The spectra of (closed) confining flux tubes in D=3+1 and D=2+1 SU(N) gauge theories

Michael Teper (Oxford) - Lattice 2016

- D=2+1, fundamental flux

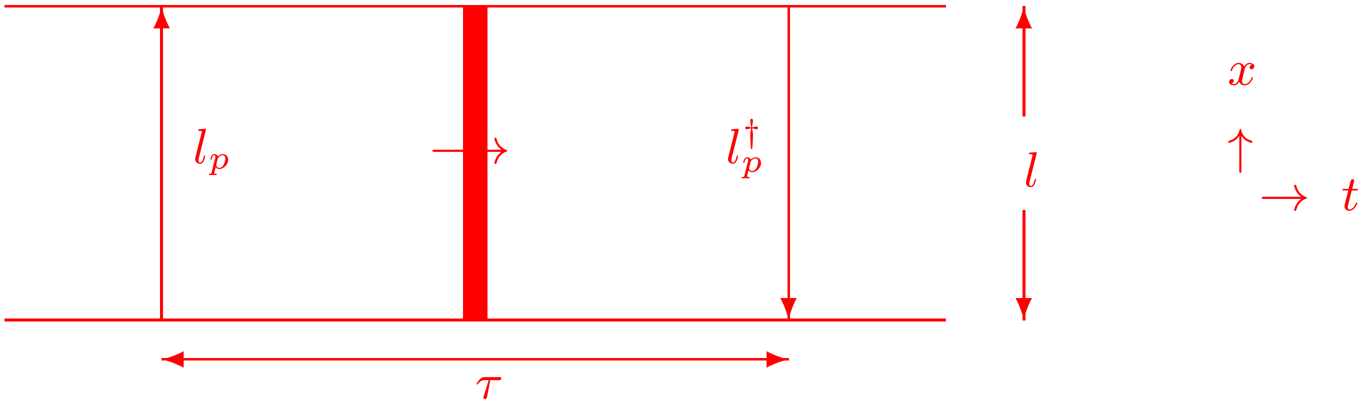
- D=2+1, higher rep flux

- D=3+1, fundamental flux

calculate the energy spectrum of a confining flux tube winding around a spatial torus of length l, using correlators of $p_\perp = 0$ Polyakov loops (Wilson lines):

$$\langle l_p^\dagger(\tau) l_p(0)\rangle = \sum_n c_n(l) e^{-E_n(l)\tau} \overset{\tau\to\infty}{\propto} \exp\{-E_0(l)\tau\}$$

in pictures



a flux tube sweeps out a cylindrical $l \times \tau$ surface $S \cdots$ integrate over these world sheets with an effective string action $\propto \int_{cyl=l\times\tau} dS e^{-S_{eff}[S]}$

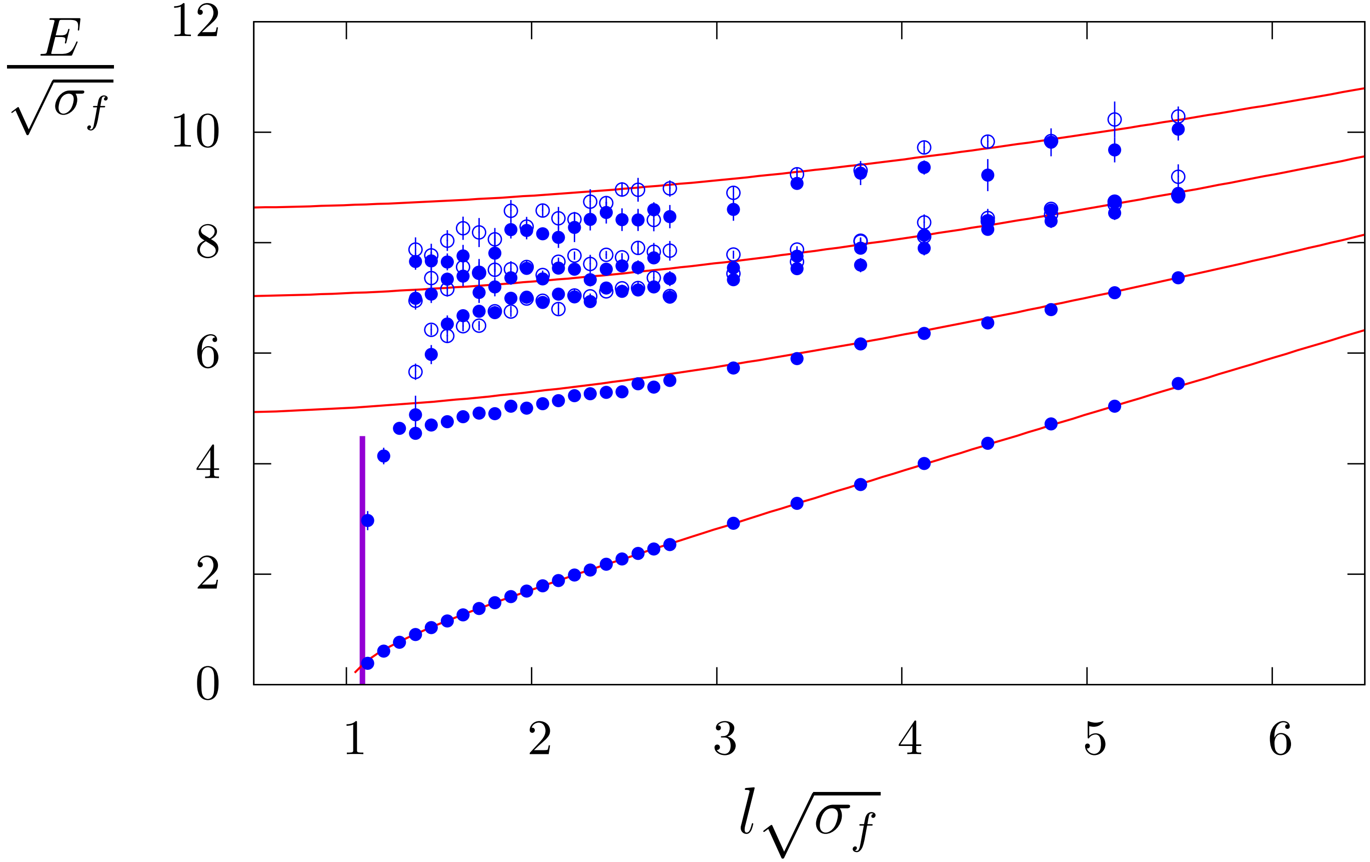
Lattice calculations from:

D=2+1, f: A.Athenodorou,B.Bringoltz,MT: 1103.5854
D=3+1, f: AA,BB,MT: 1007.4720 and AA,MT: in preparation
D=2+1, $\not{f}$: AA,MT: 1303.5946
D=2+1, f: AA,MT: 1602.07634
also: AA,BB,MT: 0709.0693, 0812.0334; BB,MT: 0802.1490
also open strings etc: Torino group – Caselle, Gliozzi, ...

