

**$|V_{us}|$ FROM INCLUSIVE DETERMINATIONS
BASED ON HADRONIC τ DECAY DATA**

Kim Maltman, York University, Toronto, Canada

with R.J. Hudspith, R. Lewis, J. Zanotti and

R. J. Hudspith, T. Izubuchi, R. Lewis, H. Ohki, C. Lehner + \dots (RBC/UKQCD)

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OUTLINE

- *The inclusive flavor-breaking FESR V_{us} determination*
 - *Background, including a resolution of the long-standing $> 3\sigma$ low $|V_{us}|$ puzzle*
 - *Results + current experimental limitations*
- *A new lattice+inclusive us $V+A$ τ data approach*

BASICS: HADRONIC τ DECAYS IN THE SM

- $R_{ij;V/A} \equiv \Gamma[\tau \rightarrow \nu_\tau \text{ hadrons}_{ij;V/A}(\gamma)] / \Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e(\gamma)]$
- With $y_\tau \equiv s/m_\tau^2$, flavor ij decays in SM [Tsai PRD4 (1971) 2821]

$$\frac{dR_{ij;V+A}}{ds} = \frac{12\pi^2 |V_{ij}|^2 S_{EW}}{m_\tau^2} [1 - y_\tau]^2 \tilde{\rho}_{ij;V+A}(s)$$

$$\tilde{\rho}_{ij;V+A}(s) \equiv \left[(1 + 2y_\tau) \rho_{ij;V+A}^{(J=1)}(s) + \rho_{ij;V+A}^{(J=0)}(s) \right]$$

THE INCLUSIVE FB τ $|V_{us}|$ DETERMINATION

- Context

$ V_{us} $	Source
0.2258(9)(?)	3-family unitarity, HT14 $ V_{ud} $
0.2231(4) _{exp} (7) _{latt}	$K_{\ell 3}$, 2+1+1 lattice $f_+(0)$
0.2250(4) _{exp} (9) _{latt}	$\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$, lattice f_K/f_π
0.2176(19) _{exp} (10?) _{th}	Inclusive FB τ FESR (Passemar CKM14)

- Basic tool: FESRs (*Cauchy's Thm*)

$$\int_{s_{th}}^{s_0} ds w(s) \rho(s) = \frac{-1}{2\pi i} \oint_{|s|=s_0} ds w(s) \Pi(s)$$

experimental data OPE

$[s_0 > 0$, analytic $w(s)$, kinematic-singularity-free $\Pi]$

- General inclusive FB τ FESRs for $|V_{us}|$

- FESRs for $\Pi_{ud-us;V+A}^{(J)}(Q^2)$, $\rho_{ud-us;V+A}^{(J)}(s)$

- Experiment: $|V_{ij}|^2 \rho_{ij;V/A}^{(0+1)}(s)$ from $dR_{ij;V/A}/ds$,
[small external us $J=0$ “subtraction”; SM “kinematic weight” w_τ]

- $R_{ij;V/A}^w(s_0)$: re-weighted $R_{ij;V/A}$ analogue

$$R_{ij;V/A}^w(s_0) \sim \int_{th}^{s_0} ds \frac{dR_{ij;V/A}}{ds} \frac{w(s/s_0)}{w_\tau(s/m_\tau^2)}$$

- FB differences $\delta R^w(s_0) \equiv \frac{R_{ud;V+A}^w(s_0)}{|V_{ud}|^2} - \frac{R_{us;V+A}^w(s_0)}{|V_{us}|^2}$

- FESR, OPE for $\delta R^w(s_0)$, input $|V_{ud}| \Rightarrow$

$$|V_{us}| = \sqrt{\frac{R_{us;V+A}^w(s_0)}{\frac{R_{ud;V+A}^w(s_0)}{|V_{ud}|^2} - [\delta R^w(s_0)]^{OPE}}}$$

- Self-consistency: $|V_{us}|$ independent of s_0, w

- **The conventional implementation** [Gamiz et al. JHEP03(2003)060]

