Systematic study of operator dependence in nucleus calculation at large quark mass

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for PACS Collaboration

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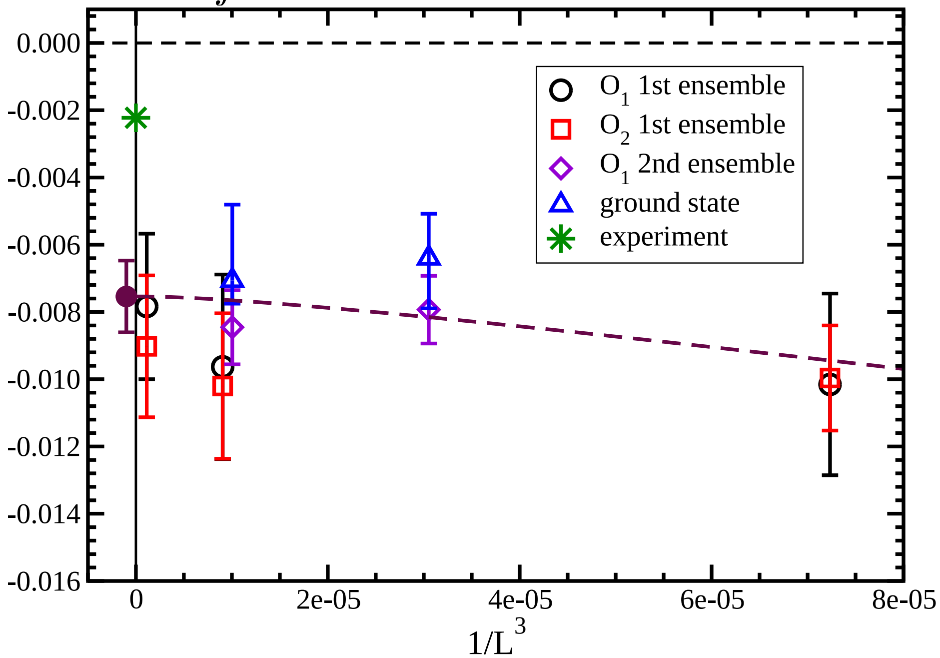
Outline

- Introduction
- Simulation parameter
- Calculation method of nuclei in lattice QCD
- Preliminary result of $N_f = 2 + 1$ $m_\pi = 0.7$ GeV
- Preliminary result of $N_f = 0$ $m_\pi = 0.8$ GeV
- Summary and future work

Introduction

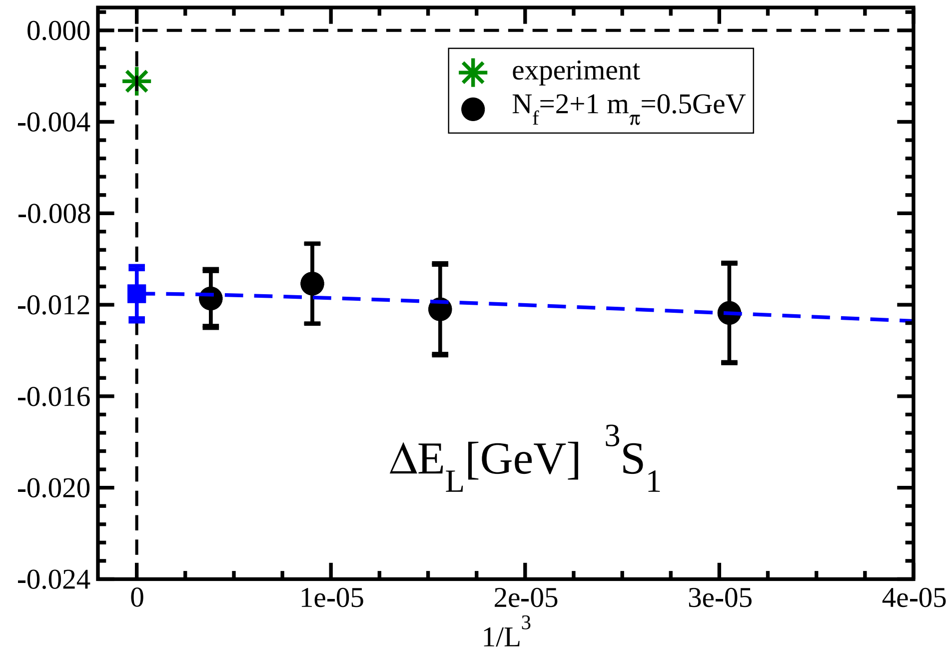
Energy shift ΔE_{NN} in NN ^{3}S$_1$ channel from Exponetial source

$N_f = 0$ $m_\pi = 0.8$ GeV



PRD84:054506(2011)

$N_f = 2+1$ $m_\pi = 0.5$ GeV



PRD86:074514(2012)

other previous works: '15 TY *et al.* $N_f = 2+1$ $m_\pi = 0.3$ GeV [PRD92:014501(2015)]

'12 NPLQCD, '15 CalLat $N_f = 3$ $m_\pi = 0.81$ GeV; '15 NPLQCD $N_f = 2+1$ $m_\pi = 0.45$ GeV

Nucleus calculated from exponential and gaussian sources

Introduction

Energy shift ΔE_{NN} in NN $^3\text{S}_1$ channel from Exponetial source

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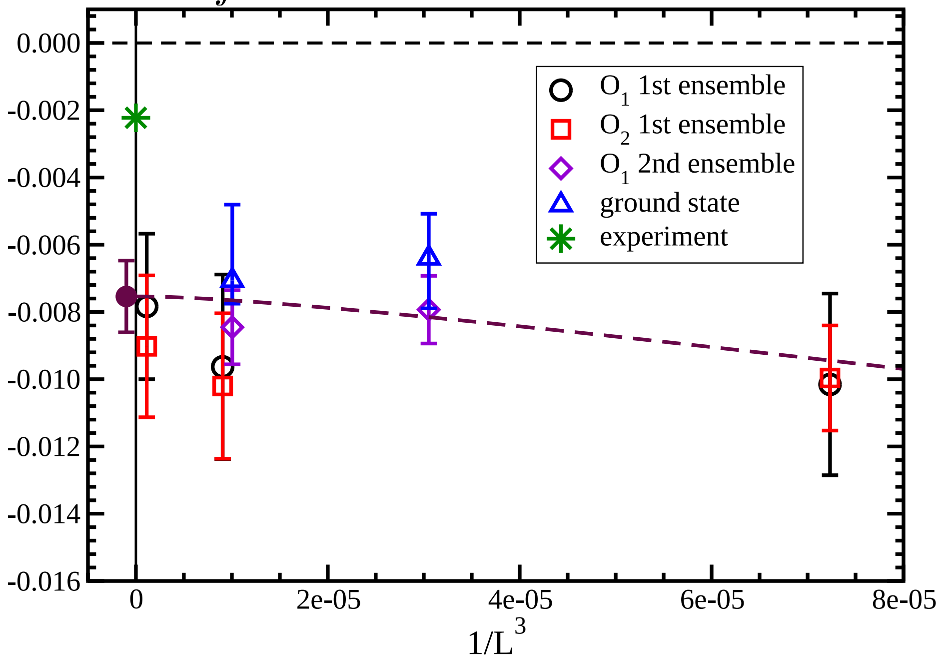

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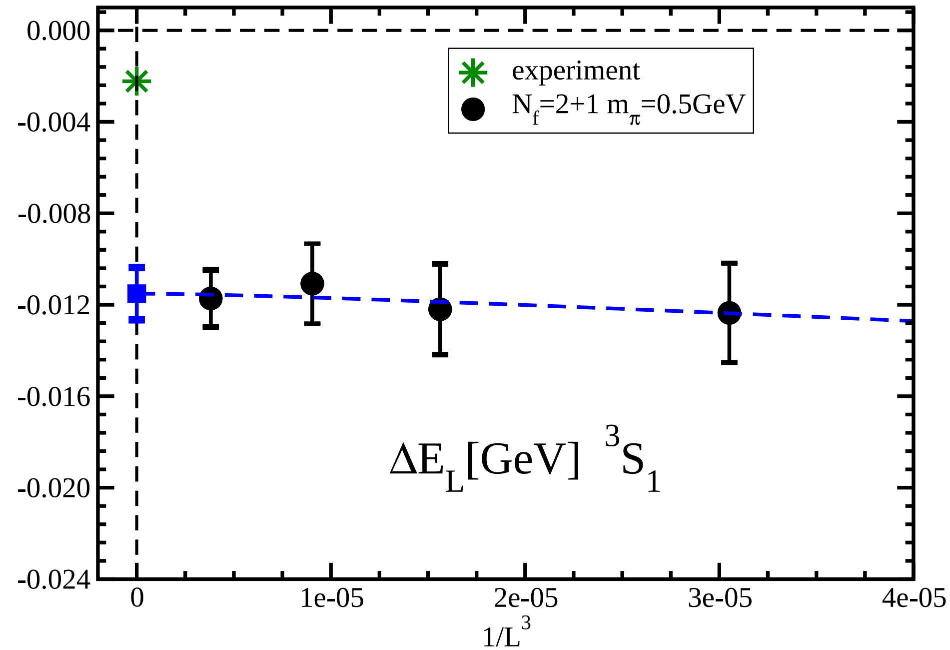

