Lattice 2016
Southampton, July 24-30, 2016

Phase structure analysis of CP(N-1) model using Tensor renormalization group

Hikaru Kawauchi
with
Shinji Takeda
Kanazawa University

July 25, 2016

Outline

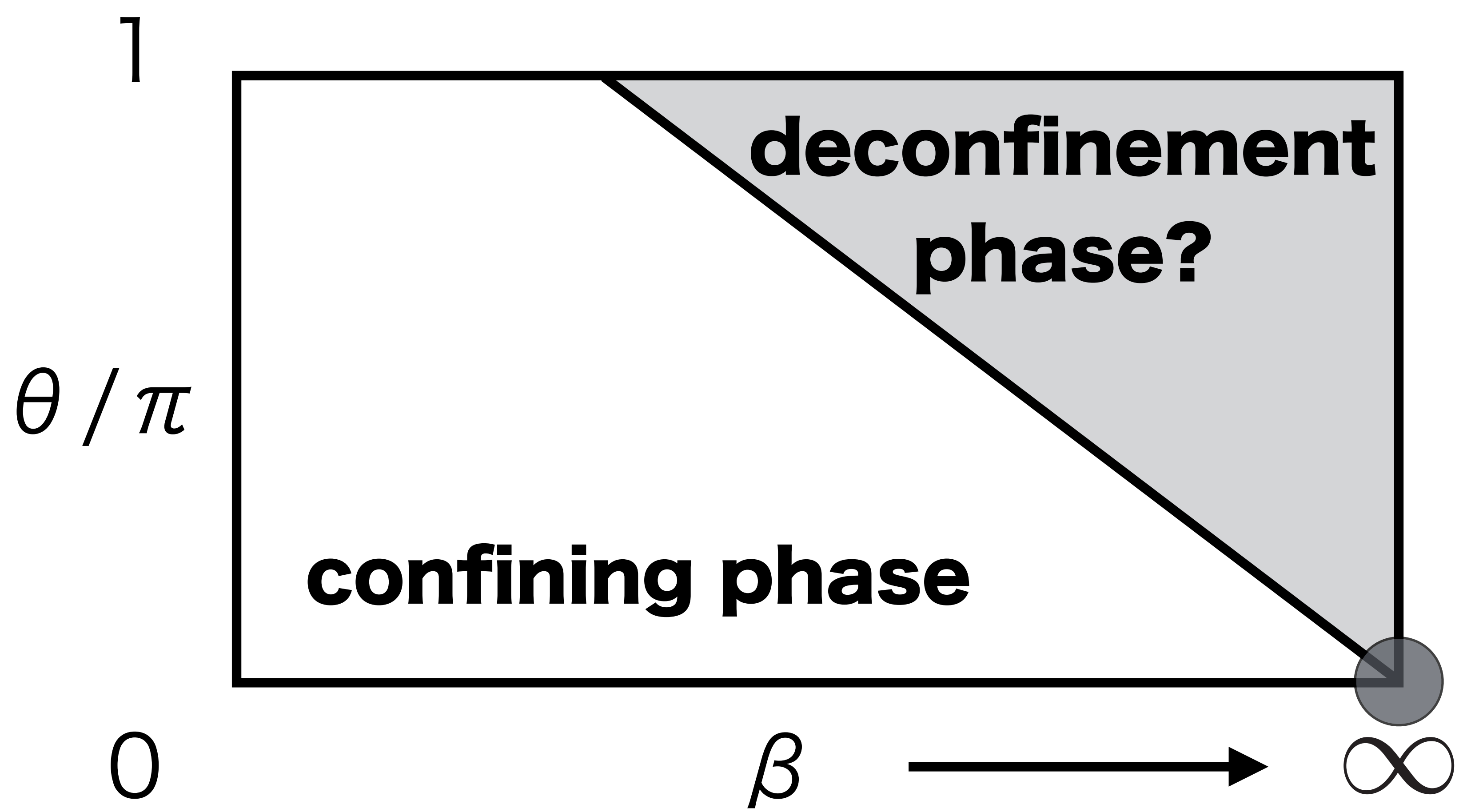
- Introduction
- Tensor Renormalization Group (TRG)
- CP(N-1) model and Numerical results
- Summary

Introduction

- The two dimensional CP(N-1) model including the θ term is a toy model of QCD.
- The phase structure is left vague because of the sign problem.
- One possible way to avoid the sign problem is to simply abandon Monte Carlo simulation.
- We apply the Tensor Renormalization Group method to analyze this phase structure.

Phase structure of CP(N-1) model?

Schierholz suggested a possibility that θ becomes 0 in the continuum limit. G. Schierholz, Nucl. Phys. B, Proc. Suppl. 37A, 203 (1994).

