Baryon interactions in lattice QCD: the direct method vs. the HAL QCD potential method

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Ref. TI for HAL Coll., *"Mirage in Temporal Correlation functions for Baryon-Baryon Interactions in Lattice QCD"*, [arXiv:1607.06371], PoS(Lattice2015) 089, [arXiv:1511.05246].

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1. Baryon interactions from lattice QCD
2. Direct measurement vs HAL QCD method
   - Formalisms
   - Direct Measurement
   - HAL QCD Measurement
3. Origin of Fake Signal in Direct Method
4. Summary

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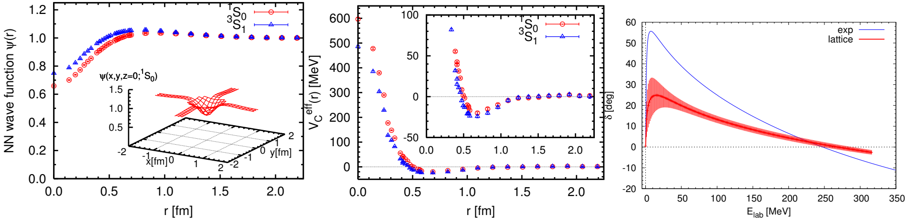
2 Methods for Hadron Interaction from Lattice QCD

QCD ▶ **Hadron Interaction** ▶ Nuclear Physics

1. **Lüscher's finite volume method** — Lüscher '86, '91
   energy shift of two-particle in **"box"** ▶ phase shift

$$\Delta E_L = 2\sqrt{k^2+m^2} - 2m \implies k\cot\delta(k) = \frac{1}{\pi L}\sum_{\boldsymbol{n}\in\mathbb{Z}^3}\frac{1}{|\boldsymbol{n}|^2-(kL/2\pi)^2}$$

2. **HAL QCD method** — Ishii-Aoki-Hatsuda '07
   NBS wave function ▶ potential ▶ phase shift

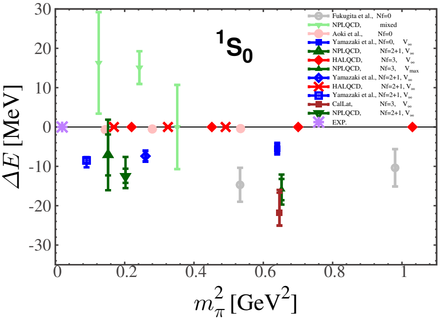


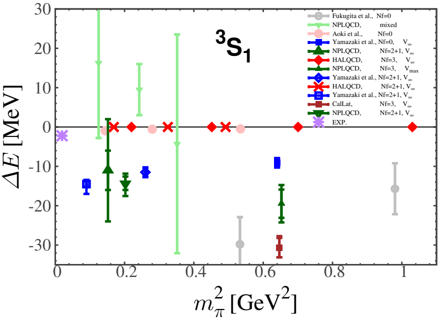
NN Interactions from Lattice QCD

	Lüscher		HAL QCD	phys. point
dineutron ($^1S_0$)	**bound**	$\Leftrightarrow$	**unbound**	**unbound**
deuteron ($^3S_1$)	**bound**	$\Leftrightarrow$	**unbound**	**bound**

$\Rightarrow$ **inconsistencies** between two methods, which is correct?

- Today we will clarify the origin of this puzzle





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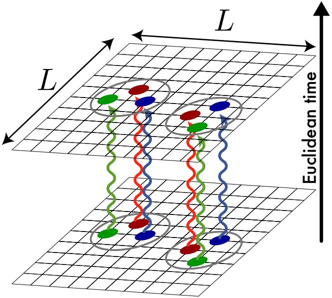
Lüscher's Finite Volume Method

- "**energy shift**" **in finite box** L^3

$$\Delta E_L = E_{BB} - 2m_B = 2\sqrt{k^2 + m_B^2} - 2m_B$$

$\Rightarrow$ **phase shift** $\delta(k)$

$$k \cot \delta(k) = \frac{1}{\pi L} \sum_{\boldsymbol{n} \in \mathbb{Z}^3} \frac{1}{|\boldsymbol{n}|^2 - (kL/2\pi)^2}$$



$\Uparrow$ **THEORY**

$\Downarrow$ **PRACTICE** — "Direct Method"

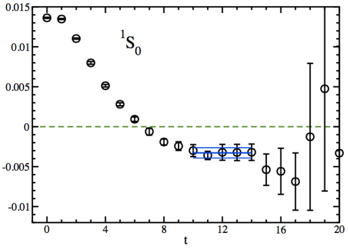
- **measure**: plateau in **effective mass**

$$\Delta E_{\rm eff}(t) = \log \frac{R(t)}{R(t+1)} \to \Delta E_L$$

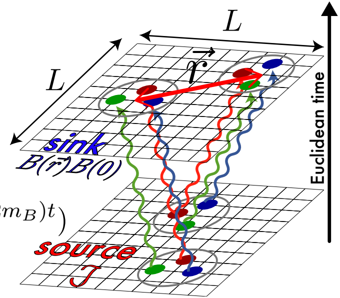
$$R(t) = \frac{G_{BB}(t)}{\{G_B(t)\}^2} \to \exp\left[-\left(E_{BB} - 2m_B\right)t\right]$$

with $G_{BB}(t)(G_B(t))$: BB(B) correlators

- NN(^{1}S$_0$) (Yamazaki et al. '12)



Time-dependent HAL QCD Method



■ **Nambu-Bethe-Salpeter wave function**

$$R(\vec{r},t) \equiv \frac{\langle 0|T\{B(\vec{x}+\vec{r},t)B(\vec{x},t)\}\bar{\mathcal{J}}(0)|0\rangle}{\{G_B(t)\}^2}$$

$$= \sum_n A_n \psi_n(\vec{r}) e^{-(E_n-2m_B)t} + \mathcal{O}(e^{-(E_{\rm th}-2m_B)t})$$

■ with **elastic** saturation $R(r,t)$ satisfies

$$\left[\frac{1}{4m_B}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right] R(\vec{r},t) = \int d\vec{r'} U(\vec{r},\vec{r'}) R(\vec{r'},t)$$