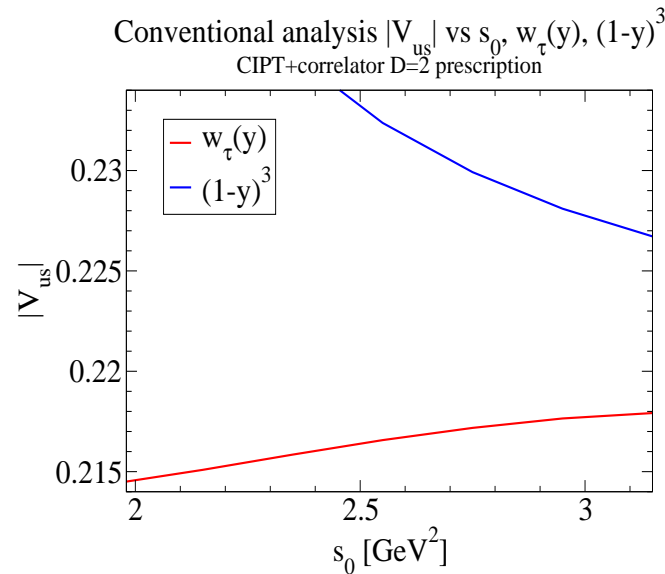
- $s_0 = m_\tau^2, w = w_\tau$ only [spectral integrals from inclusive ud, us BFs, **but no self-consistency tests**]
- w_τ degree 3 \Rightarrow OPE to $D = 8$
- Strong assumptions re $D = 6$ (VSA), $D = 8$ (~ 0)

- **Conventional implementation tests** [KM et al. arXiv:1511.08514]

- Variable $s_0 \leq m_\tau^2$ $|V_{us}|$ stability checks
- Targeted $D = 6, 8$ assumptions test: $y = (s/s_0)$,
 $w_\tau(y) = 1 - 3y^2 + 2y^3$ c.f. $\hat{w}(y) = 1 - 3y + 3y^2 - y^3$

D=8

D=6



- Slow $D = 2$ convergence also a potential issue

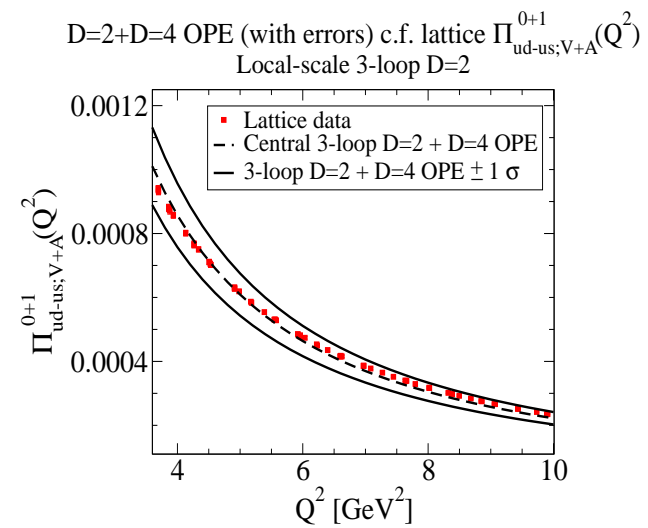
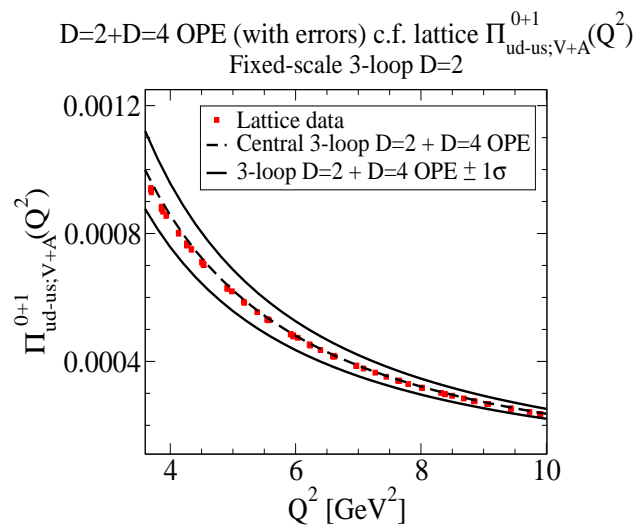
$$\left[\Delta \Pi_\tau(Q^2) \right]_{D=2}^{OPE} = \frac{3}{2\pi^2} \frac{\bar{m}_s^2}{Q^2} \left[1 + \frac{7}{3} \bar{a} + 19.933 \bar{a}^2 + 208.746 \bar{a}^3 + \dots \right]$$

$$\overline{MS} \text{ running } \bar{a} = \frac{\alpha_s(Q^2)}{\pi}, \quad \bar{m}_s = m_s(Q^2), \quad \bar{a}(m_\tau^2) > 0.1$$