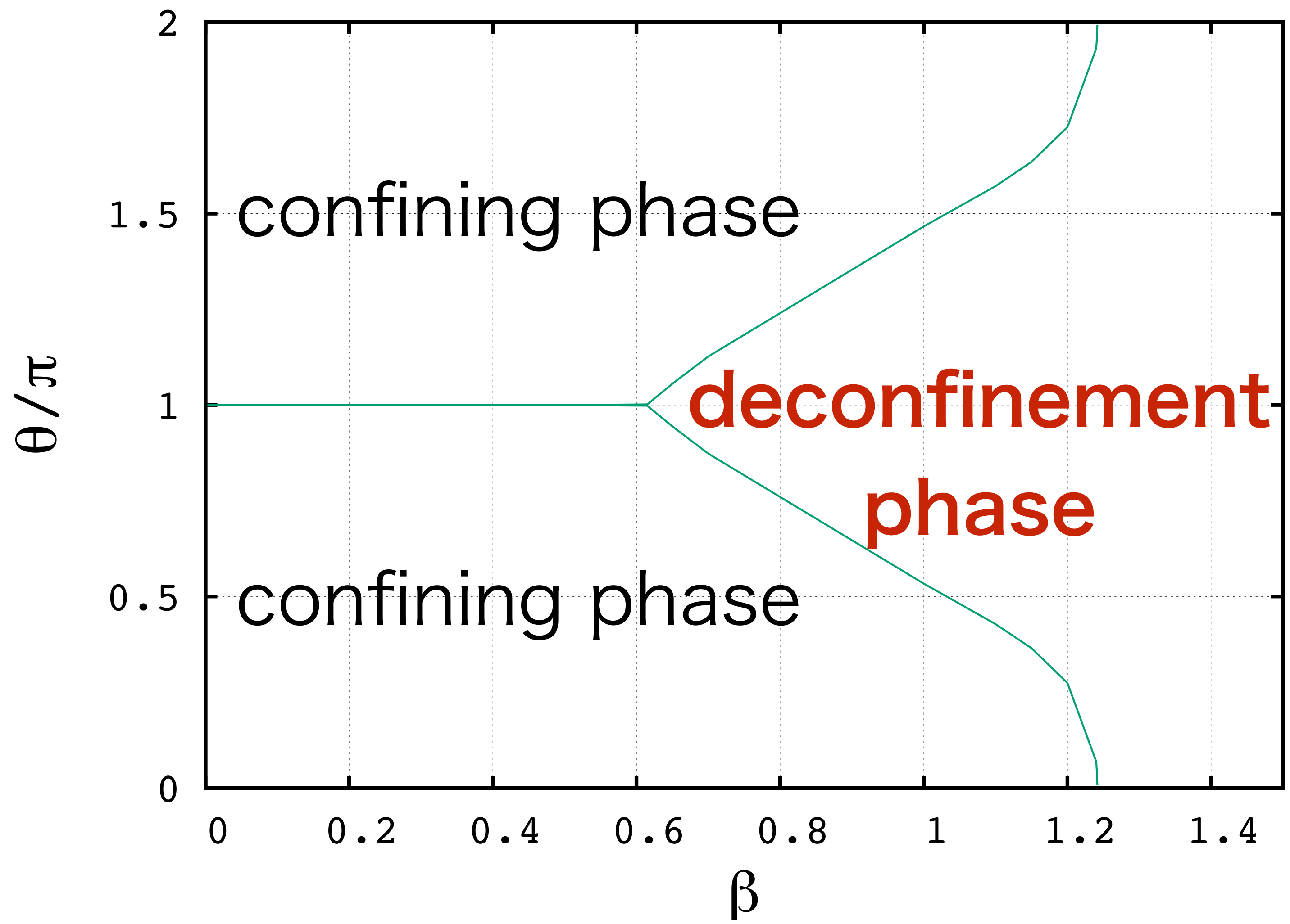


Other researches indicated that the deconfinement phase boundary may appear due to the statistical errors.

J. C. Plefka and S. Samuel, Phys. Rev. D 56, 44 (1997).
M. Imachi, S, Kanou, and H. Yoneyama, Prog. Theor. Phys. 102, 653 (1999).

Strong coupling analysis of the phase structure of the CP(1) model

J. C. Plefka and S. Samuel, Phys. Rev. D 55, 3966 (1997).



Deconfinement phase appears, but strong coupling analysis is not appropriate for large β region.

