Luescher's finite volume test for two-baryon systems with attractive interactions

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LATTICE 2016

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with T. Doi (Riken) and T. Iritani (StonyBrook U.) for HAL QCD collaboration

Hadrons to Atomic nuclei

HAL

from Lattice QCD

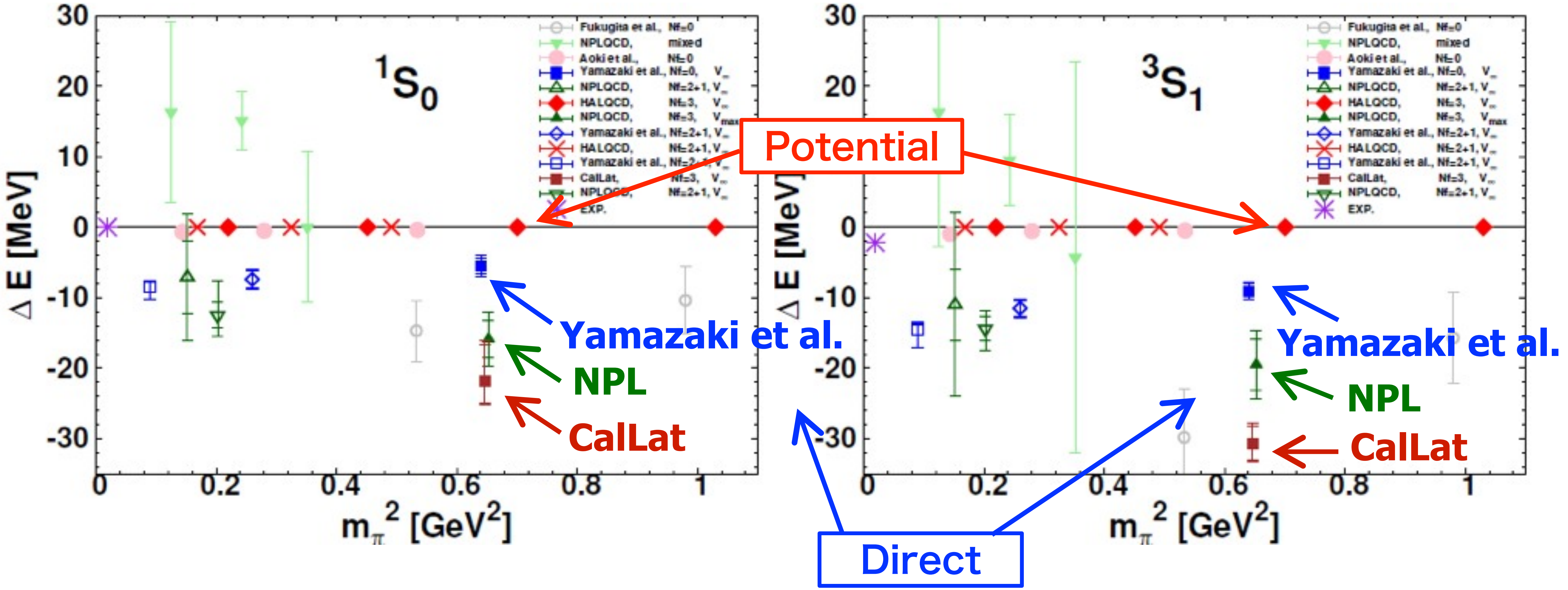
A previous talk by T. Iritani in this session.

Motivation

Direct vs Potential : NN systems

Reviewed in T. Doi PoS LAT2012,009 (+ updates)

“di-neutron” “deuteron”

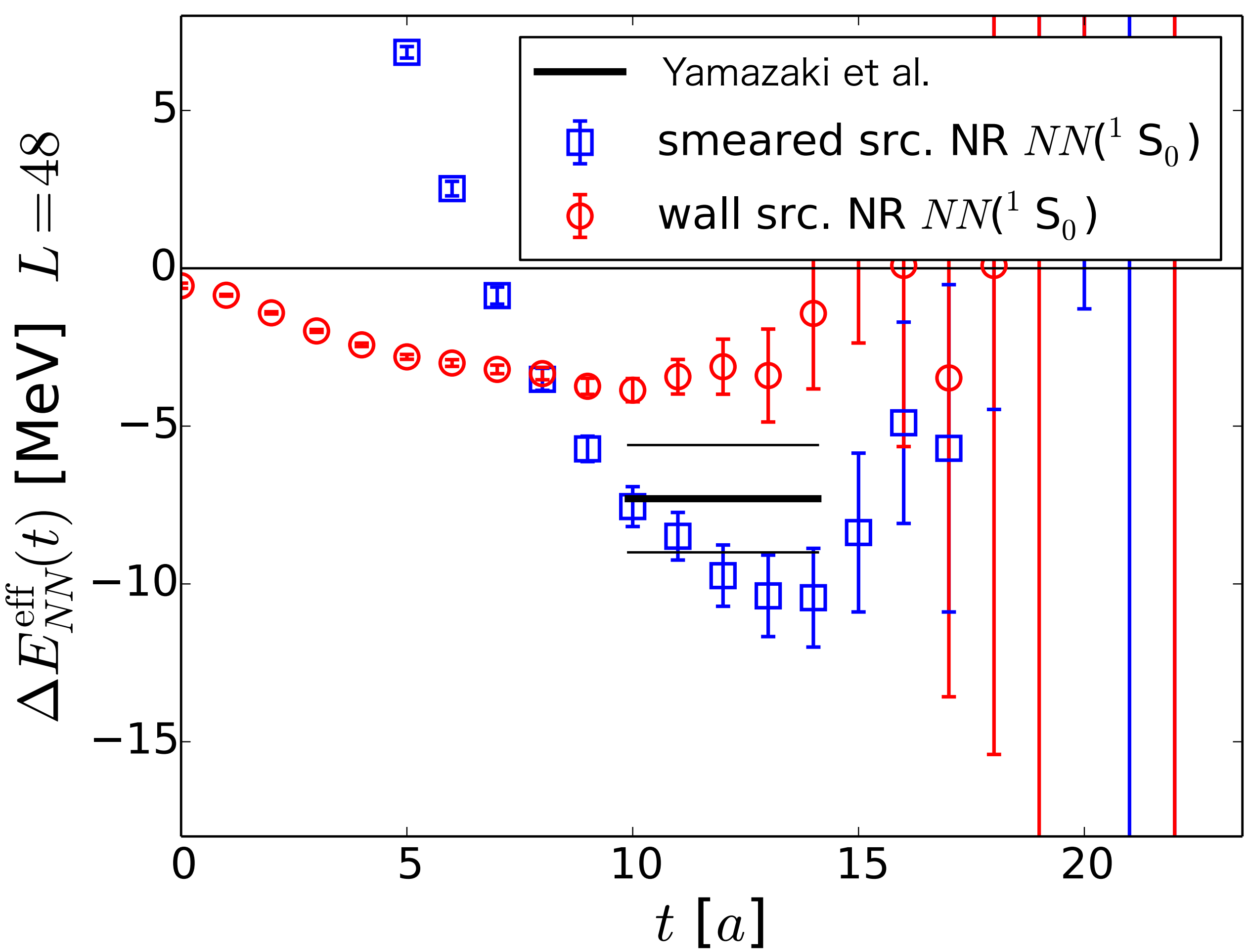


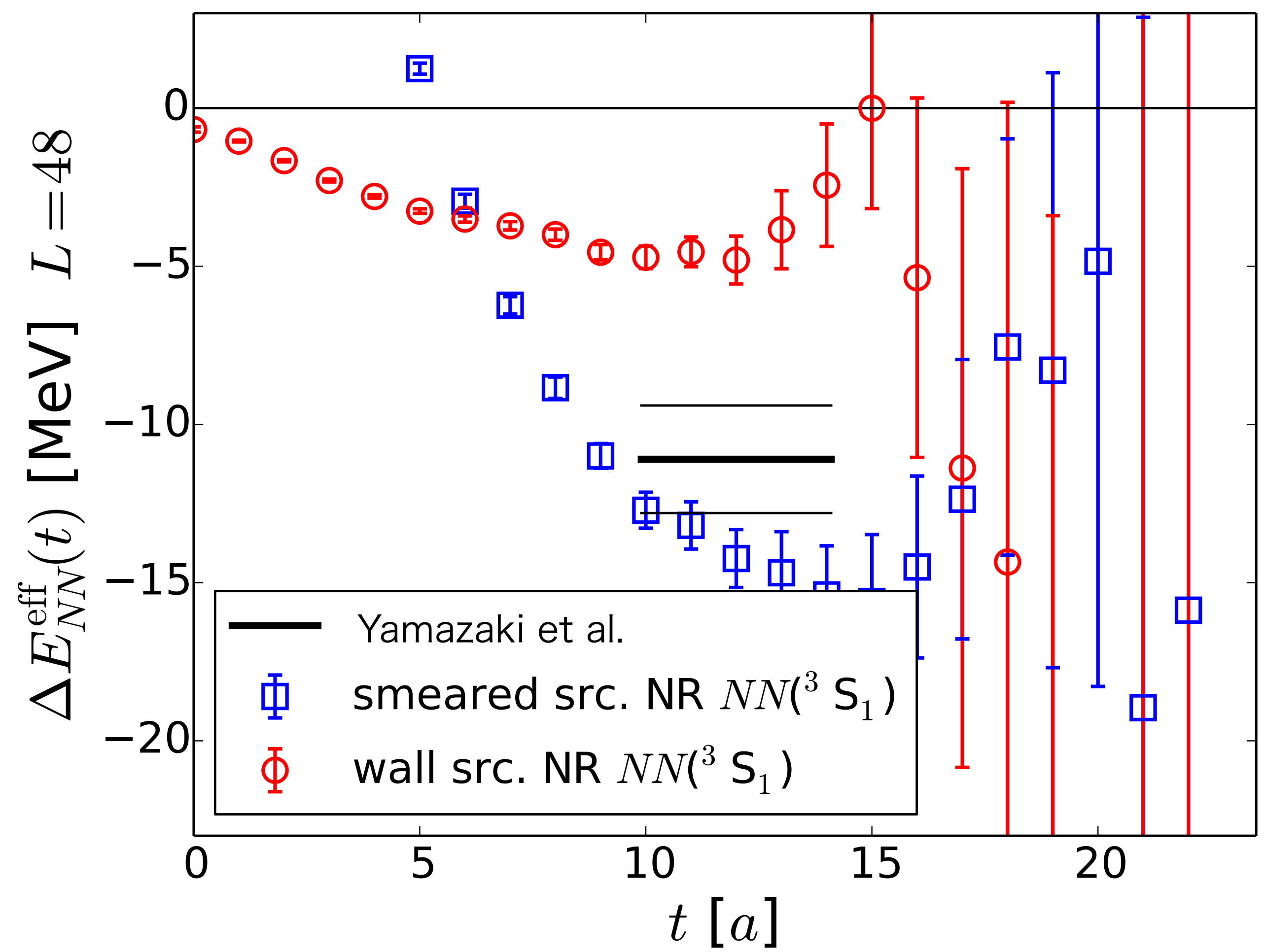
Potential method (HALQCD) : unbound

Direct method (Yamazaki et al./NPL/CalLat): bound

Fake plateau problem (direct method)

A previous talk by T. Iritani





Plateaux from wall and smeared sources disagree.

One (or both) of them is fake, but we can not judge if they are fake or not.

need a method to see a reliability of data from one source without others.

This talk

Finite volume test

Finite volume formula

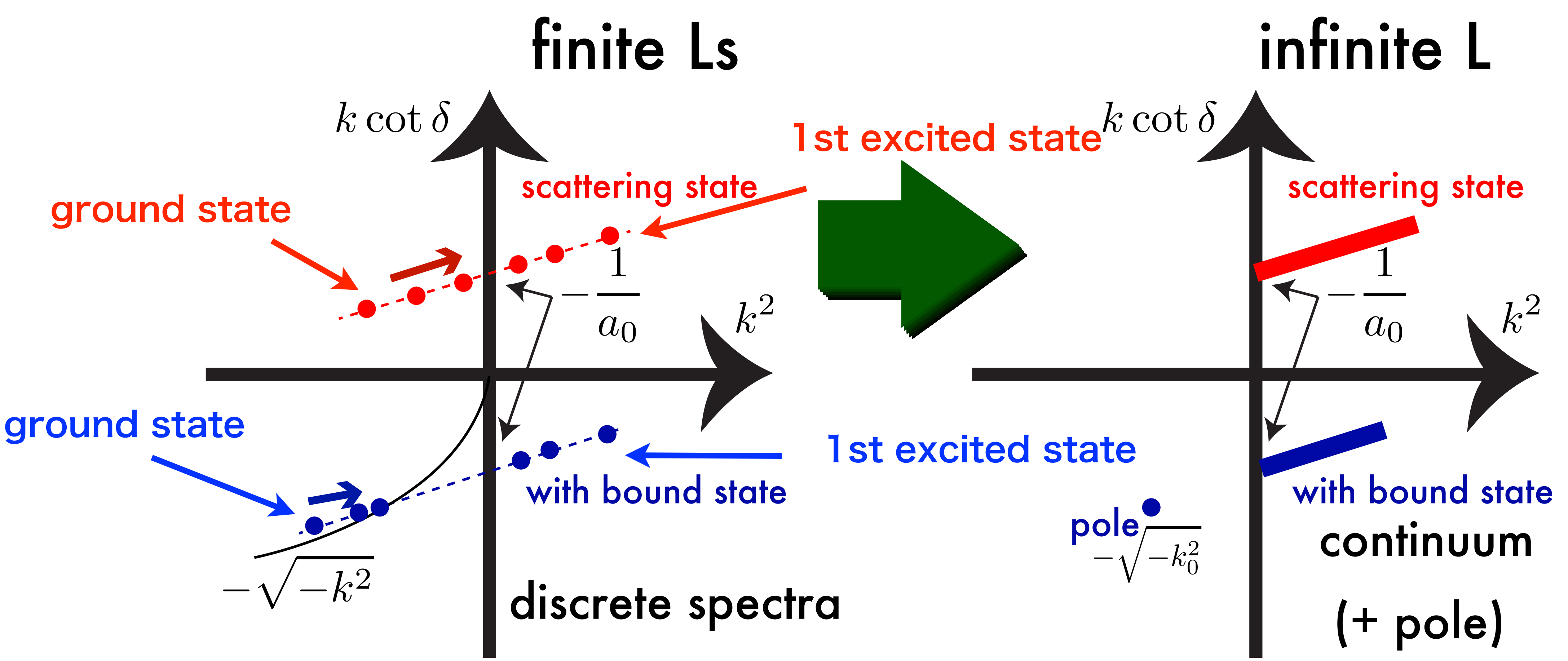
S-wave

Lüscher, NPB354(1991)531

$$\boxed{k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{\vec{n}^2 - q^2},} \qquad q = \frac{kL}{2\pi}, \qquad \Delta E = 2\sqrt{k^2 + m^2} - 2m$$

