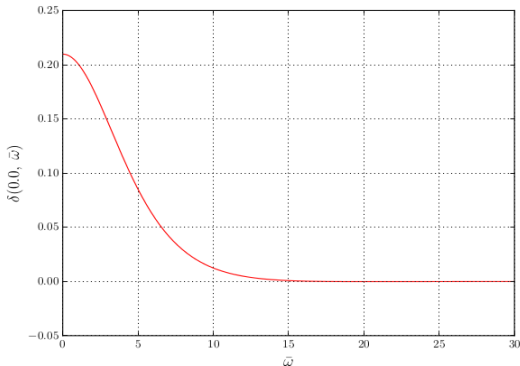
$$b_i(\bar{\omega}) = \frac{\sum_j W_{ij}^{-1} R_j}{\sum_{ij} R_i W_{ij}^{-1} R_j},$$

$$W_{ij} = \int d\omega K(x_i, \omega) (\omega - \bar{\omega})^2 K(x_j, \omega), \quad R_i = \int d\omega K(x_i, \omega)$$

- Regularization by the covariance matrix S_{ij} :

$$W_{ij} \rightarrow \lambda W_{ij} + (1 - \lambda) S_{ij}, \quad 0 < \lambda < 1$$

Resolution function $\delta(0, \omega)$ ($T/T_c = 1$, $\lambda = 0.001$)



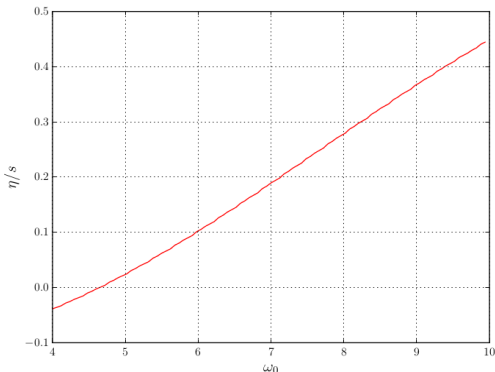
- Width of the resolution function $\omega/T \sim 4$
- Hydrodynamical approximation works up to $\omega/T < \pi$
- Problem: large contribution from ultraviolet tail ($\sim 50\%$)

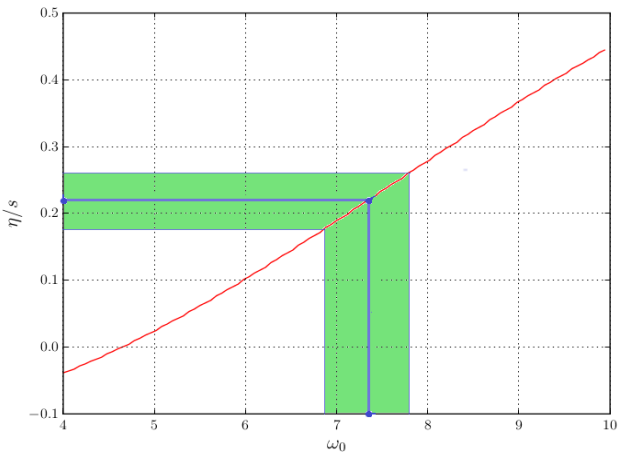
Solution:

- Take ultraviolet contribution in the form:

$$\rho_{ultr} = A\rho_{lat}(\omega)\theta(\omega - \omega_0)$$

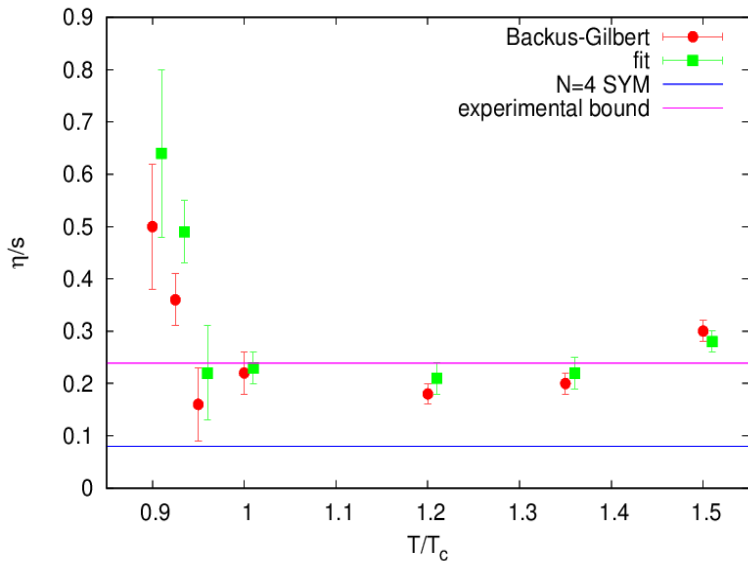
- Determine the value of the A from the $C(t)$ at small t
- Subtract ultraviolet contribution and obtain η/s as a function of ω_0

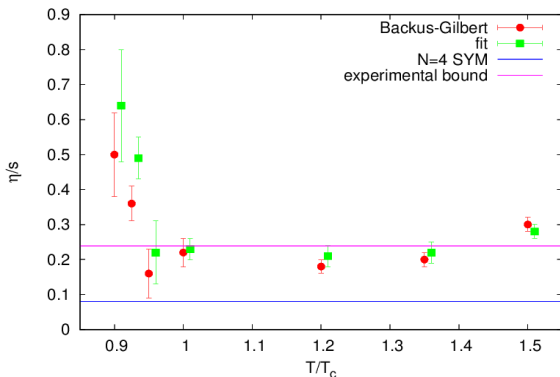




- For $T/T_c = 1$. $\omega_0/T = 7.33 \pm 0.47$
- $\eta/s = 0.22 \pm 0.04$

Preliminary results





Conclusion:

- We calculated η/s for set of temperatures $T/T_c \in (0.9, 1.5)$
- Applied fitting procedure and Backus-Gilbert method for the SF
- η/s is close to N=4 SYM and in agreement with experiment