Effective string theory:

Luscher, Symanzik, Weisz: early '80s – $O(1/l)$ universal Luscher correction
Luscher, Weisz: 2004: $O(1/l^3)$ – (sometimes) universal term (also Drummond)
O. Aharony+Karzbrun, 0903.1927; +Field 1008.2636 +Klinghoffer 1008.2648; +Field,Klinghoffer 1111.5757; +Dodelson 1111.5758: +Komargodski: 1302.6257 – all universal corrections
S.Dubovsky, R. Flauger, V. Gorbenko 1203.4932, 1205.6805, 1301.2325, 1404.0037, +PC,AM,SS 1411.0703 SD,VG 1511.01908 – medium l and integrability
see also Torino group – Gliozzi, Tateo et al, ...

SU(6), p=0; P=+,●, P=-,○. $a\sqrt{\sigma} \simeq 0.086$,D=2+1



Vertical line is deconfining length. Solid curves are NG predictions.

Nambu-Goto ‘free string’ theory

$$\mathcal{Z} = \int \mathcal{D}S e^{-\kappa A[S]}$$

massless ‘phonons’ carry momentum and produce energy gaps:

$$E^2(l) = (\sigma\, l)^2 + 8\pi\sigma \left(\frac{N_L + N_R}{2} - \frac{D-2}{24} \right) + p^2 .$$

$p = 2\pi q/l$ momentum along string;
$n_L(k), n_R(k)$ = number left,right moving ‘phonons’ of momentum $2\pi k/l$:
$N_{L,R} = \sum_{k>0} n_{L,R}(k)k$ = sum left and right ‘phonon’ momenta:

$$Parity = (-1)^{number\ phonons}; \qquad p = 2\pi(N_L - N_R)/l$$

Note: $E(l) \neq \sigma l$+energy free phonons : i.e. the $D = 1 + 1$ phonon field theory is *not* a free field theory.

for long strings expand NG in powers of $1/\sigma l^2$: e.g.

$$E_0(l) = \sigma l \left(1 - \frac{\pi(D-2)}{3}\frac{1}{\sigma l^2}\right)^{1/2} = \sigma l - \frac{\pi(D-2)}{6l} + O(1/l^3) \qquad l^2\sigma \geq 3/\pi(D-2)$$

similarly for excited states once $l^2\sigma \geq 8\pi n$

Universal terms for any S_{eff}:

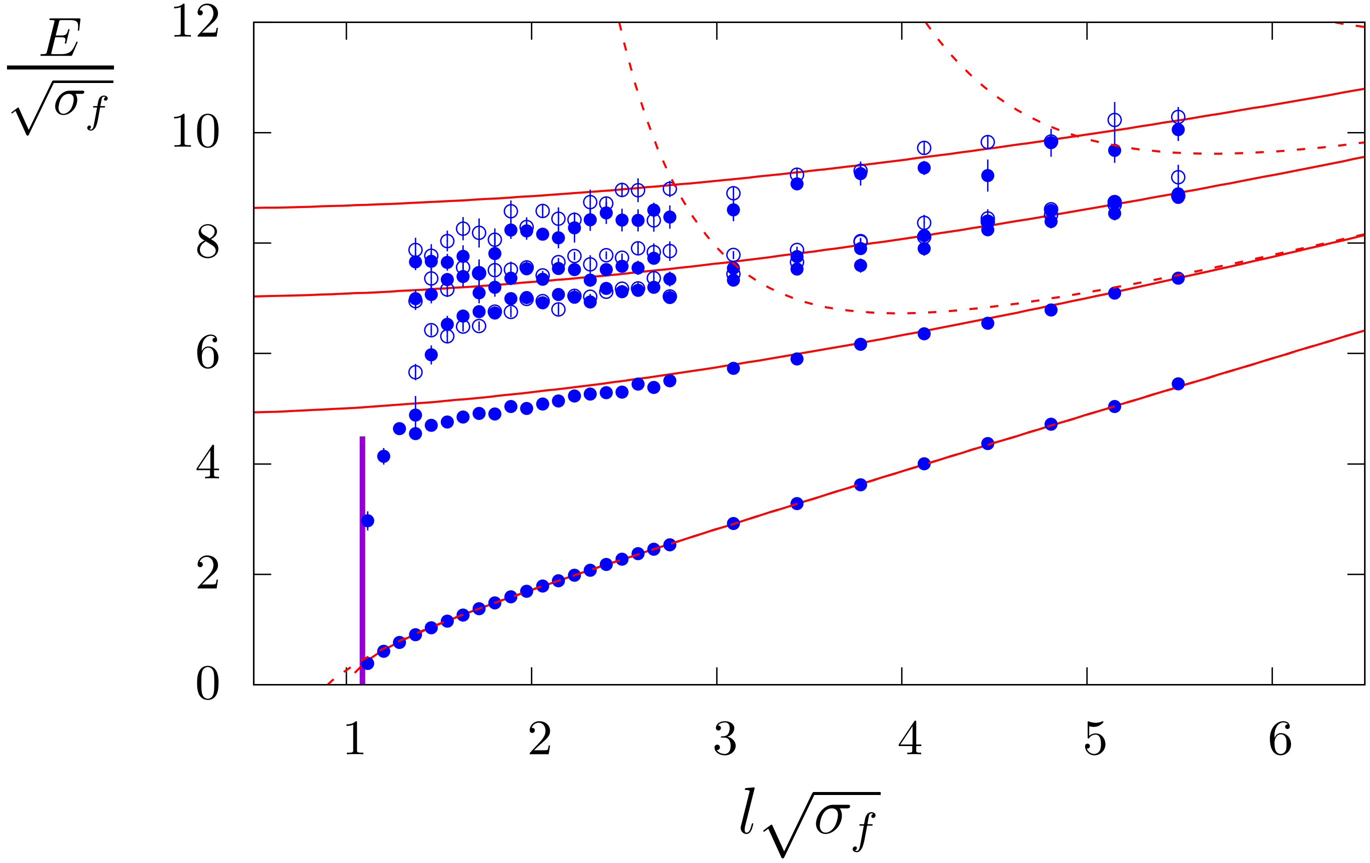
$$E_0(l) \stackrel{l\to\infty}{=} \sigma l - \frac{\pi(D-2)}{6l} - \frac{\{\pi(D-2)\}^2}{72}\frac{1}{\sigma l^3} - \frac{\{\pi(D-2)\}^3}{432}\frac{1}{\sigma^2 l^5} + O\left(\frac{1}{l^7}\right)$$

- ◦ $O\left(\frac{1}{l}\right)$ Luscher correction, $\sim$ 1980
- ◦ $O\left(\frac{1}{l^3}\right)$ Luscher, Weisz; Drummond, $\sim$ 2004
- ◦ $O\left(\frac{1}{l^5}\right)$ Aharony et al, $\sim$ 2009-10