- OPE/lattice $\Pi_{ud-us:V+A}^{(0+1)}(Q^2)$ comparison

- Lightest $m_\pi \sim 300 \text{ MeV}$, $1/a = 2.38 \text{ GeV}$, $m_\pi L \sim 4.1$, $32^3 \times 64 \times 2 + 1$ RBC/UKQCD ensemble
- Tight cylinder cut for continuum correlator behavior
- Excellent lattice/ $D = 2 + 4$ OPE match for fixed scale, 3-loop $D = 2$, $Q^2 \sim 4 - 10 \text{ GeV}^2$

- Confirms non-negligible $D > 4$, $Q^2 < 4 \text{ GeV}^2$
- Conventional OPE error estimates VERY conservative despite slow $D = 2$ convergence [FIG]



A NEW FB FESR IMPLEMENTATION

- Theory side
 - No $D > 4$ assumptions: effective condensates $C_{D>4}$ from fits to data (requires variable s_0)
 - 3-loop-truncated FOPT $D = 2$, standard $D = 2 + 4$ error estimates [as per lattice]
 - C_{2N+2} , $|V_{us}|$ from $w_N(y) = 1 - \frac{y}{N-1} + \frac{y^N}{N-1}$ FESR
 - $|V_{us}|$ from different w_N as self-consistency check

- Experimental input

- Updated/corrected 2013 ALEPH for ud $V+A$

- us $V+A$ from sum over exclusive modes

- * K : $K_{\mu 2}$

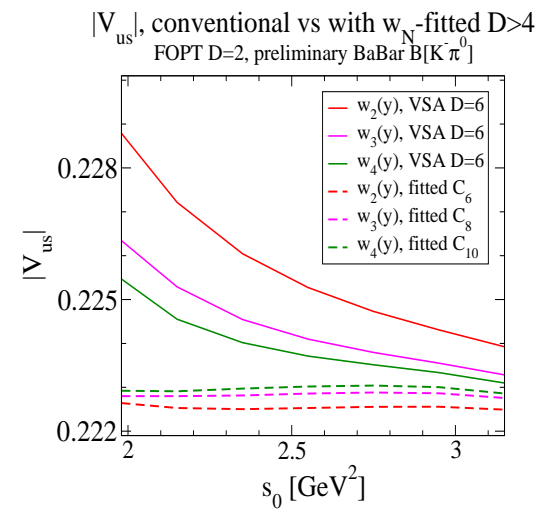
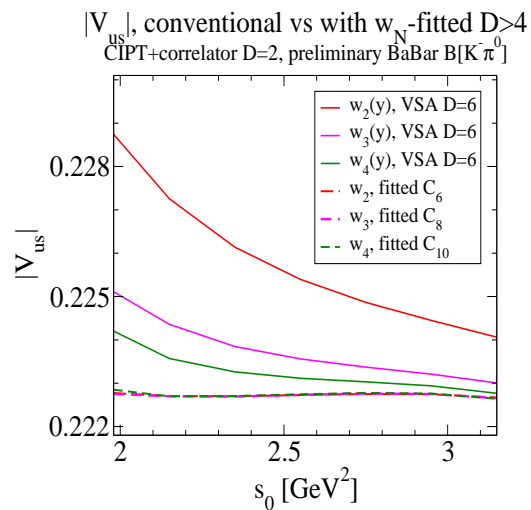
- * $K\pi$, $K^-\pi^+\pi^-$, $\bar{K}^0\pi^-\pi^0$: BaBar, Belle unit-normalized distributions, BFs

- [Note: HFAG $B[K^-\pi^0\nu_\tau] = 0.00433(15)$ (BaBar dominated) c.f. preliminary BaBar (Adametz) thesis $0.00500(15)$ (recommended by BaBar)]

- * Remaining (“residual modes”) from 1999 ALEPH (note: some MC)

NEW IMPLEMENTATION RESULTS

- Unphysical s_0 - $w(y)$ -dependence problems resolved. E.g., for Adametz $B[K^- \pi^0 \nu_\tau]$ input



