Hadron Structure

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Outline

Nucleon charges:

g_A: benchmark quantity.

g_S, g_T $\rightarrow$ searching for BSM signals in precision β decay experiments.

Isovector nucleon form factors:

Electromagnetic: $G_E(Q^2)$, $G_M(Q^2)$ $\rightarrow$ proton radius puzzle, large Q^2 behaviour.

Axial: $G_A(Q^2)$ $\rightarrow$ neutrino oscillation experiments.

Isoscalar nucleon form factors:

$G^s_{E,M}(Q^2)$, $G^s_A(Q^2)$ $\rightarrow$ contributions of strangeness to properties of nucleon.

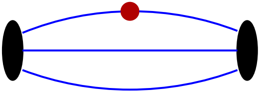
Pion form factor: electromagnetic, $f^{\pi}_{+}(Q^2)$ $\rightarrow$ large Q^2 behaviour to be studied at JLab Hall C to 6 GeV^2.

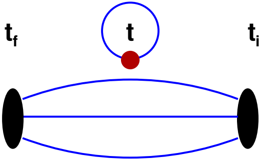
Nucleon sigma terms: $\sigma_{\pi N}$, σ_s, $\sigma_{c,b,t}$ $\rightarrow$ predicting dark matter scattering cross-sections.

Nucleon quark momentum fraction. EM transition form factors of resonances.

Summary/Outlook. Many results are preliminary and can change!

General considerations: $\langle N|\bar{q}\Gamma q|N\rangle$



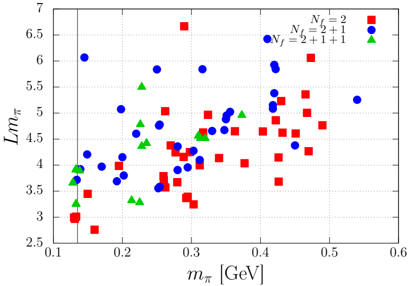


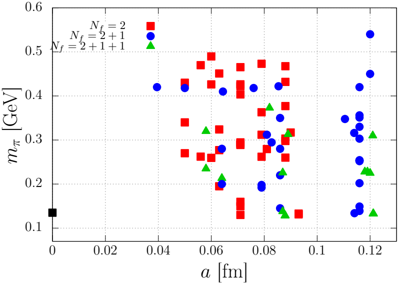
(Isospin symmetric limit) Isovector combinations only connected. Isoscalar also disconnected.

Systematics:

- Excited state pollution.
- Renormalisation+ improvement: for $\vec{p} = \vec{p}\,' = 0$/some operators/actions $c_{\mathcal{O}} = 0$ or $b_{\mathcal{O}} = 0$
  $\mathcal{O}^{\overline{\text{MS}}}(\mu) = Z^{\overline{\text{MS}},latt}(a\mu)\left[(1 + b_{\mathcal{O}}\, am_q)\mathcal{O}^{latt} + ac_{\mathcal{O}}\mathcal{O}_1^{latt}\right]$
- Volume: exponentially suppressed $\sim e^{-Lm_\pi}$, $Lm_\pi > 4$.
- Discretisation effects: $\mathcal{O}(a)$ or $\mathcal{O}(a^2)$.
- Physical point extrapolation: chiral pert. (inspired) $m_\pi \to m_\pi^{phys}$.

Ensembles used for hadron structure 3pt functions





$N_f = 2$: RQCD (RQCD/QCDSF), ETMC, Mainz (CLS), QCDSF

$N_f = 2+1$: LHPC (BMW-c), RBC/UKQCD, Mainz (CLS), RQCD (CLS), JLQCD, PACS, χQCD (RBC/UKQCD), NME (JLab/W&M), QCDSF/UKQCD/CSSM

$N_f = 2+1+1$: PNDME (MILC), HPQCD (MILC), ETMC

Nucleon isovector charges and form factors

Charged currents ($n \to p$): $\bar{u}\Gamma d$

Neutral currents ($p \to p$): $\bar{u}\Gamma u$, $\bar{d}\Gamma d$, $\bar{s}\Gamma s$

Isospin limit: $\langle p|\bar{u}\Gamma d|n\rangle = \langle p|\bar{u}\Gamma u - \bar{d}\Gamma d|p\rangle = \langle n|\bar{d}\Gamma d - \bar{u}\Gamma u|n\rangle$

Also: $\Gamma = \gamma_\mu$, $\langle p|\bar{u}\gamma_\mu d|n\rangle = \langle p|J_{em}|p\rangle - \langle n|J_{em}|n\rangle$ etc.

Isovector form factors:

$$\Gamma = \gamma_\mu \qquad F_1^v(Q^2)\gamma_\mu + \frac{F_2^v(Q^2)}{2m_N}\sigma_{\mu\nu}Q^\nu \qquad \xrightarrow{Q^2\to 0} 1$$

$$\Gamma = \gamma_\mu\gamma_5 \qquad G_A^v(Q^2)\gamma_\mu\gamma_5 - i\gamma_5\frac{\tilde{G}_P^v(Q^2)}{2m_N}Q_\mu \qquad \longrightarrow g_A$$

$$\Gamma = \sigma_{\mu\nu} \qquad G_T^v(Q^2)\sigma_{\mu\nu} \qquad \longrightarrow g_T$$