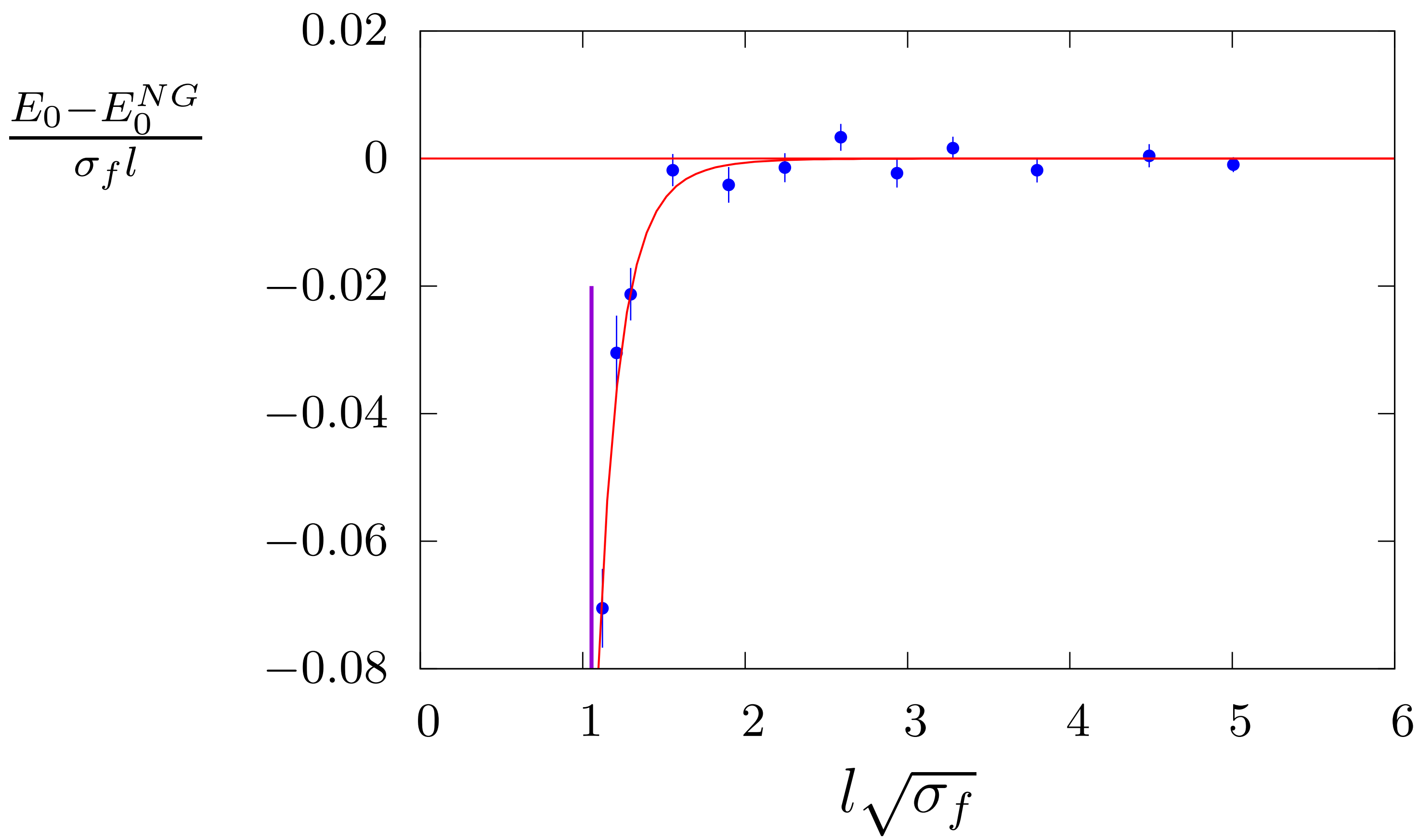
and similar results for $E_n(l)$, but only to $O(1/l^3)$ in $D = 3+1$

– identical to NG expansion up to explicit $O(1/l^7)$ corrections in $D = 2+1$; extra $O(1/l^5)$ universal correction in $D = 3+1$

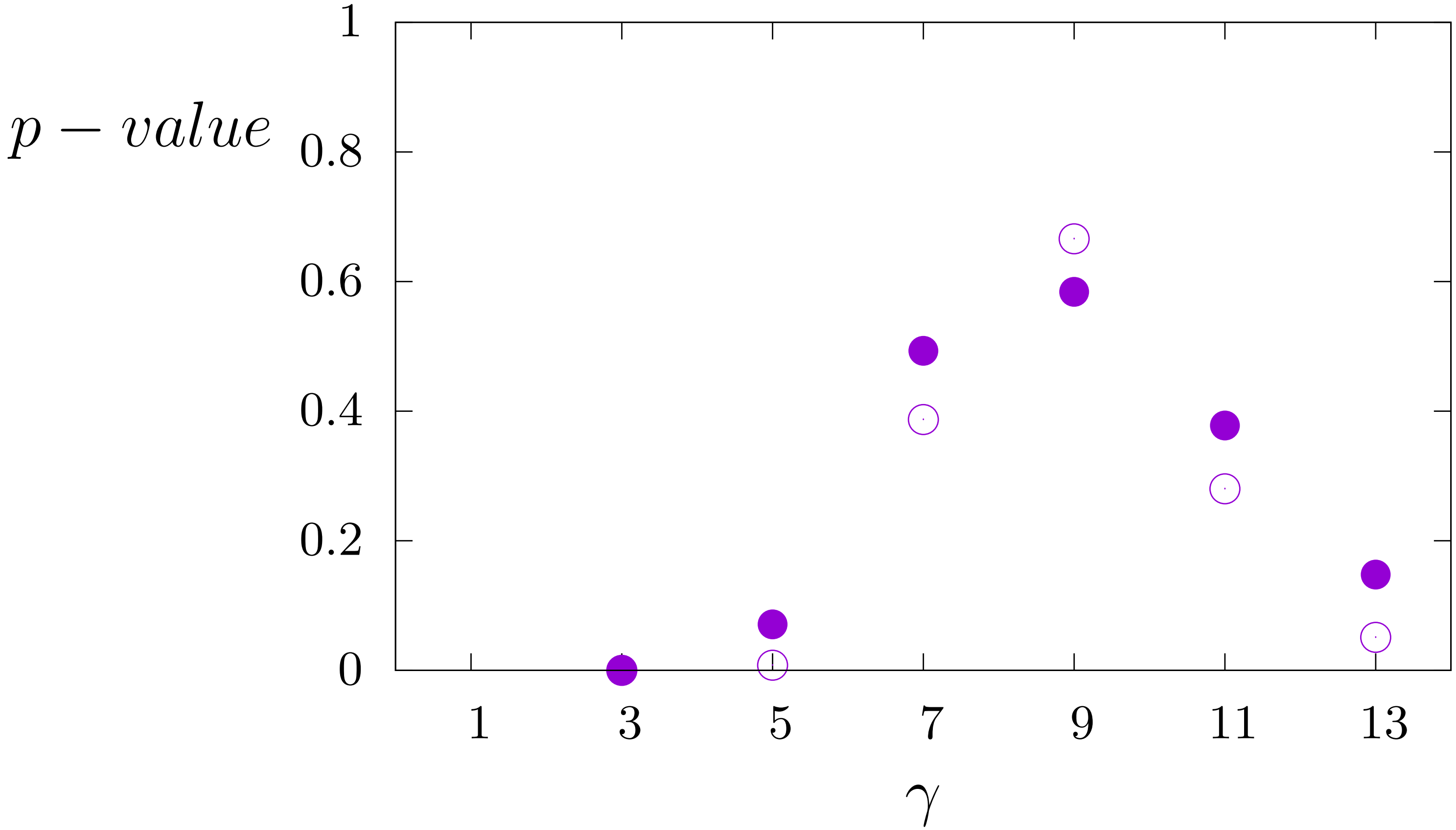
SU(6), p=0; P=+,●, P=-,○. $a\sqrt{\sigma} \simeq 0.086$,D=2+1