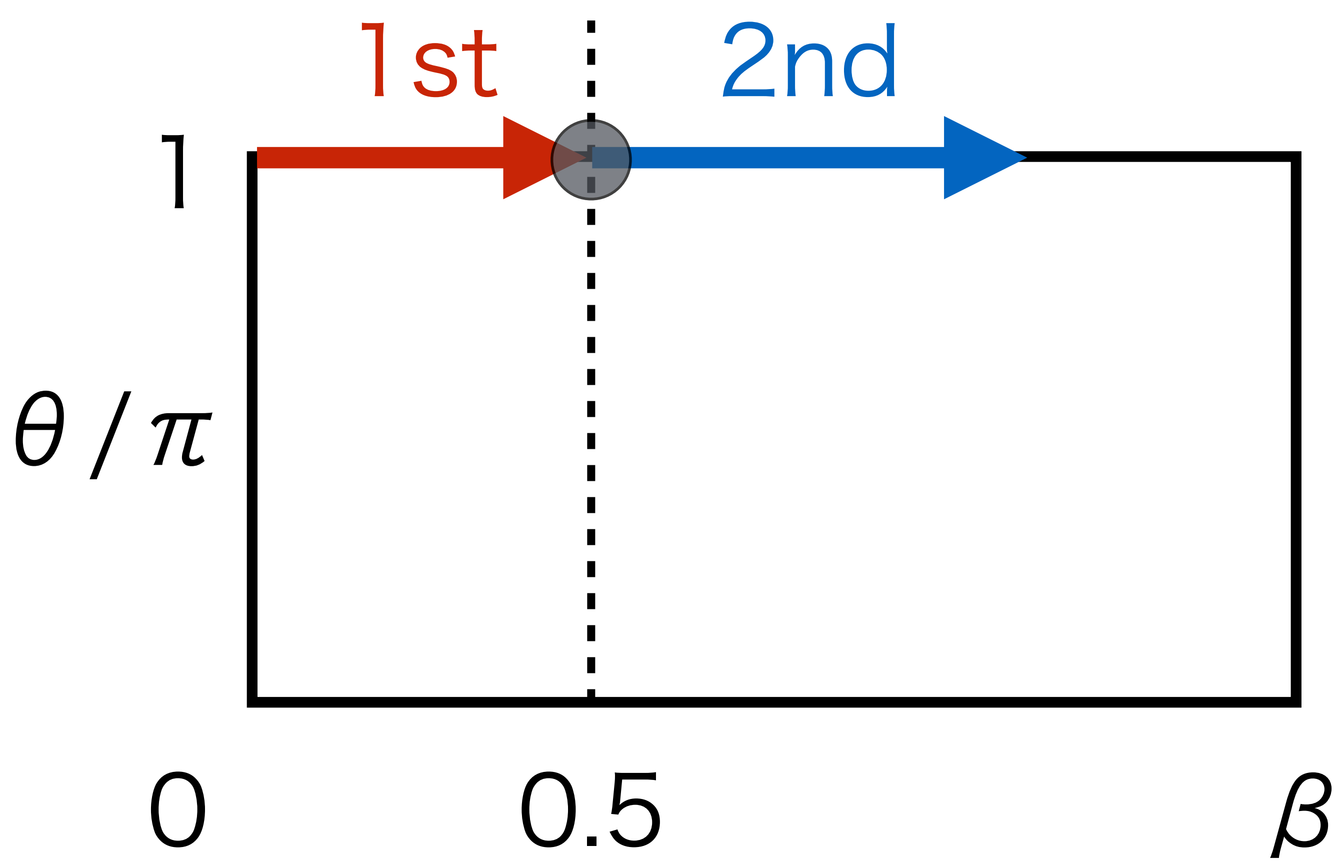
Monte Carlo analysis of the phase structure of CP(1) model

V. Azcoiti, G. Di Carlo, and A. Galante, Phys. Rev. Lett. 98, 257203 (2007).

Haldane conjecture: 2d O(3) model at $\theta = \pi$ is gapless.

F. D. M. Haldane, Phys. Lett. A 93, 464 (1983).
F. D. M. Haldane, Phys. Rev. Lett. 50, 1153 (1983).

→ How about CP(1) model?



The order of phase transition changes around β=0.5.

Tensor Renormalization Group (TRG)

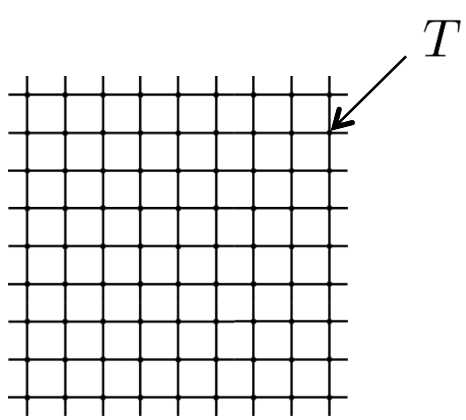
Michael Levin and Cody P. Nave, Phys. Rev. Lett. 99, 120601 (2007).

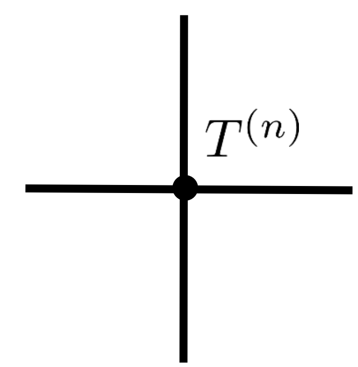
1. Tensor network representation

$$Z = \mathrm{Tr}\, e^{-\beta H} \quad \Longrightarrow \quad Z = \mathrm{Tr}_{\{x,y\}} \prod_i T_{x_i x'_i y_i y'_i \cdots}$$

2. Reduce the number of tensors

$$Z = \mathrm{Tr}[TT\cdots T] \to \cdots \xrightarrow{\text{coarse graining}} \cdots \to Z \simeq \mathrm{Tr}[T^{(n)}]$$





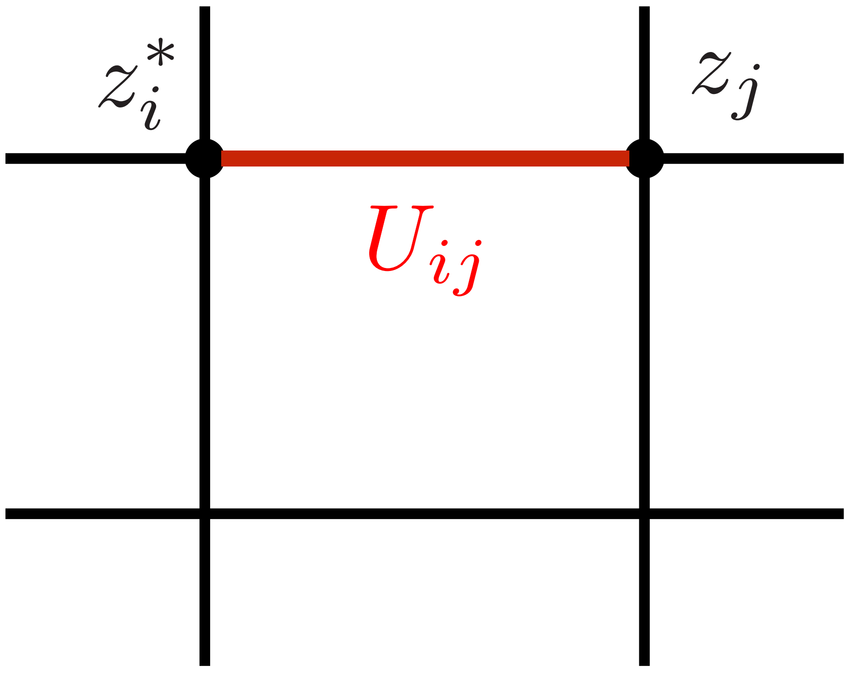
Truncate the bond dimensions according to the hierarchy of the singular values of the tensors.