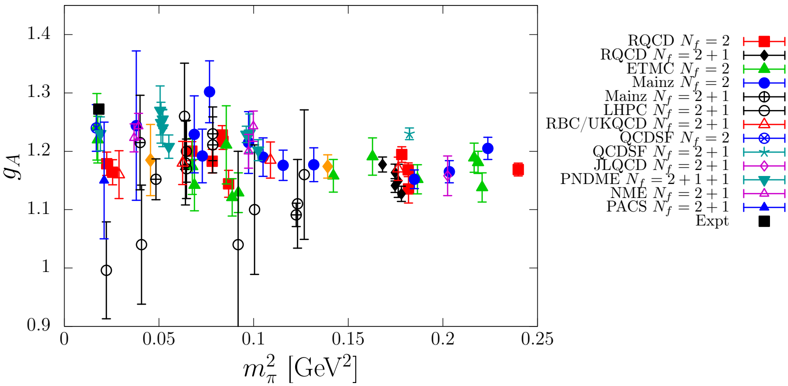
$$\Gamma = \mathbb{1} \qquad G_S^v(Q^2)\mathbb{1} \qquad \longrightarrow g_S$$

$$\Gamma = \gamma_5 \qquad G_P^v(Q^2)\gamma_5 \qquad \longrightarrow g_P$$

Isovector charges $g_A = \Delta u - \Delta d$

β-decay, $g_A/g_V = 1.2723(23)$ PDG 2015.

Benchmark quantity sensitive to systematics.

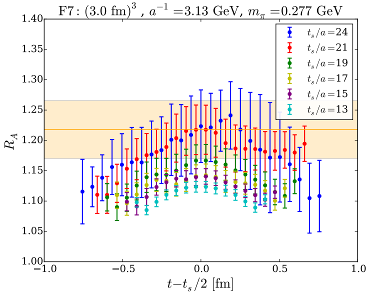


Presented 2016:

PNDME, NME, Mainz, RQCD, ETMC, PACS, χQCD, QCDSF, ...

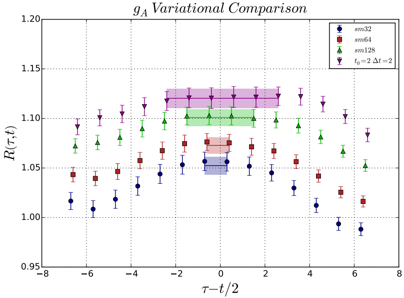
Excited state contamination.

Mainz:[von Hippel,1605.00564]



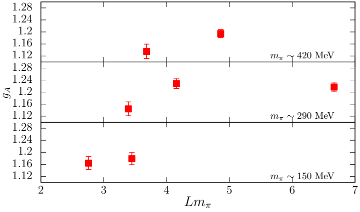
Vary interpolator

CSSM [Dragos,1606.03195]



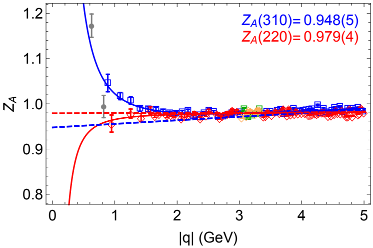
Finite volume

RQCD [Bali,1412.7336]



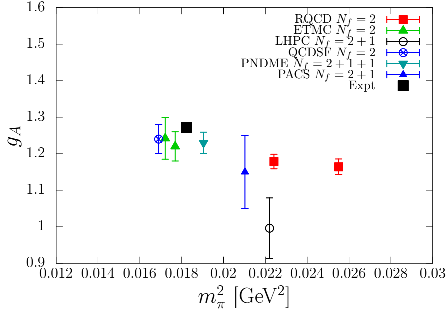
Renormalisation

PNDME [Bhattacharya,1606.07049].

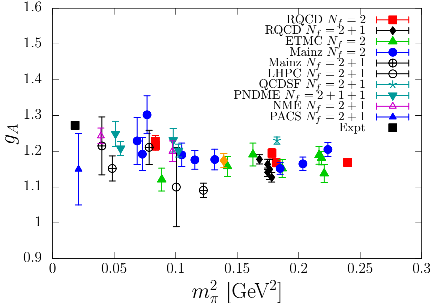


Isovector charges g_A

Several $m_\pi < 165$ MeV results.



Impose $Lm_\pi > 4$, $a < 0.1$ fm

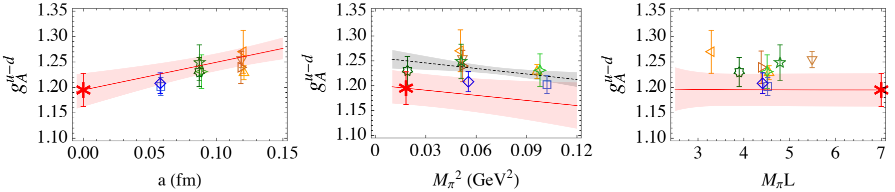


ETMC: [Alexandrou,Mon,15:15] $N_f = 2$ twisted mass fermions, $Lm_\pi = 3$. Increased statistics on 1507.04936, 579 configs $\times$ 16 measurements, $g_A = 1.22(3)(2)$ - systematics from fitting.
PACS: [Kuramashi,Thu,16:30] $N_f = 2+1$ NP clover, stout smeared links, $m_\pi = 145$ MeV, $a = 0.085$ fm, 146 configs $\times$ 64 measurements, $t_f - t_i$=1.3 fm, $Lm_\pi = 6$
PNDME: [Gupta,Thu,17:50]

Isovector charges g_A

PNDME: 1606.07049: $N_f = 2+1+1$ MILC HISQ, $a = 0.06 - 0.12$ fm Valence clover (tree-level, tadpole improved) $m_\pi = 135 - 315$ MeV

Final result: $g_A = 1.195(33)(20)$ [Gupta,Thu,17:50].