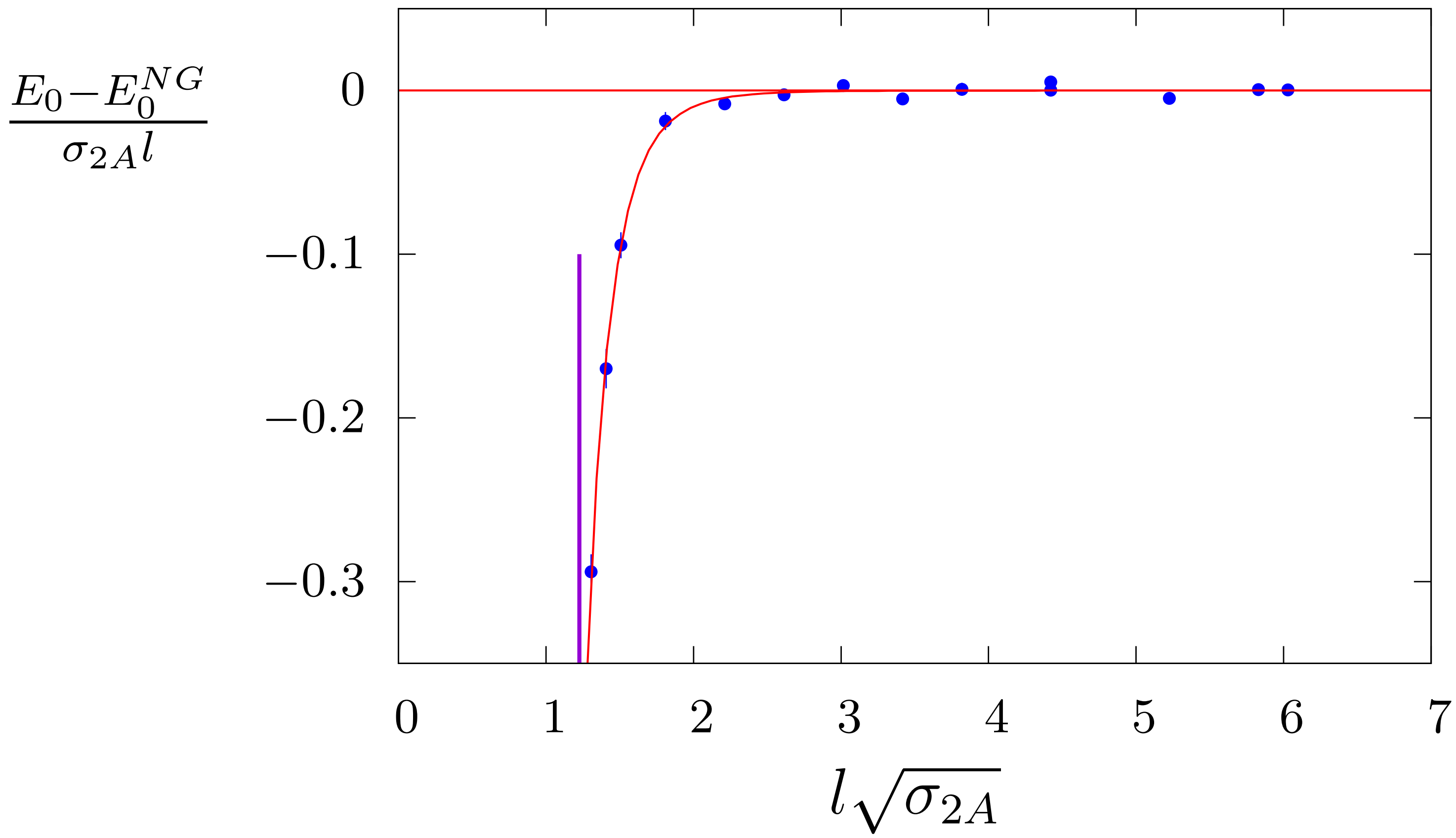
Solid curves are NG; dashed ones are universal terms up to $O(1/l^5)$.



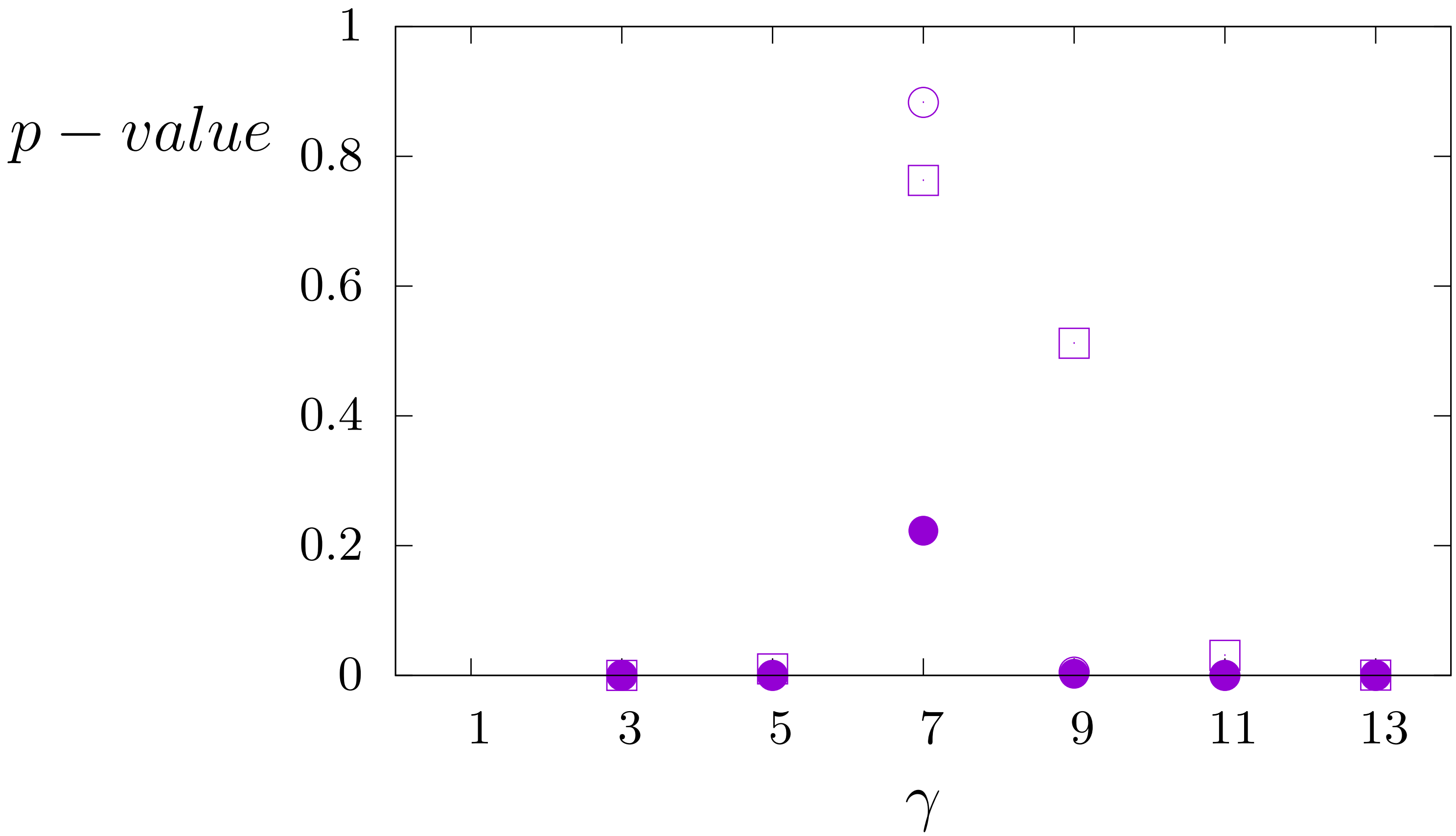
Best fits to SU(4) $k = 1$ ground state energy with Nambu-Goto plus a $O(1/l^7)$ correction.