CP(N-1) model in two dimensions

Tensor Network Representation

$$z^{a*}z^{a} = 1,\ a = 1, \cdots, N$$
$$U_{ij} = \exp\{iA_{ij}\}$$

$$Z = \int DzDz^{*}DU \mathrm{e}^{-(S-i\theta Q)}$$
$$= \int \prod_{i} dz_i dz_i^{*} \prod_{<i,j>} dU_{i,j}\ \mathrm{e}^{\beta N \sum_{i,j}[z_i^{*}\cdot z_j U_{i,j} + z_j^{*}\cdot z_i U_{i,j}^{\dagger}] + i\frac{\theta}{2\pi}\sum_p q_p}.$$



CP(N-1) model in two dimensions

Tensor Network Representation

$$\boxed{\begin{aligned} &z^{a*}z^a = 1,\ a = 1,\cdots,N \\ &U_{ij} = \exp\{iA_{ij}\} \end{aligned}}$$

$$\begin{aligned} Z &= \int DzDz^*DU \mathrm{e}^{-(S-i\theta Q)} \\ &= \int \prod_i dz_i dz_i^* \prod_{<i,j>} dU_{i,j}\ \underline{\mathrm{e}^{\beta N \sum_{i,j}[z_i^* \cdot z_j U_{i,j} + z_j^* \cdot z_i U_{i,j}^\dagger] + i\frac{\theta}{2\pi}\sum_p q_p}}. \end{aligned}$$

Expand the weight with new integers $(s, t, u, v, \cdots \in \mathbb{Z})$

CP(N-1) model in two dimensions

Tensor Network Representation

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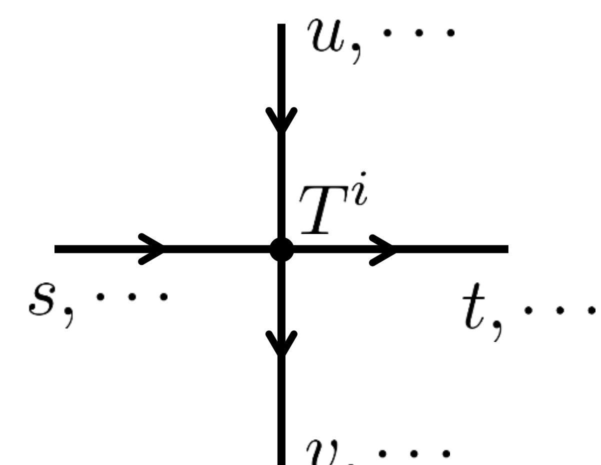
Expand the weight with new integers $(s, t, u, v, \cdots \in \mathbb{Z})$

↓

Integrate out old d.o.f

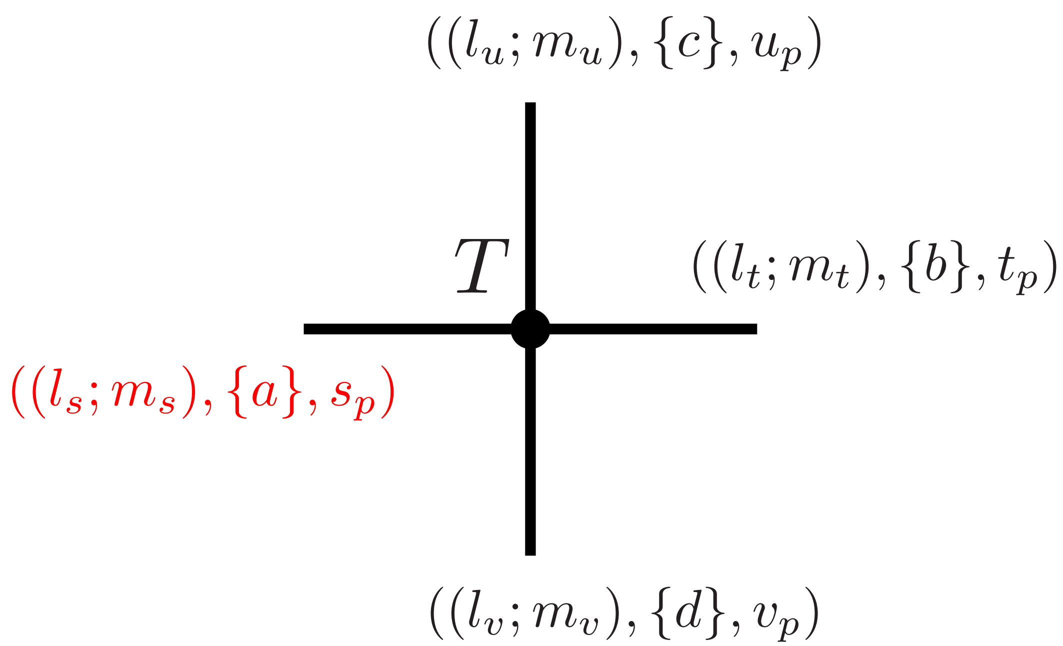
↓

Tensor $T_{s,t,u,v,\cdots}$



Tensor network representation of CP(N-1) model

H. K. and S. Takeda, Phys. Rev. D 93, 114503 (2016).



l_s, m_s: non-negative integers
$\{a\}=\{a_1, \cdots, a_{l+m}\}$ $(a_n=1, \cdots, N)$
s_p: integer