▶ **"potential"** using velocity expansion $U(r,r') \simeq V(r)\delta(r-r')$

$$V(\vec{r}) = \frac{1}{4m_B}\frac{(\partial/\partial t)^2 R(\vec{r},t)}{R(\vec{r},t)} - \frac{(\partial/\partial t)R(\vec{r},t)}{R(\vec{r},t)} - \frac{H_0 R(\vec{r},t)}{R(\vec{r},t)}$$

▶ **This method does not require the ground state saturation.**

Difficulties in Multi-Baryons

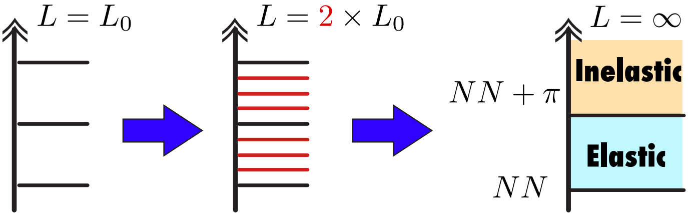
- Lüscher's method requires **ground state saturation**

$$G_{NN}(t) = c_0 \exp(-E_0^{(NN)} t) + c_1 \exp(-E_1^{(NN)} t) + \cdots \simeq c_0 \exp(-E_0^{(NN)} t)$$

- S/N problem: [mass number A] $\times$ [light quark] $\times$ [$t \to \infty$]

$$S/N \sim \exp\left[-A \times (m_N - (3/2)m_\pi) \times t\right]$$

- **smaller gap of scattering state:** $\Delta E \sim \vec{p}^{\,2}/m \sim \mathcal{O}(1/L^2)$



Contamination of Scattering State and Fake Plateau

Example

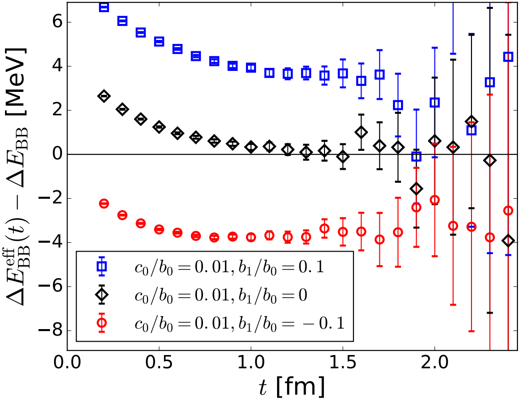
$$R(t) = b_0 e^{-\Delta E_{\rm BB} t} + b_1 e^{-\delta E_{\rm el} t} + c_0 e^{-\delta E_{\rm inel} t}$$

with $\delta E_{\rm el} - \Delta E_{\rm BB} = 50$ MeV $\sim \mathcal{O}(1/L^2)$, $\delta E_{\rm inel} - \Delta E_{\rm BB} = 500$ MeV $\sim \mathcal{O}(\Lambda_{\rm QCD})$

- g.s. saturation
  $\Delta E^{\rm eff}_{\rm BB}(t) - \Delta E_{\rm BB} \to 0$
- elastic saturation $t \sim 1$ fm
- few % of contamination

$\Rightarrow$ **"mirage"** of plateau around $t \sim 1 - 1.5$fm

much larger t for true g.s.



$\Rightarrow$ a true ground state can be checked by **quark source dependence**

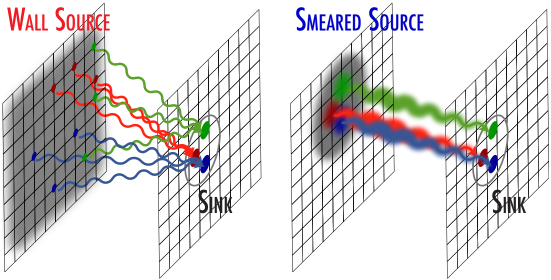
▶ **HAL QCD** — scattering state are not noises, but **signals**

Lattice Setup: Wall Source and Smeared Source

☐ **ΞΞ interaction** from both direct and HAL QCD methods

☐ CHECK **2 quark sources** — mixture of excited states are different

- **wall source**
  standard of HAL QCD
- **smeared source**
  standard of direct method†



■ setup — $2+1$ improved Wilson + Iwasaki gauge†

- lattice spacing: $a = 0.08995(40)$ fm, $a^{-1} = 2.194(10)$ GeV
- lattice volume: $32^3 \times 48$, $40^3 \times 48$, $48^3 \times 48$, and $64^3 \times 64$

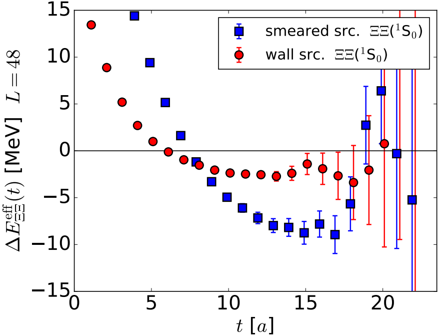
$m_\pi = 0.51$ GeV, $m_N = 1.32$ GeV, $m_K = 0.62$ GeV, $m_\Xi = 1.46$ GeV

† Yamazaki-Ishikawa-Kuramashi-Ukawa, arXiv:1207.4277.

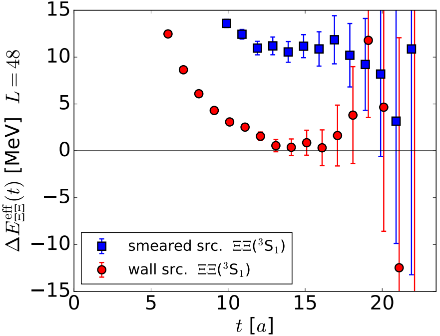
Energy Shift of ΞΞ: Smeared Src. vs. Wall Src.