NME: [Gupta,Thu,17:50] $N_f = 2+1$ JLab/W&M tadpole clover, $a = 0.114$ and 0.080 fm, $m_\pi = 200$, 315 MeV.

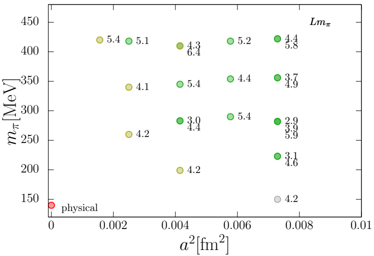
Isovector charges g_A

CLS: $N_f = 2+1$ NP clover

Physical point along:

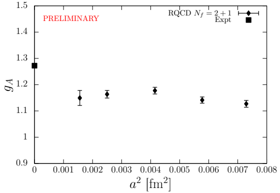
$2m_l + m_s =$const. and
m_s=const.

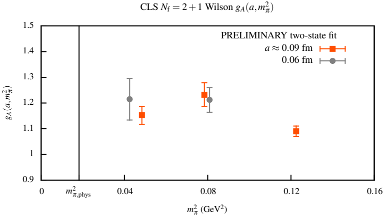
$a = 0.039 - 0.085$ fm



RQCD: [Bali,Fri,16:50] Had. Spec. & Int.

Mainz: [Harris,Thu,16:50]





Improved axial vector currents: QCDSF [Perlt,Tue,17:50], χQCD [Yi-Bo,Mon,14:35]
Feynman-Hellmann theorem: [Walker-Loud,Mon,16:45].

Isovector charges: g_S and g_T

BSM contributions to β decay.

$$\mathcal{L}_{\rm CC} = -\frac{G_F^{(0)} V_{ud}}{\sqrt{2}} \left[\epsilon_S \ \bar{e}(1-\gamma_5)\nu_\ell \cdot \bar{u}d + \epsilon_T \ \bar{e}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell \cdot \bar{u}\sigma^{\mu\nu}(1-\gamma_5)d \right]$$

Studied in planned precision β decay expts. + LHC $pp \to e\nu + X$

Estimate of g_T:

Transverse spin $g_T = \int dx[\delta u(x) - \delta d(x)]$

Phenomenological estimates from fits to SIDIS data $g_T \approx 1$, $\Delta g_T / g_T \gtrsim 25\%$.

Estimate of g_S:

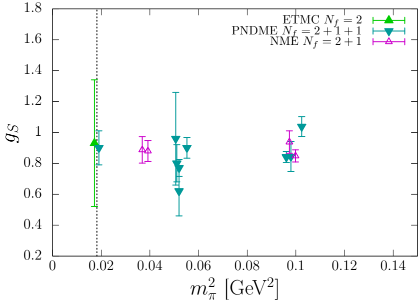
CVC relation $\partial_\mu(\bar{u}\gamma_\mu d) = -i(m_u - m_d)\bar{u}d$ applied to $\langle p(p_{\rm f})|\bar{u}\gamma_\mu d|n(p_{\rm i})\rangle$

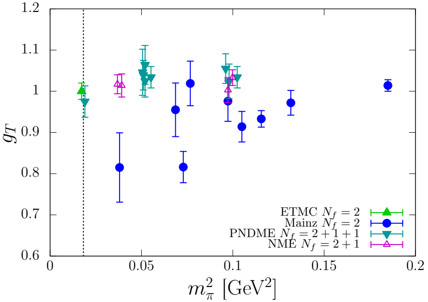
Forward limit: $(m_u - m_d) g_S = (m_p - m_n)^{QCD}$.

[Gonzalez-Alonso,1309.4434]:

Lattice estimates of $m_u - m_d$ and $(m_p - m_n)^{QCD} \to g_S = 1.02(11)$.

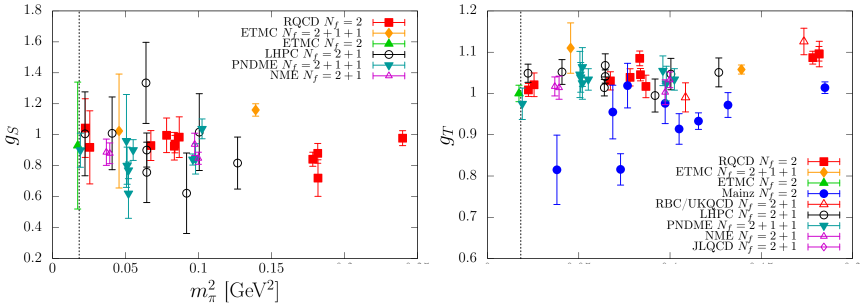
Isovector charges: g_S and g_T





PNDME: [Gupta,Thu,17:50], NME: [Gupta,Thu,17:50], Mainz: [von Hippel,Wed,9:40], ETMC: [Alexandrou,Mon,15:15]

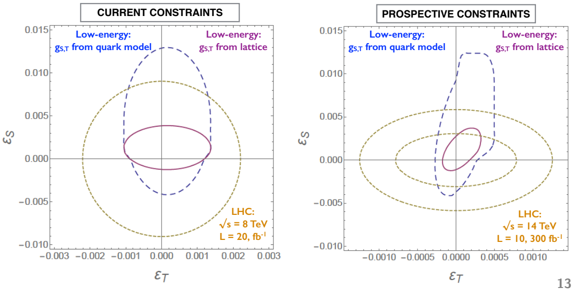
Isovector charges: g_S and g_T



PNDME
[Bhattacharya,1606.07049]:
continuum+chiral extrap

$$g_S^{u-d} = 0.97(12)(6)$$

$$g_T^{u-d} = 0.987(51)(20)$$



Aim for precision: improving statistics

- Increase sampling per configuration.
- More sources/volume averaging/low modes.
- Cost mitigated by using e.g. All-mode-averaging (AMA) techniques [Blum,1208.4349].