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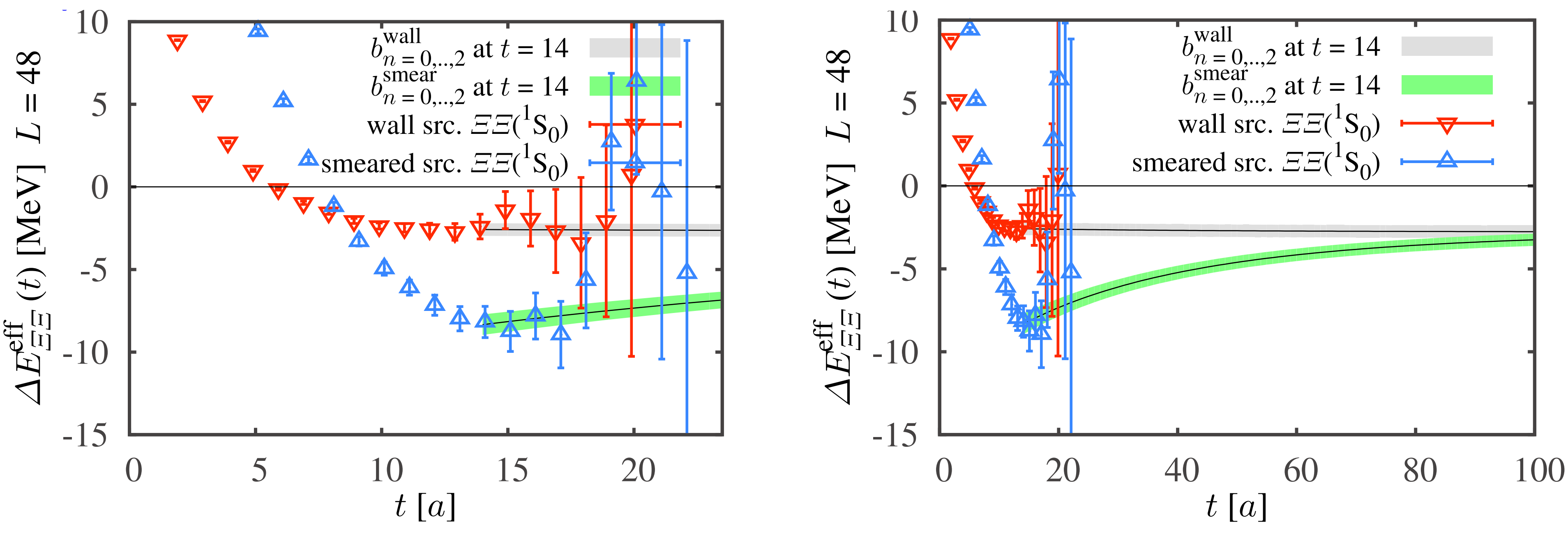
Nucleus calculated from exponential and gaussian sources

HALQCD: Wall source gives correct ΔE, but others are incorrect

Introduction

HALQCD: Wall source gives correct ΔE, but others are incorrect



figures from slide by Iritani at INT workshop "Nuclear Physics from Lattice QCD"

Solid curves

expected t dependence of effective ΔE by HALQCD potential

$\rightarrow$ Wall source is almost flat.

$\rightarrow$ ΔE of Wall source $\approx$ ΔE from HALQCD potential

Introduction

Purpose

Investigate difference of ΔE_{NN} between Exponential and Wall sources at large m_π ($m_\pi \geq 0.7$ GeV) in $N_f = 2+1$ and $N_f = 0$

Why large m_π and $N_f = 0$?

1. high precision calculation

   pilot study of $N_f = 0$ $m_\pi = 0.8$ GeV [PRD84:054506(2011)]

   decided not to adopt wall source

   $\Leftarrow$ hard to obtain ΔE_{NN} in plateau region

2. no qualitative difference from smaller m_π and $N_f = 2+1$ in our previous studies

Simulation parameter

$N_f = 2+1$ $m_\pi = 0.7$ GeV $\beta = 1.90$, $a^{-1} = 2.194$ GeV

Iwasaki gauge + non-perturbative $O(a)$-improved Wilson fermion actions

same action as '10 PACS-CS, PRD86:074514(2012); PRD92:014501(2015)

$N_f = 0$ $m_\pi = 0.8$ GeV $\beta = 2.416$, $a^{-1} = 1.541$ GeV

Iwasaki gauge + tadpole imporved Wilson fermion actions

same action as '02 CP-PACS, PRD81:111504(R)(2010); PRD84:054506(2011)

Focus on NN ^{3}S$_1$ channel

N_f	L	T	source	N_{meas}
2+1	32	48	Exp	204800
			Wall	110400
0	16	64	Exp$_1$	1600000
			Exp$_2$	3200000
			Wall	1920000

Exp and Wall sources with point sink (each N with $p = 0$)

All results are preliminary.

Computational resources (HPCI System Research Project: hp160124)
COMA (U. of Tsukuba), FX10 and Reedbush (U. of Tokyo), Tatara (Kyushu U.)

Calculation method of multi-nucleon bound state

Traditional method

nucleon

$$C_N(t) \;=\; \langle 0|N(t)\overline{N}(0)|0\rangle = \sum_n \langle 0|N|n\rangle\langle n|\overline{N}|0\rangle e^{-E_n^N t} \xrightarrow[t\gg 1]{} A_0^N\, e^{-m_N t}$$

NN channel

$$C_{NN}(t) \;=\; \langle 0|O_{NN}(t)\overline{O}_{NN}(0)|0\rangle = \sum_n \langle 0|O_{NN}|n\rangle\langle n|\overline{O}_{NN}|0\rangle e^{-E_n^{NN} t}$$
$$\xrightarrow[t\gg 1]{} A_0^{NN}\, e^{-E_0^{NN} t}$$

Ratio of correlation functions

$$R(t) = \frac{C_{NN}(t)}{(C_N(t))^2} \xrightarrow[t\gg 1]{} A\, e^{-\Delta E_{NN} t}, \quad \Delta E_{NN} = E_0^{NN} - 2m_N$$

Important condition

$C_N(t)$ and $C_{NN}(t)$ in $t \gg 1$ are written by each ground state

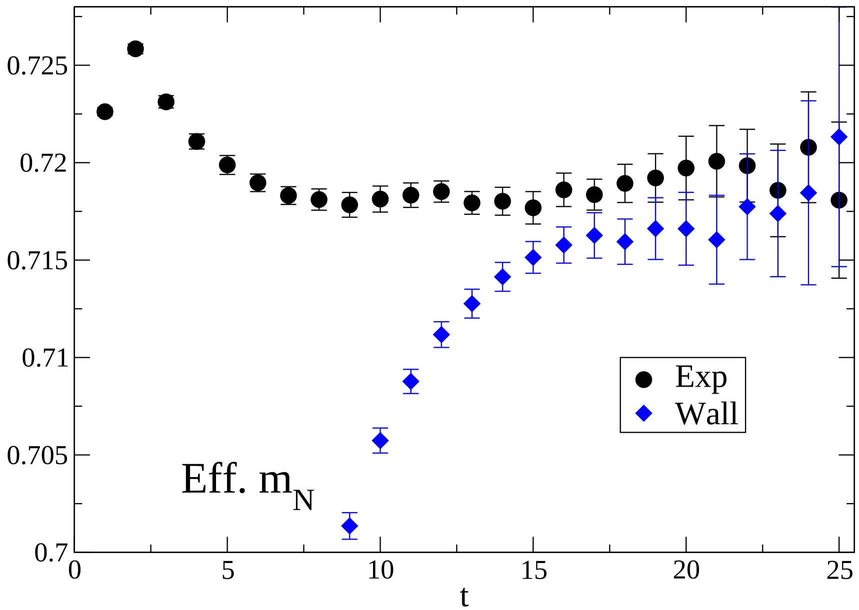
$\rightarrow \Delta E_{NN}$ obtained from plateau region for $C_N(t)$ and $C_{NN}(t)$