$\Delta E_L^{\rm eff}(t) \longrightarrow \Delta E_L$??? — depends on quark source (smeared or wall)

$\Xi\Xi(^1\mathrm{S}_0)$ at $48^3 \times 48$



$\Xi\Xi(^3\mathrm{S}_1)$ at $48^3 \times 48$



- source dependence suggests these plateaux are **"fake" signal**

$L \to \infty$	Smeared src.	Wall source
$\Delta E_{\Xi\Xi}(^1\mathrm{S}_0)$	< 0 **bound**	$\simeq 0$ **unbound**
$\Delta E_{\Xi\Xi}(^3\mathrm{S}_1)$	> 0 **unphysical**	$\simeq 0$ **unbound**

cf. $\Delta E < 0 \Longrightarrow$ binding or $\Delta E = 0 \Longrightarrow$ scattering

Generalized Sink Operator

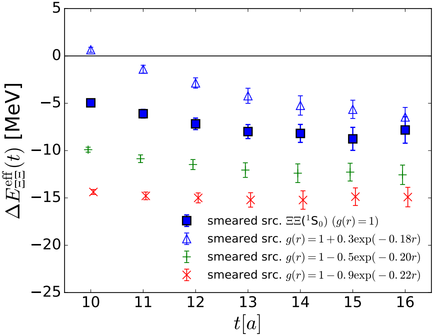
$$C^{(g)}_{\Xi\Xi}(t) = \sum_{\vec{r}} g(|\vec{r}|) \sum_{\vec{R}} \langle \Xi(\vec{R}+\vec{r},t)\Xi(\vec{R},t)\overline{\mathcal{J}_{\Xi\Xi}}(t=0)\rangle \to \exp(-E_{\Xi\Xi}t)$$

$\Rightarrow$ g.s. energy does not depend on $g(r)$

- $g(r) = 1$: standard sink operator
- $g(r) = 1 + A\exp(-Br)$: exp-type projection

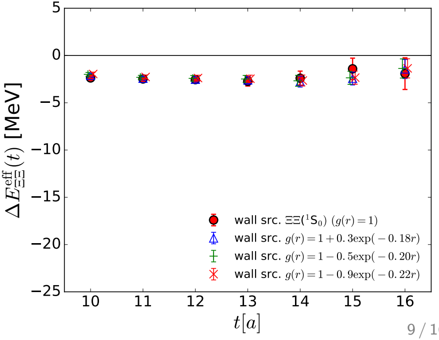
Smeared Src.

one can make any **"fake plateau"**



Wall Src.

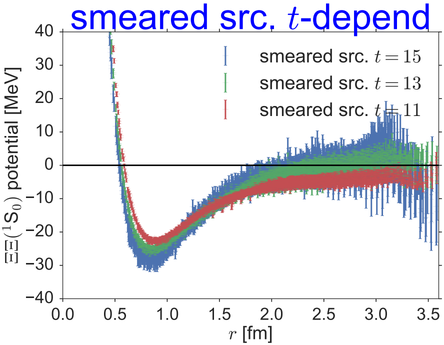
"stable"

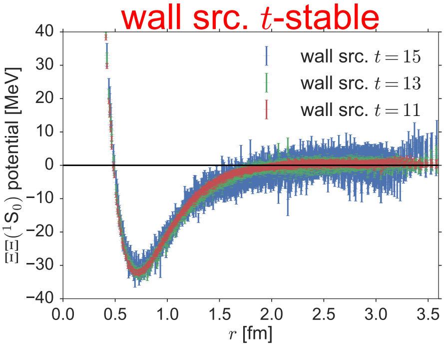


HAL: Potential of $\Xi\Xi(^1S_0)$ Smeared Src. vs Wall Src.

NBS wavefunction: $R^{\rm smear}(r,t)$ or $R^{\rm wall}(r,t)$

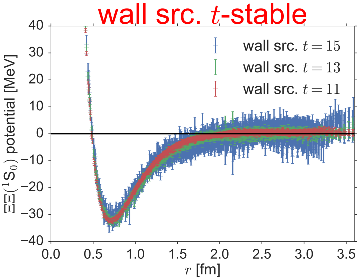
$$V_c(r) = \frac{1}{4m}\frac{(\partial^2/\partial t^2)R(r,t)}{R(r,t)} - \frac{(\partial/\partial t)R(r,t)}{R(r,t)} - \frac{H_0 R(r,t)}{R(r,t)}$$

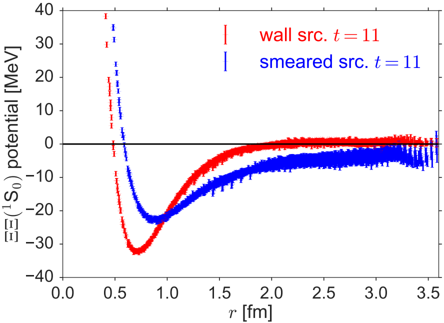


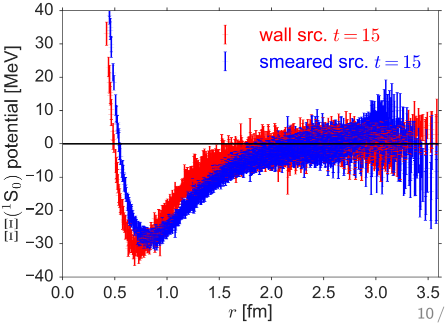


HAL: Potential of $\Xi\Xi(^1S_0)$ Smeared Src. vs Wall Src.

- **wall src.** — good convergence
- **smeared src.** — t-dep.
- **smeared src.** $\longrightarrow$ **wall src.** for large t







Residual Diff. of Pot.: Next Leading Order Correction

Derivative expansion: $U(r,r') = \{V_0(r) + V_1(r)\nabla^2\}\delta(r-r')$ (for ${}^1\mathrm{S}_0$)

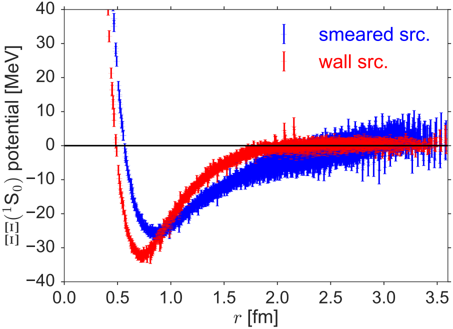
$$\left[\frac{1}{4m}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right] R(r,t) = \int d^3r' U(r,r') R(r',t)$$

$$\simeq V_0(r)R(r,t) + V_1(r)\nabla^2 R(r,t) + \cdots$$

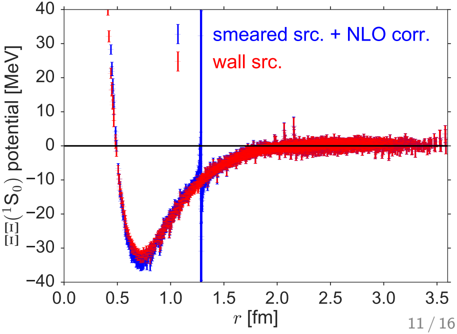
R^{smear} and R^{wall} $\Longrightarrow$ $V_0(r)$ and $V_1(r)$