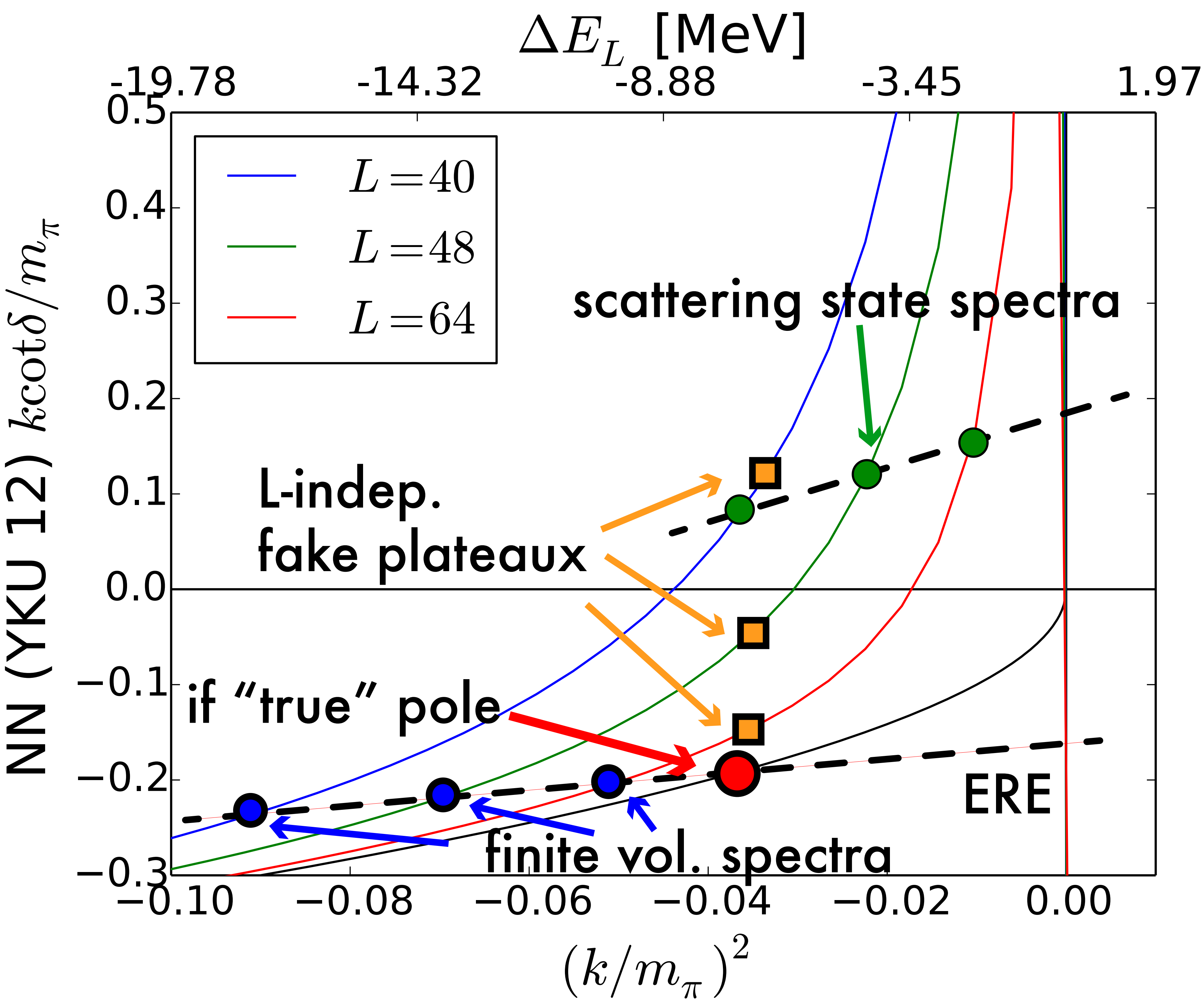
attractive interaction $\Delta E < 0$ ➡ $k^2 < 0$ ➡ $\delta(k)$ at $k^2 < 0$ **?**

➡ $\boxed{\text{analytic continuation of } \delta(k) \text{ at } k^2 < 0}$



One can check lattice data at finite volume from ERE behaviors.

ERE(Effective Range Expansion) $k\cot\delta(k) = \frac{1}{a_0} + \frac{r_0}{2}k^2 + \cdots$

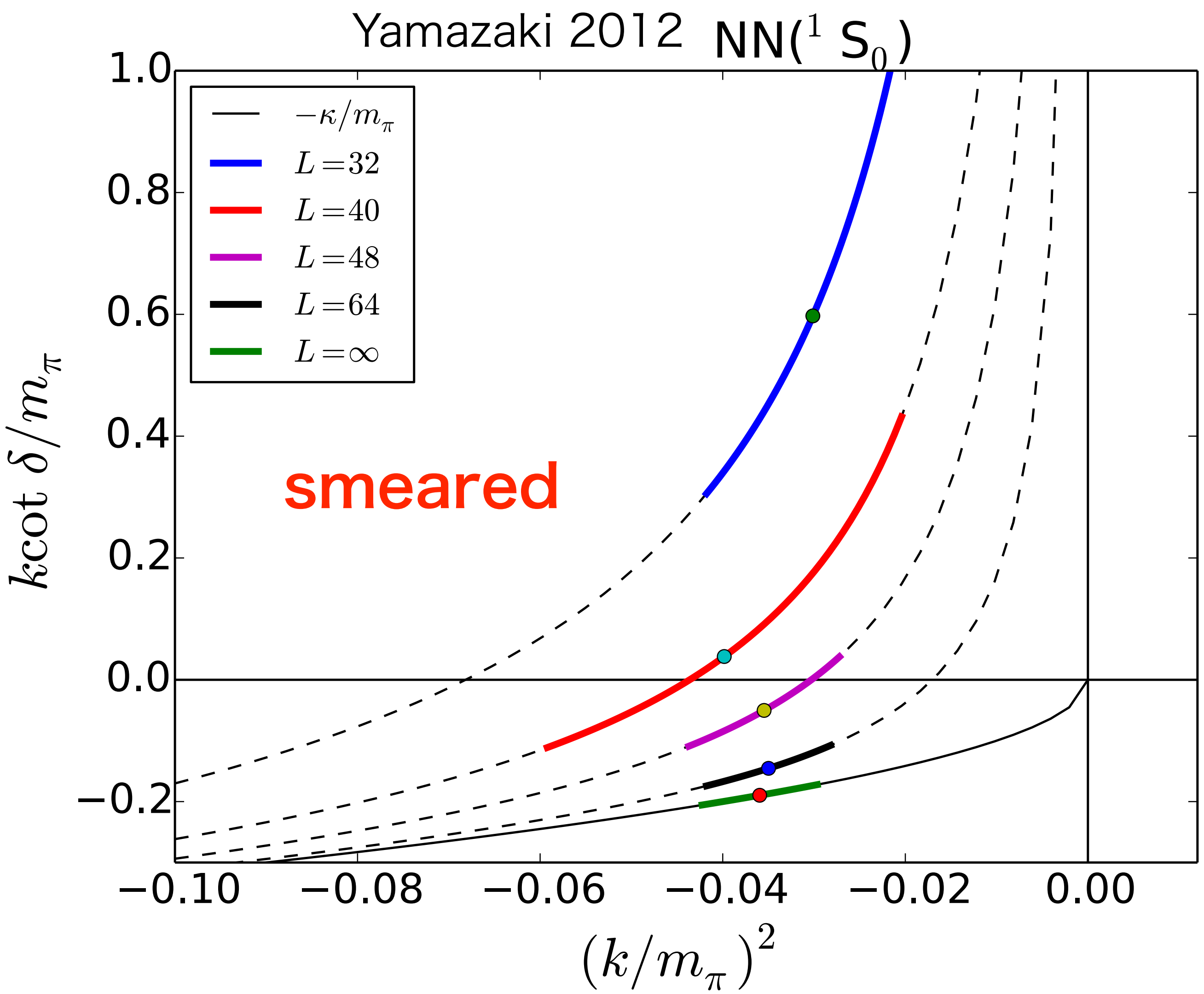


Results

Yamazaki et al. 2012 : PRD86(2012)074514

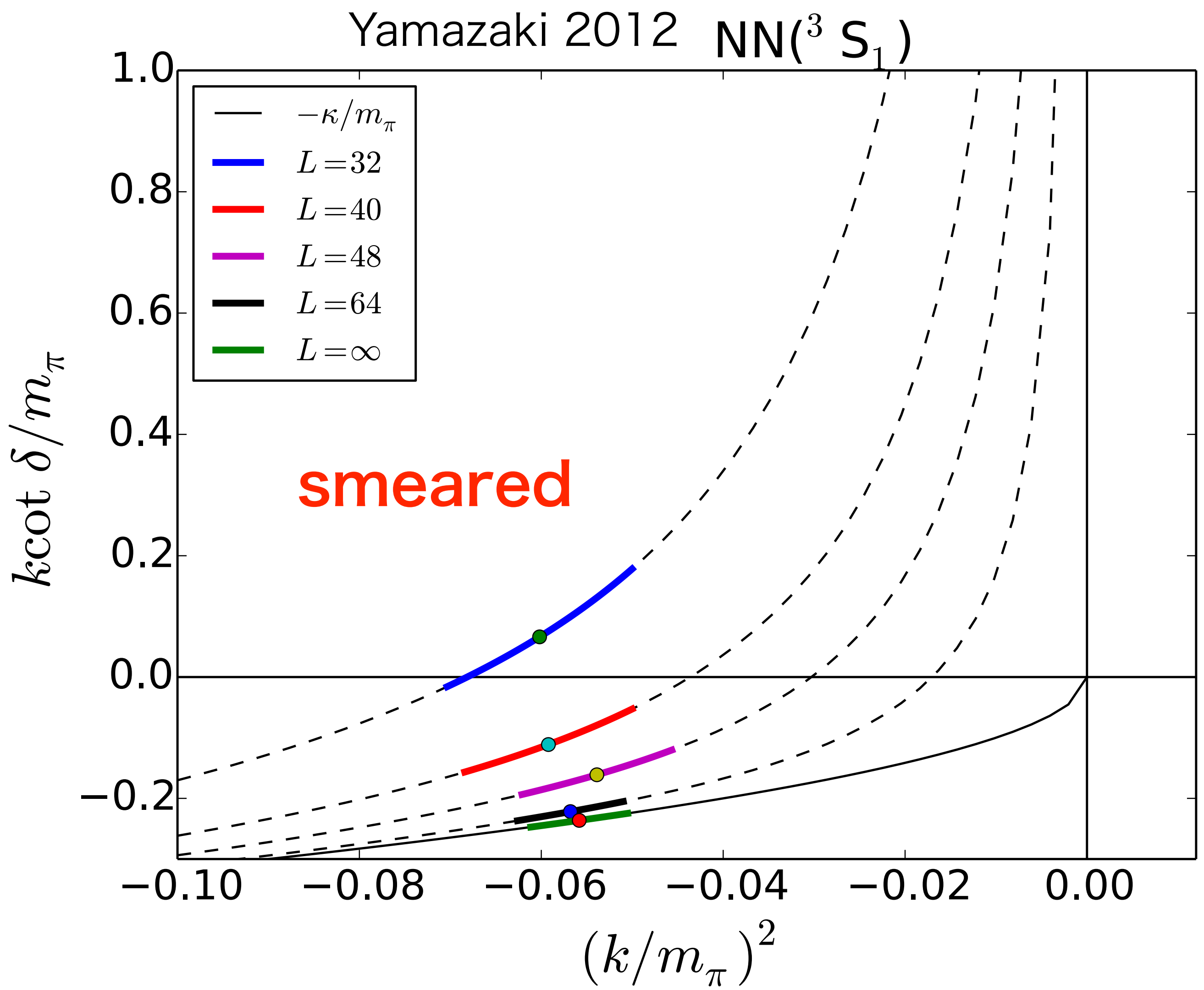
$$N_f = 2+1,\ a \simeq 0.09\ \mathrm{fm},\ m_\pi \simeq 510\ \mathrm{MeV}$$