AMA (truncating the solver [Bali,0910.3970]):

$$\langle C \rangle^{AMA} = \frac{1}{n} \sum_{source\ i=1}^{n} \langle C_{x_i}^{approx} \rangle + \langle C_{x_0}^{exact} - C_{x_0}^{approx} \rangle$$

Efficiency depends on the solver + smearing/contraction overhead.

NME: [Yoon,1602.07737]: Multigrid solver [Babich,1005.3043] $\rightarrow$ factor of $\sqrt{2}$ reduction in errors for fixed cost at $m_\pi = 312$ MeV.

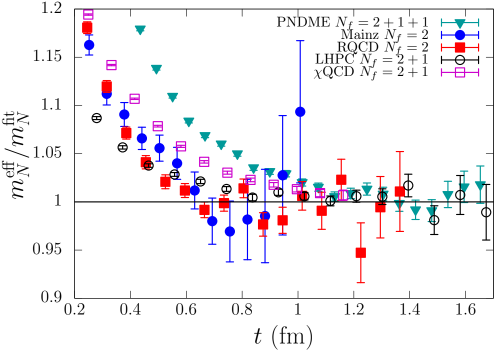
Cost effectiveness improves as m_π decreases [Bhattacharya,1606.07049].

Mainz: [von Hippel,1605.00564]: Locally deflated SAP-preconditioned solver (OpenQCD) $\rightarrow$ factor of 2 reduction in errors.

Aim for precision: excited state contamination

Keeping statistical errors small → small sink-source separation → excited state contamination.

$m_\pi \sim 280 - 320$ MeV



More smearing, pro: smaller $t_f - t_i$, con: more expensive ($a \to 0$), larger errors.

Strategies:

(A) multiple $t_f - t_i$, single smearing → 2-state fits, summation method, ...

(B) single $t_f - t_i$, multiple smearings → variational method.

Investigated in CSSM: [Dragos,1606.03195] and NME: [Yoon,1602.07737].

Also on excited states: [Hansen,Fri,14:20], [Walker-Loud,Mon,16:45]

Nucleon isovector form factors

$$\langle p(\mathbf{p}_\mathrm{f})|V_\mu^{u-d}|p(\mathbf{p}_\mathrm{i})\rangle = \bar{u}_p(\mathbf{p}_\mathrm{f})\left[F_1^v(Q^2)\gamma_\mu + \frac{F_2^v(Q^2)}{2m_N}\sigma_{\mu\nu}Q^\nu\right]u_p(\mathbf{p}_\mathrm{i})$$

$$\langle p(\mathbf{p}_\mathrm{f})|A_\mu^{u-d}|p(\mathbf{p}_\mathrm{i})\rangle = \bar{u}_p(\mathbf{p}_\mathrm{f})\left[G_A^v(Q^2)\gamma_\mu - i\frac{\tilde{G}_P^v(Q^2)}{2m_N}Q_\mu\right]\gamma_5 u_p(\mathbf{p}_\mathrm{i})$$

Sachs ff.:

$$G_E^v(Q^2) = F_1^v(Q^2) - \frac{Q^2}{4m_N^2}F_2^v(Q^2), \quad G_M^v(Q^2) = F_1^v(Q^2) + F_2^v(Q^2)$$

Forward limit:

$$G_E^v(0) = F_1^v(0) = 1, \quad G_A^v(0) = g_A, \quad G_M^v(0) = 1 + F_2^v(0) = \mu^{p-n} = 1 + \kappa^{p-n} = 4.79$$

Shape at low Q^2, $\langle r_X^2\rangle = -6\frac{dG_X(Q^2)}{dQ^2}$: different probe $\to$ different radius.

$$G_X(Q^2) = G_X(0)\left[1 - \frac{1}{6}\langle r_X^2\rangle Q^2 + \dots\right]$$

Nucleon isovector form factors

Challenges for the lattice:

Achieving low Q^2, (conventionally) $\rightarrow$ large L, $ap = (2\pi n/L)$.

Very sensitive to m_π: radii diverge as $m_\pi \rightarrow 0$.

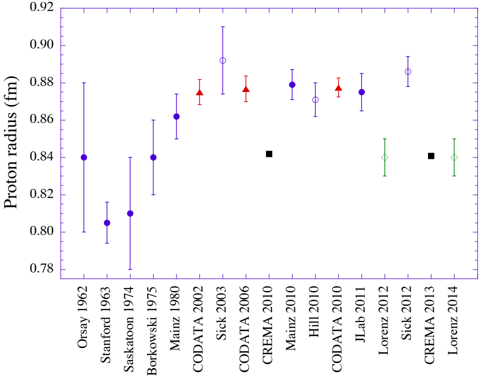
Parameterising $\rightarrow$ dipole form, z expansion etc $\rightarrow$ $\langle r_E^2 \rangle$, $\langle r_A^2 \rangle$.

Extrapolation $\rightarrow$ $\tilde{G}_P^v(0)$, $G_M^v(0)$, $\langle r_M^2 \rangle$.