- $|V_{us}|$ increased by ~ 0.0020 with fitted $C_{D>4}$

- Significant impact of HFAG 2014 → preliminary BaBar Adametz thesis $B[K^- \pi^0 \nu_\tau]$ (3-weight averages)

$$|V_{us}| = 0.2200(23)_{exp}(5)_{th} \quad (HFAG)$$

$$|V_{us}| = 0.2228(23)_{exp}(5)_{th} \quad (Adametz)$$

- Adametz $B[K^- \pi^0 \nu_\tau]$ input w -independence example

Weight	$ V_{us} $ CIPT+corr $D = 2$	$ V_{us} $ FOPT $D = 2$
w_2	0.2227(23)	0.2225(23)
w_3	0.2227(23)	0.2228(23)
w_4	0.2227(23)	0.2230(23)

- New implementation, updated $B[K^- \pi^0 \nu_\tau]$ combination completely resolves old $> 3\sigma$ low $|V_{us}|$ puzzle

- Very favorable (~ 0.0005) theory error
- us spectral integral uncertainty dominates current error
- E.g., error budget, 3-weight, Adametz $B[K^-\pi^0\nu_\tau]$, 3-loop-truncated FOPT $D = 2$ fit

Source	$\delta V_{us} $ (w_2 FESR)	$\delta V_{us} $ (w_3 FESR)	$\delta V_{us} $ (w_4 FESR)
$\delta\alpha_s$	0.00001	0.00004	0.00004
$\delta m_s(2 \text{ GeV})$	0.00017	0.00019	0.00019
$\delta\langle m_s \bar{s}s \rangle$	0.00035	0.00035	0.00035
$\delta(\text{long corr})$	0.00009	0.00009	0.00009
ud exp	0.00027	0.00028	0.00028
us exp	0.00226	0.00227	0.00227

- *Theory error* \Rightarrow *competitive with $K_{\ell 3}$, $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$*
with sufficient us experimental error improvement
- us experimental uncertainties currently BF dominated
- BFs more easily improved experimentally than exclusive mode dR/ds distribution contributions
- Near-term low-multiplicity mode progress likely
[Combined BaBar, Belle (+ Belle II) effort on spectral functions from existing B-factory data, future Belle II data under way]
- **However** sub-0.5% $|V_{us}|$ needs sub-% $R_{us;V+A}^w$ error

- Exclusive us mode w_N spectral integral contributions

Relative exclusive mode $R_{us:V+A}^w$ contributions

Wt	s_0 [GeV ²]	K	$K\pi$	$K\pi\pi$ (B-factory)	Other
w_2	2.15	0.496	0.426	0.062	0.010
	3.15	0.360	0.414	0.162	0.065
w_3	2.15	0.461	0.446	0.073	0.019
	3.15	0.331	0.415	0.182	0.074
w_4	2.15	0.441	0.456	0.082	0.021
	3.15	0.314	0.411	0.194	0.081

- “Other”: 1999 ALEPH data/MC, $\sim 25\%$ error

\Rightarrow “sufficient improvement” includes experimentally (much) more challenging higher-multiplicity modes

A PROMISING τ -BASED ALTERNATIVE

- Work with J. Hudspith, T. Izubuchi, R. Lewis, H. Ohki, C. Lehner + ... (RBC/UKQCD)
- Basic idea: generalized dispersion relations for products of combination $\tilde{\Pi}$ of $J = 0, 1$ us $V+A$ polarizations with weights having poles at Euclidean Q^2
 - $\tilde{\Pi}(Q^2)$: polarization sum with spectral function $\tilde{\rho}(s)$ (experimental $dR_{us;V+A}/ds$ distribution)
 - Theory: Lattice us 2-point function data (no OPE)
 - Weights tunable, allow suppression of larger-error, higher-multiplicity us spectral contributions

More on the lattice-inclusive us τ approach

- $|V_{us}|^2 \tilde{\rho}(s)$ from experimental $dR_{us;V+A}/ds$ distribution

$$\tilde{\rho}_{us;V+A}(s) \equiv \left(1 + 2\frac{s}{m_\tau^2}\right) \rho_{us;V+A}^{(J=1)}(s) + \rho_{us;V+A}^{(J=0)}(s)$$

(experiment, no continuum us $J = 0$ subtraction)