$\Delta E_{NN}(^1S_0) \simeq -7.4(1.3)\ \mathrm{MeV}$



strange behaviors

$\Delta E_{NN}(^3S_1) \simeq -11.5(1.1)\ \mathrm{MeV}$



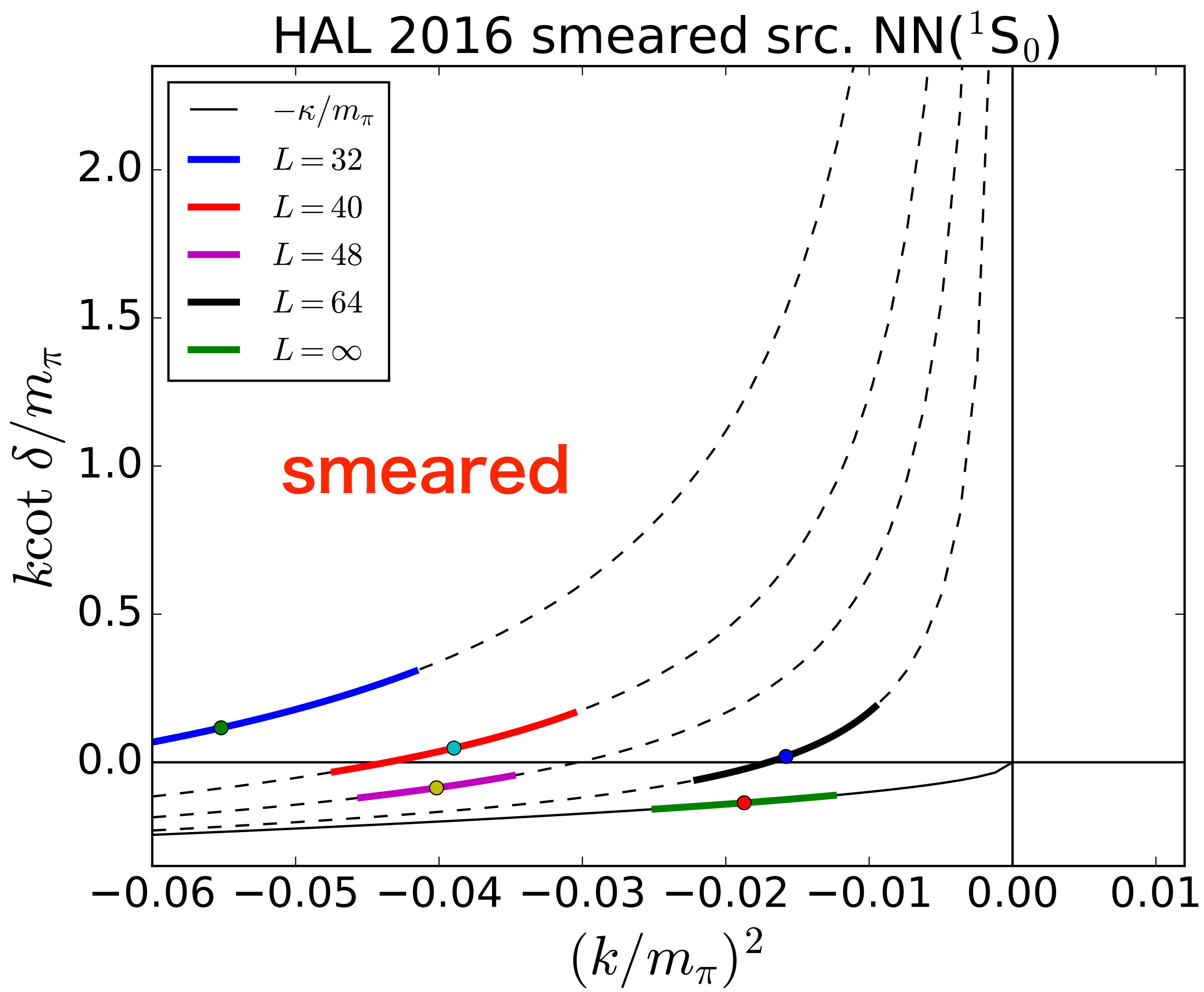
strange behaviors

HALQCD 2016

Data from a previous talk by T. Iritani

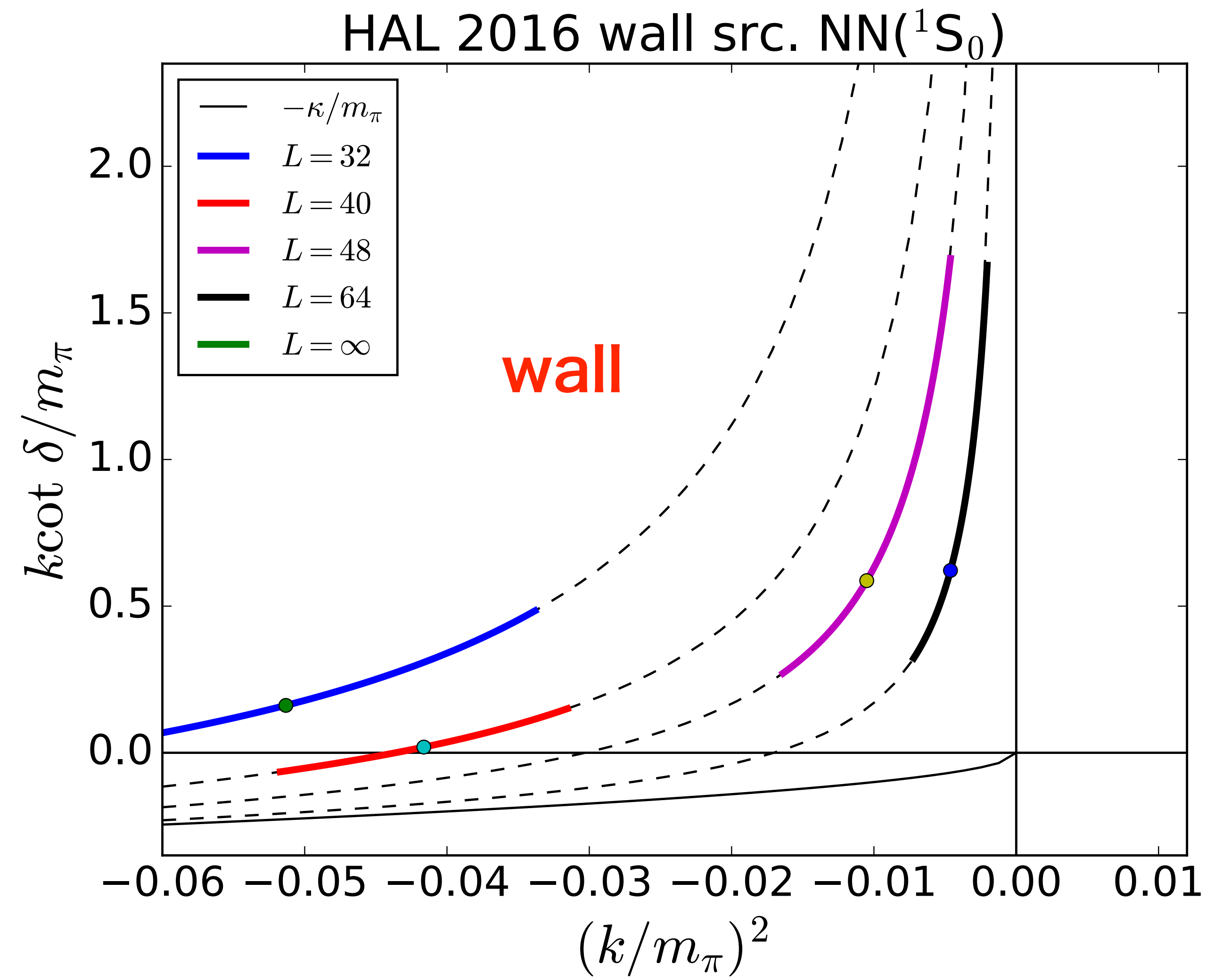
same ensembles of Yamazaki et al. 2012

$$\Delta E_{NN}(^1S_0) \simeq -3.9(1.3)\ \mathrm{MeV}$$



strange behaviors

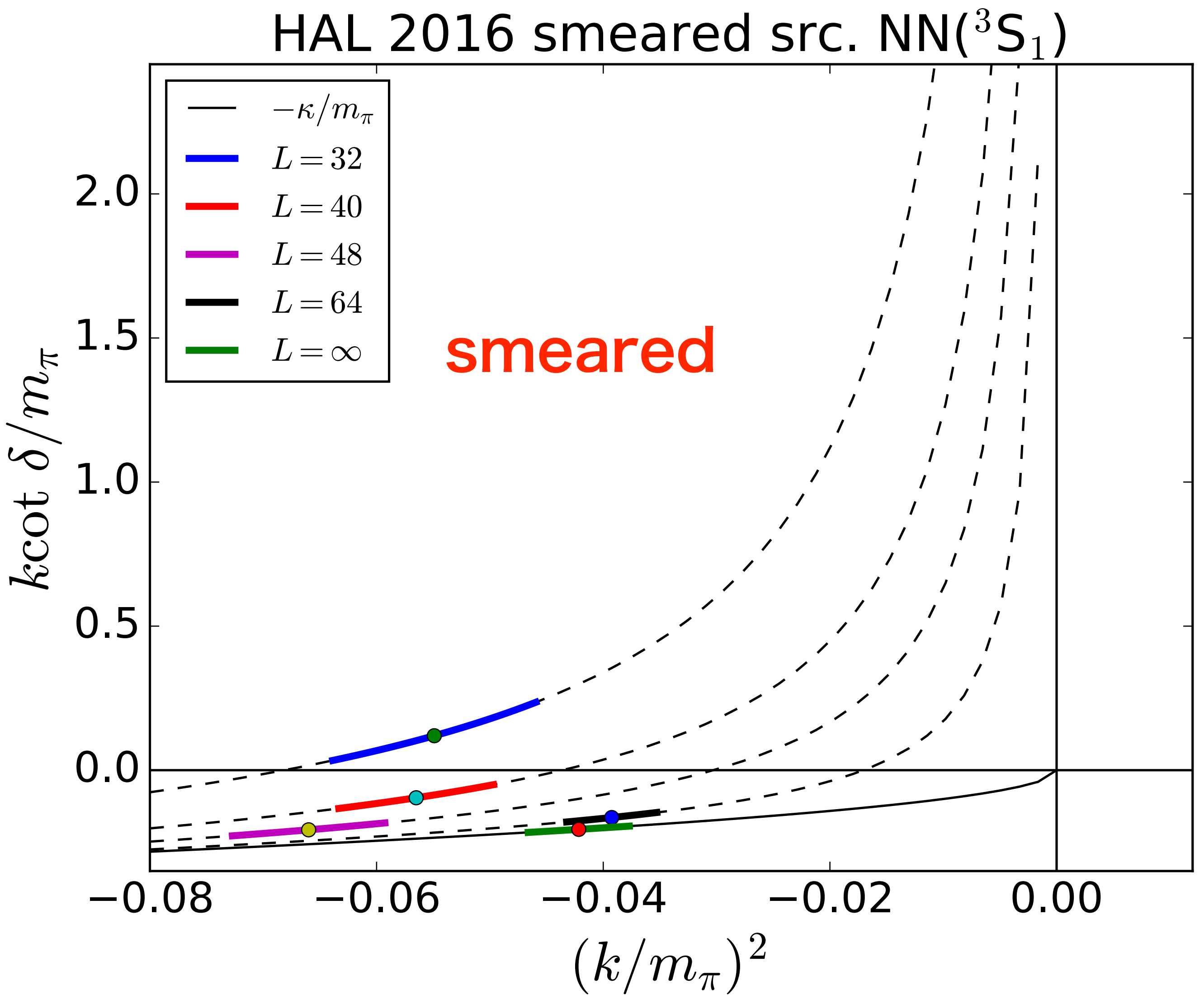
$$\Delta E_{NN}(^1S_0) \simeq -0.7(0.8)\ \mathrm{MeV}$$