$I_n(x)$: modified Bessel functions of the first kind

$$T_{stuv} \equiv T_{((l_s;m_s),\{a\},s_p)((l_t;m_t),\{b\},t_p)((l_u;m_u),\{c\},u_p)((l_v;m_v),\{d\},v_p)}$$

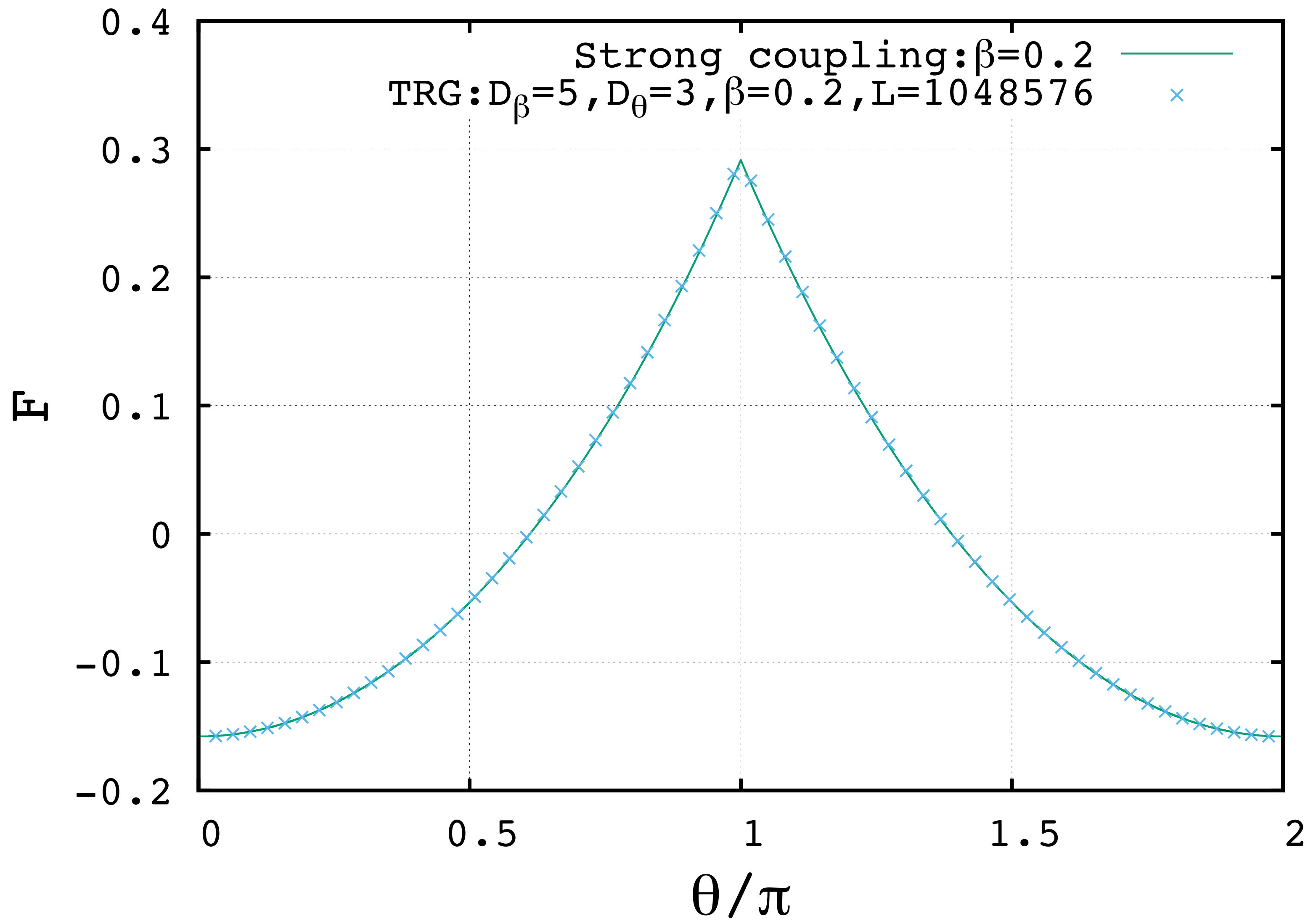
$$\propto \underline{\sqrt{I_{N-1+l_s+m_s}(2N\beta)}} \times \underline{\frac{2\sin\frac{\theta+2\pi s_p}{2}}{\theta+2\pi s_p}}$$