- Associated (kinematic-singularity-free) polarization

$$\tilde{\Pi}_{us;V+A}(Q^2) \equiv \left(1 - 2\frac{Q^2}{m_\tau^2}\right) \Pi_{us;V+A}^{(J=1)}(Q^2) + \Pi_{us;V+A}^{(J=0)}(Q^2)$$

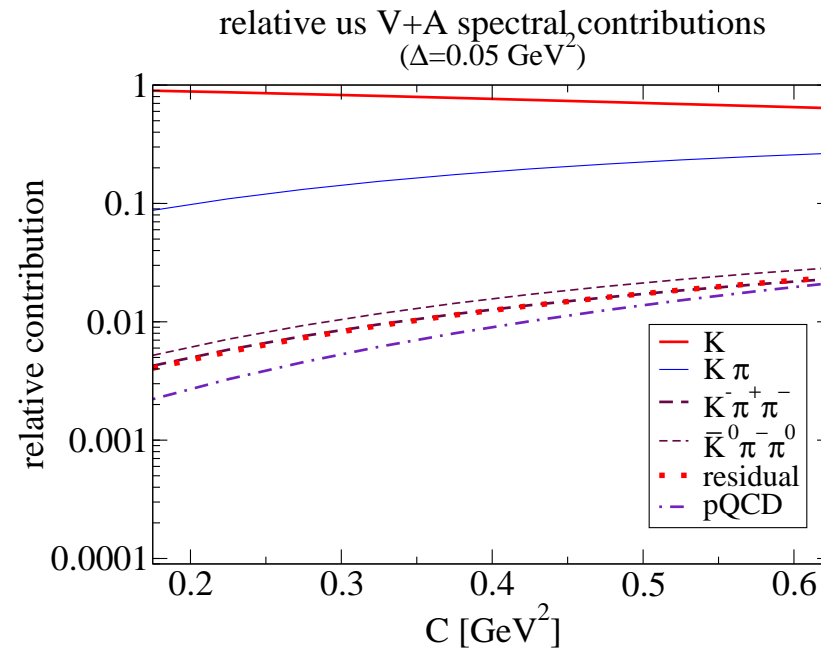
- $\tilde{\rho}_{us;V+A}(s) \sim s$ as $s \rightarrow \infty$

- For weights $w_N(s) \equiv \frac{1}{\prod_{k=1}^N (s+Q_k^2)}$, $N \geq 3$, obtain convergent, unsubtracted 'dispersion relation'

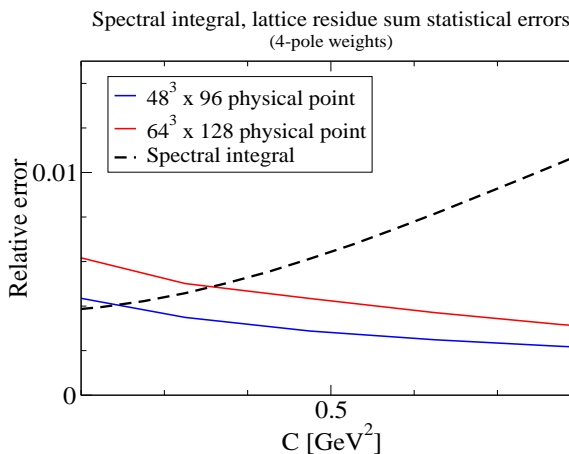
$$\int_{th}^{\infty} ds w_N(s) \tilde{\rho}_{us;V+A}(s) = \sum_{k=1}^N \frac{\tilde{\Pi}_{us;V+A}(Q_k^2)}{\prod_{j \neq k} (Q_j^2 - Q_k^2)}$$

- Lattice data for $\tilde{\Pi}_{us;V+A}(Q_k^2)$ on RHS
- LHS from experimental $dR_{us;V+A}/ds$, up to $|V_{us}|^2$
- $w_N(s)$: rapid fall-off if all $Q_k^2 < 1 \text{ GeV}^2$
 \Rightarrow **$K, K\pi$ dominate LHS, near-endpoint multi-particle, $s > m_\tau^2$ contributions strongly suppressed**
- Optimization: increasing $\{Q_k^2\}$ decreases RHS lattice error, increases LHS experimental error