strange behaviors

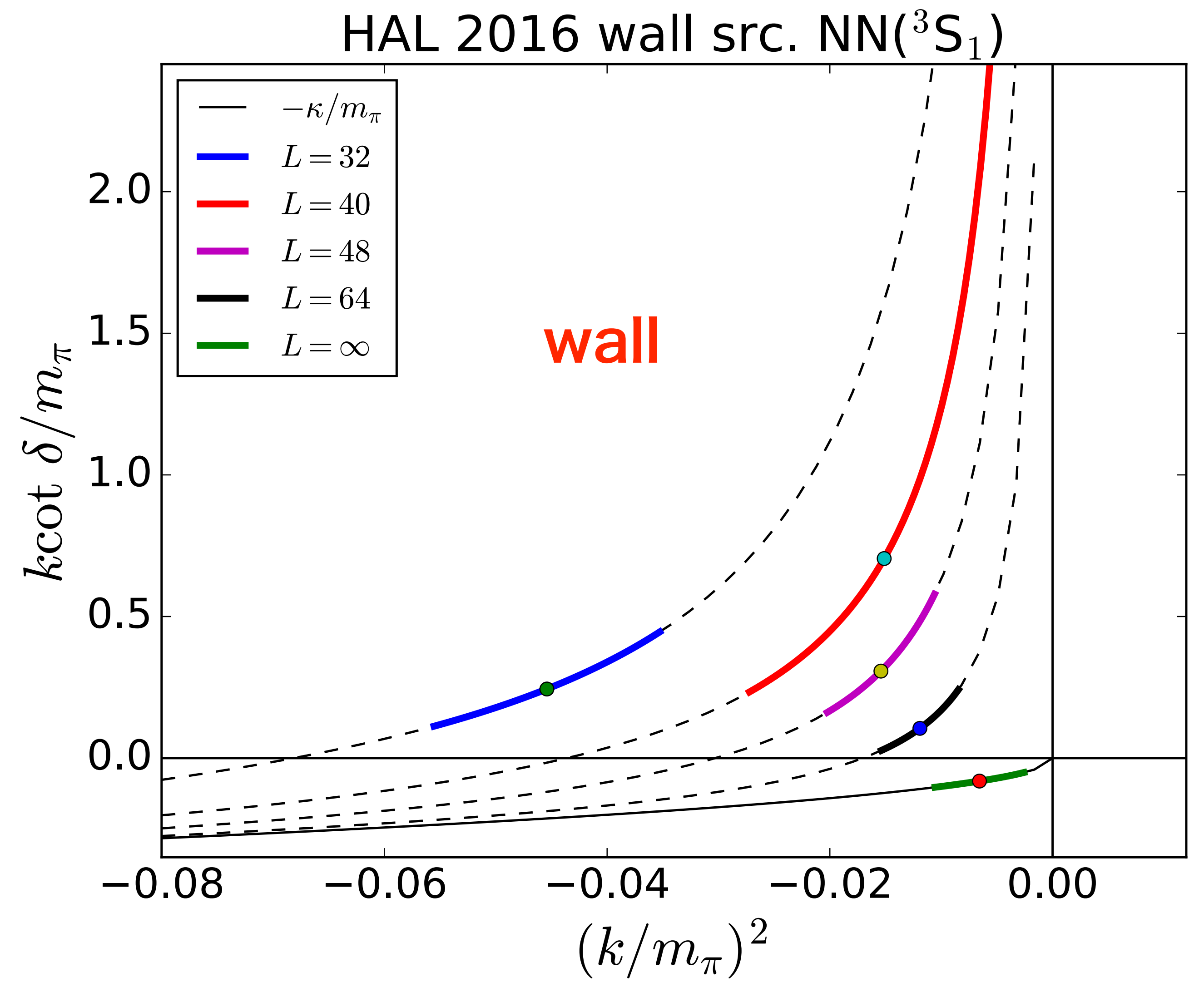
except two largest volumes

$\Delta E_{NN}(^3S_1) \simeq -8.7(0.9)$ MeV



strange behaviors

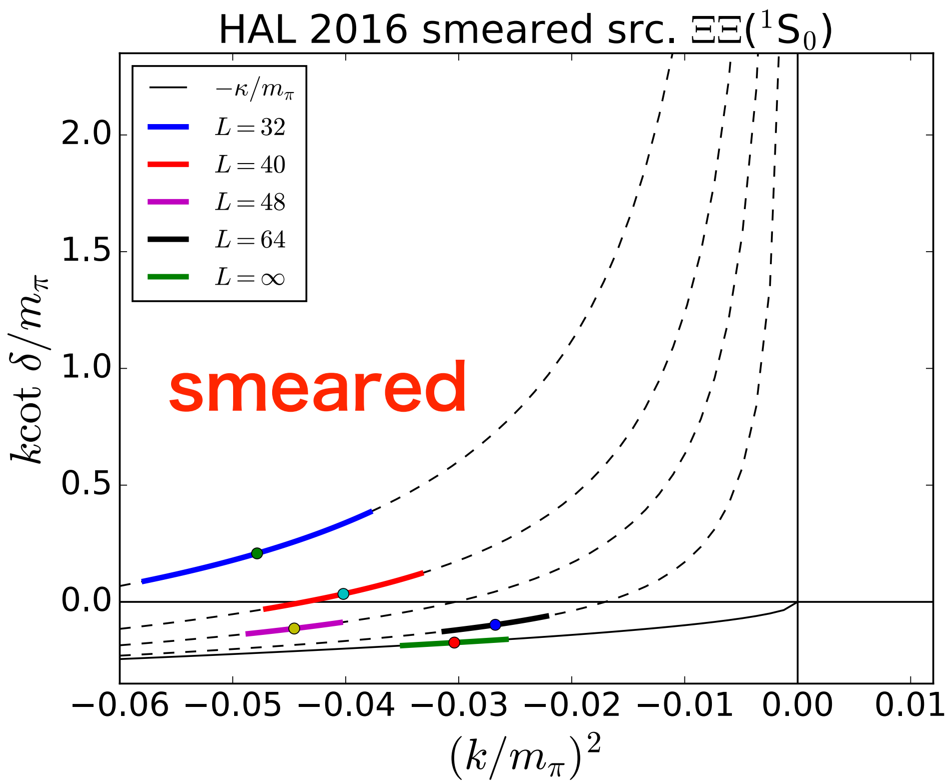
$\Delta E_{NN}(^3S_1) \simeq -1.4(0.8)$ MeV



strange behaviors

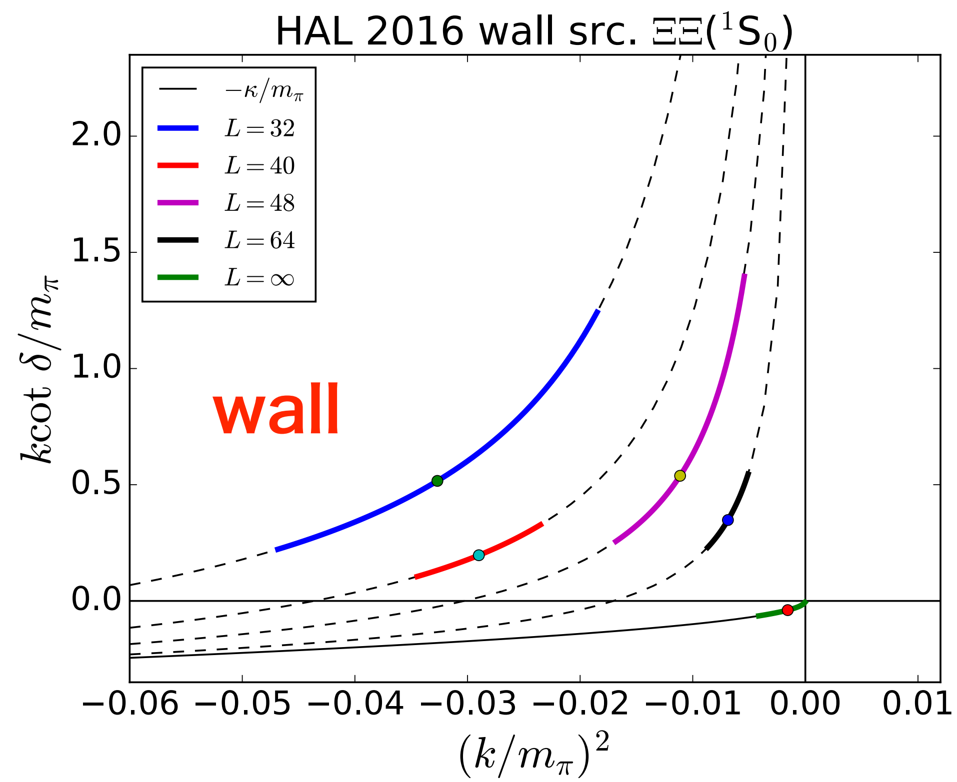
finite volume tests suggest signals for NN bound states are fake.

$\Delta E_{\Xi\Xi}(^1S_0) \simeq -5.4(0.8)$ MeV



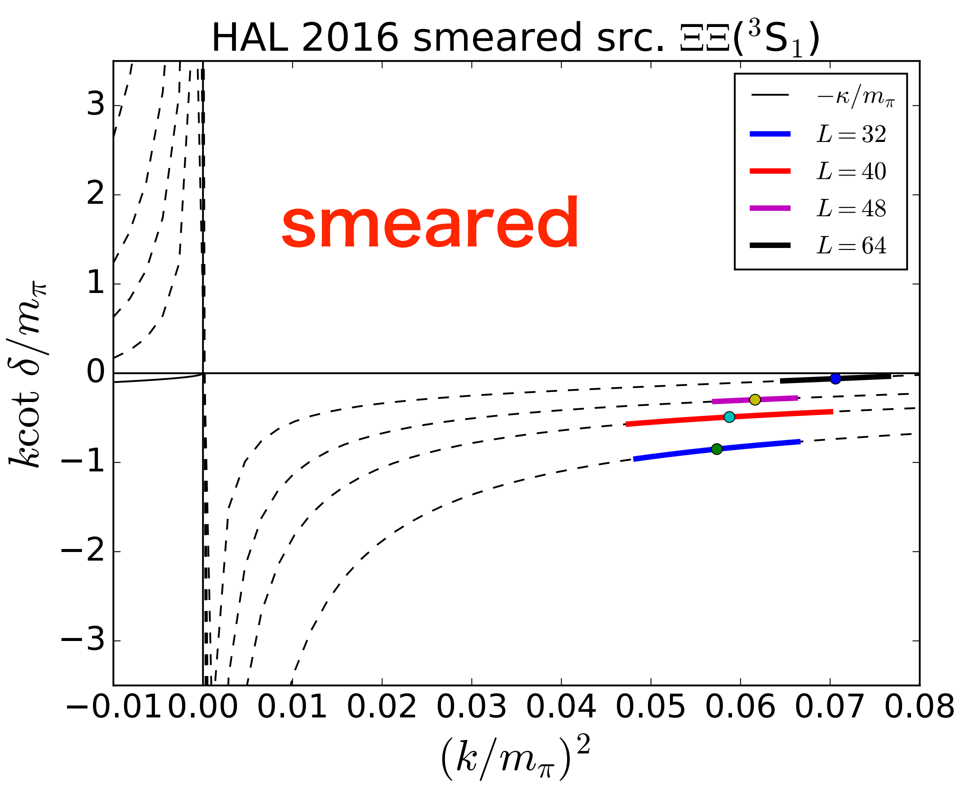
strange

$\Delta E_{\Xi\Xi}(^1S_0) \simeq -0.3(0.5)$ MeV



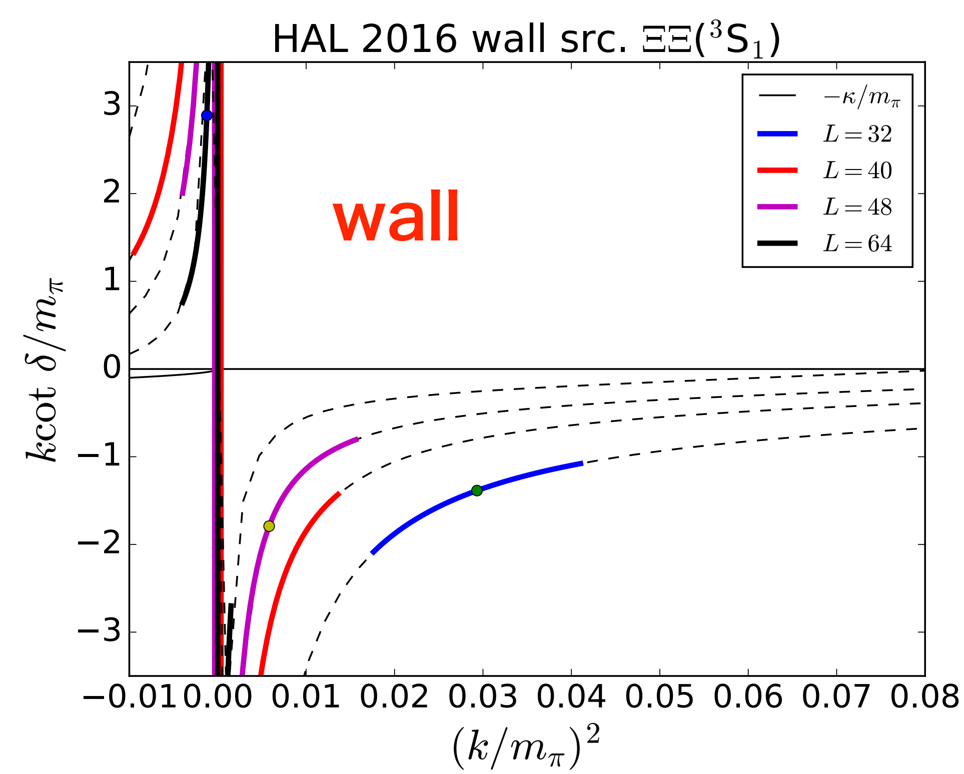
possible

$\Delta E_{\Xi\Xi}(^3S_1) \simeq 12.2(0.9)$ MeV



incorrect

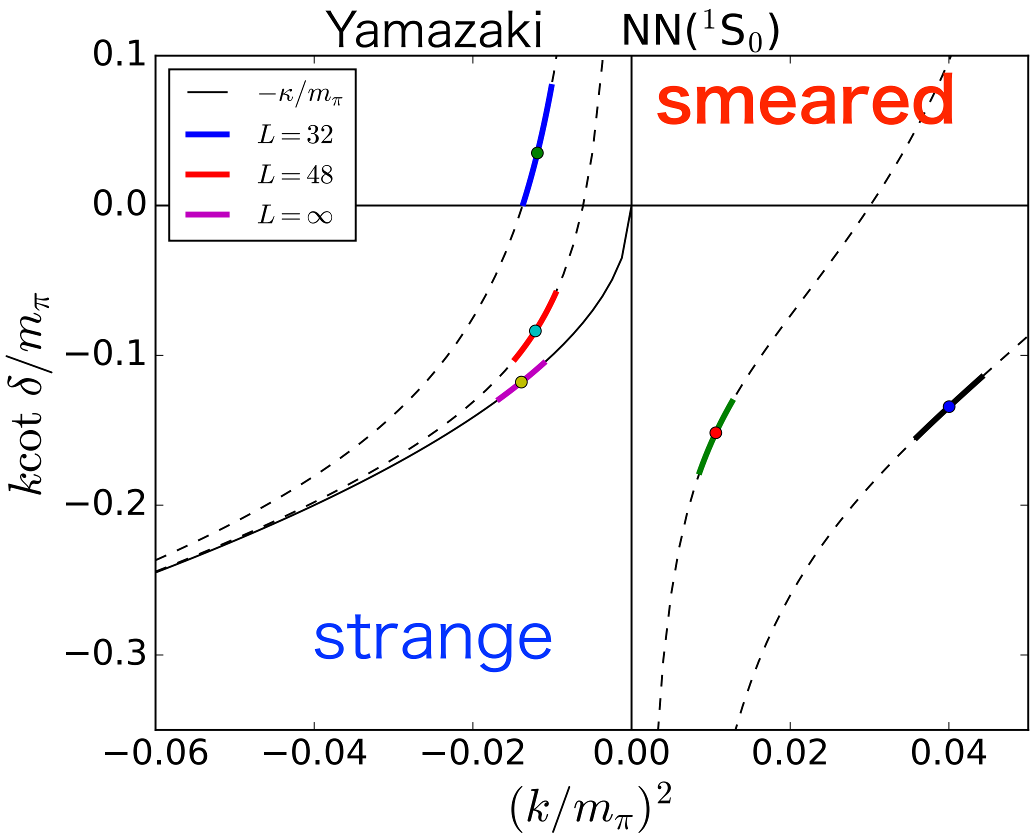
$\Delta E_{\Xi\Xi}(^3S_1) \simeq -0.9(0.6)$ MeV