$R(t) = C_{NN}(t)/(C_N(t))^2$ in $N_f = 2+1$

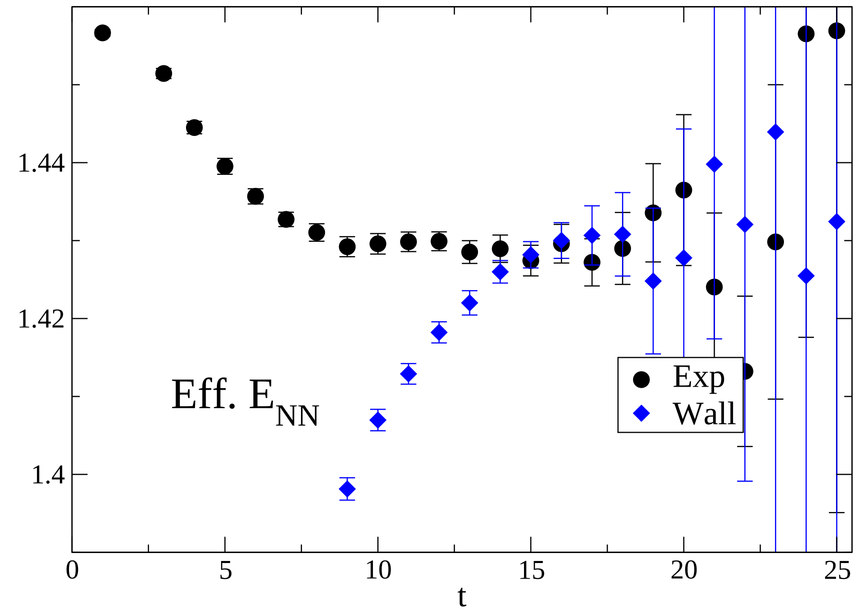
Effective mass and energy

Preliminary result: $N_f = 2+1$ $m_\pi = 0.7$ GeV

Denominator $C_N(t)$



Numerator $C_{NN}(t)$ in $^3S_1$



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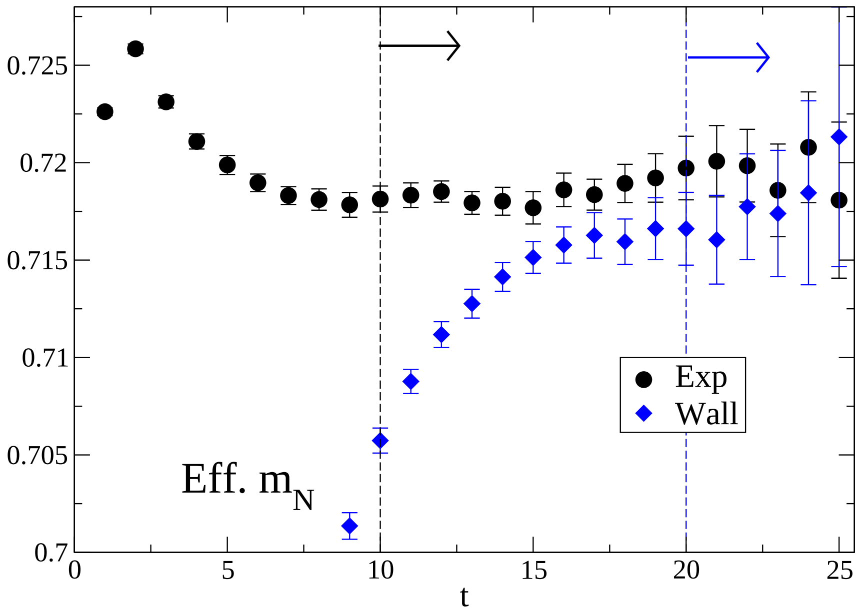

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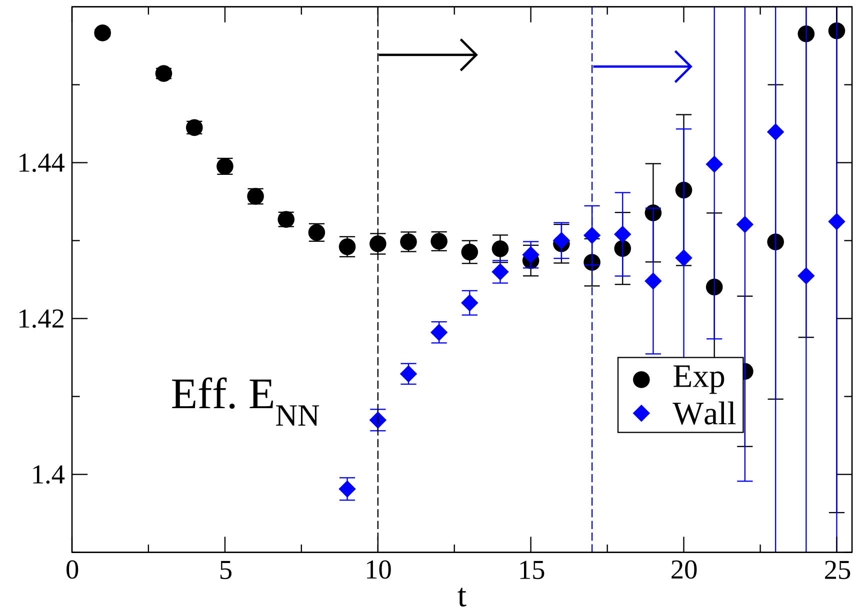
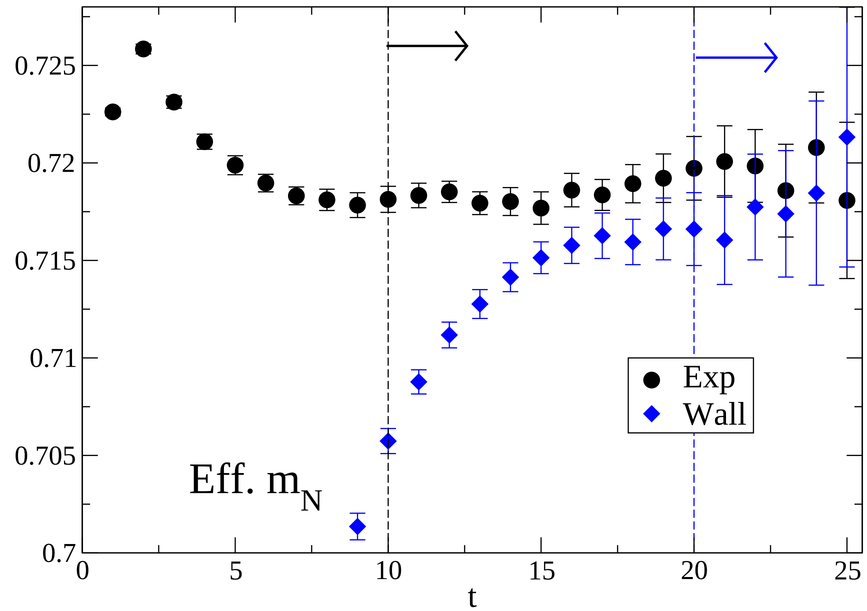
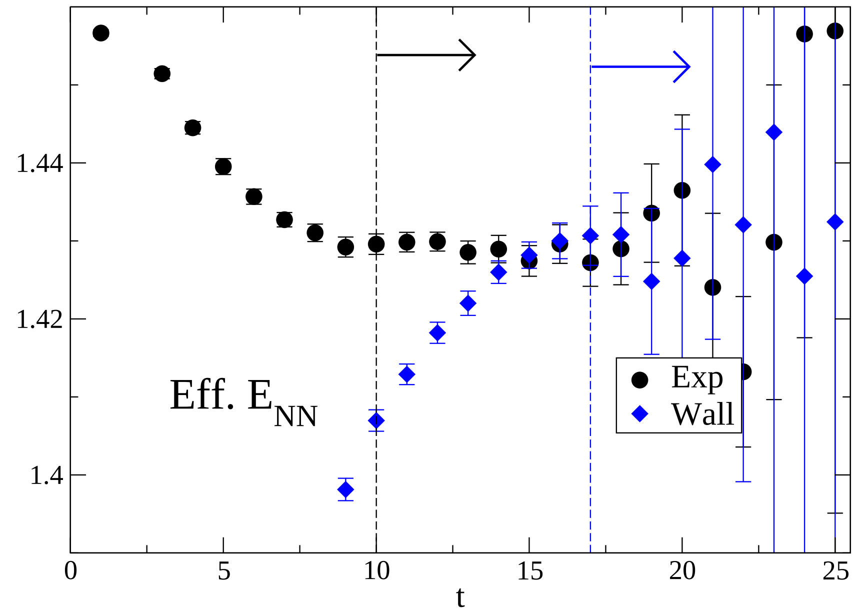


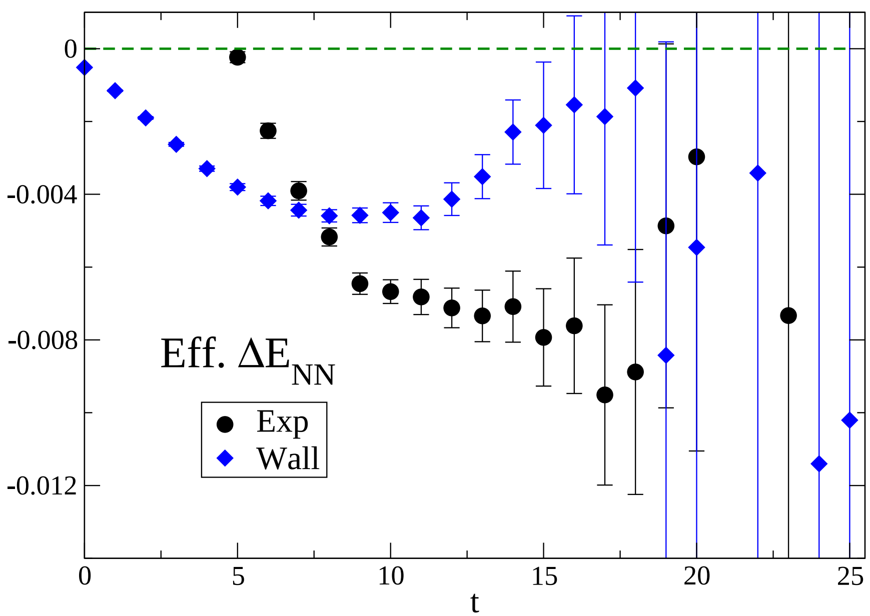
vertical dashed line : plateau starts expressed by t_{plateau}