▶ **HAL method works** — quark src. independent w/o g.s. saturation

☐ Leading order approximation



☐ Next leading order correction



HAL meets Lüscher: Energy Shift from Potential

- HAL QCD works well **w/o g.s. saturation problem**

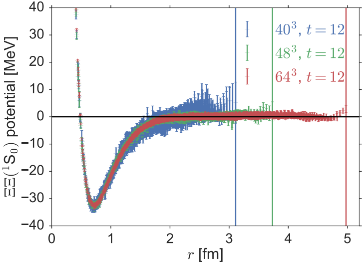
use potential $\Longrightarrow$ true **"energy shift"** in finite volume

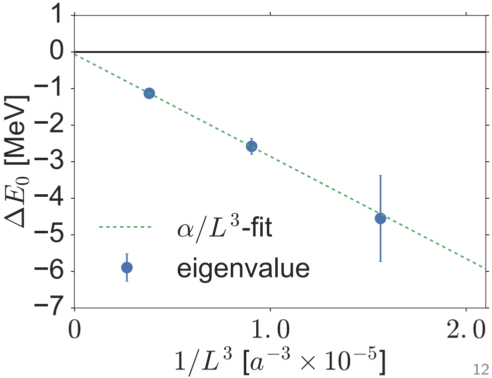
▶ Eigenequation in finite box L^3 with HAL QCD potential $V(\vec{r})$

$$[H_0 + V]\,\psi = \Delta E \psi$$

□ eigenvalue $\Delta E_0 \propto 1/L^3 \longrightarrow 0 \Longrightarrow$ **scattering** by **Lüscher's formula**

- potential $V(r)$





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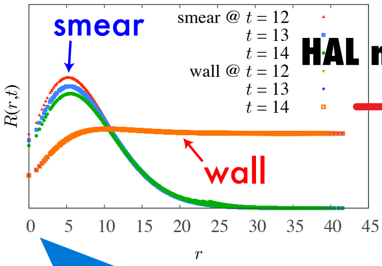
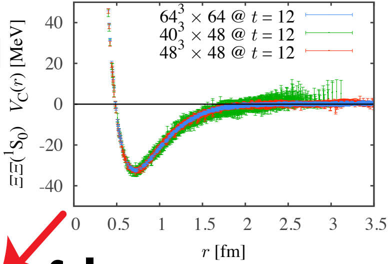
Wavefunction, Potential, Eigenvalues and Eienfunctions

NBS wave function



HAL method

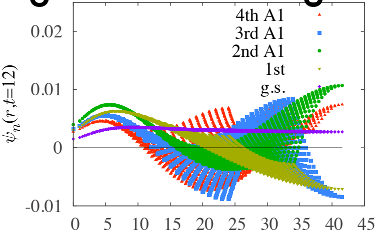
Potential



Solve $[H_0 + V]\psi = E\psi$

feed back

Eigenvalues & Eigenfunctions



decomposition
projection

ground state

& excited states

(elastic scattering)

Excited States in Wavefunction

▶ R-corr. decomposition by energy eigenmodes ◀ from **HAL pot.**

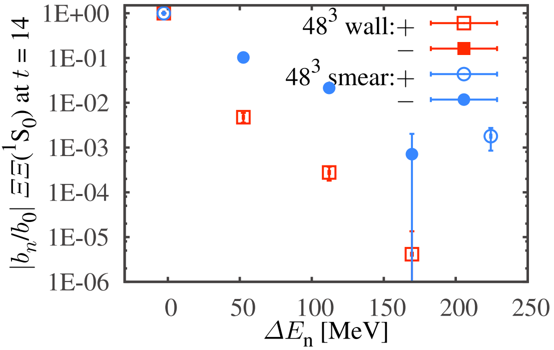
$$R^{\mathrm{wall/smear}}(\vec{r},t) = \sum_n a_n^{\mathrm{wall/smear}} \Psi_n(\vec{r},t) \exp\left(-\Delta E_n t\right)$$

$$\therefore R(\vec{p}=0,t) = \sum_r R(\vec{r},t) = \sum_n b_n^{\mathrm{wall/smear}} e^{-\Delta E_n t}$$

☐ ex. **1st excited state**

- **wall source**
  $b_1/b_0 \ll 0.01$
- **smeared source**
  $b_1/b_0 \simeq -0.1$
- with energy gap
  $E_1 - E_0 \simeq 50$ MeV
  for $L^3 = 48^3$

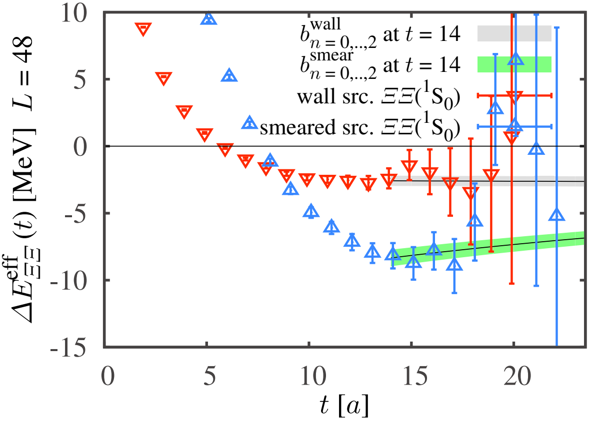
"contamination" of excited states b_n/b_0



Origin of Fake Plateau — Contamination of Excited States

$$\Delta E_{\mathrm{eff}}(t) \equiv \log \frac{R(p=0,t)}{R(p=0,t+1)} = \log \frac{\sum_n b_n \exp\left(-\Delta E_n t\right)}{\sum_n b_n \exp\left(-\Delta E_n (t+1)\right)}$$