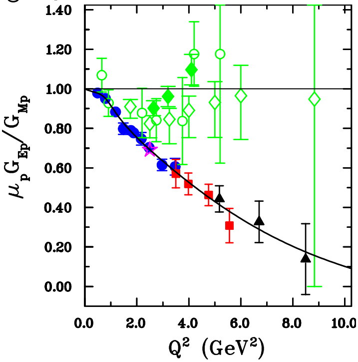
Electromagnetic form factors

[Punjabi,1503.01452]

Proton charge radius:



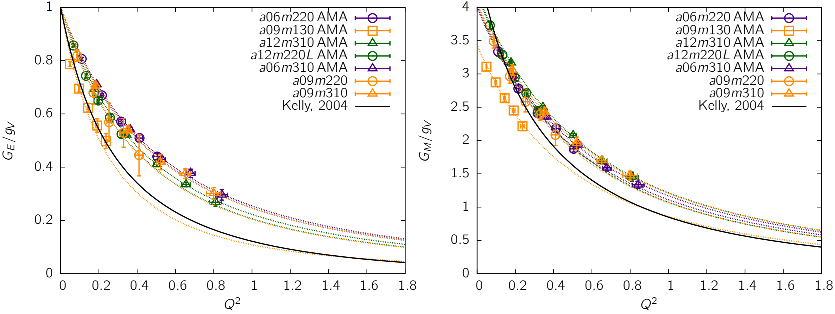
Large Q^2:



Radius: would need $< 2\%$ error with all systematics included.

Compute isovector form factors: comparing with $\langle r_E^2 \rangle^v = \langle r_E^2 \rangle^{p-n}$.

$G_E^v(Q^2)$ and $G_M^v(Q^2)$

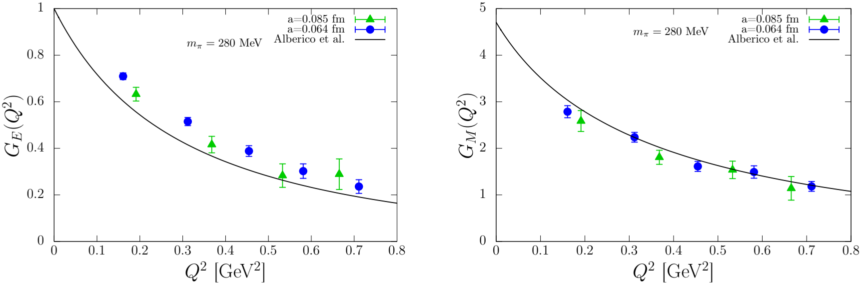


[Jang,Thu,15:00]
$\langle r_{E,M}^2 \rangle$ increase as $m_\pi \to 0$, $\langle r_E^2 \rangle$ decreases as $a \to 0$

PNDME: $N_f = 2+1+1$ MILC HISQ, $a = 0.06 - 0.12$ fm Valence clover (tree-level, tadpole improved) $m_\pi = 135 - 315$ MeV, AMA

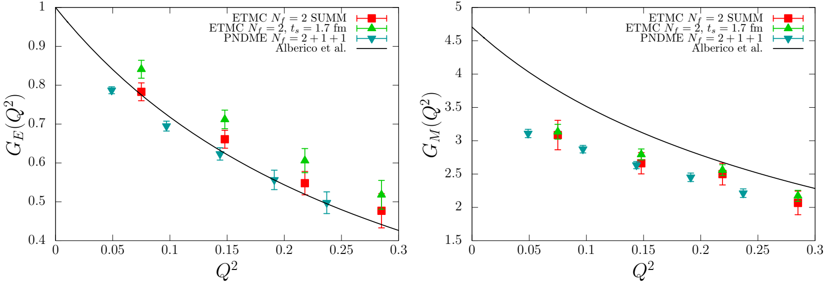
NME: $N_f = 2+1$ JLab/W&M tadpole clover, $a = 0.114$ and 0.080 fm, $m_\pi = 200$, 315 MeV, AMA. Consistency between clover+HISQ and clover-clover results.

$G_E^v(Q^2)$ and $G_M^v(Q^2)$



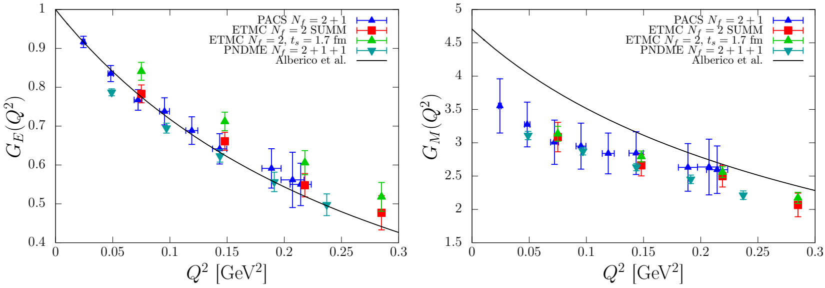
Mainz: [Harris,Thu,16:50] $N_f = 2+1$, NP clover CLS, $m_\pi = 200 - 350$ MeV, $Lm_\pi \gtrsim 4$, $a = 0.084, 0.06$ fm, $8000 - 30000$ measurements (AMA), $t_f - t_i = 0.7 - 1.3$ fm.