$R(t)$ in $N_f = 2+1$

Preliminary result: $N_f = 2+1$ $m_\pi = 0.7$ GeV







Effective energy shift ΔE_{NN}

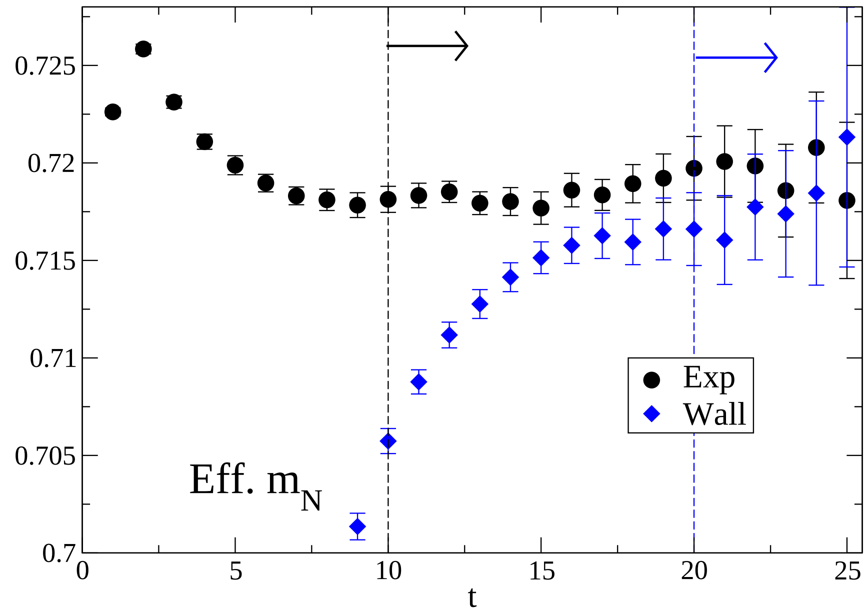
$$\log\left(\frac{R(t)}{R(t+1)}\right) \text{ with } R(t) = \frac{C_{NN}(t)}{(C_N(t))^2}$$

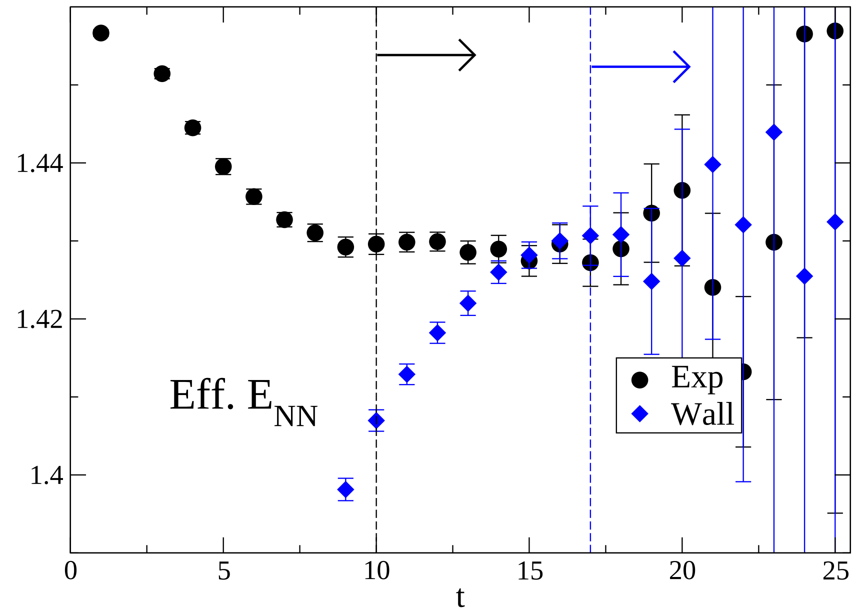
Plateau in Exp

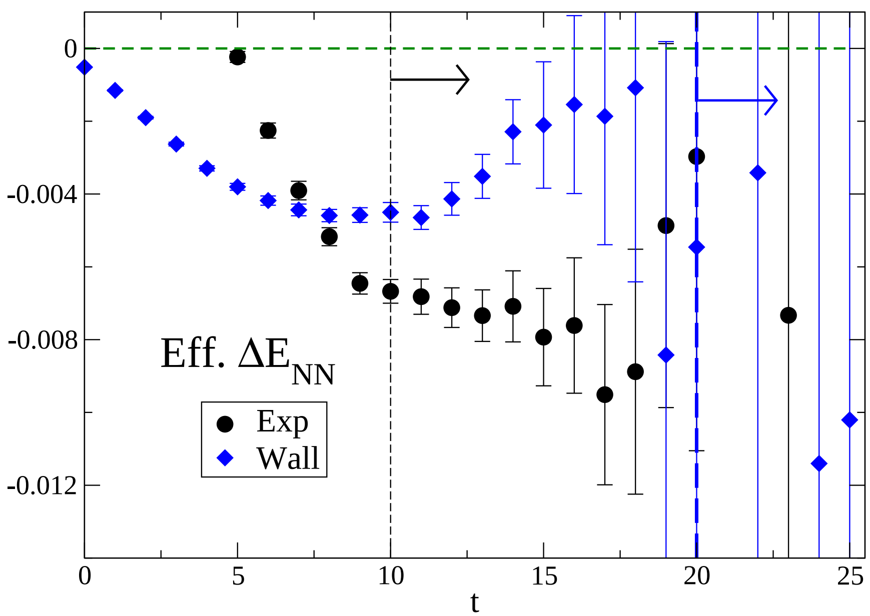
Short flat region in Wall

$R(t)$ in $N_f = 2+1$

Preliminary result: $N_f = 2+1$ $m_\pi = 0.7$ GeV







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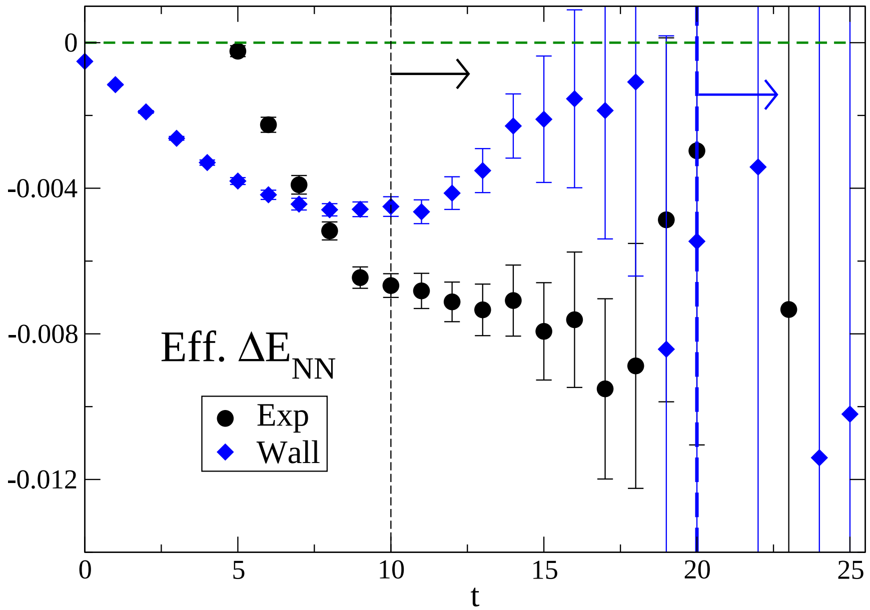
Short flat region of Wall

in $t \ll t_{\text{plateau}}$

$R(t)$ in $N_f = 2+1$

Preliminary result: $N_f = 2+1$ $m_\pi = 0.7$ GeV



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Short flat region of Wall in $t \ll t_{\text{plateau}}$
$= \Delta E_{NN}^W$

HALQCD found $\Delta E_{NN}^W \approx \Delta E_{NN}$ from HALQCD potential
$\rightarrow$ other smearing results are incorrect

Our understanding: ΔE_{NN}^W does not reach plateau region.
$\rightarrow$ to obtain true ΔE_{NN} from Wall, need much larger t behavior clearly
While, Exp smearing : ΔE_{NN} determined in $t \gtrsim t_{\text{plateau}}$

$N_f = 0$ $m_\pi = 0.8$ GeV in $L = 16$

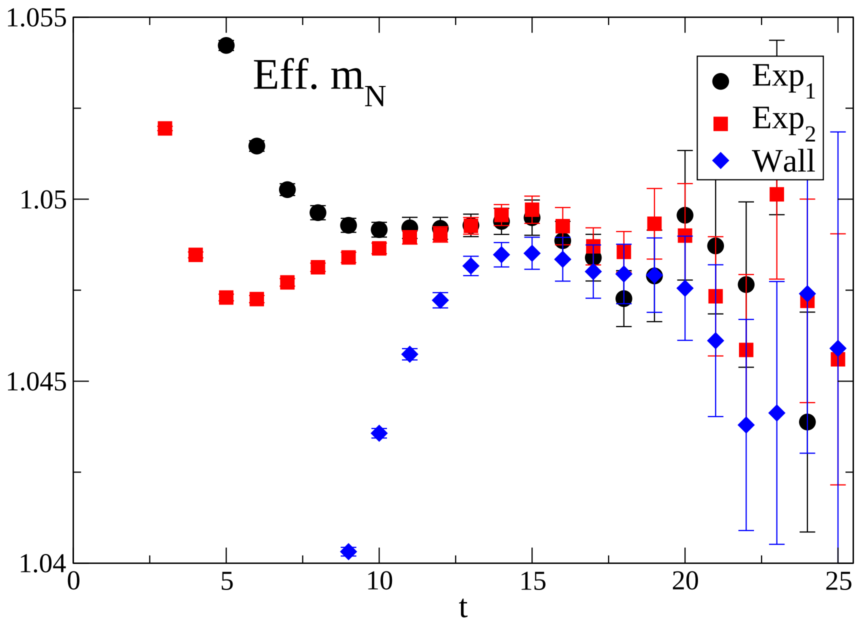
- more accurate data $\rightarrow$ closer to plateau region in wall source
- (relatively) smaller volume $\rightarrow$
  (relatively) larger overlap to bound state in wall source