- "direct measurement" — reproduced by low-lying modes†

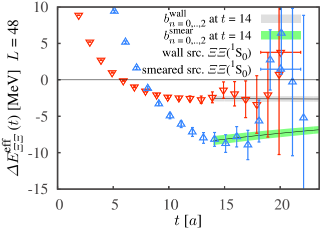


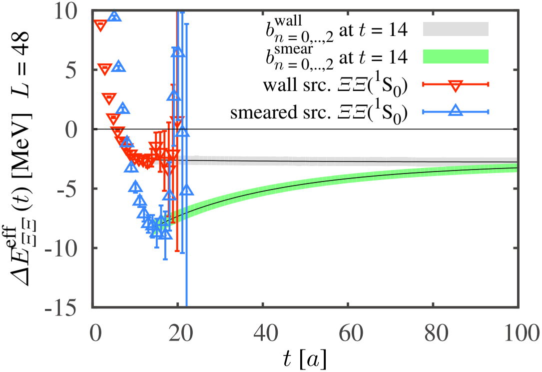
† eigenvalues ΔE_n, coefficients $b_n^{\mathrm{smear/wall}}$ for $n = 0, 1, 2$, at $t = 14$.

Origin of Fake Plateau — Contamination of Excited States

$$\Delta E_{\mathrm{eff}}(t) \equiv \log \frac{R(p=0,t)}{R(p=0,t+1)} = \log \frac{\sum_n b_n \exp\left(-\Delta E_n t\right)}{\sum_n b_n \exp\left(-\Delta E_n (t+1)\right)}$$

- ■ "direct measurement" — reproduced by low-lying modes†
- □ **g.s. saturation** of smeared source — **100 lattice units** $\sim$ **10 fm !!!**





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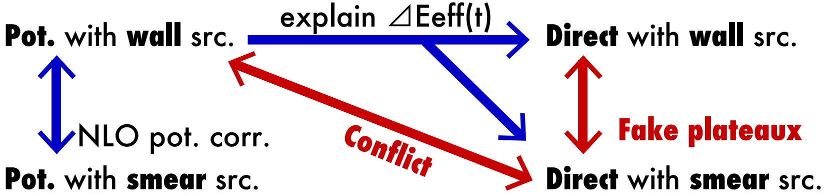
Summary: ~~Lüscher~~ Direct vs HAL QCD

- "Direct method" — **ground state saturation** is extremely difficult
  - scattering states $\Longrightarrow$ "fake plateau" ▶ **Wrong Conclusion!**
  - much smaller gap & larger noise @ phys. pt. $\Longrightarrow$ almost impossible

- HAL QCD works well **without g.s. saturation**

  HAL QCD $\Longrightarrow$ **"correct"** ΔE_L and input of **Lüscher's formula**
- **NBS corr.** + **"potential"** $\Longrightarrow$ excited states contamination and origin of **fake plateau**.

- (*even if you do not trust HAL QCD method*)
  fake plateau can be checked **by Lüscher's formula** ▶ Aoki's Talk



5 Appendix

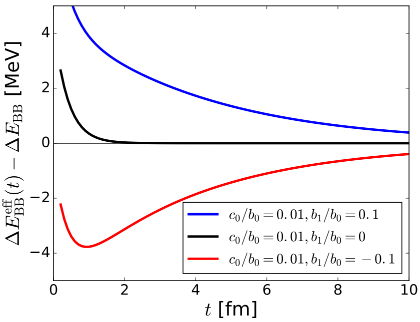
Demo: Contamination of Scattering State

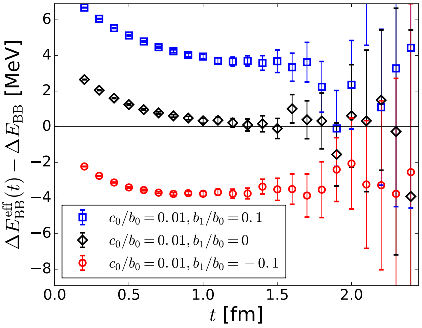
Mock up data

$$R(t) = b_0 e^{-\Delta E_{\mathrm{BB}} t} + b_1 e^{-\delta E_{\mathrm{el}} t} + c_0 e^{-\delta E_{\mathrm{inel}} t}$$

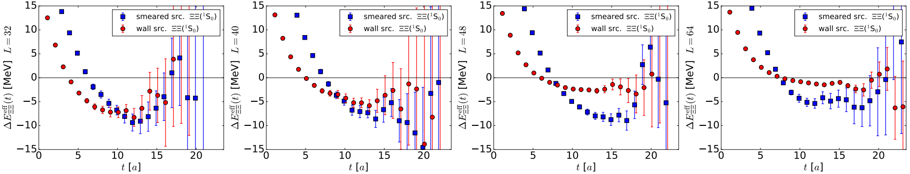
with $\delta E_{\mathrm{el}} - \Delta E_{\mathrm{BB}} = 50$ MeV $\sim \mathcal{O}(1/L^2)$, $\delta E_{\mathrm{inel}} - \Delta E_{\mathrm{BB}} = 500$ MeV $\sim \mathcal{O}(\Lambda_{\mathrm{QCD}})$

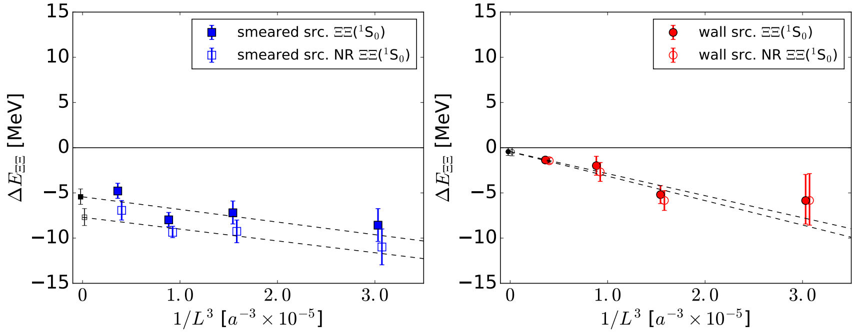
- g.s. saturation around $t \to 10$ fm
- fake plateau around $t \sim 1$ fm