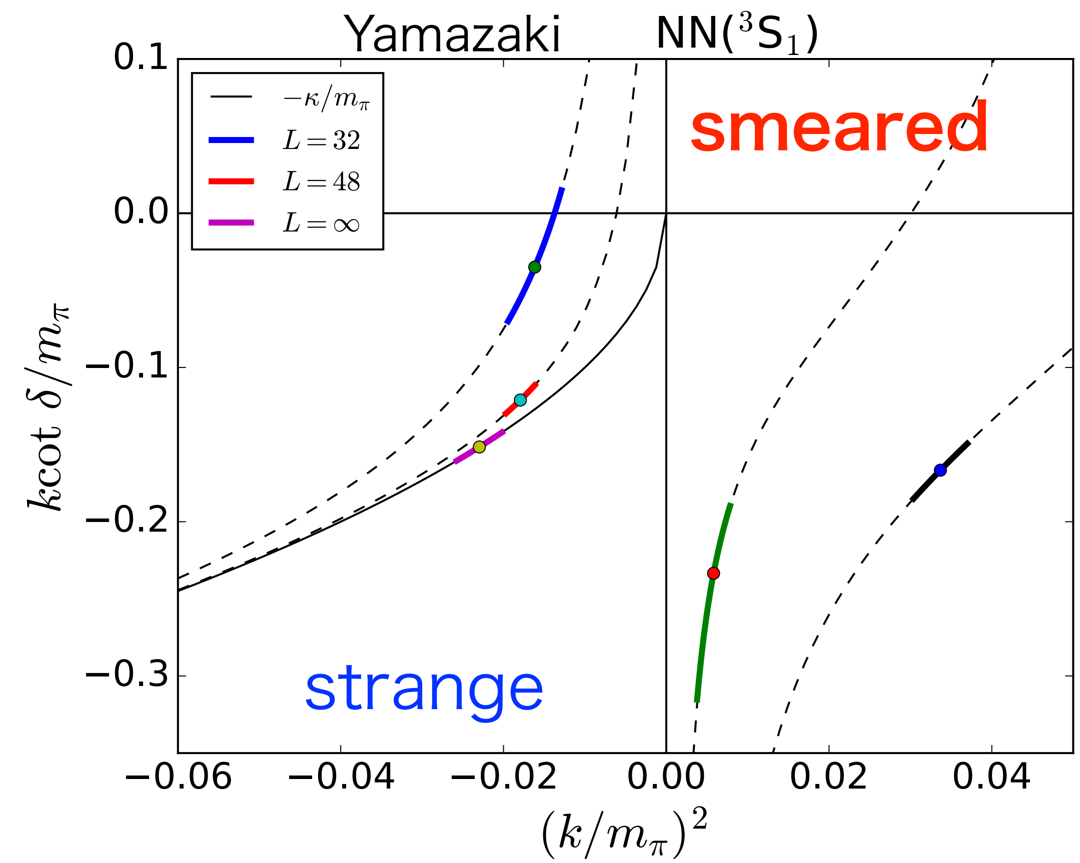


possible

Yamazaki et al. 2011 : PRD84(2011)054506

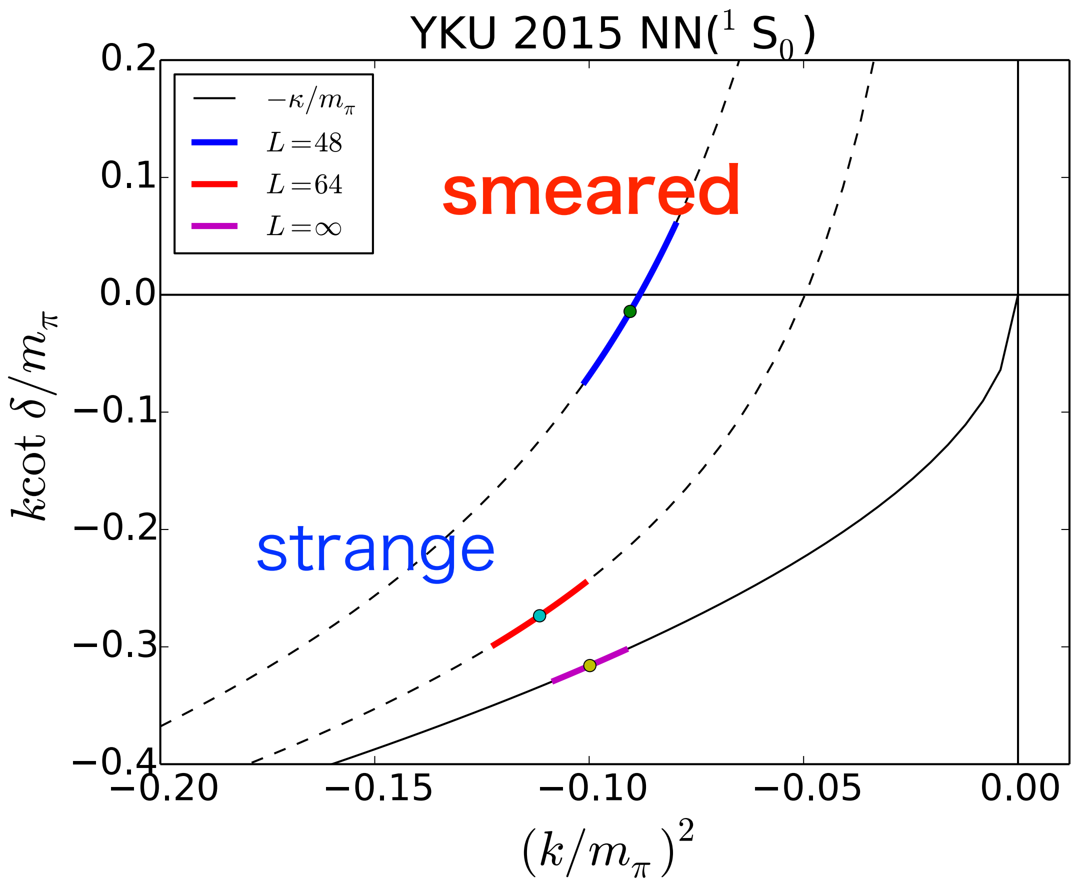
Quenched, $a \simeq 0.128$ fm, $m_\pi \simeq 800$ MeV

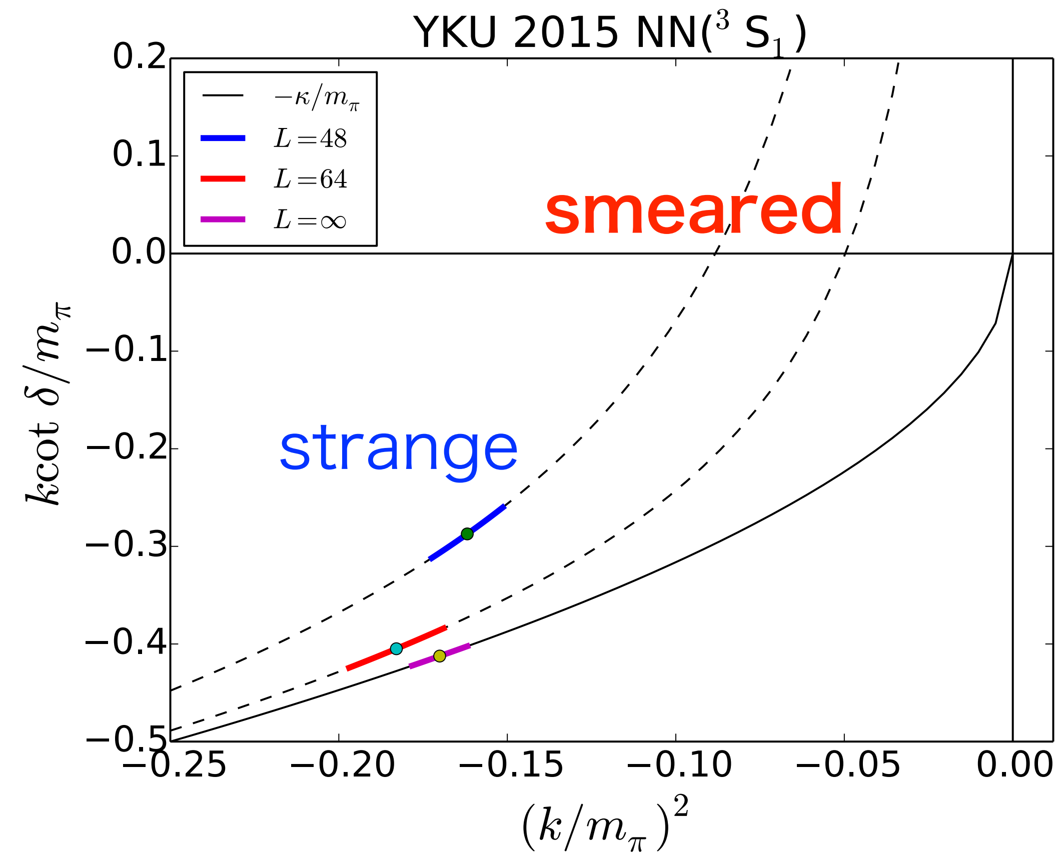




Yamazaki et al. 2015 : PRD92(2015)014501

$N_f = 2+1$, $a \simeq 0.09$ fm, $m_\pi \simeq 300$ MeV





All NN bound states from Yamazaki et al. have strange ERE behaviors

1. finite volume formula does not work (too small volumes) unlikely
2. strange ERE behaviors are correct. unlikely
3. extracted energy shifts are incorrect likely, agrees with Iritani's results

finite volume formula

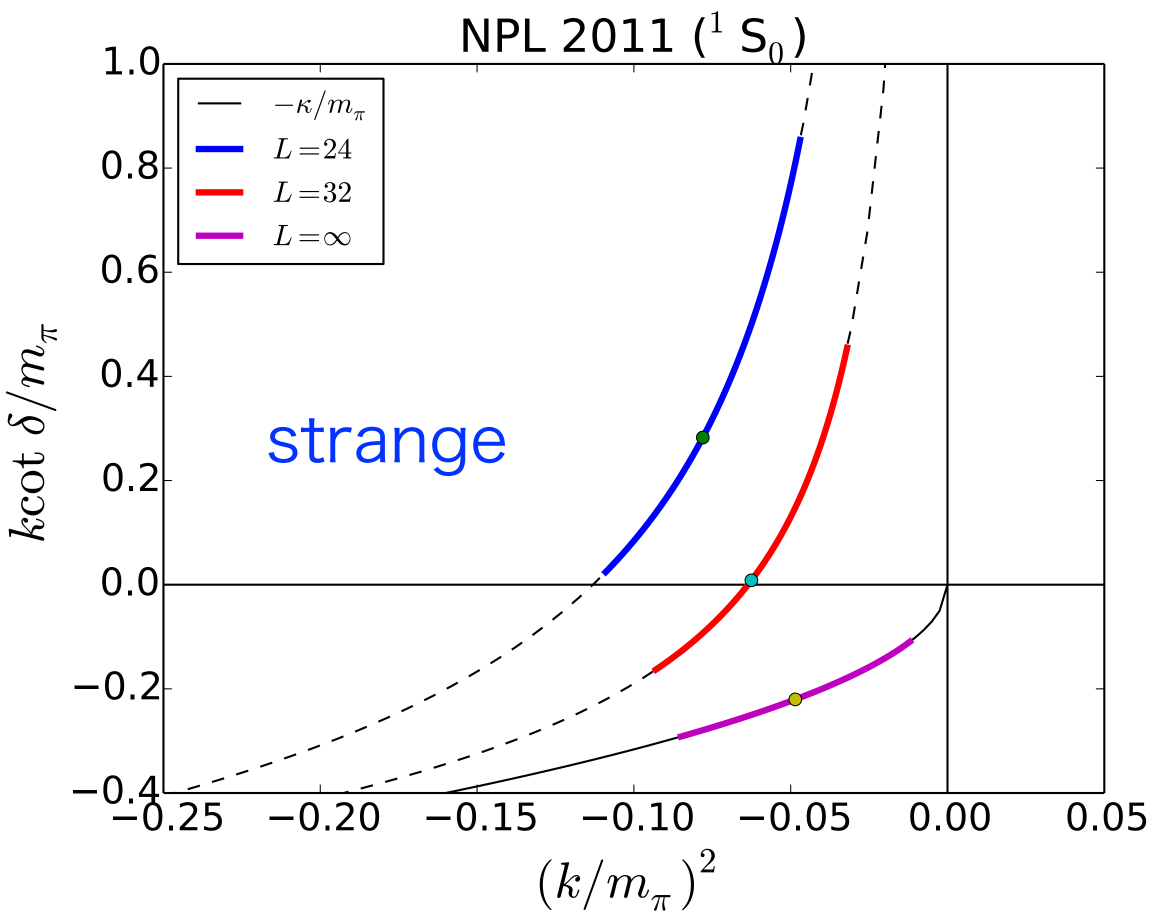
$$\boxed{k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{\vec{n}^2 - q^2} = \frac{1}{a_0} + \frac{r_0}{2} k^2 + \cdots}$$

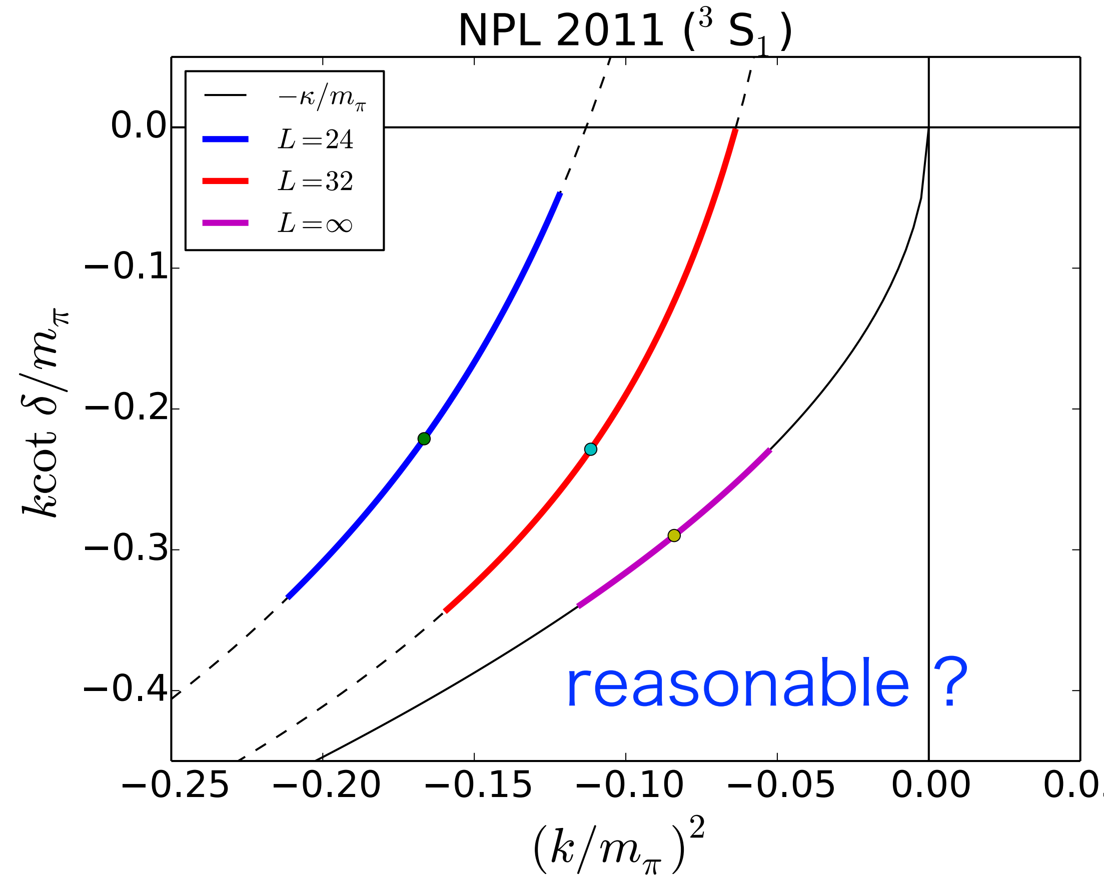
a very easy and useful test for a reliability of the extracted energy shift

How about other results ?