$R(t) = C_{NN}(t)/(C_N(t))^2$ in $N_f = 0$ at $L = 16$

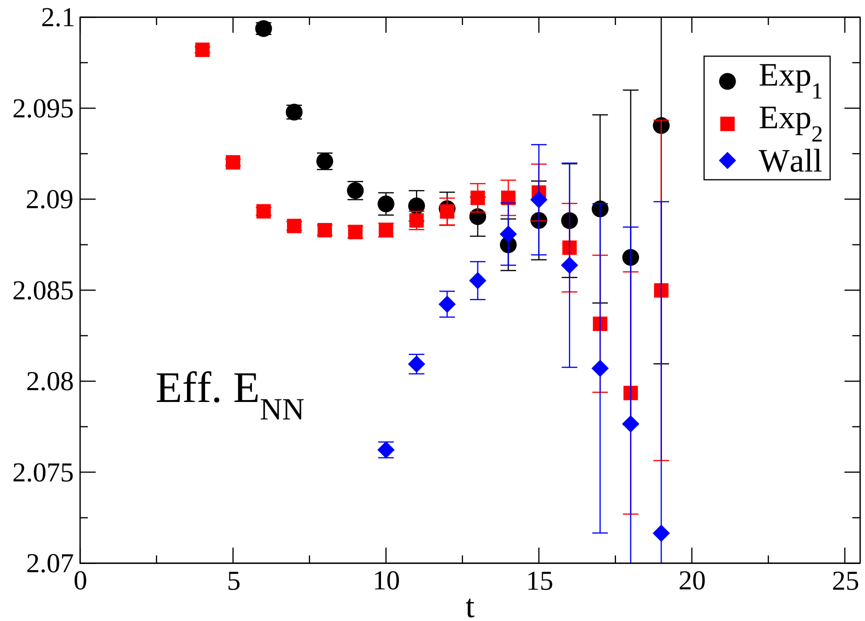
Effective mass and energy

Preliminary result: $N_f = 0$ $m_\pi = 0.8$ GeV

Denominator $C_N(t)$



Numerator $C_{NN}(t)$ in $^3S_1$



reasonably consistent plateaus from Exp_1 and Exp_2

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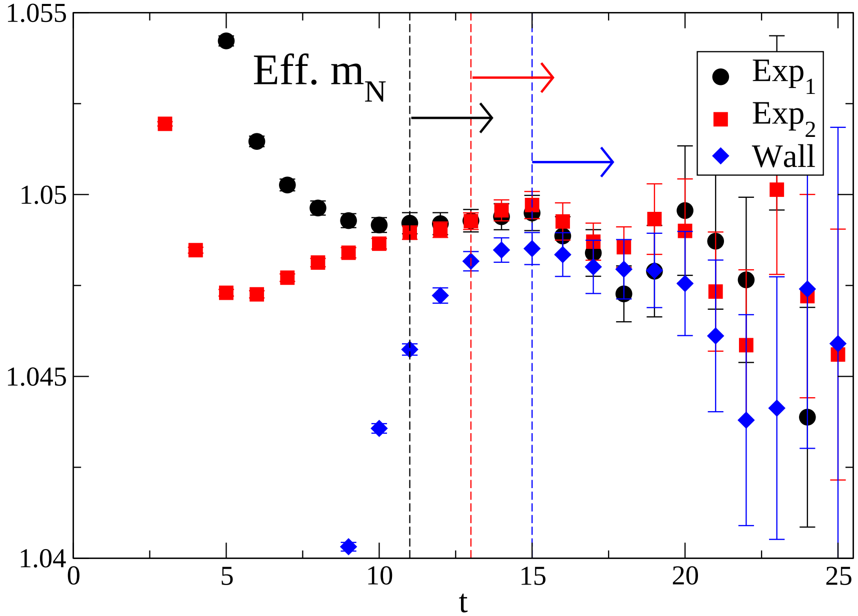

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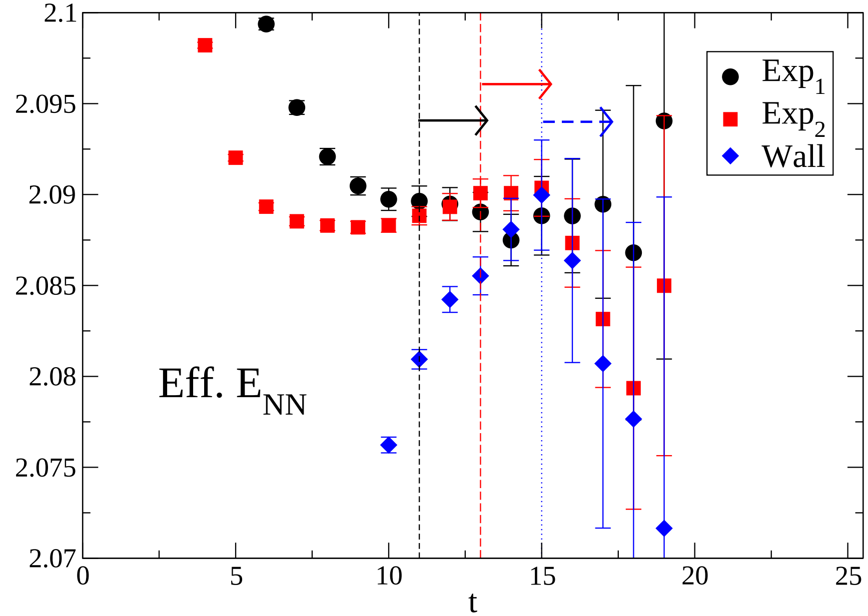
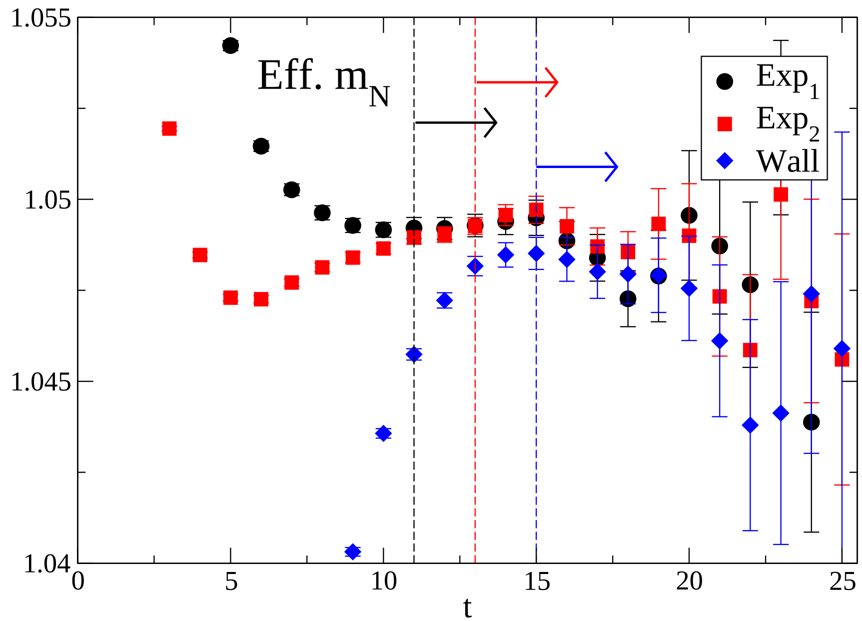


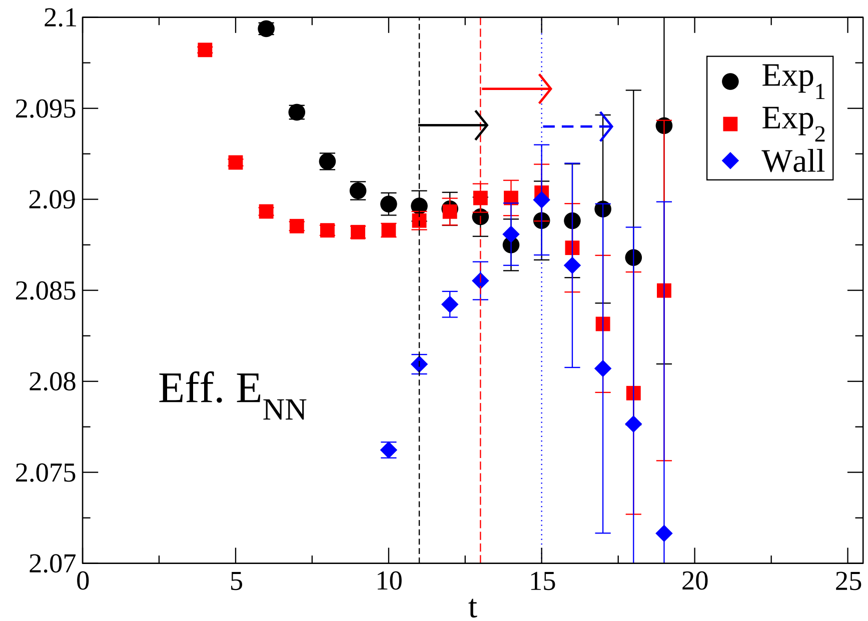
vertical dashed line : plateau starts expressed by t_{plateau}