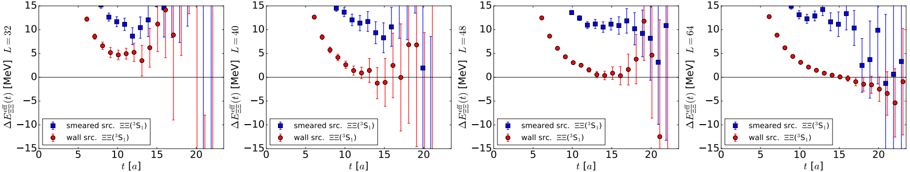
$\Xi\Xi(^1S_0)$

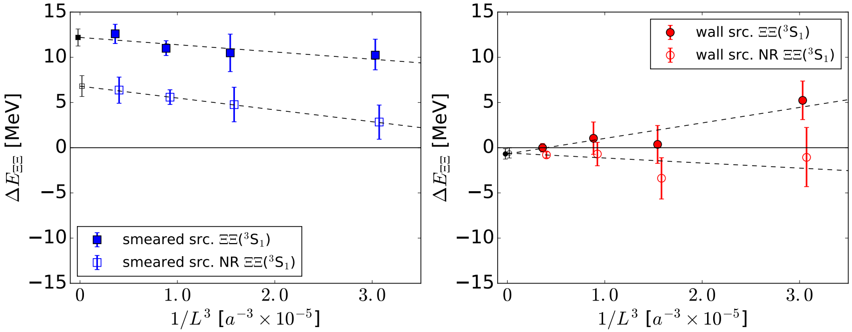




relativistic op. and non-rela. op. (NR)

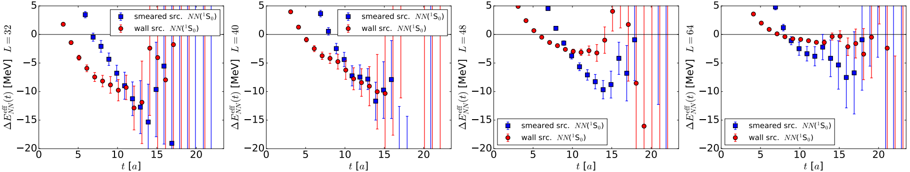
$\Xi\Xi(^3S_1)$

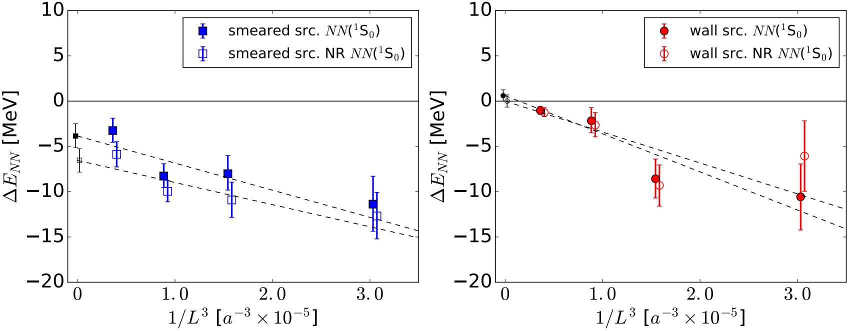




relativistic op. and non-rela. op. (NR)

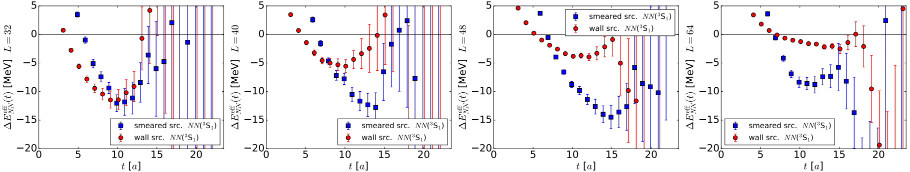
$NN(^1\mathrm{S}_0)$

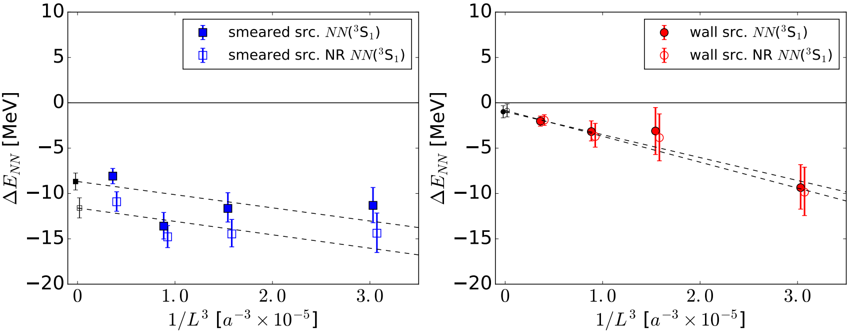




relativistic op. and non-rela. op. (NR)

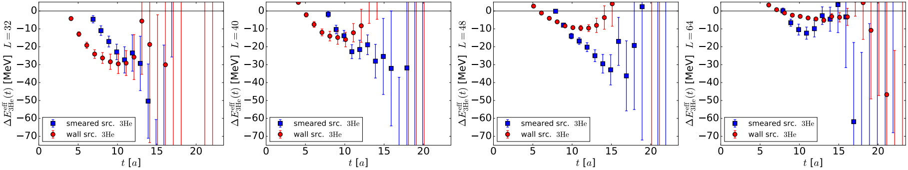
$NN(^3\mathrm{S}_1)$

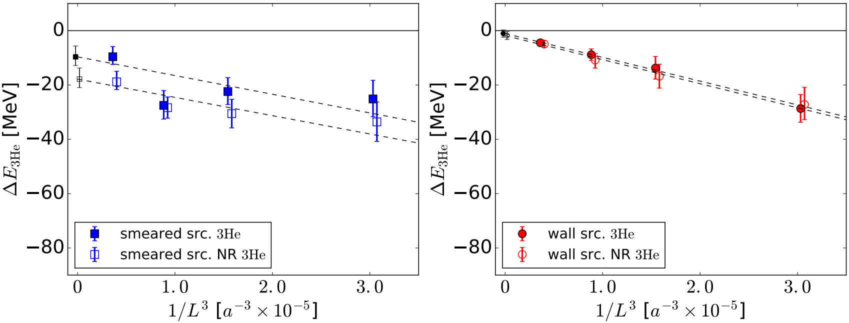




relativistic op. and non-rela. op. (NR)