NPL 2011 : PRD85(2012)054511

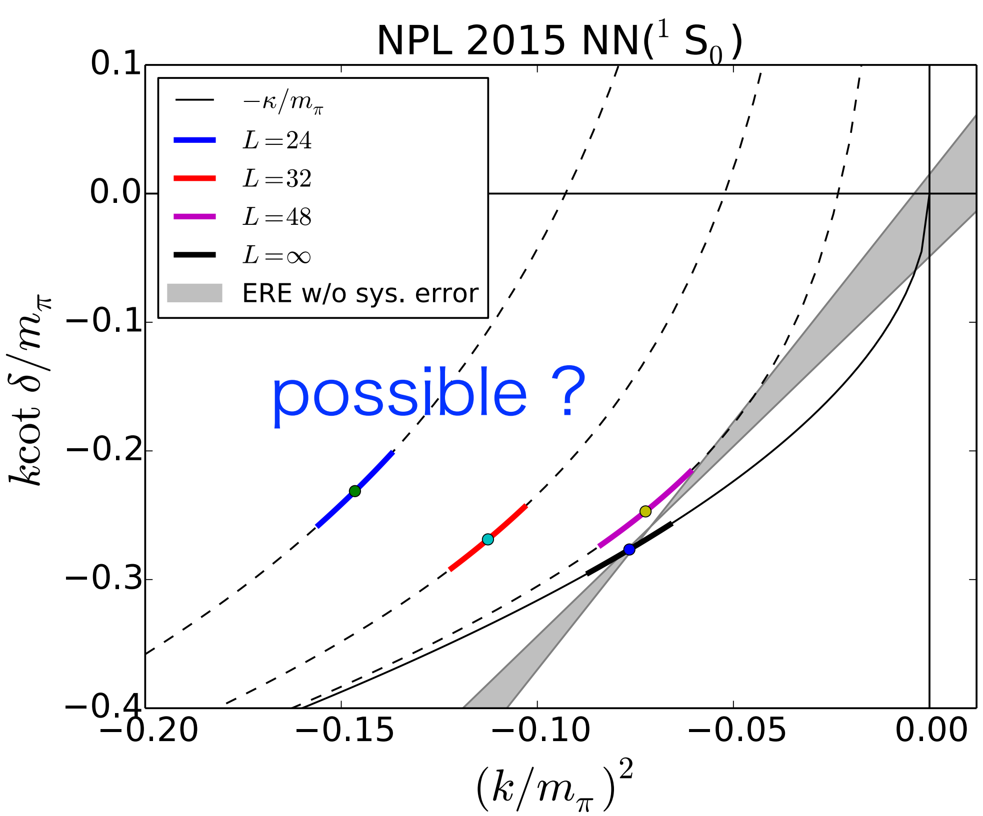
$N_f = 2+1,\ a_s \simeq 0.123$ fm, $a_s/a_t \simeq 3.5,\ m_\pi \simeq 390$ MeV