$G_E^v(Q^2)$ and $G_M^v(Q^2)$



ETMC: [Koutsou,Thu,17:10] $m_\pi = 131$ MeV, $a = 0.093$ fm, $Lm_\pi = 3$, $L = 4.5$ fm
$N_f = 2$ twisted mass with clover term, $t_f - t_i = 1.1 - 1.7$ fm, 6816-69784 measurements.
PNDME: [Jang,Thu,15:00] $m_\pi = 138$ MeV, $a = 0.09$ fm, $Lm_\pi = 3.9$, $L = 5.6$ fm

$G_E^v(Q^2)$ and $G_M^v(Q^2)$



PACS: [Kuramashi,Thu,16:30], $m_\pi = 145$ MeV, $a = 0.085$ fm, $Lm_\pi = 6$, $L = 8.1$ fm
$N_f = 2+1$ NP clover, stout smeared links, 146 configs $\times$ 64 measurements, $t_f - t_i$=1.3 fm

ETMC: [Koutsou,Thu,17:10] $m_\pi = 131$ MeV, $a = 0.093$ fm, $Lm_\pi = 3$, $L = 4.5$ fm
$N_f = 2$ twisted mass with clover term, $t_f - t_i = 1.1 - 1.7$ fm, 6816-69784 measurements.

PNDME: [Jang,Thu,15:00] $m_\pi = 138$ MeV, $a = 0.09$ fm, $Lm_\pi = 3.9$, $L = 5.6$ fm

Direct extraction of radii and anomalous magnetic moment

Applying similar methods to those used in hadronic vacuum polarisation studies.

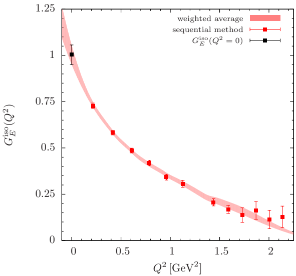
E.g. $C_i^{3pt}(t,\vec{q},\Gamma_k) = \Gamma_k \langle \mathcal{N}(\vec{p}_f,t_f) J_i^{em,u-d}(\vec{q},t) \overline{\mathcal{N}}(\vec{0},0)\rangle \propto \epsilon_{ijk} q_j G_M(Q^2)$

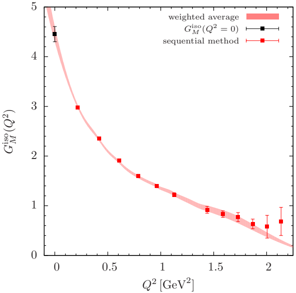
Extract $G_M(0)$ using $\lim_{q^2\to 0} \frac{\partial}{\partial q_j} C_i^{3pt}(t,\vec{q},\Gamma_k)$

ETMC: [Koutsou,Thu,17:10]:

Position space method: lattice version of $\int d^3x \, ix_j \, C_i^{3pt}(t,\vec{x},\Gamma_k)$

Tested with $C_i^{3pt}(t,\vec{q},\Gamma_i) \propto q_i G_E(Q^2)$

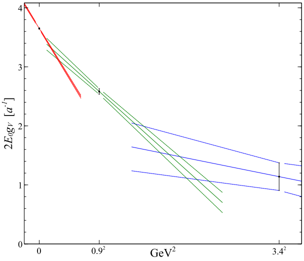




[Alexandrou,1605.07327]
$N_f = 2+1+1$,
$m_\pi = 373$ MeV,
$a = 0.08$ fm.
$G_M^v(0) = 4.45(17)(7)$,
previously 3.93(12).

Direct extraction of radii and anomalous magnetic moment

[Chang,Thu,14:40]
$N_f = 2+1$ clover with $a = 0.12$ fm, $m_\pi = 400$ MeV



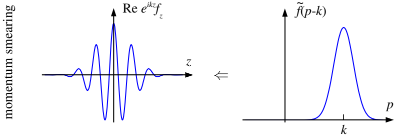
Also: LHPC [Hasan,Thu,15:20] using "Rome method" [de Divitiis,1208.5914]:

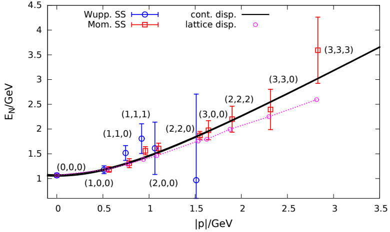
Expand correlation functions with respect to the spatial components of external momenta.

$$C[\vec{p}; U] = C^{(0)}[U] + p_k C_k^{(1)}[U] + \frac{p_h p_k}{2} C_{hk}^{(2)}[U] + \cdots \; .$$

Isovector form factors

Achieving large Q^2: momentum smearing RQCD [Bali,1602.05525]





Poster: B. Lang (RQCD) for pion. Talk: [Syritsyn,Mon,15:55]

Achieving large Q^2: Feynman-Hellmann approach CSSM/QCDSF/UKQCD [Chambers,Mon,17:05]

Also:

Variational method for form factors including parity projection for $\vec{p} \neq \vec{0}$ [Stokes,Phys.Rev.D92,114506] and [Stokes,Fri,15:40] $\rightarrow$ extract EM form factors for first three states of $P = \pm$ nucleon at $m_\pi = 515$ MeV.

Nucleon axial form factor $G_A(Q^2)$

Previously, [Lin,0802.0863], [Yamazaki,0904.2039], [Bratt,1001.3620], [Bali,1412.7336]

Needed for neutrino oscillation experiments:

Charged current quasielastic (CCQE) neutrino-nucleus interaction must be known to high precision.

Connecting quark - nucleon level: $G_A(Q^2)$ form factor.
nucleon - nucleus level: nuclear model.

Traditionally: information on $G_A(Q^2)$ extracted from expt. using dipole fit:

$$G_A(Q^2) = \frac{g_A}{(1 + \frac{Q^2}{{M_A}^2})^2} \qquad \langle r_A^2 \rangle = \frac{12}{{M_A}^2}$$

World average (pre 1990) from ν scattering $M_A = 1.026(21)$ GeV.