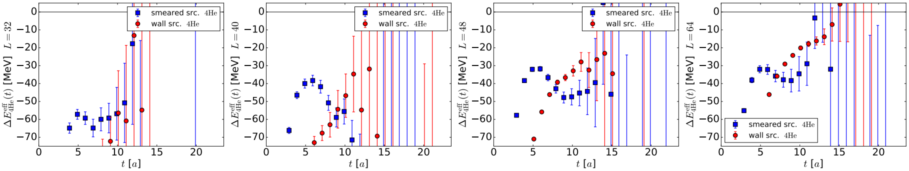
Triton

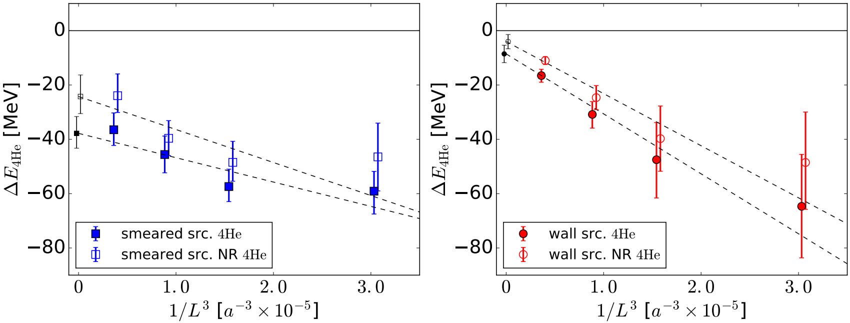




relativistic op. and non-rela. op. (NR)

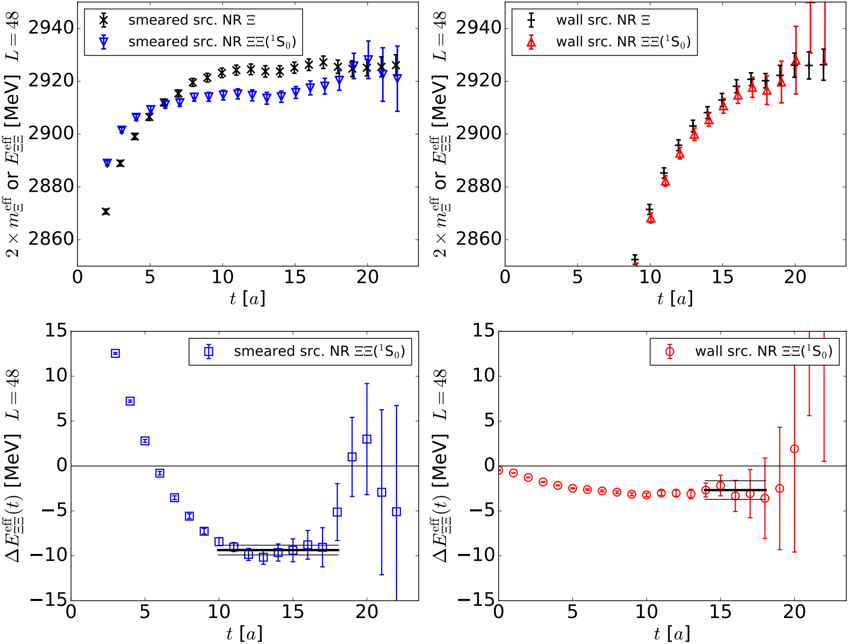
Helium





relativistic op. and non-rela. op. (NR)

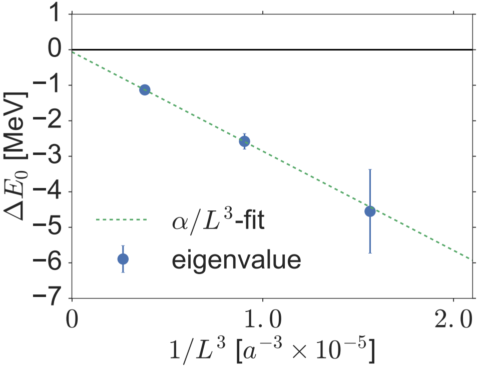
$\Delta E_{\rm eff}(t) = E^{\rm eff}_{\Xi\Xi}(t) - 2m^{\rm eff}_{\Xi}(t)$: Smeared Src. vs. Wall Src.



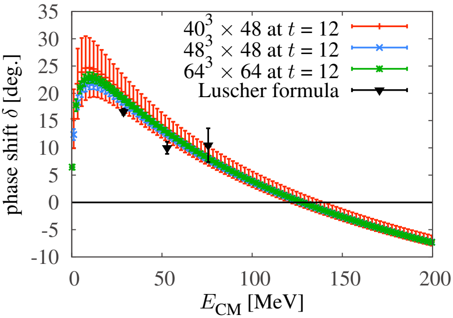
$\Xi\Xi(^1S_0)$ is Unbound at $m_\pi = 510$ MeV

$$k \cot \delta(k) = \frac{1}{\pi L} \sum_{n \in \mathbf{Z}^3} \frac{1}{|n|^2 - (kL/2\pi)^2}, \qquad \Delta E = 2\sqrt{m^2 + k^2} - 2m$$