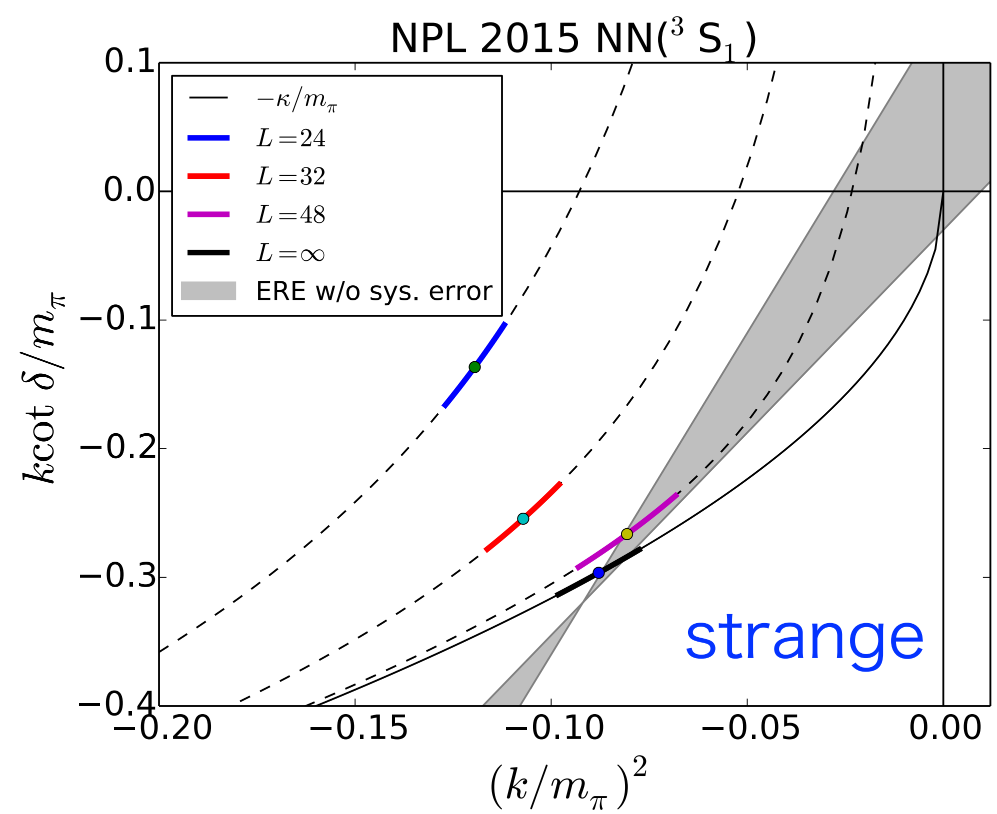




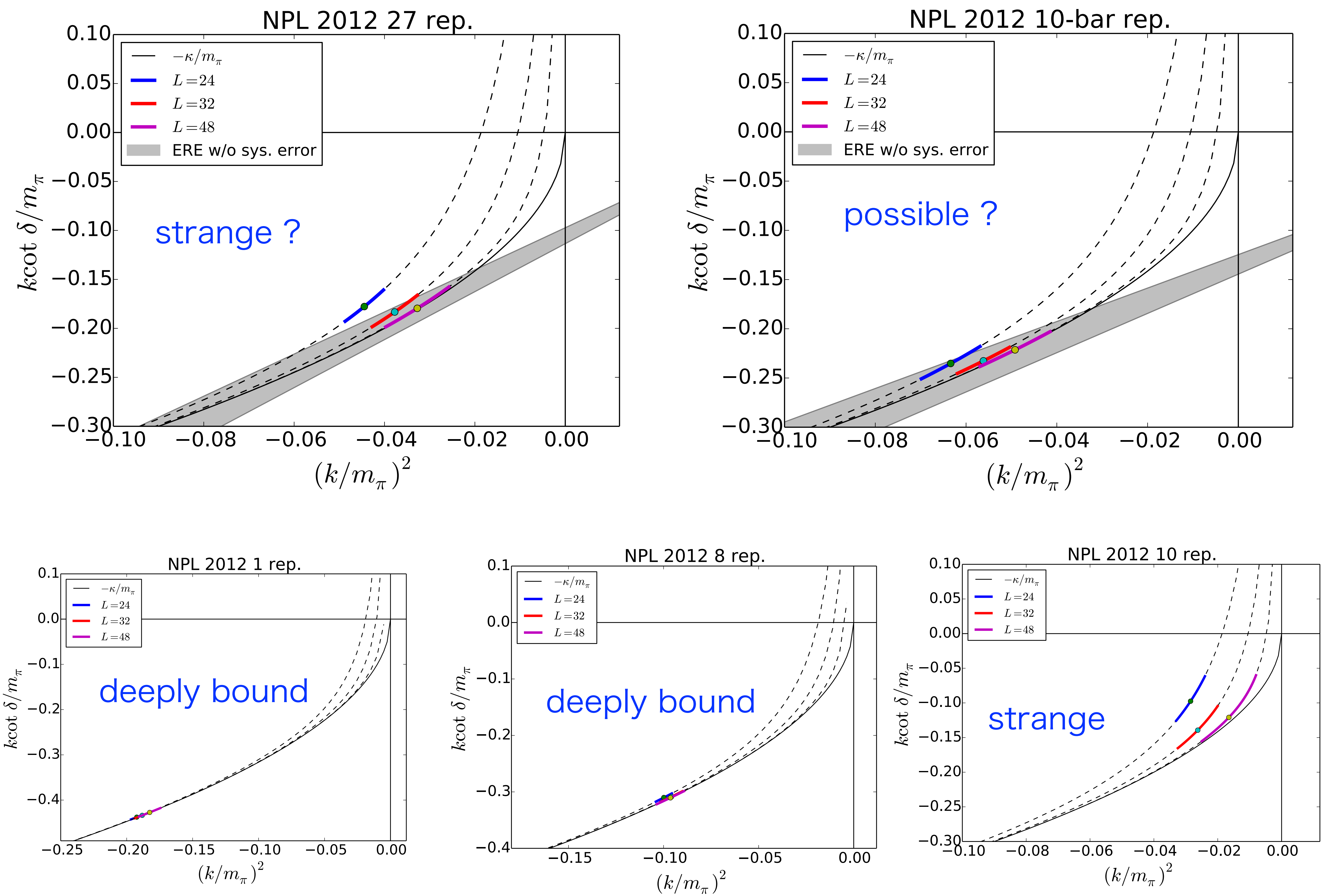
NPL 2015 : PRD92(2015)114512

$N_f = 2+1,\ a \simeq 0.1167$ fm, $m_\pi \simeq 450$ MeV

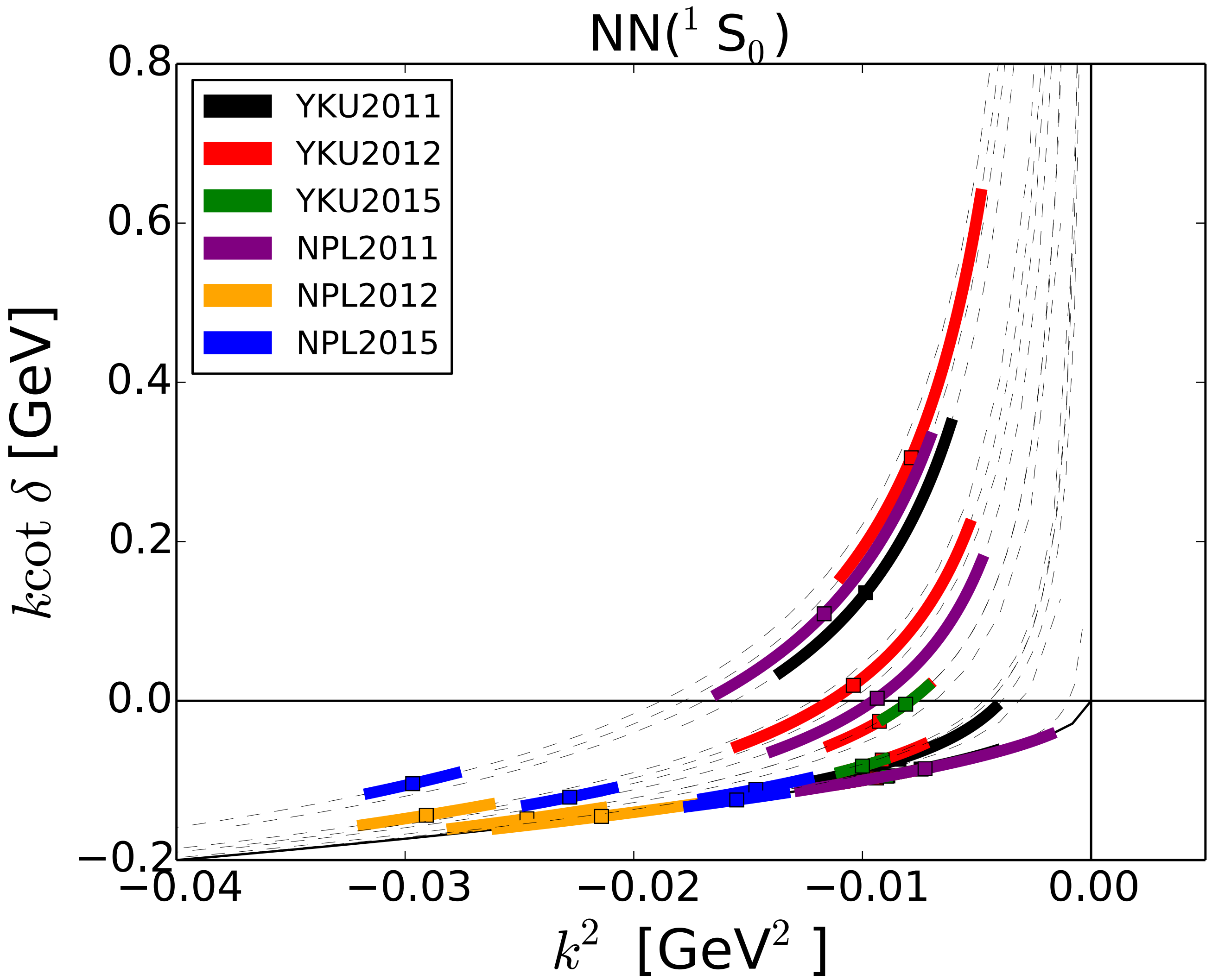


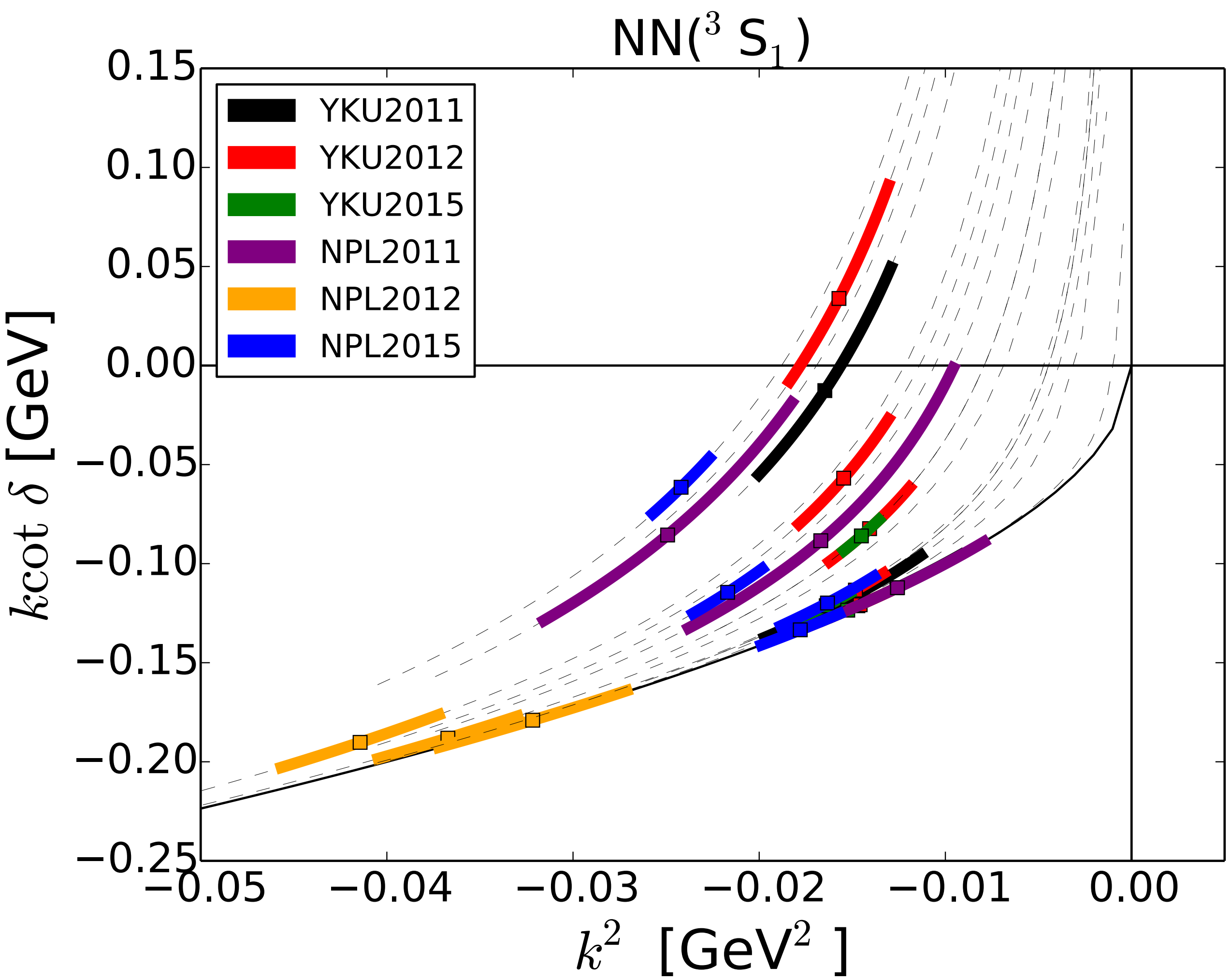


NPL 2012 : PRC88(2013)024003 $N_f = 3$ (SU(3) limit), $a \simeq 0.145$ fm, $m_{\mathrm{PS}} \simeq 800$ MeV



Summary Plots

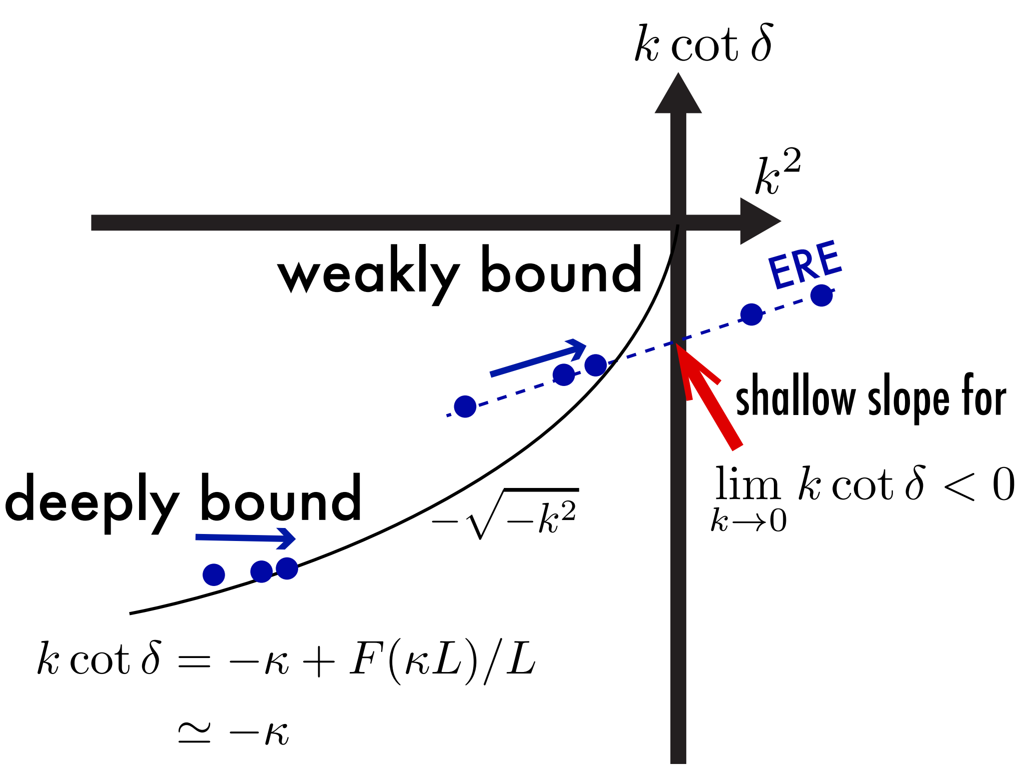




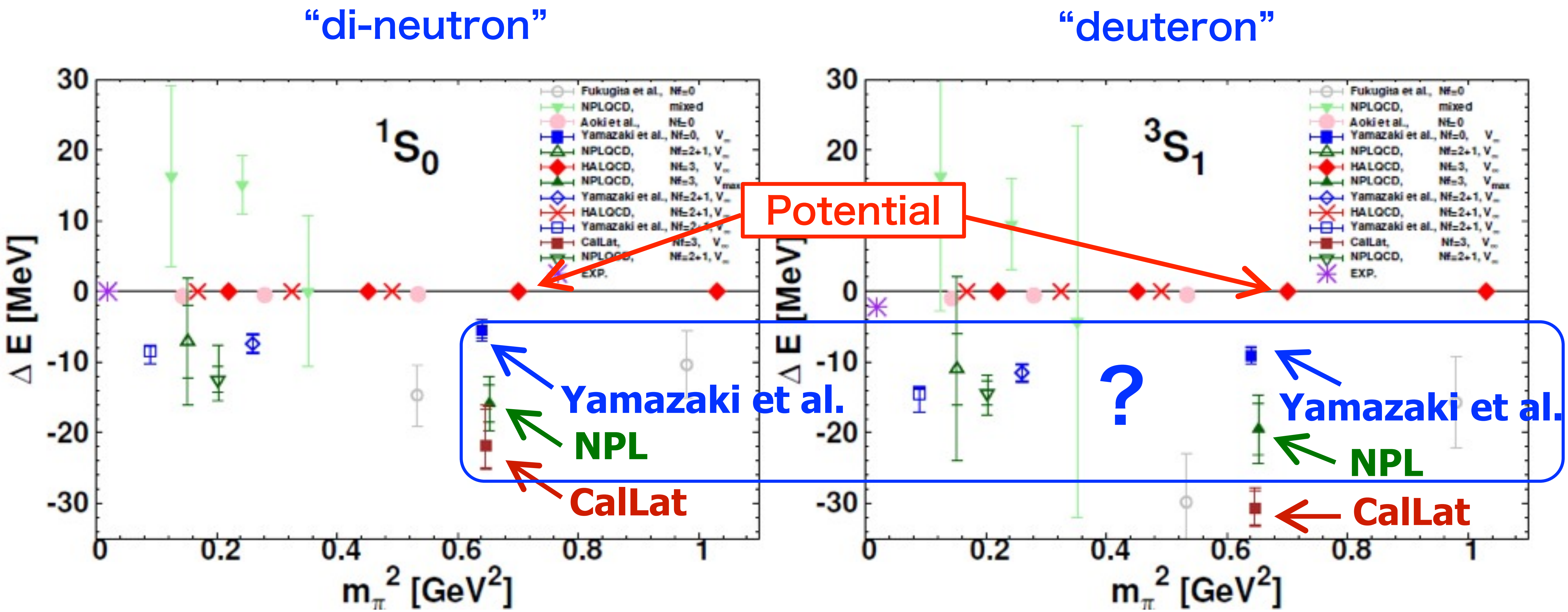
Conclusion and Discussion

- Finite volume formula give a useful test for the bound states.
  - Yamazaki et al.: very strange behaviors (fail the test)
    - confirmed by HAL smeared data.
  - NPL: some pass, the other fail the test. (Not conclusive)
    - necessary test but can not guarantee the correctness.
    - need further checks (wall vs. smeared, variational method)
- finite volume test is mandatory for the bound state search in lattice QCD
- the formula should be used for the infinite volume extrapolation
  - using LO (NLO) ERE

$$\boxed{k\cot\delta(k) = \frac{1}{\pi L}\sum_{\vec{n}\in\mathbb{Z}^3}\frac{1}{\vec{n}^2 - q^2} = \frac{1}{a_0} + \frac{r_0}{2}k^2 + \cdots}$$



Direct vs Potential : NN systems



Potential method (HALQCD) : unbound

Direct method (Yamazaki et al./NPL/CalLat): ~~bound~~

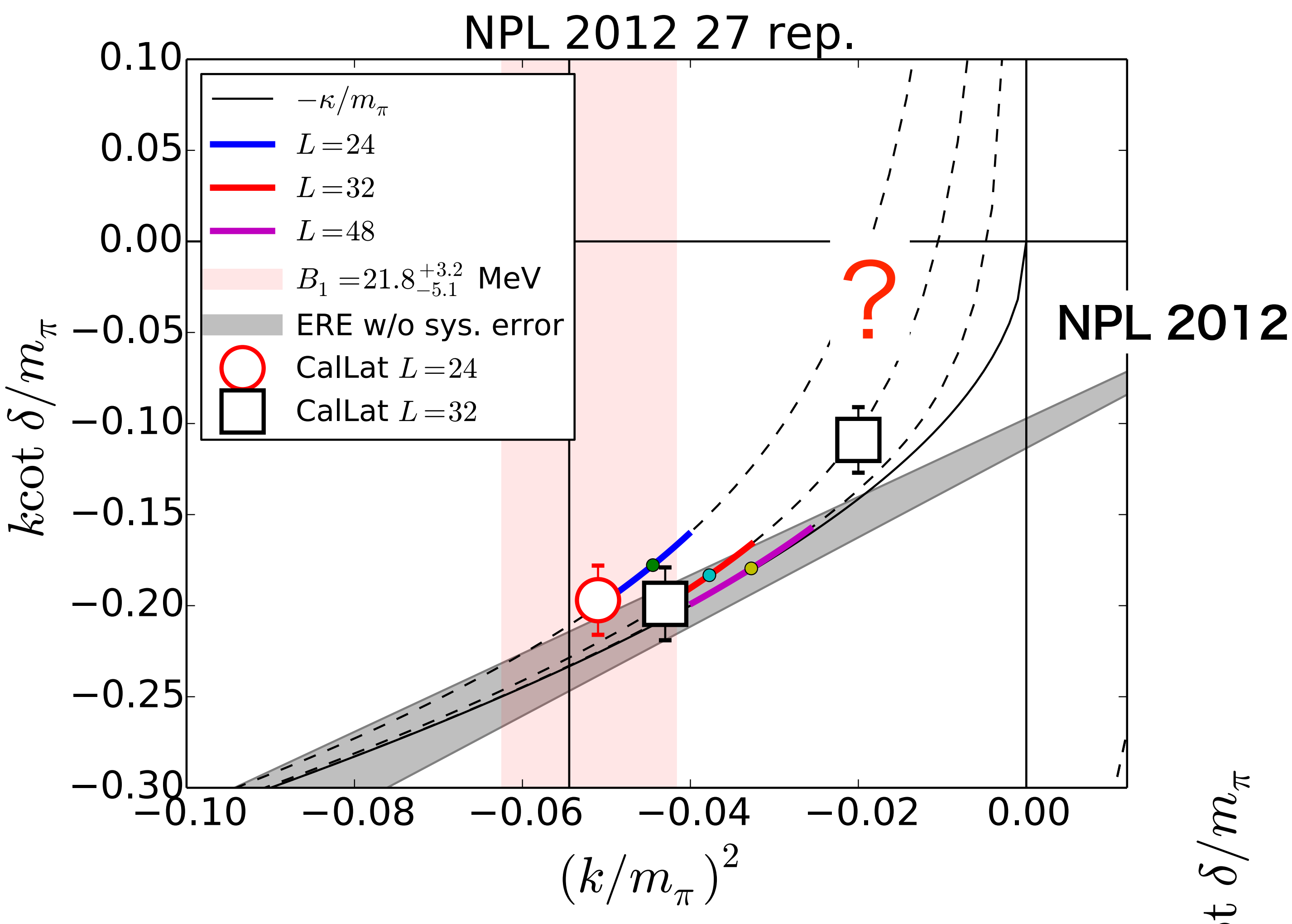
questionable

CalLat2015:arXiv:1508.00886[hep-lat]