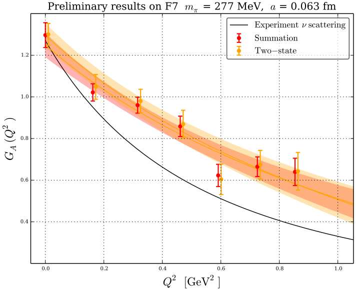
Overconstrained form: different measurements, different M_A.

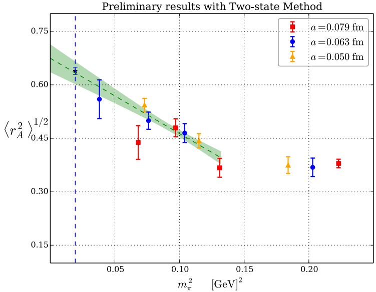
Lower energy expts: e.g. MiniBooNE: $M_A = 1.35(17)$ GeV
[Aguilar-Arevalo,1002.2680]

Systematics being explored including new analysis of old expt data:
$\langle r_A^2 \rangle = 0.46(22)$ fm^2 $\rightarrow$ $M_A = 1.01(24)$ GeV from z-expansion [Meyer,1603.03048].

Nucleon axial form factor $G_A(Q^2)$

Mainz: [von Hippel,Wed,9:40], SM params & renorm.



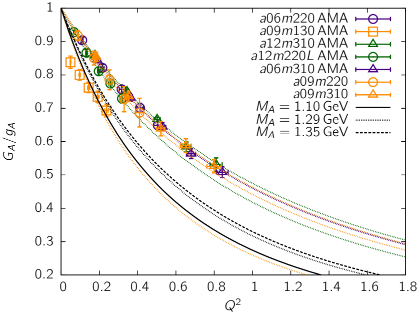


$N_f = 2$ CLS NP clover, $m_\pi = 195 - 450$ MeV, $Lm_\pi \gtrsim 4$, $t_f - t_i \sim 0.6 - 1.4$ fm

$\langle r_A^2 \rangle$ extracted using z-expansion.

Expt: dipole form with $M_A = 1.02$.

Nucleon axial form factor $G_A(Q^2)$



[Jang,Thu,15:00]

Small dependence of $\langle r_A^2 \rangle$ on lattice spacing (dipole fit).

PNDME: $N_f = 2 + 1 + 1$ MILC HISQ, $a = 0.06 - 0.12$ fm Valence clover (tree-level, tadpole improved) $m_\pi = 135 - 315$ MeV

NME: $N_f = 2 + 1$ JLab/W&M tadpole clover, $a = 0.114$ and 0.080 fm, $m_\pi = 200$, 315 MeV. Consistency between clover+HISQ and clover-clover results.

Nucleon axial form factor $G_A(Q^2)$

Also:

RBC/UKQCD: [Ohta,Mon,17:25] $N_f = 2 + 1$ domain wall

PACS: [Kuramashi,Thu,16:30] $N_f = 2 + 1$ NP clover, stout smeared links

FERMILAB: [Meyer,Thu,18:10] $N_f = 2 + 1 + 1$ HISQ

ETMC: [Koutsou,Thu,17:10] $N_f = 2$ twisted mass

Nucleon strangeness electric and magnetic form factors

Expt: extracted from parity violating $e - N$ scattering, interference between γ and Z exchange.

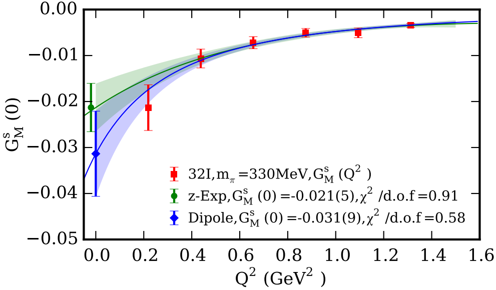
Last year: LHPC [Green,1505.01803], $N_f = 2 + 1$ BMW-c clover ensembles, clover valence fermions, $m_\pi = 317$ MeV, $a = 0.11$ fm.

This year: χQCD [Sufian,1606.07075], $N_f = 2 + 1$, RBC/UKQCD domain wall ensembles,overlap valence fermions, $a = 0.08, 0.11$ fm, $m_\pi = 139$, 300 and 330 MeV + non-unitary values.

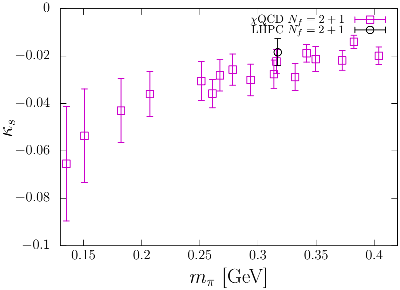
Shown $m_\pi = 330$ MeV

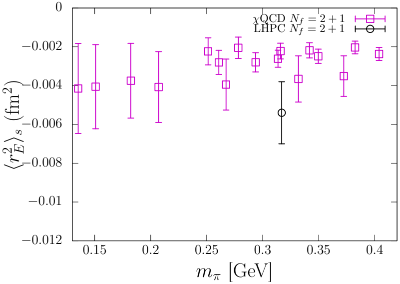
[Liu,Mon,17:45]

Expt: e.g. HAPPEX [1107.0913]
$G_M^s(0.62\, GeV^2) = -0.070(67)$



Nucleon strangeness electric and magnetic form factors





LHPC [Green,1505.01803]

χQCD [Sufian,1606.07075]

$\langle r_E^2 \rangle_s = -0.0067(10)(17)(15)\text{fm}^2$

$\langle r_M^2 \rangle_s = -0.018(6)(5)(5)\text{fm}^2$

$\mu_s = -0.022(4)(4)(6)\mu_N$

$\langle r_E^2 \rangle_s = -0.0046(21)(02)(09)(04)\,\text{fm}^2$

$\mu_s = -0.073(17)(04)(06)(04)\,\mu_N$

Isoscalar axial form factor

Renormalisation: non-singlet currents: $Z^{ns} = Z^{ns}(a)$.