Best fits to SU(4) $k = 1$ ground state energy using Nambu-Goto with a $O(1/l^{\gamma})$ correction: p-value for all $l \in [13, 60]$, $\bullet$, and for $l \in [13, 18]$, $\circ$, versus γ.



Best fits to SU(4) $k = 2A$ ground state energy with Nambu-Goto plus a $O(1/l^7)$ correction. Vertical line indicates the deconfining transition.



Best fits to SU(4) $k = 2A$ ground state energy using Nambu-Goto with a $O(1/l^{\gamma})$ correction: p-value for all $l \in [13, 60]$, $\bullet$, and for $l \in [13, 18]$, $\circ$, versus γ. Also fits $l \in [14, 18]$, $\square$, that exclude the shortest flux tube.

this and $SU(6)$ and $SU(8) \Longrightarrow$

$$\gamma \geq 7$$

confirming prediction of universal terms through $O(1/l^5)$

BUT: why such good agreement with NG for excited states at smaller l?

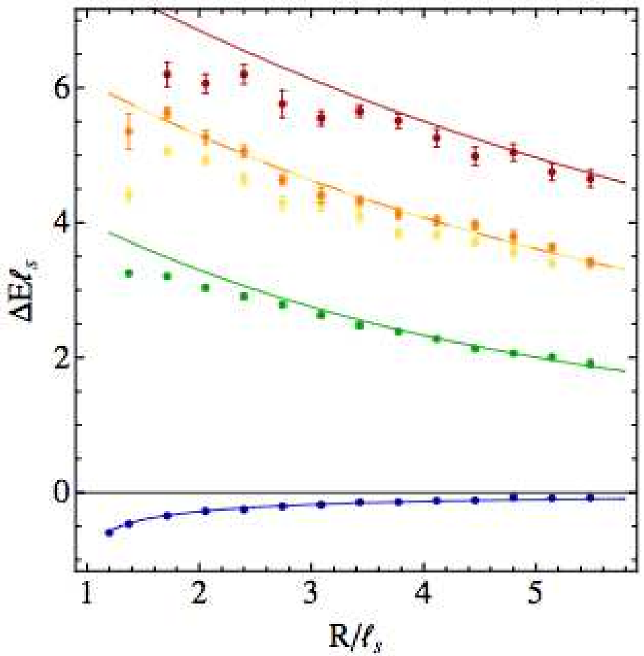
$D = 1 + 1$ phonon field theory is approximately integrable (Dubovsky et al)

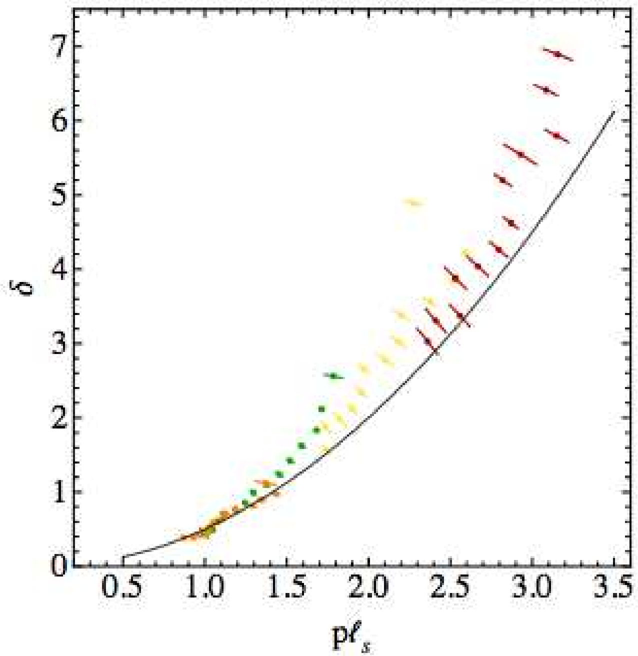
$\Longrightarrow$

and $\delta_{GGRT} = s/8\sigma$ in Thermodynamic Bethe Ansatz ($\sim$ Luscher finite V)

leads to the finite volume spectrum :