➡ truncate the bond dimensions to $D_\beta \times D_\theta$

Free energy density of CP(1) model

TRG method vs. Strong coupling analysis

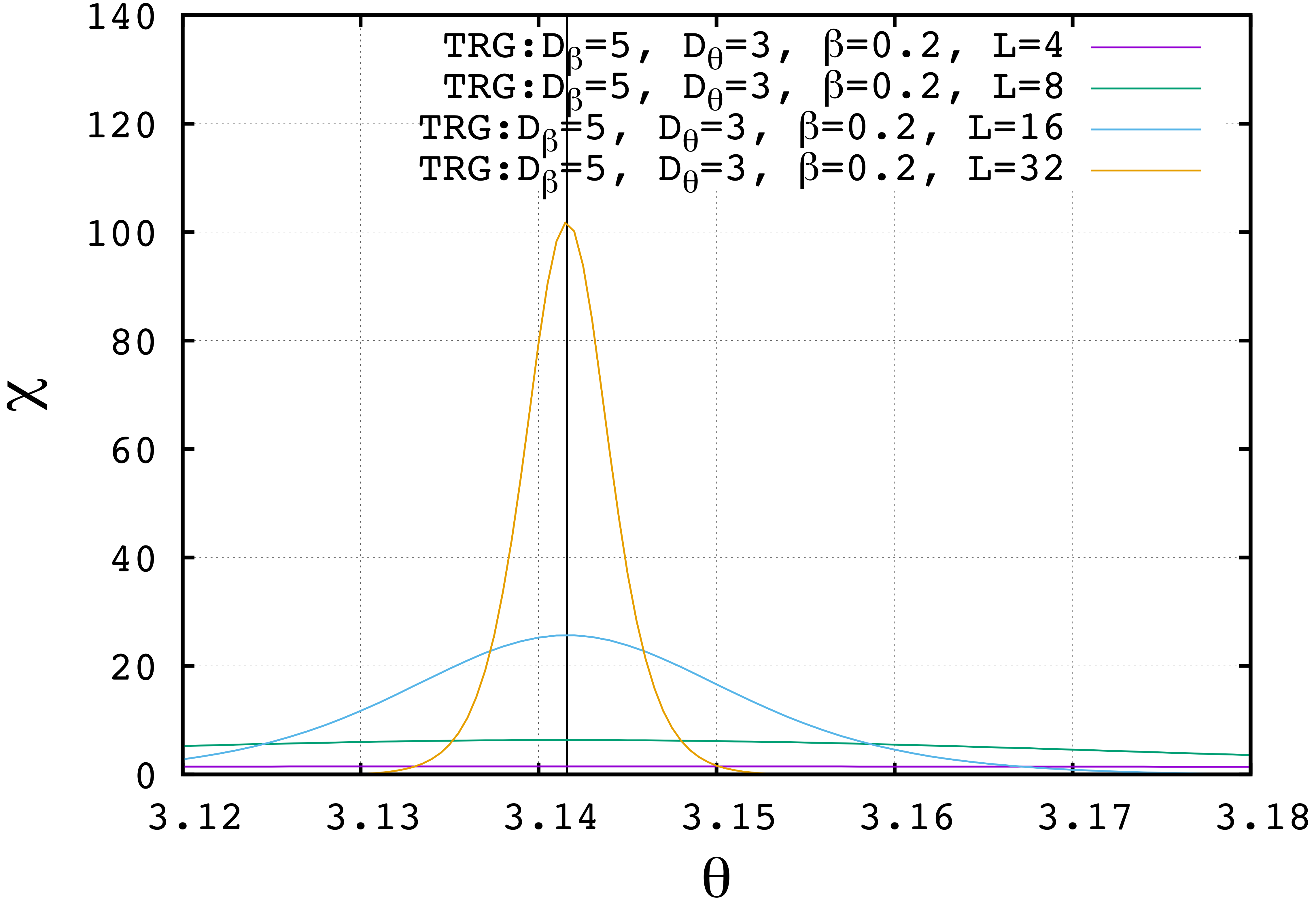
J. C. Plefka and S. Samuel, Phys. Rev. D 55, 3966 (1997).



