Luescher’s finite volume test for two-baryon systems with attractive interactions

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with T. Doi (Riken) and T. Iritani (StonyBrook U.) for HAL QCD collaboration

A previous talk by T. Iritani in this session.
Motivation
Direct vs Potential : NN systems

Reviewed in T. Doi PoS LAT2012,009 (+ updates)

Potential method (HALQCD) : unbound
Direct method (Yamazaki et al./NPL/CalLat): bound
Fake plateau problem (direct method)

Plateaux from wall and smeared sources disagree.
One (or both) of them is fake, but we can not judge if they are fake or not.

need a method to see a reliability of data from one source without others.

This talk
Finite volume formula

S-wave

\[ k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{\vec{n}^2 - q^2}, \quad q = \frac{kL}{2\pi}, \quad \Delta E = 2\sqrt{k^2 + m^2} - 2m \]

attractive interaction  \( \Delta E < 0 \)  \( k^2 < 0 \)  \( \delta(k) \) at \( k^2 < 0 \)  \( ? \)

analytic continuation of \( \delta(k) \) at \( k^2 < 0 \)
One can check lattice data at finite volume from ERE behaviors.

ERE (Effective Range Expansion)

\[ k \cot \delta(k) = \frac{1}{a_0} + \frac{r_0}{2} k^2 + \cdots \]

\[ \Delta E_L \, [\text{MeV}] \]

\[ \text{scattering state spectra} \]

\[ \text{L-indep. fake plateaux} \]

\[ \text{if "true" pole} \]

\[ \text{finite vol. spectra} \]

\[ (k/m_\pi)^2 \]
Results
$N_f = 2 + 1$, $a \simeq 0.09$ fm, $m_\pi \simeq 510$ MeV

$\Delta E_{NN}(^1S_0) \simeq -7.4(1.3)$ MeV

$\Delta E_{NN}(^3S_1) \simeq -11.5(1.1)$ MeV

same ensembles of Yamazaki et al. 2012

\[ \Delta E_{NN}(^{1}S_0) \simeq -3.9(1.3) \text{ MeV} \]

\[ \Delta E_{NN}(^{1}S_0) \simeq -0.7(0.8) \text{ MeV} \]
\[ \Delta E_{NN}(^{3}S_{1}) \simeq -8.7(0.9) \text{ MeV} \]

\[ \Delta E_{NN}(^{3}S_{1}) \simeq -1.4(0.8) \text{ MeV} \]

finite volume tests suggest signals for NN bound states are fake.
\[ \Delta E_{\Xi \Xi}(^1S_0) \simeq -5.4(0.8) \text{ MeV} \]

\[ \Delta E_{\Xi \Xi}(^3S_1) \simeq 12.2(0.9) \text{ MeV} \]

\[ \Delta E_{\Xi \Xi}(^1S_0) \simeq -0.3(0.5) \text{ MeV} \]

\[ \Delta E_{\Xi \Xi}(^3S_1) \simeq -0.9(0.6) \text{ MeV} \]
Figure 1: Plot showing the transition probability $k^{\alpha} \delta / m_\pi$ as a function of $(k/m_\pi)^2$ for $N_f = 2 + 1$ with $a \simeq 0.09$ fm, $m_\pi \simeq 300$ MeV.

Figure 2: Plot showing the transition probability $k^{\alpha} \delta / m_\pi$ as a function of $(k/m_\pi)^2$ for $N_f = 2 + 1$ with $a \simeq 0.128$ fm, $m_\pi \simeq 800$ MeV.
All NN bound states from Yamazaki et al. have strange ERE behaviors

1. finite volume formula does not work (too small volumes)  unlikely
2. strange ERE behaviors are correct.  unlikely
3. extracted energy shifts are incorrect  likely, agrees with Iritani’s results

finite volume formula

\[ k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{\vec{n}^2 - q^2} = \frac{1}{a_0} + \frac{r_0}{2} k^2 + \cdots \]

a very easy and useful test for a reliability of the extracted energy shift

How about other results?
$N_f = 2 + 1$, $a_s \simeq 0.123$ fm, $a_s/a_t \simeq 3.5$, $m_\pi \simeq 390$ MeV

$N_f = 2 + 1$, $a \simeq 0.1167$ fm, $m_\pi \simeq 450$ MeV
$N_f = 3$ (SU(3) limit), $a \simeq 0.145$ fm, $m_{PS} \simeq 800$ MeV.

Strange?

Deeply bound

Deeply bound

Strange
Summary Plots

\[ NN^{(1\ S_0)} \]

\[ k^2 \ [\text{GeV}^2] \]

\[ k \cot \delta \ [\text{GeV}] \]

\[ NN^{(3\ S_1)} \]

\[ k^2 \ [\text{GeV}^2] \]

\[ k \cot \delta \ [\text{GeV}] \]
Conclusion and Discussion
- Finite volume formula give a useful test for the bound states.
- Yamazaki et al.: very strange behaviors (fail the test)
  - confirmed by HAL smeared data.
- NPL: some pass, the other fail the test. (Not conclusive)
  - necessary test but can not guarantee the correctness.
  - need further checks (wall vs. smeared, variational method)
- finite volume test is mandatory for the bound state search in lattice QCD
- the formula should be used for the infinite volume extrapolation
  - using LO (NLO) ERE

\[
k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{\vec{n}^2 - q^2} = \frac{1}{a_0} + \frac{r_0}{2} k^2 + \cdots
\]
Direct vs Potential : NN systems

“di-neutron”

Potential

Yamazaki et al.
NPL
CalLat

“deuteron”

Yamazaki et al.
NPL
CalLat

Potential method (HALQCD) : unbound
Direct method (Yamazaki et al./NPL/CalLat): bound
questionable
$N_f = 3$ (SU(3) limit), $a \simeq 0.145$ fm, $m_{PS} \simeq 800$ MeV

same as NPL 2012

$NN(^1S_0)$

second negative energy state?
(second bound state?)
incompatible with NPL ERE?
$NN(^3S_1)$

NPL and CalLat seems incompatible.

Second bound state in CalLat?

NLO is large?

Large effective range?
Expectation at physical point

\[ L \sim 8 \text{ fm} \]

\[ \Delta E_L \text{ [MeV]} \]

\[ \frac{k \cot \delta}{m_\pi} \]

\[ \left( \frac{k}{m_\pi} \right)^2 \]

- Line for \( L = 96 \) (K-conf.)
- \( ^3S_1:\text{ERE} \) with \( a_0 = 5.4 \text{ fm}, r_0 = 1.8 \text{ fm} \)
- \( ^1S_0:\text{ERE} \) with \( a_0 = -23.7 \text{ fm}, r_0 = 2.7 \text{ fm} \)