SU(6), lowest p=0 P=+ states $\qquad$ δ=extracted phase shift





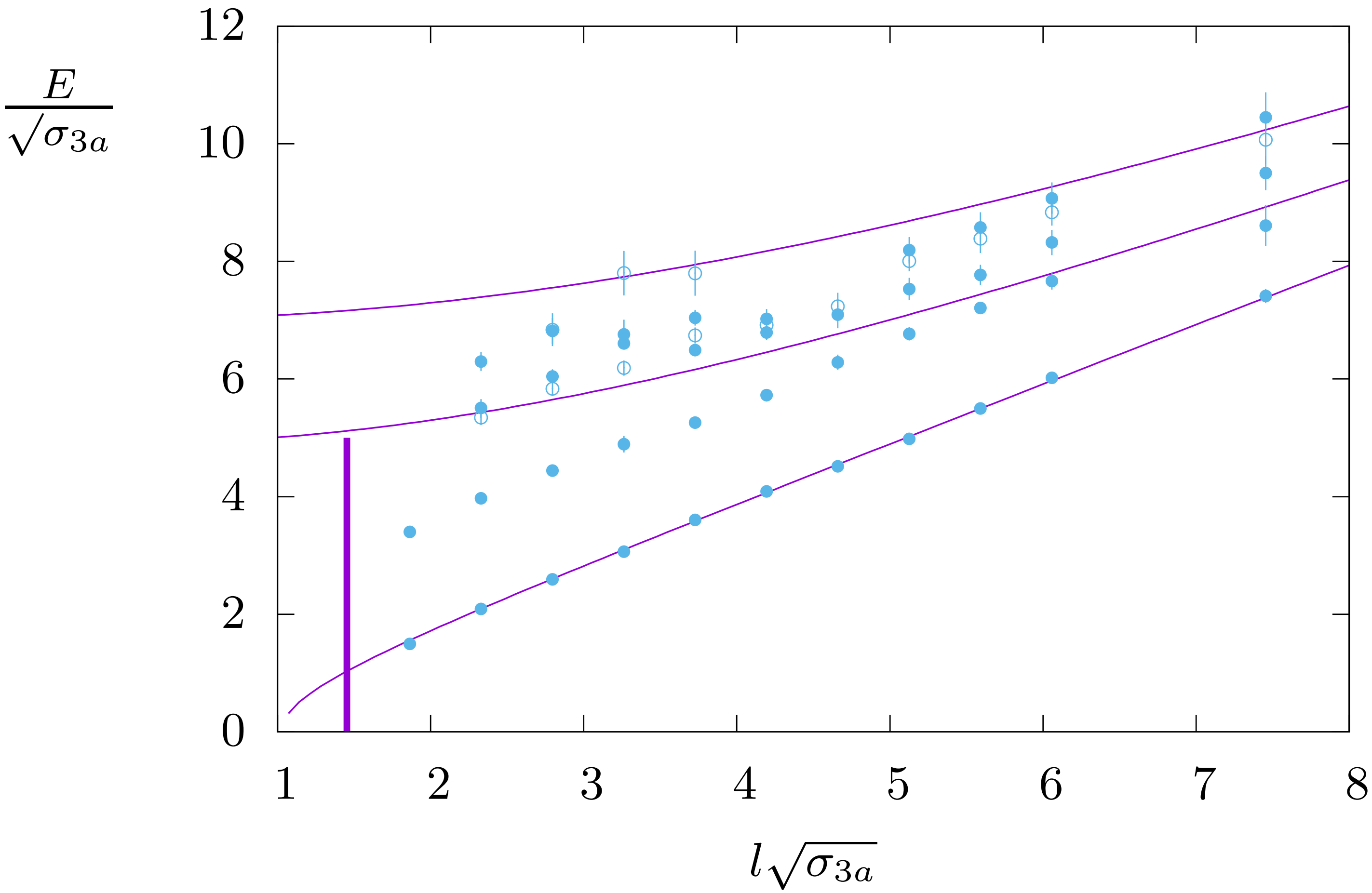
$\Delta E = E - \sigma l$, $R = l$, $l_s = 1/\sqrt{\sigma}$; $\qquad$ δ-curve GGRT phase shift.

So, massless phonons describe the flux tube spectrum down to small l ...

BUT where are the massive modes, e.g. when $l \sim$ width flux tube?

$\Longrightarrow$

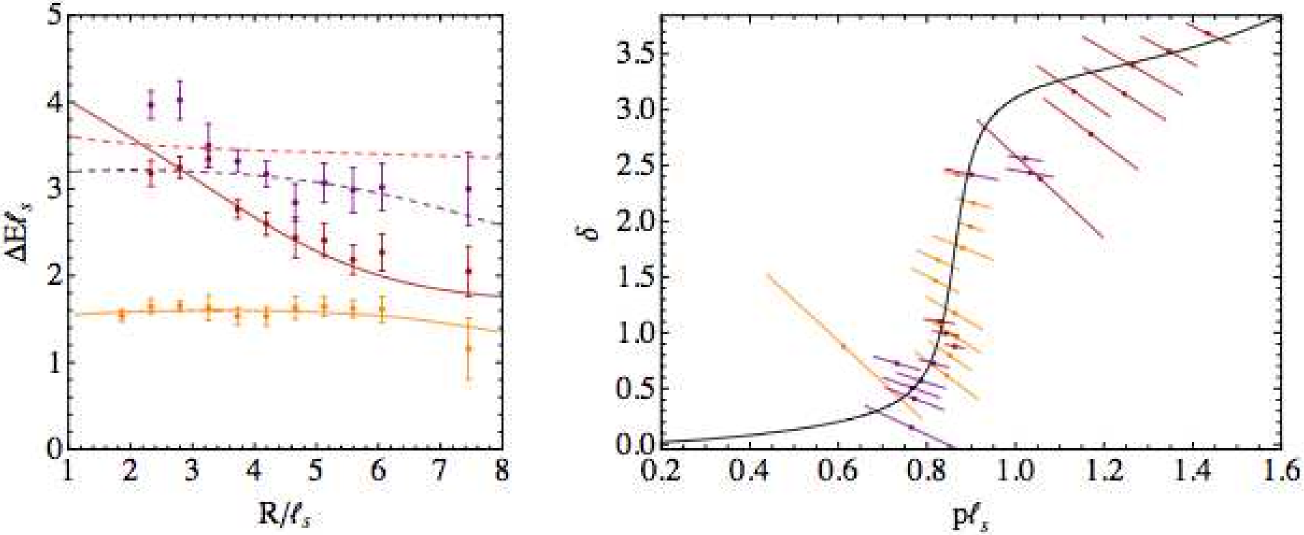
go to k-strings where we know there *must* be massive modes associated with binding of the k fundamentals



SU(6): $k = 3A$ ground state and lowest excited states with $p = 0$ and $P = \pm$, $\bullet, \circ$; solid curves are NG predictions.

$E_1(l) - E_0(l) \simeq \mu$ ind of l : massive mode?

TBA analysis (Dubovsky et al) : spectrum $\longrightarrow$ δ=extracted phase shift