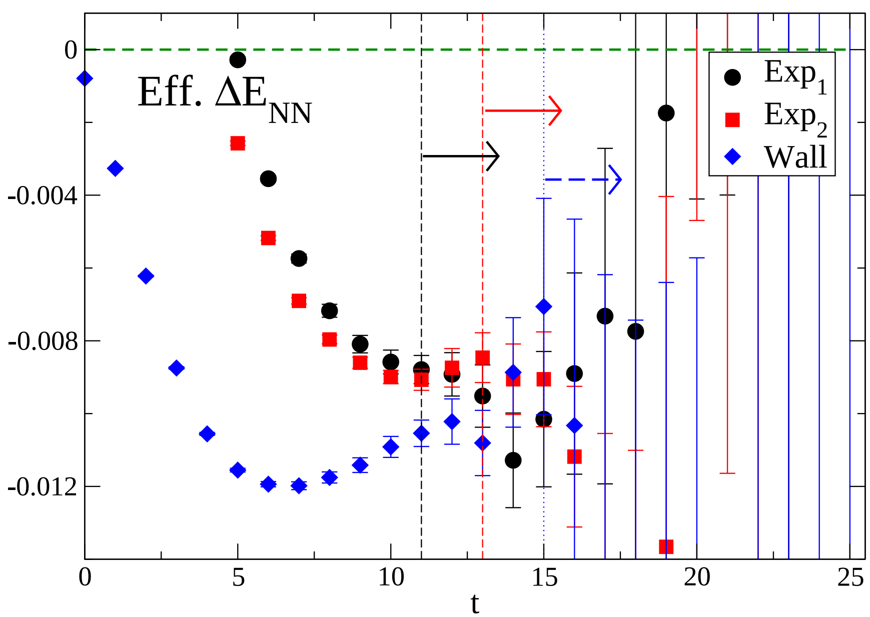
though plateau of E_{NN} not clear in Wall

$R(t)$ in $N_f = 0$ at $L = 16$

Preliminary result: $N_f = 0$ $m_\pi = 0.8$ GeV







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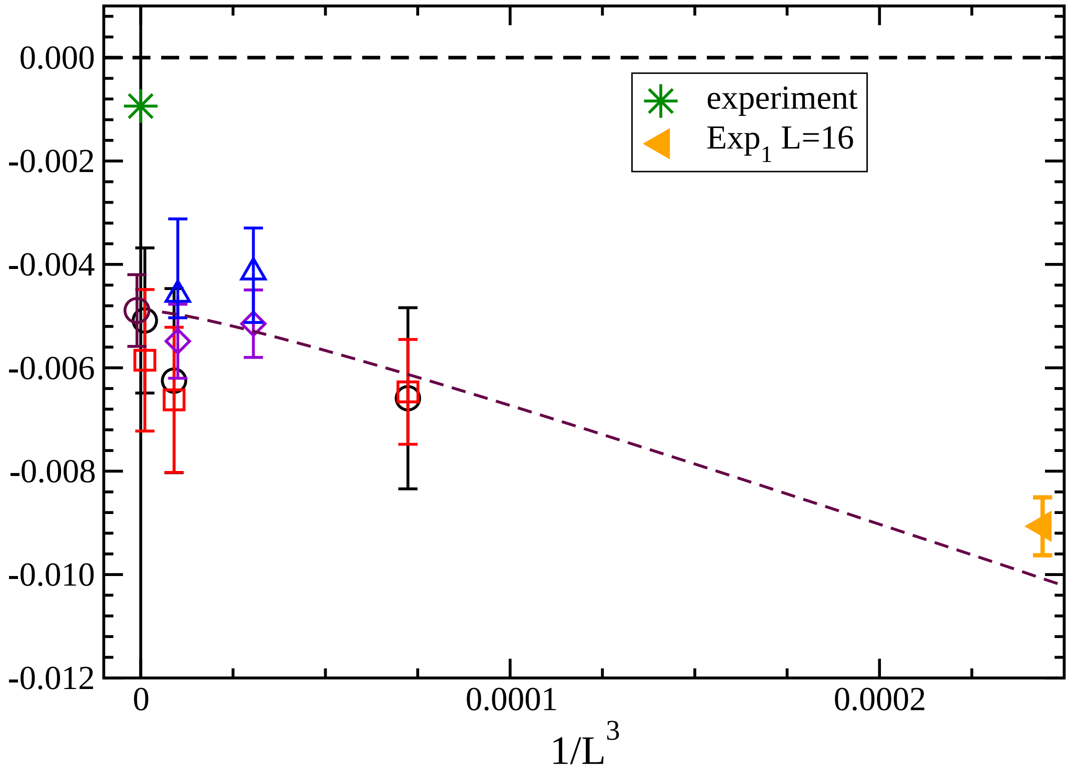
Plateau in $\text{Exp}_{1,2}$

No plateau before t_{plateau} in Wall

Wall approaches to plateau of $\text{Exp}_{1,2}$

Volume dependence of ΔE_{NN} from Exp in $N_f = 0$

Preliminary result: $N_f = 0$ $m_\pi = 0.8$ GeV



open symbols and dashed line : previous result PRD84:054506(2011)

Exp$_1$ result not inconsistent with fit curve of previous work

including finite volume effect of binding energy

'04 Beane *et al.*, '06 Sasaki & TY

Volume dependence of effective ΔE_{NN}

of wall source in $N_f = 0$ $m_\pi = 0.8$ GeV

Expected behavior of wall source in larger volume

- Harder to obtain clear effective ΔE_{NN} in plateau region

  1. plateau of effective $m_N \to$ larger t
  2. plateau of effective $E_{NN} \to$ larger t

     larger overlap to 1st excited state ($p \sim 0$ scattering state)

     maybe similar to $\pi(p)\pi(-p)$ correlator with $p = 2\pi/L$

     [CP-PACS, PRD67 (2003) 014502]

     $\Rightarrow$ relatively smaller overlap to ground state (bound state)

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- Effective $\Delta E_{NN} > 0$ in middle t

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     $\Rightarrow$ relatively smaller overlap to ground state (bound state)

- Effective $\Delta E_{NN} > 0$ in middle t

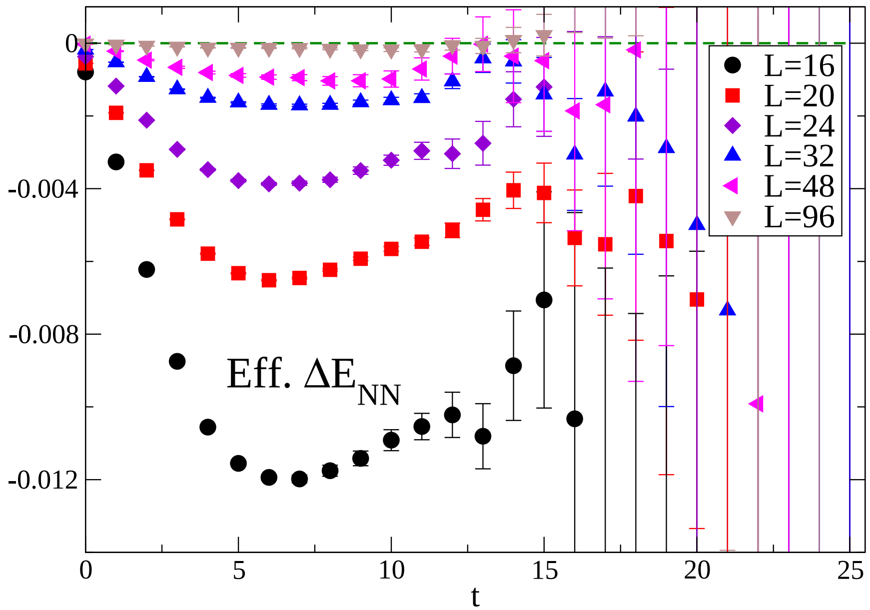
Bound state exists $\Rightarrow$ $\Delta E_{NN} > 0$ in 1st excited state ($a_0 < 0$)

Large enough volume

region of t	small	middle	large
effective ΔE_{NN}	negative	positve	negative
dominat state in C_{NN}	several	1st excited	ground

Volume dependence of wall source in $N_f = 0$

Preliminary result + pilot study of PRD84:054506(2011): $N_f = 0$ $m_\pi = 0.8$ GeV



Clear volume dependence + bowl like structure in small t

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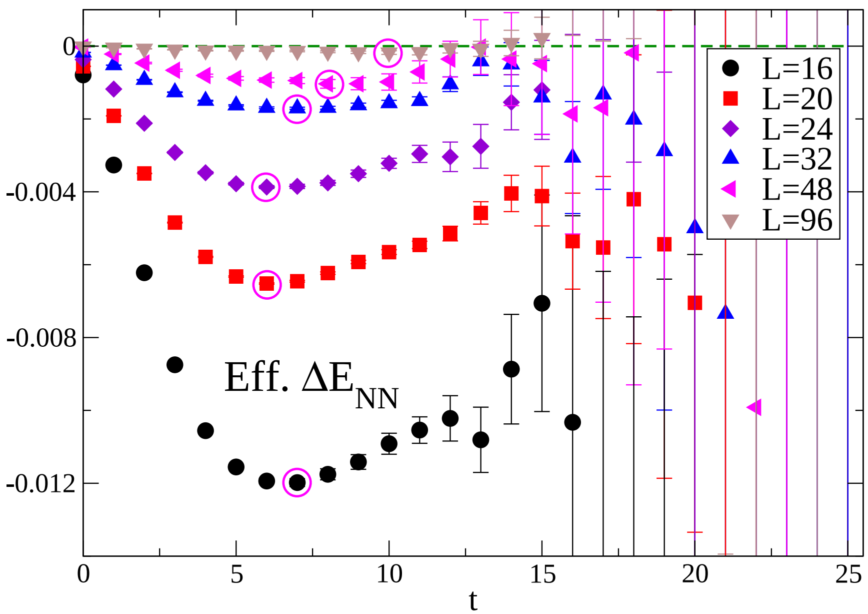