volume dep. of ΔE_0

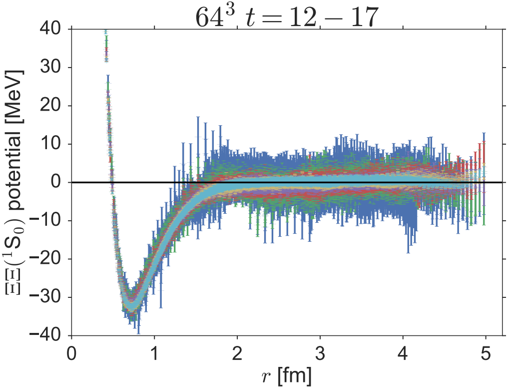


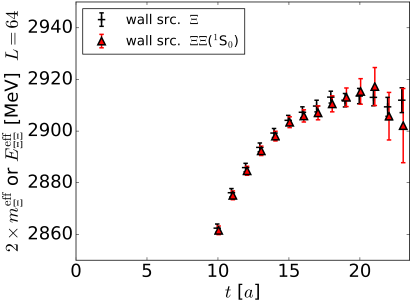
phase shift δ



t-depenence of Potential

t-dependence of Wall Src. potential is stable





Time-dependent HAL QCD Method

- space-time correlation function

$$R(\vec{r},t) \equiv \left\langle 0|T\{B(\vec{x}+\vec{r},t)B(\vec{x},t)\}\bar{\mathcal{J}}(0)|0\right\rangle / \{G_B(t)\}^2$$
$$= \sum_n A_n \psi_n(\vec{r}) e^{-(E_n - 2m_B)t} + \mathcal{O}(e^{-(E_{\rm th}-2m_B)t})$$

□ each $\psi_n(\vec{r})e^{-E_n t} \equiv \langle 0|T\{B(\vec{x}+\vec{r},t)B(\vec{x},t)\}|2B,n\rangle$ satisfies

$$\left[\frac{k_n^2}{m_B} - H_0\right]\psi_n(\vec{r}) = \int d\vec{r'} U(\vec{r},\vec{r'})\psi_n(\vec{r'})$$

with non-local interaction kernel $U(\vec{r},\vec{r'})$

- R-corr. satisfies t-dep. Schrödinger-like eq. with **elastic** saturation

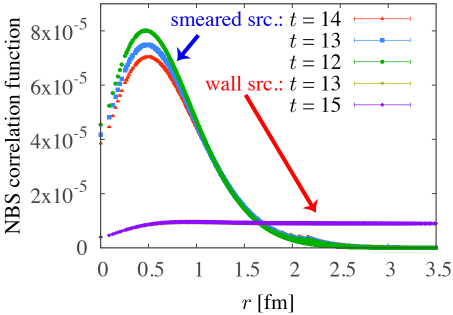
$$\left[\frac{1}{4m_B}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right] R(\vec{r},t) = \int d\vec{r'} U(\vec{r},\vec{r'}) R(\vec{r'},t)$$

▶ **"potential"** using velocity expansion $U(r,r') \simeq V(r)\delta(r-r')$

$$V(\vec{r}) = \frac{1}{4m_B}\frac{(\partial/\partial t)^2 R(\vec{r},t)}{R(\vec{r},t)} - \frac{(\partial/\partial t)R(\vec{r},t)}{R(\vec{r},t)} - \frac{H_0 R(\vec{r},t)}{R(\vec{r},t)}$$

▶ **This method does not require the ground state saturation.**

HAL: Wave Function and $\Xi\Xi(^1S_0)$ Potential $V_c(\vec{r})$



- ○ **wall src.** — weak t-dep.
- ○ **smeared. src.** — strong t-dep.

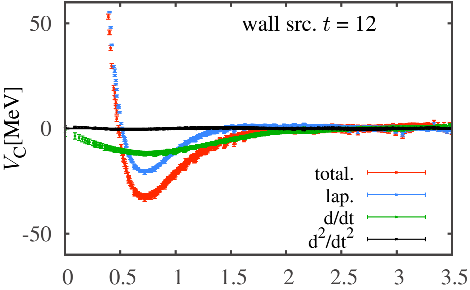
$\Rightarrow$ contribution of excited states

- ● time-dep. HAL method works well

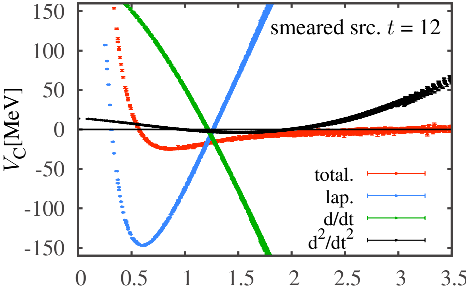
$\Rightarrow$ $\mathcal{O}(100)$ MeV of cancellation

$$V_c(\vec{r}) = -\frac{H_0 R}{R} - \frac{(\partial/\partial t)R}{R} + \frac{(\partial/\partial t)^2 R}{4mR}$$

■ **wall src.**



□ **smeared src.**



Next Leading Order of Derivative Expansion

Derivative expansion: $U(r, r') = \{V_0(r) + V_1(r)\nabla^2\}\delta(r - r')$ (for $^1S_0$)

$$\left[\frac{1}{4m}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right] R(r,t) = \int d^3r' U(r,r') R(r',t)$$

$$\therefore \frac{1}{4m}\frac{(\partial^2/\partial t^2)R}{R} - \frac{(\partial/\partial t)R}{R} - \frac{H_0 R}{R} = V_0(r) + V_1(r)\frac{\nabla^2 R(r,t)}{R(r,t)} \equiv \tilde{V}_{\text{eff}}(r,t)$$

- R^{smear} and R^{wall}

$$\begin{cases} V_0(r) + V_1(r)\nabla^2 R^{\text{smear}}/R^{\text{smear}} = \tilde{V}_{\text{eff}}^{\text{smear}}(r, t_{\text{smear}}) \\ V_0(r) + V_1(r)\nabla^2 R^{\text{wall}}/R^{\text{wall}} = \tilde{V}_{\text{eff}}^{\text{wall}}(r, t_{\text{wall}}), \end{cases}$$

- LO $V_0(r)$ and NLO $V_1(r)$ potentials are given by

$$V_1(r) = \frac{\tilde{V}_{\text{eff}}^{\text{smear}}(r, t_{\text{smear}}) - \tilde{V}_{\text{eff}}^{\text{wall}}(r, t_{\text{wall}})}{\nabla^2 R^{\text{smear}}/R^{\text{smear}} - \nabla^2 R^{\text{wall}}/R^{\text{wall}}}$$

$$V_0(r) = \tilde{V}_{\text{eff}}^{\text{smear}}(r, t_{\text{smear}}) - V_1(r)\frac{\nabla^2 R^{\text{smear}}}{R^{\text{smear}}}.$$