- “Tuning” impact example, $N = 4$ [$\Delta = 0.05 \text{ GeV}^2$ pole spacing, $C = \text{midpoint}$, pole location interval]



- Spectral integral, residue-sum statistical errors: $N = 4$, $\Delta = 0.05 \text{ GeV}^2$; RBC/UKQCD physical-pt ensembles



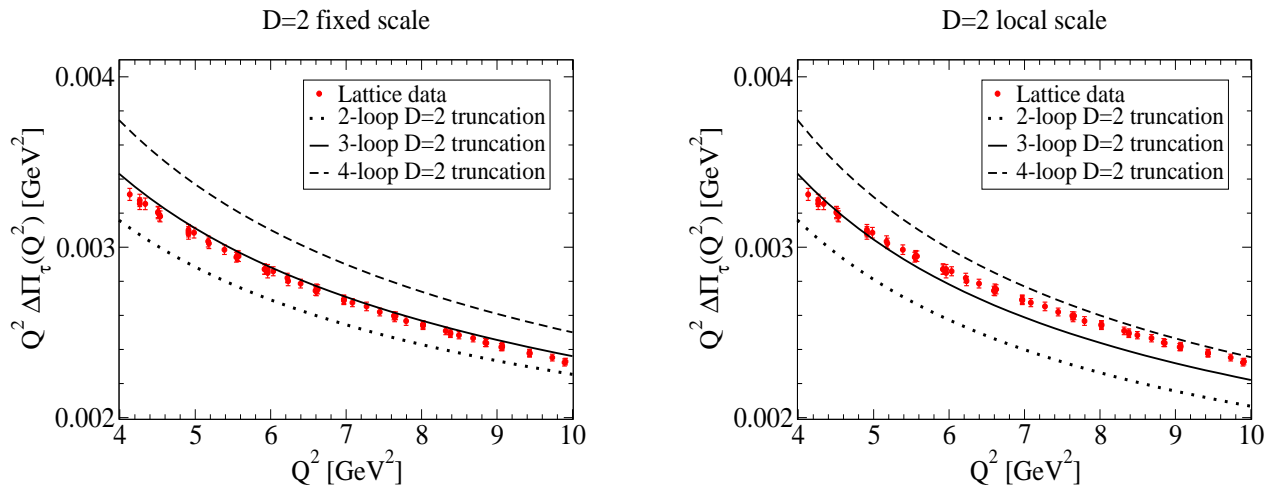
- expt+stat $|V_{us}|$ error below ~ 0.0008 appears feasible
- Optimization/systematics (FV, continuum limit, small $m_{\ell,s}$ mis-tuning corrections) in progress [H. Ohki talk]

SUMMARY

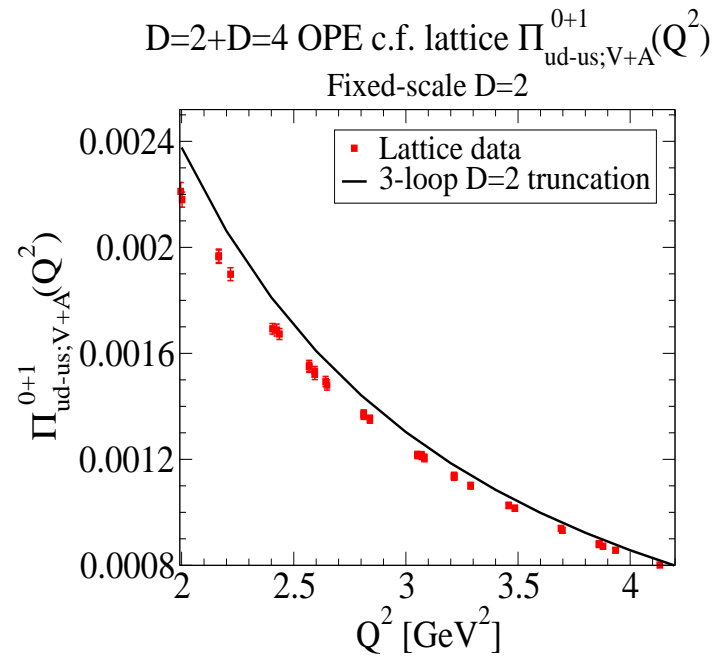
- Old 3σ low inclusive FB τ FESR $|V_{us}|$ problem resolved
 - Alternate, no-assumptions implementation: $|V_{us}|$ higher by ~ 0.0020 , compatible with other determinations
 - Near-term improvements feasible through improvements in us exclusive mode BFs
 - Highly favorable theoretical error situation
 - However, for competitive $|V_{us}|$ need improvements to old ALEPH higher-multiplicity, low-statistics data [unlikely in the near-term]