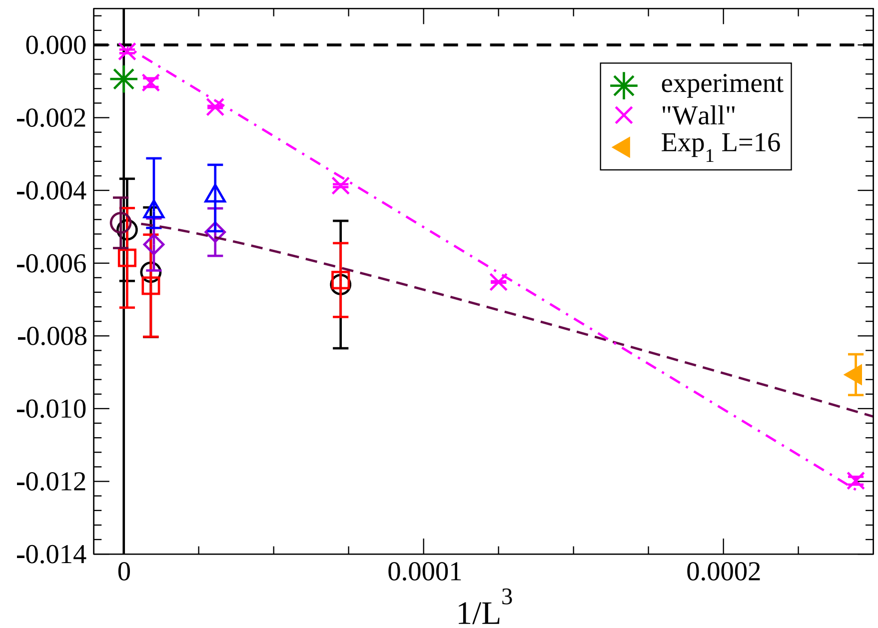

Clear volume dependence + bowl like structure in small t

Pick up smallest value in small t region on each volume

$\rightarrow$ Similar to $1/L^3$ behavior

Volume dependence of wall source in $N_f = 0$

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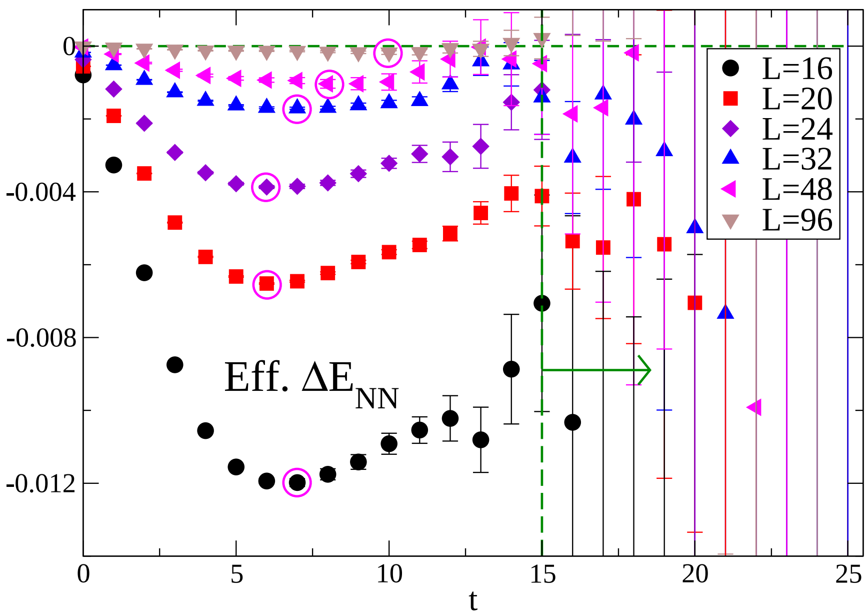
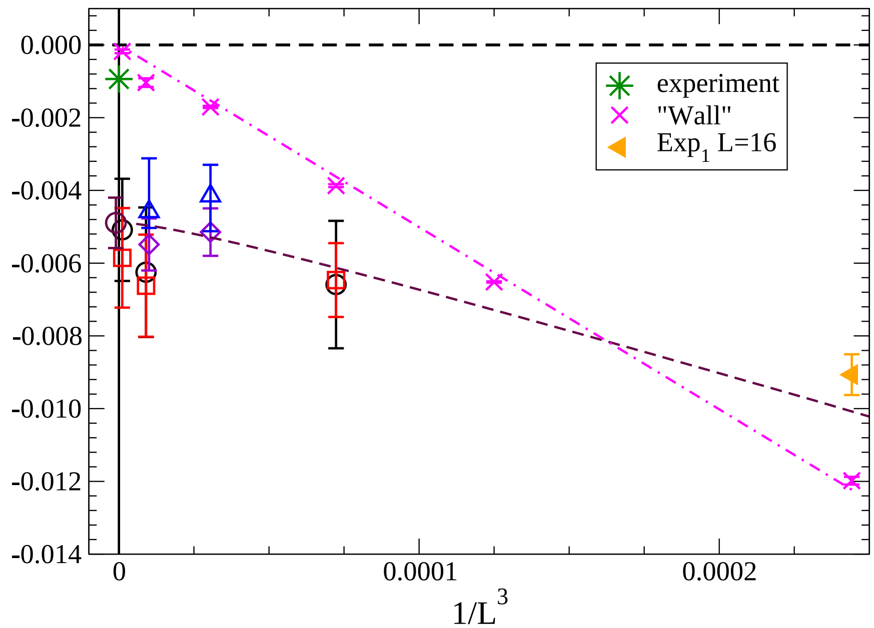




Clear volume dependence + bowl like structure in small t

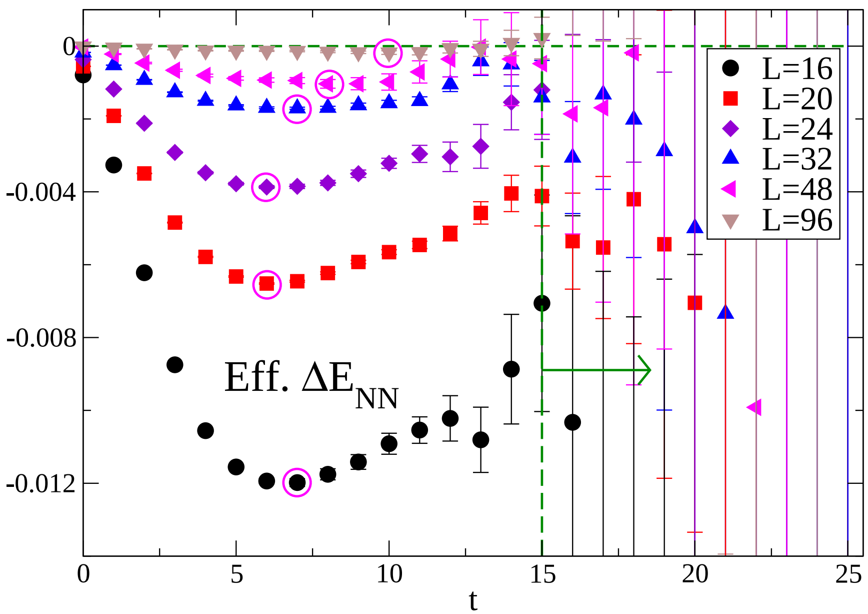
Pick up smallest value in small t region on each volume

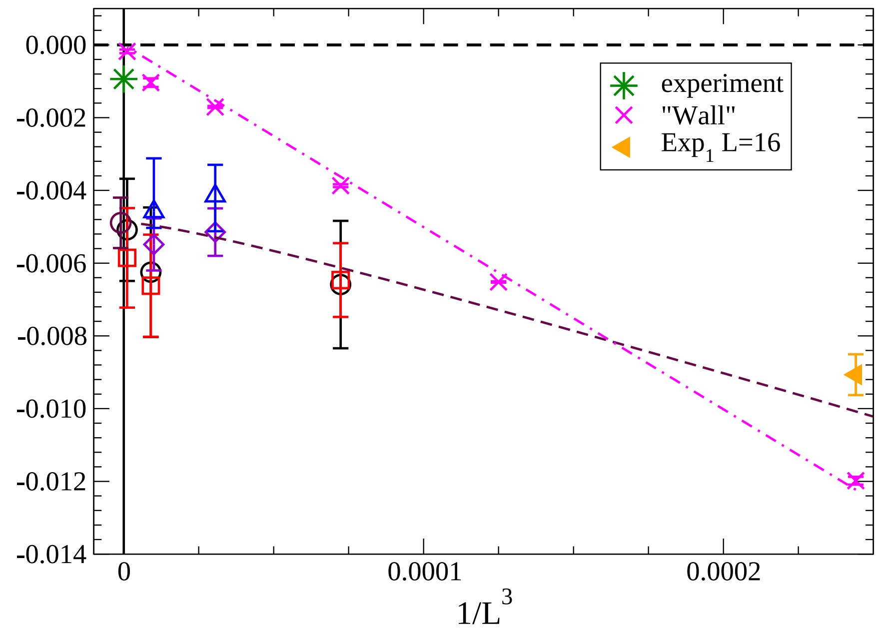
$\rightarrow$ Similar to $1/L^3$ behavior $\Leftarrow$ Not ΔE_{NN} of ground state

Ground state should be obtained from $t > t_{\text{plateau},L=16}$.