$\Longrightarrow$ resonant state with $\mu \sim m_G/2$

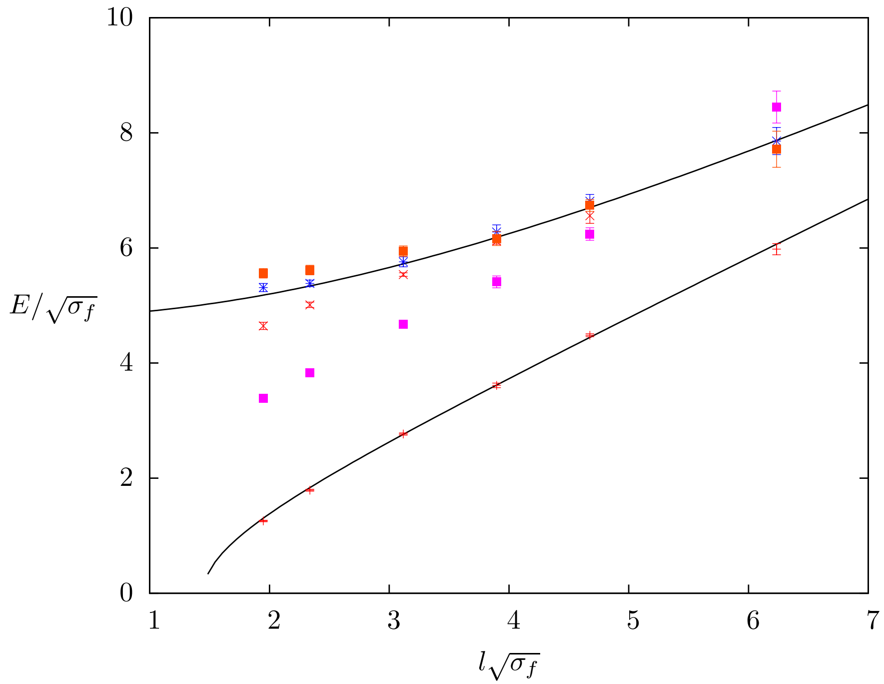
D=3+1 : fundamental flux in SU(3) with $a\sqrt{\sigma} \simeq 0.20,\ 0.13$

phonons have $J = \pm 1$ and (when free) $p = 2\pi k/l$: a^{+}_{k}, a^{-}_{k}

flux along x: $P_t : y, z \rightarrow y, -z$ i.e. $a^{+}_{k} \rightarrow a^{-}_{k}$

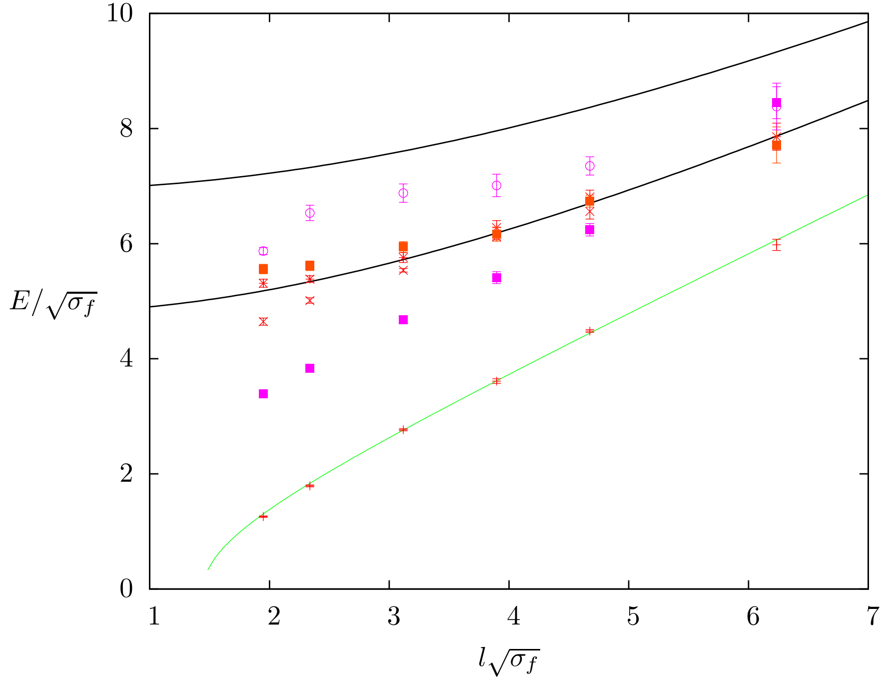
flux along x: $P_l : x \rightarrow -x$ and C i.e. $a^{+}_{k} \rightarrow a^{+}_{-k}$

$p = 0$, ground and first excited energy levels (NG: $N_L = N_R = 1$)

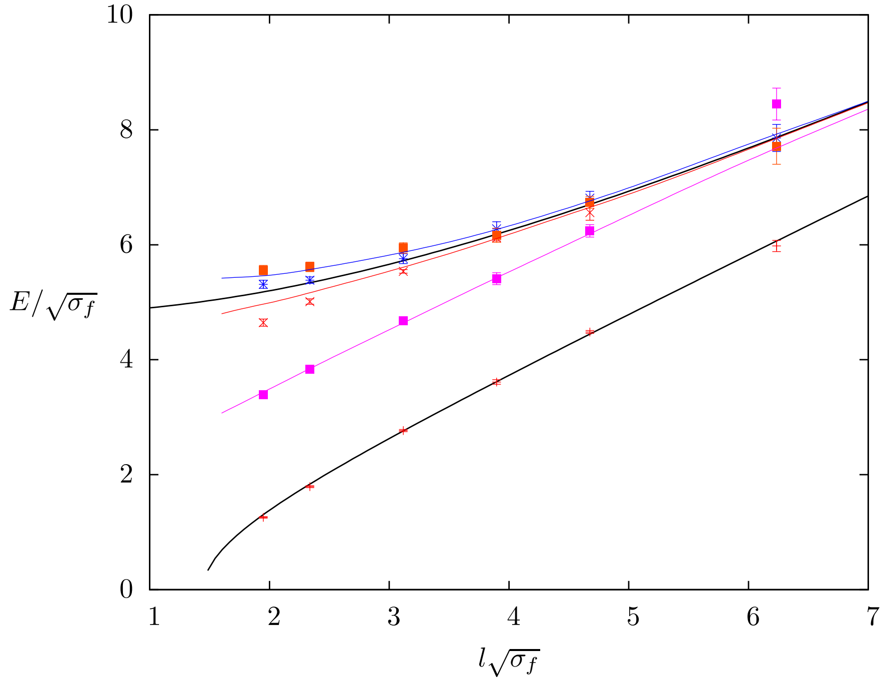


purple 0^{--}; red 0^{++}; orange, blue $J = 2$.

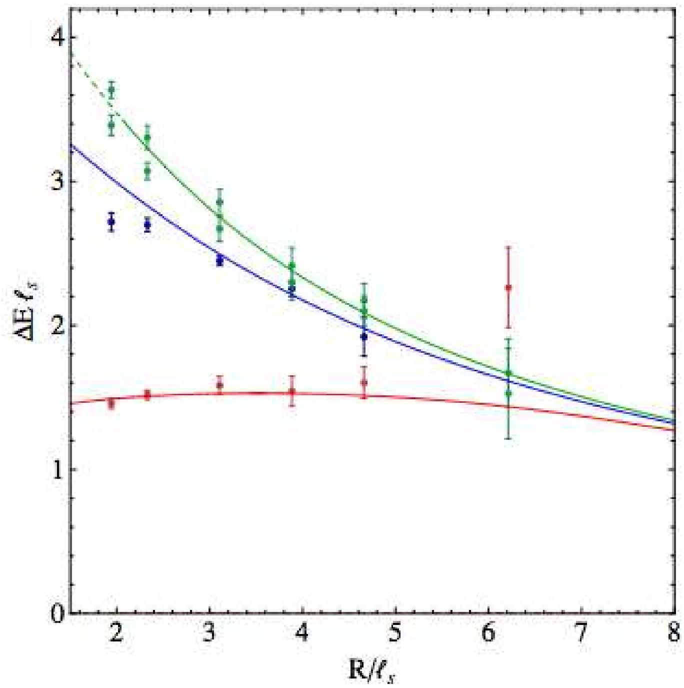
as above but with next excited 0^{--} as well