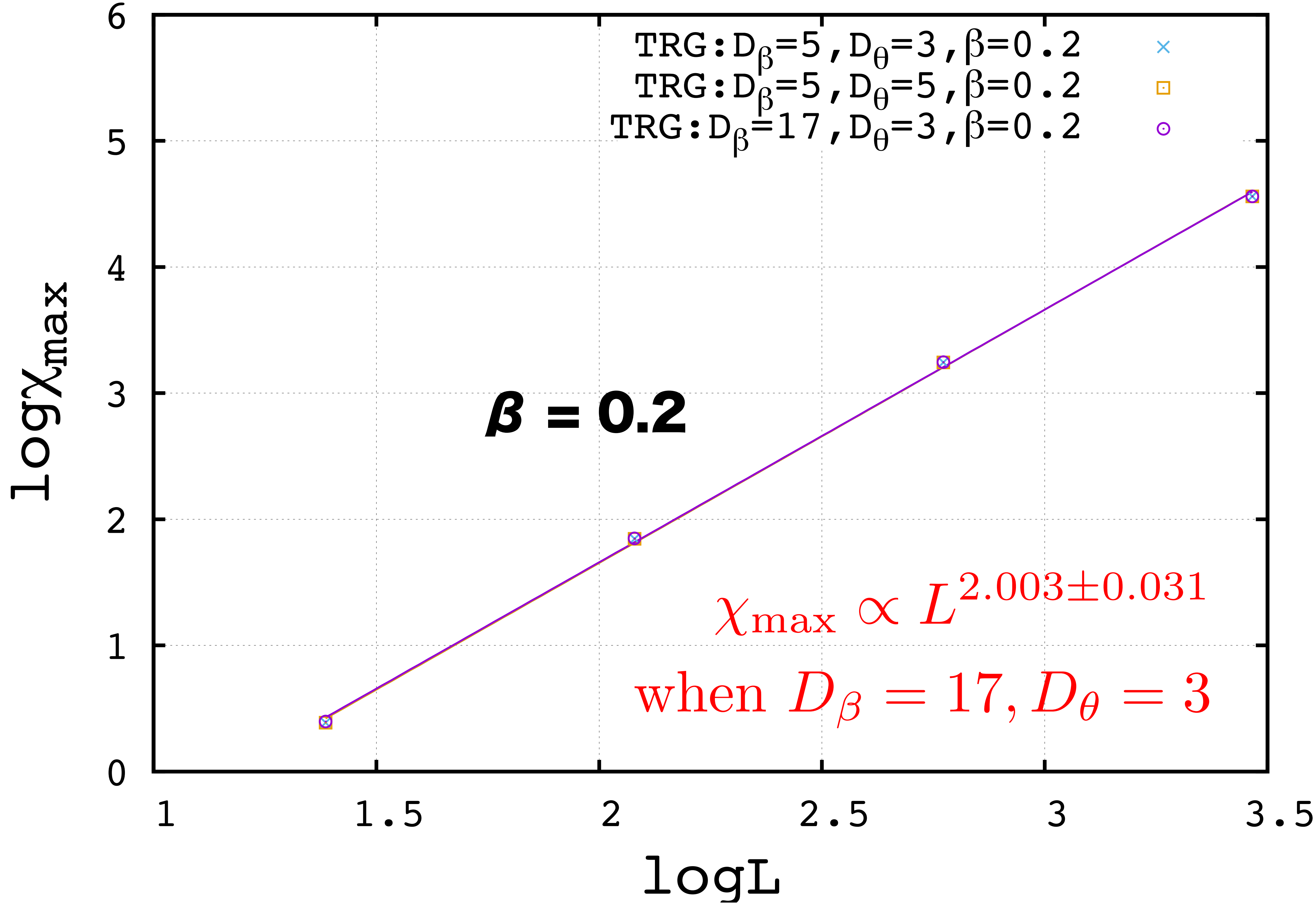
$\beta = 0.2$

$L = 2^{20}$

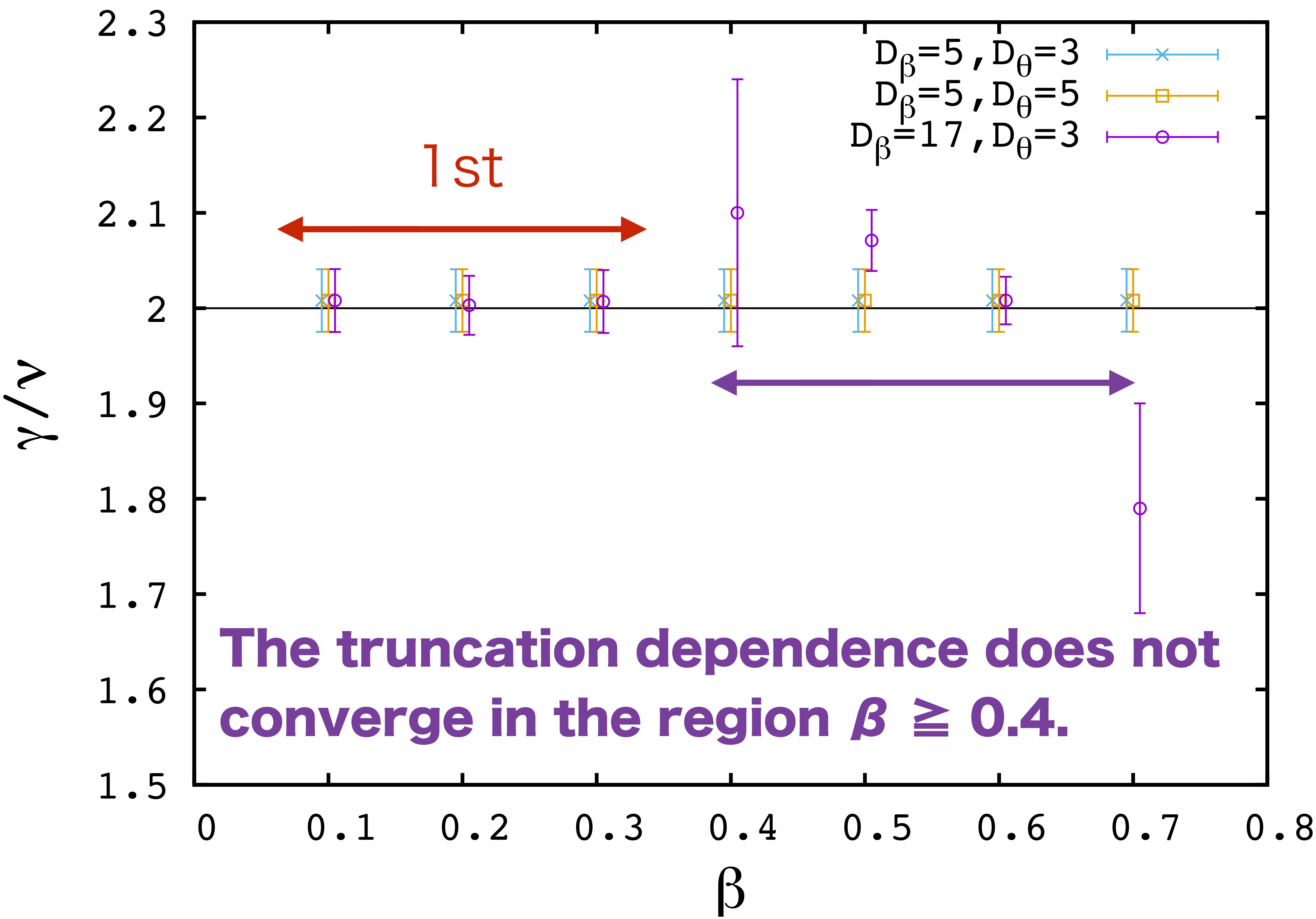
V dependence of susceptibility χ



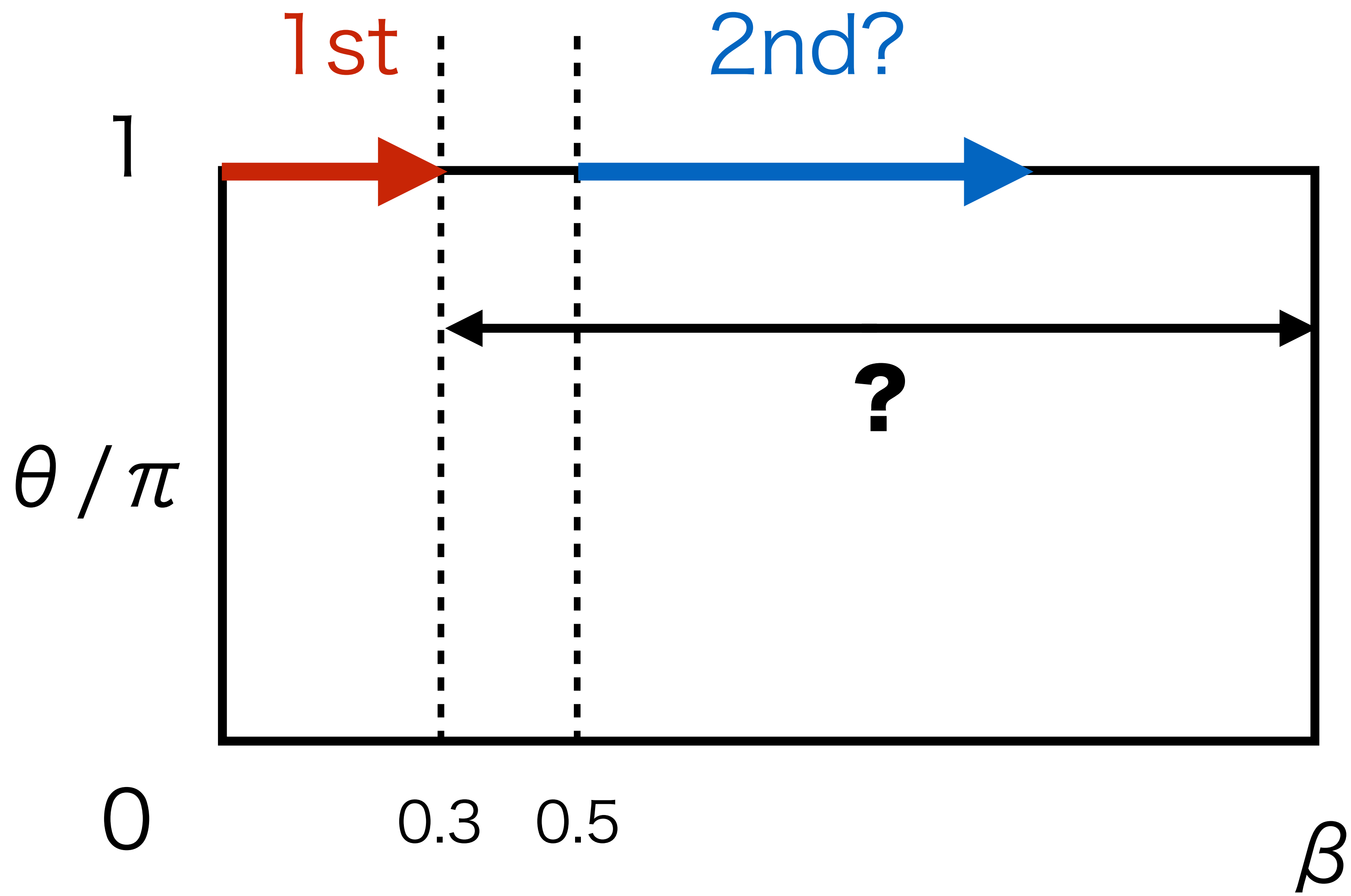
V dependence of the peak of susceptibility χ



Truncation dependence of $\chi_{\max} \propto L^{\frac{\gamma}{\nu}}$



Phase structure analysis of CP(1) model using TRG



TRG does not work well especially in critical region.

→ Tensor Network Renormalization (TNR)

G. Evenbly and G. Vidal, Phys. Rev. Lett. 115, 180405 (2015).

Summary

- We analyze the phase structure of CP(1) model including the θ term using TRG.
- We reconfirm that the order of the phase transition at $\theta = \pi$ is 1st order until β=0.3.
- Truncation dependence of the susceptibility arises for $\beta \geqq 0.4$.
- Next, we will apply TNR method to the critical region.