Volume dependence of wall source in $N_f = 0$

Preliminary result + pilot study of PRD84:054506(2011): $N_f = 0$ $m_\pi = 0.8$ GeV





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Upward behavior in larger volume might be caused by

1st excited state contribution

Summary and future work

Exponential and wall sources in NN $^3\mathrm{S}_1$ channel

$$N_f = 2+1\ m_\pi = 0.7\ \text{GeV and}\ N_f = 0\ m_\pi = 0.8\ \text{GeV}$$

Wall source

- hard to observe plateau of m_N and $E_{NN} \to \Delta E_{NN}$
- effective ΔE_{NN} approaches exponetial result in $L = 16$
  $\to$ will be consistent with each other in plateau region
- bowl like structure in effective ΔE_{NN} before plateau region

Exponential source

- possible to determine ΔE_{NN} in plateau region for $C_N(t)$ and $C_{NN}(t)$
  (by tuning smearing parameters in principle)

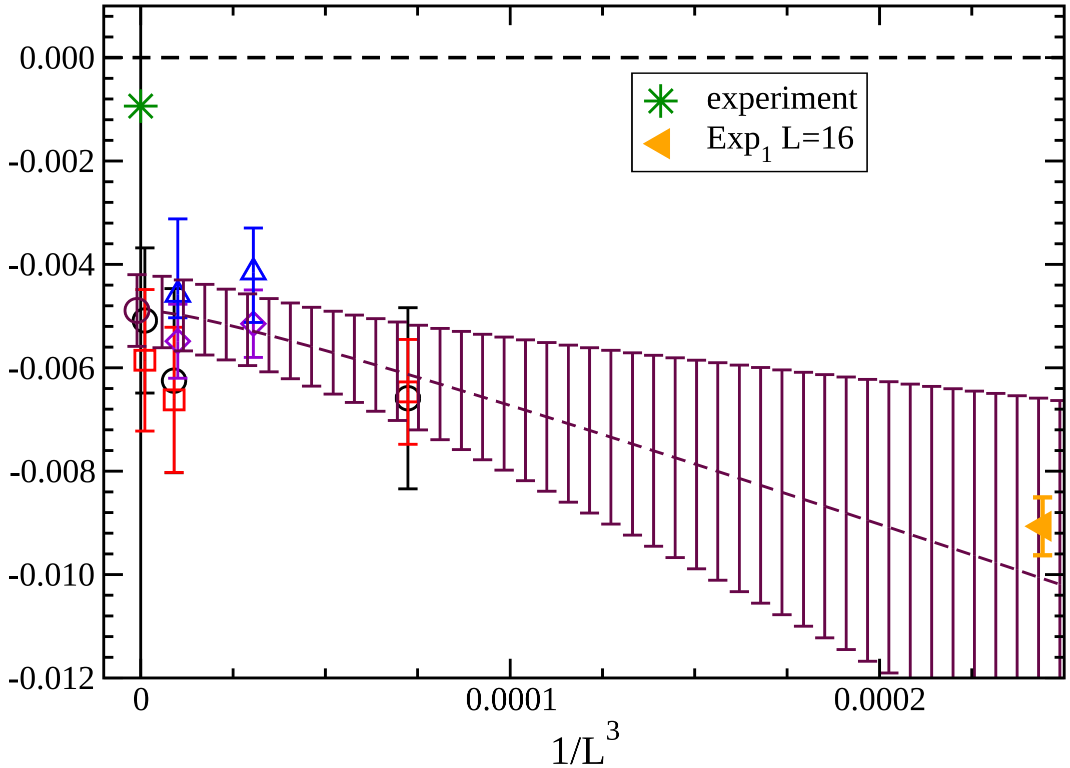
Need further investigations

check consistency of ΔE_{NN} from exponential and wall sources,
more sophisticated calculation (GEVP),
observing effective $\Delta E_{NN} > 0$ in wall source(?), $\cdots$

Back up

Volume dependence of ΔE_{NN} in $N_f = 0$

Preliminary result: $N_f = 0$ $m_\pi = 0.8$ GeV



open symbols and dashed line : previous result PRD84:054506(2011)

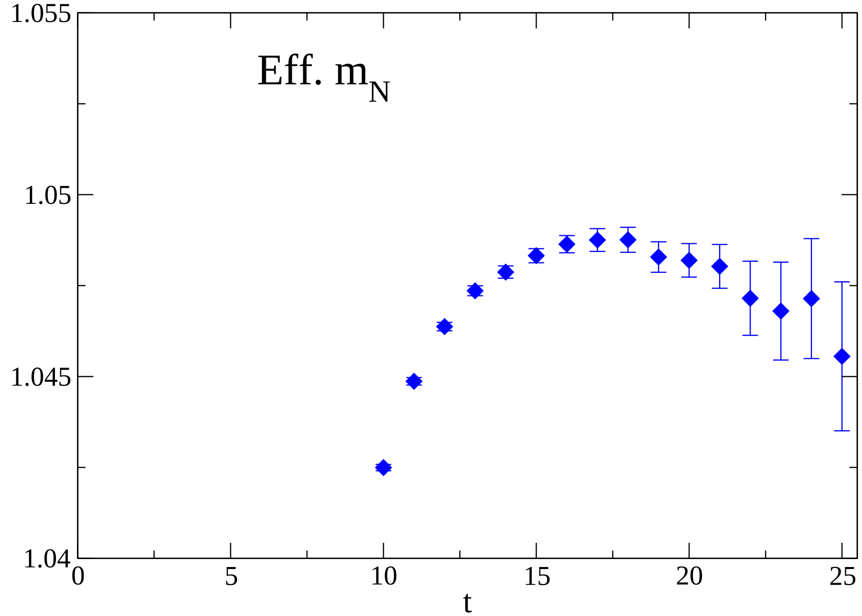
Exp$_1$ result not inconsistent with fit curve of previous work

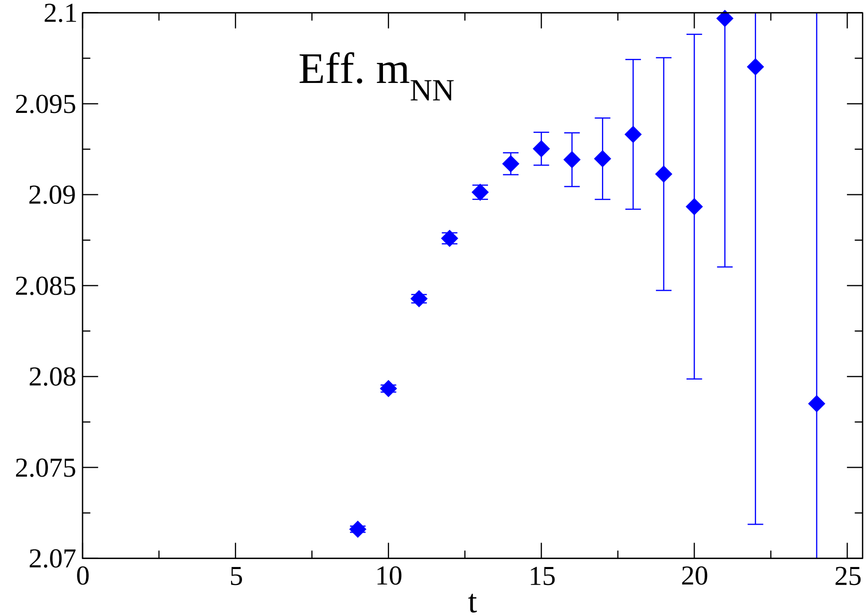
including finite volume effect of binding energy

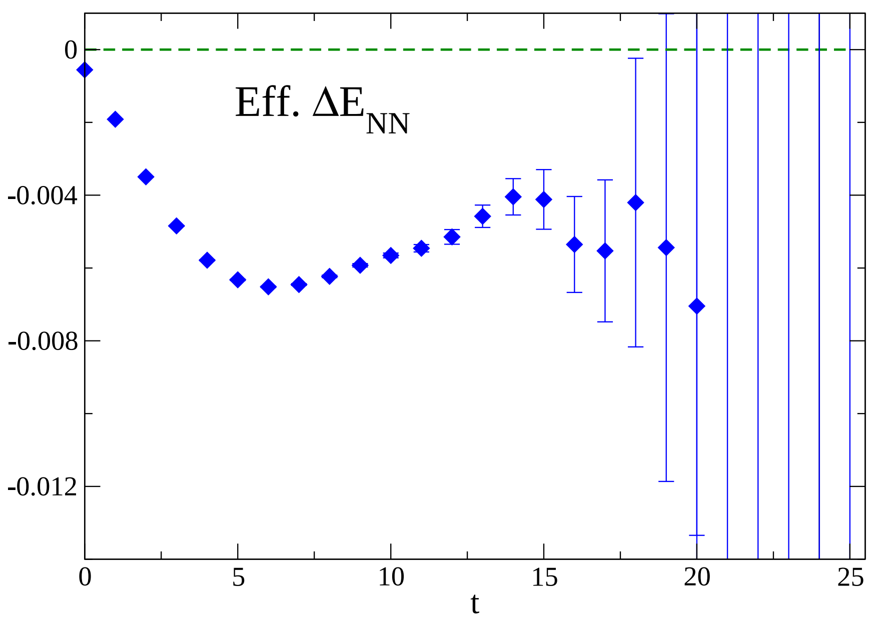
'04 Beane *et al.*, '06 Sasaki & TY

$R(t)$ in $N_f = 0$ at $L = 20$

Preliminary result: $N_f = 0$ $m_\pi = 0.8$ GeV







Effective energy shift ΔE

$$\log\left(\frac{R(t)}{R(t+1)}\right) \text{ with } R(t) = \frac{C_{NN}(t)}{(C_N(t))^2}$$