as above but with axion in theory fit to 0^{--} and 0^{++}

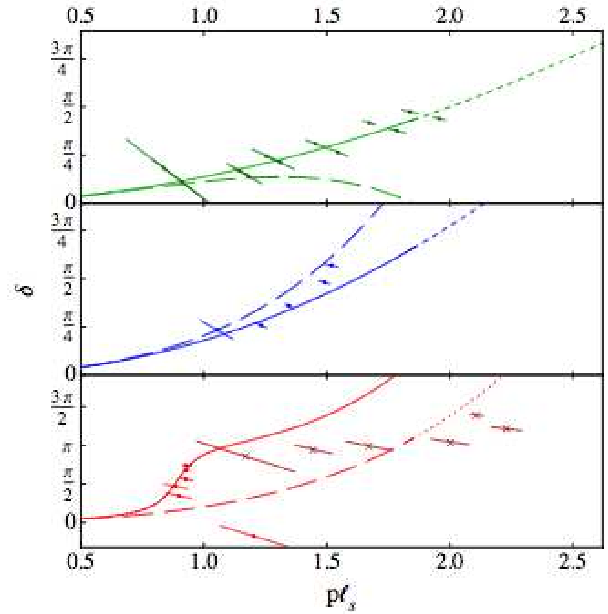


with world-sheet 0^- resonance and other lines = TBA + δ_{PS}



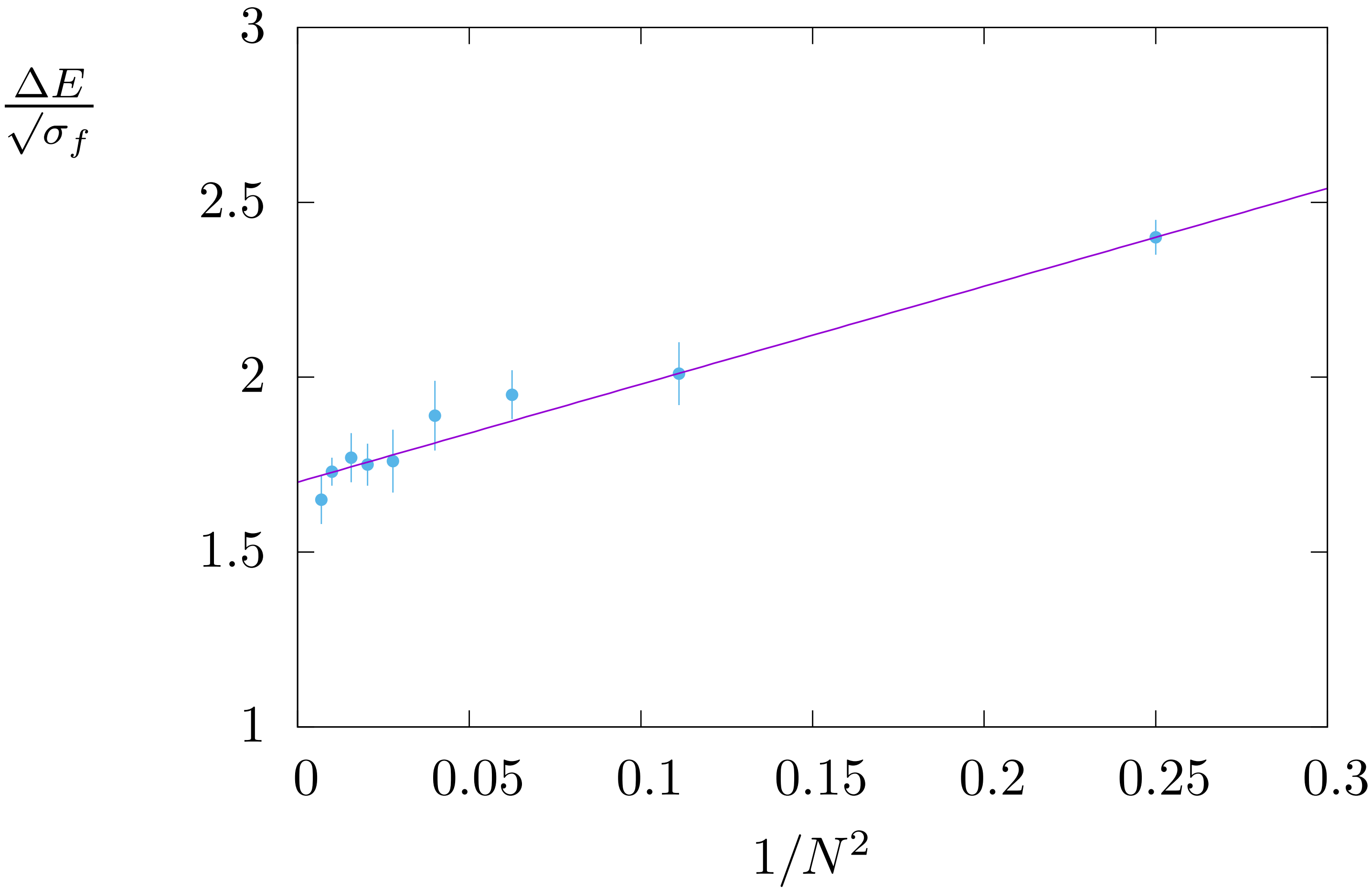
$J = 0, P = +/-$ are blue/red. $J = 2$ are green $\Delta E = E - \sigma l$;

Phase shift from: $J = 2$ top; $J = 0^+$ middle; $J = 0^-$ bottom



solid line: prediction with axion. dashed line: prediction without axion

N-dependence of axion resonance 'mass' (preliminary) SU(2) – SU(12)



Note: $M_A \overset{N\to\infty}{\simeq} 0.5 M_{G,0^{++}}$

Conclusions

• the remarkably simple spectrum of confining flux tubes uncovered through lattice calculations, has motivated powerful theoretical developments in understanding both long (universality ...) and shorter (near-integrability ...) flux tubes within effective string and world sheet frameworks

• in $D = 2+1$ lattice calculations are now able to test convincingly expectations about the power of l at which non-universal terms first appear

• TBA analysis of $D = 1+1$ world sheet theory $\Longrightarrow$ in $D = 2+1$ massive resonance associated with k-string binding *and* in $D = 3+1$ massive 0^{--} resonance in fundamental flux tube spectrum, nicely explained by a topological (self-intersection) 'axionic' field *and* both masses are $\mu \sim m_{0^{++}}/2$

• Lack of other massive modes in fundamental flux tube (e.g. intrinsic flux tube width) suggests these modes are heavy/weakly coupled $\Longrightarrow$ dynamics of flux tubes remarkably simple to an excellent approximation. (But need to do $D = 3+1$ better.)