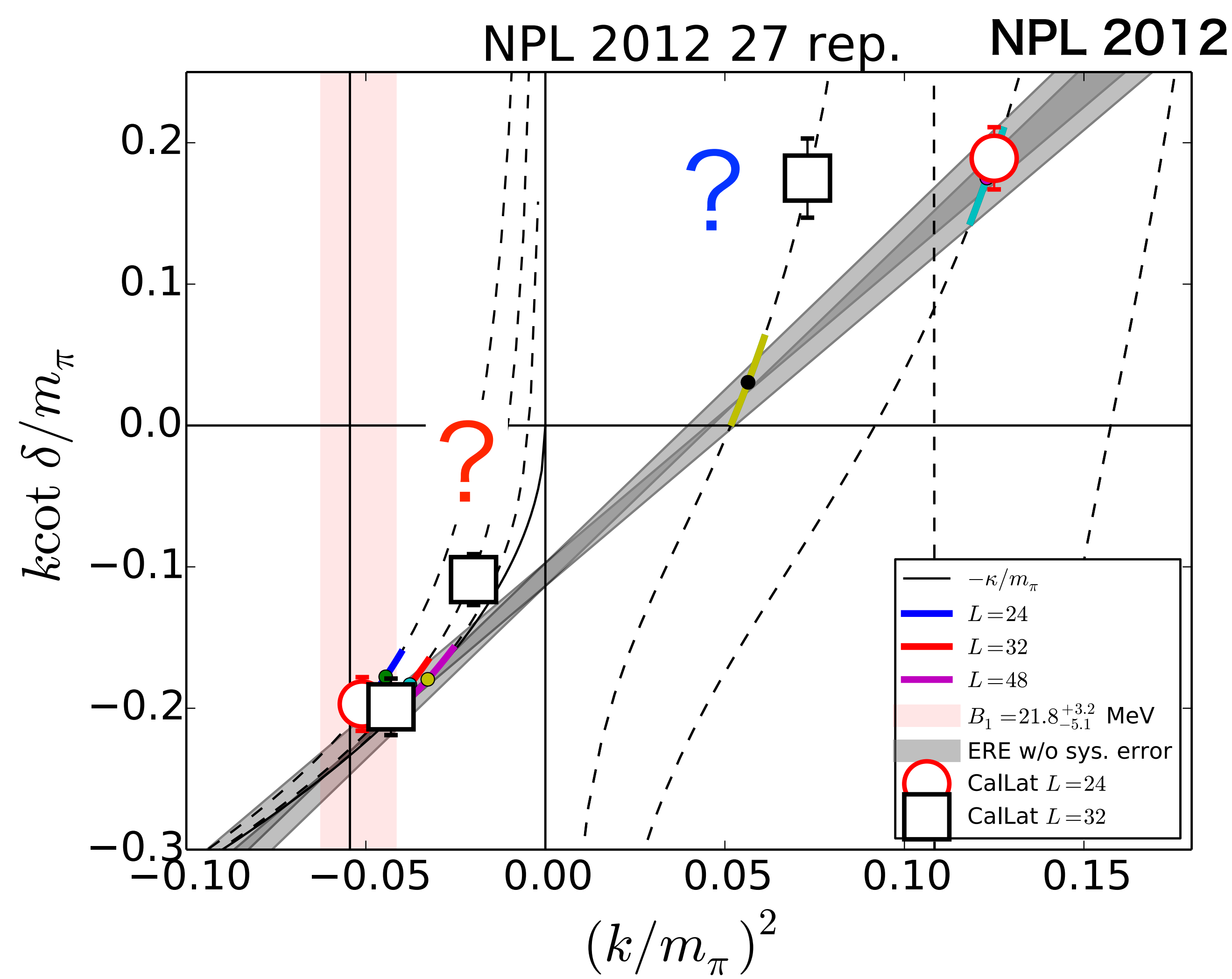
$N_f = 3$ (SU(3) limit), $a \simeq 0.145$ fm, $m_{\mathrm{PS}} \simeq 800$ MeV

same as NPL 2012

$NN(^1S_0)$



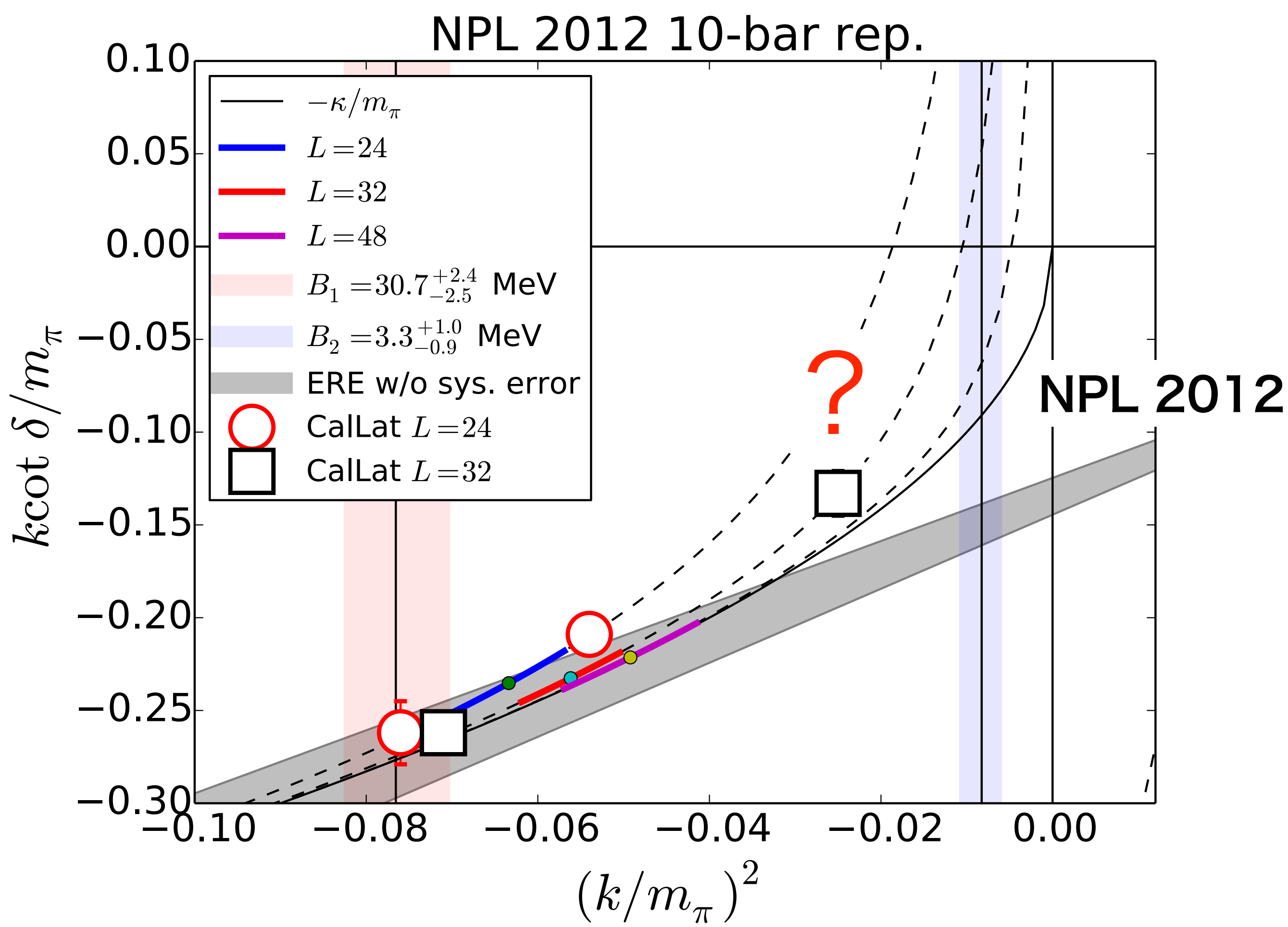
incompatible with NPL ERE?

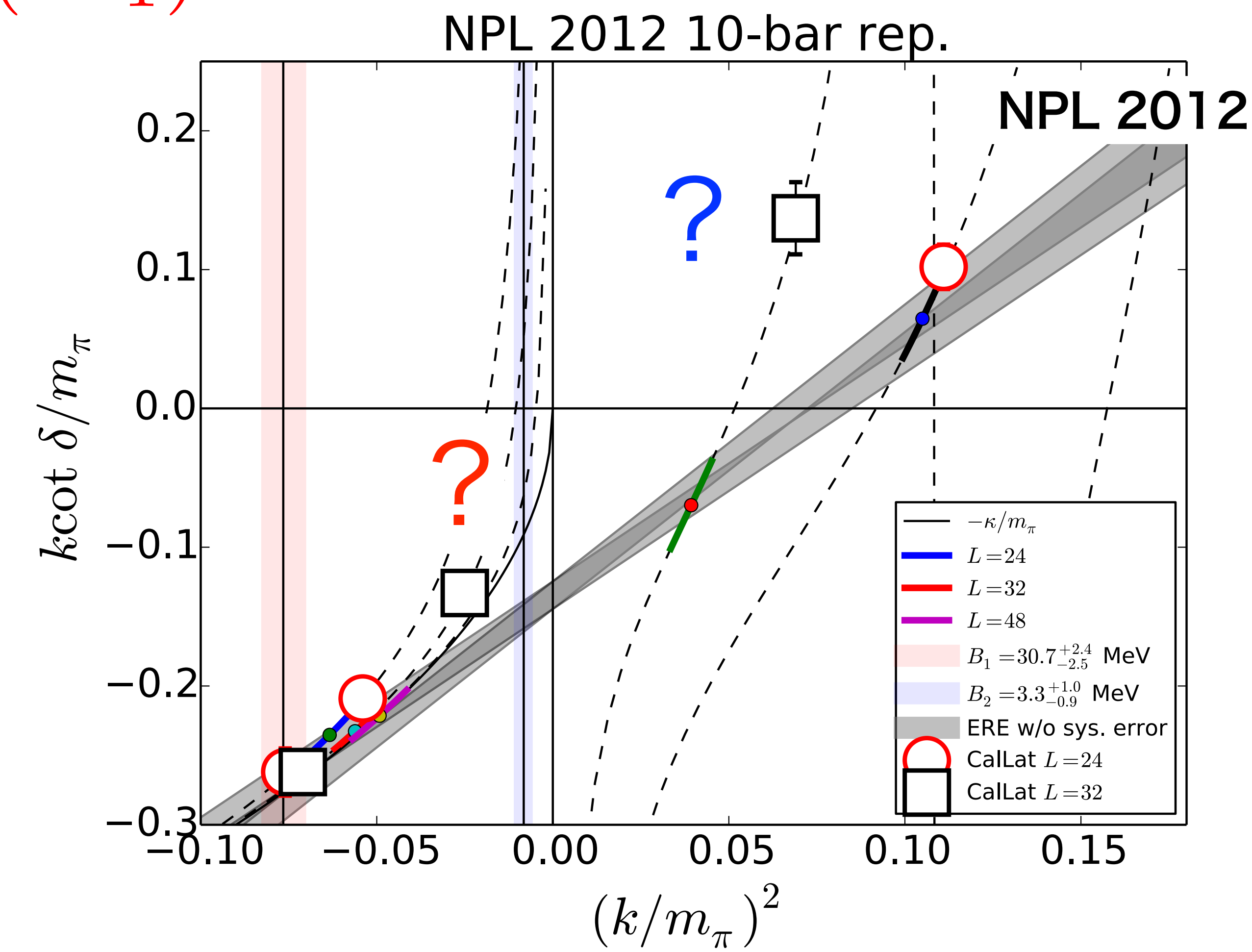


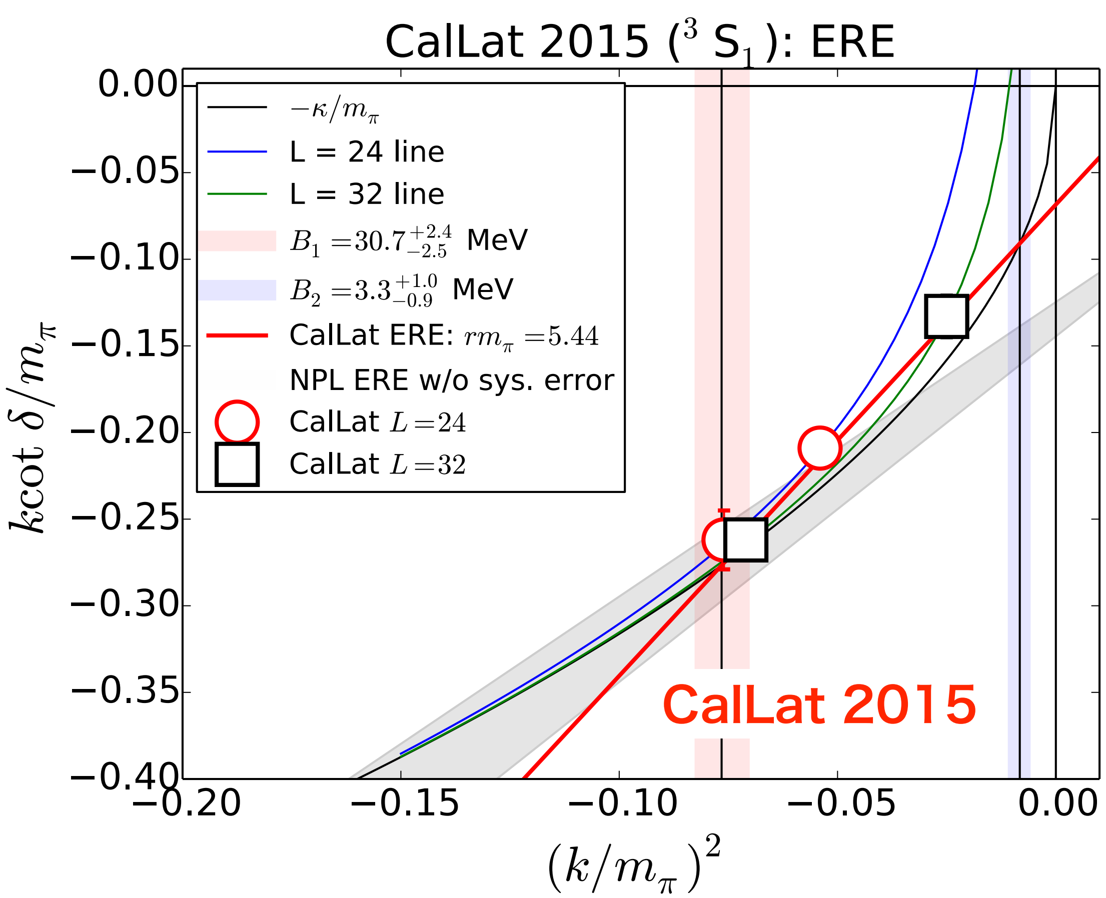
second negative energy state ?
(second bound state ?)

incompatible with NPL ERE ?

$NN(^3S_1)$







NPL and CalLat seems incompatible.

Second bound state in CalLat ?

NLO is large ?

Large effective range ?

Expectation at physical point

$L \simeq 8$ fm