- Advantage of new lattice-inclusive $us V + A \tau$ approach
 - Theory:
 - * Lattice in place of OPE; no (albeit mildly model-dependent) $us J = 0$ continuum subtraction; improvement through increased statistics
 - * Easily parasitic on lattice a_μ effort
 - Spectral integrals:
 - * Theory errors still small for weights strongly suppressing higher multiplicity contributions
 - * Strong $K, K\pi$ dominance of spectral integral
 - * Significant experimental improvements possible through just improved $K\pi$ BFs, distributions

BACKUP SLIDES



- Higher Q^2 : best (excellent) lattice vs $D = 2 + 4$ OPE match for 3-loop-truncated, fixed-scale $D = 2$
- Fixed scale suggests FOPT for FESR $D = 2$



- Onset of $D > 4$ contributions below $\sim 4 \text{ GeV}^2$

OPE, SPECTRAL INPUT

- PDG, FLAG, HPQCD input for $D = 2, 4$ OPE
- ud V+A spectral data from ALEPH 2013
- us V+A spectral data from sum over exclusive modes [$> 90\%$ of B_{us}^{TOT} from $K_{\ell 2}$, Belle, BaBar $K\pi$, $K\pi\pi$, $3K$ results; residual: 1999 ALEPH]
- Here, for brevity, $K\pi$ normalization including preliminary BaBar $B[\tau \rightarrow K^- \pi^0 \nu_\tau]$ update (Adametz thesis)

MORE ON THE us DATA

- K pole via $f_K|V_{us}|$ from $K_{\ell 2}$
- Rather precise unit-normalized $K^-\pi^0$, $\bar{K}^0\pi^-$, $K^-\pi^+\pi^-$, $\bar{K}^0\pi^-\pi^0$, $3K$ distributions from Belle, BaBar (main uncertainties from BFs)
- K , B-factory modes over 90% of B_{us}^{TOT}
- Residual us exclusive mode contributions from 1999 ALEPH data, covariances

THE EXPERIMENTAL $K\pi$ BF SITUATION

- HFAG 2014 $B[(K^-\pi^0 + \bar{K}^0\pi^-)\nu_\tau] = 0.0126$
- HFAG 2014 $B[K^-\pi^0\nu_\tau] = 0.00433(15)$ value \rightarrow preliminary BaBar (Adametz thesis) result $0.00500(15)$ yields $B[(K^-\pi^0 + \bar{K}^0\pi^-)\nu_\tau] = 0.0133$
- Central $B[(K^-\pi^0 + \bar{K}^0\pi^-)\nu_\tau]$ from $K_{\ell 3}$, dispersive analysis expectations [ACLP13] also 0.0133
- 0.07% difference “small” but represents $\sim 2.4\%$ of B_{us}^{TOT} , hence $\sim 1.2\%$ increase in $|V_{us}|$

Results for $|V_{us}|$ for current HFAG 2014 $K\pi$ BFs

Weight	$ V_{us} $ CIPT+corr $D = 2$	$ V_{us} $ FOPT $D = 2$
w_2	0.21985(230)	0.21966(230)
w_3	0.21985(231)	0.21966(231)
w_4	0.21985(231)	0.22009(231)

Error budget, existing $K\pi$ BFs

Source	$\delta V_{us} $ (w_2 FESR)	$\delta V_{us} $ (w_3 FESR)	$\delta V_{us} $ (w_4 FESR)
$\delta\alpha_s$	0.00001	0.00003	0.00005
$\delta m_s(2 \text{ GeV})$	0.00017	0.00018	0.00020
$\delta\langle m_s \bar{s}s \rangle$	0.00034	0.00034	0.00034
$\delta(\text{long corr})$	0.00009	0.00009	0.00009
ud exp	0.00027	0.00027	0.00027
us exp	0.00229	0.00229	0.00230

Stability of $|V_{us}|$ with fitted C_{2N+2} input, existing $K\pi$ BF normalization