Singlet currents: (cont.) $U_A(1)$ anomaly means $Z^s = Z^s(\mu, a)$.

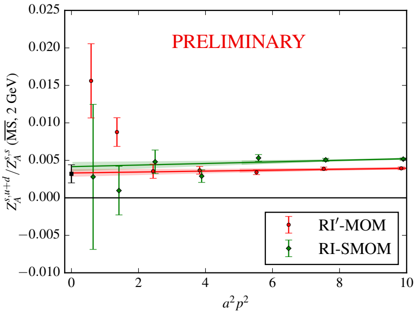
Deviation $z(\mu, a) = Z^s(\mu, a) - Z^{ns}(a)$ is $O(\alpha^2)$ in pert. theory.

Leads to mixing matrix (no $\mathcal{O}(a)$):

$$\begin{pmatrix} \Delta u(\mu) + \Delta d(\mu) \\ \Delta s(\mu) \end{pmatrix} = \begin{pmatrix} Z^{ns} + \frac{1}{3}z & \frac{1}{3}z \\ \frac{1}{3}z & Z^{ns} + \frac{1}{3}z \end{pmatrix} \begin{pmatrix} \Delta u(a) + \Delta d(a) \\ \Delta s(a) \end{pmatrix}$$

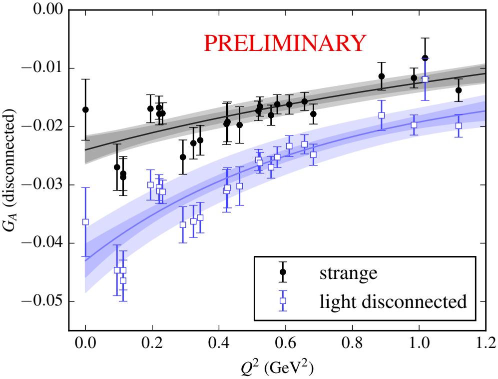
LHPC: non-perturbative determination via Rome-Southampton method. [Green,Thu,15:40]
$m_\pi = 317$ MeV, $a = 0.11$ fm.

Also RQCD $N_f = 2$
[Piemonte,Wed,9:20],
SM params & renorm.



Isoscalar axial and induced pseudoscalar form factors

LHPC: $N_f = 2 + 1$ BMW-c clover ensembles, clover valence fermions, $m_\pi = 317$ MeV, $a = 0.11$ fm.



- Data points: statistical errors only. Fits using z-expansion, stat.+sys. error shown.
- G_A^s is small.

Pion electromagnetic form factors

Expt: JLab Hall C Fpi12, charged pion form factor to 6 GeV^2.

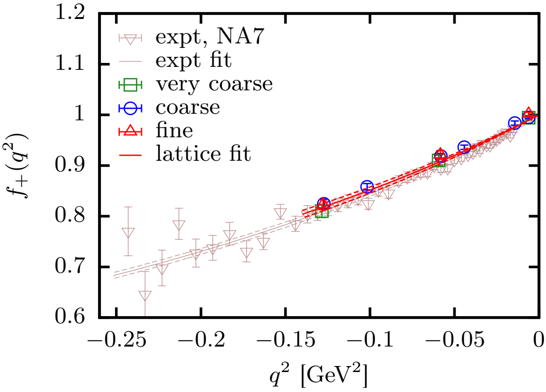
HPQCD [Koponen,1511.07382]: $N_f = 2+1+1$ MILC HISQ, $a = 0.088 - 0.15$ fm, $m_\pi \sim 133$ MeV, $Lm_\pi = 3.3 - 3.9$, $t_f - t_i = 1.4 - 2.3$ fm.

Twisted b.c.: q^2 down to $-0.006\ \text{GeV}^2$ $\langle r_E^2 \rangle = 0.403(18)(6)\ \text{fm}^2$

Fit:

$$f_+(q^2) = \frac{1}{(1 - \langle r^2 \rangle q^2/6)}$$

modified for disc. effects
+ mistuning m_q^{sea},
$m_\pi \neq m_\pi^{phys}$.

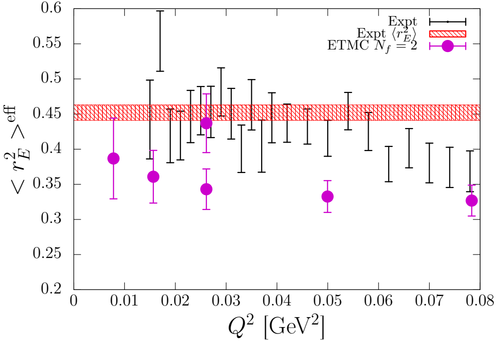


Pion electromagnetic form factors

ETMC: [Kostrzewa,Fri,15:00] $N_f = 2$ clover twisted mass, $m_\pi \sim 131$ MeV, $a = 0.093$ fm, $Lm_\pi = 3$ and 4.

Twisted b.c.: Q^2 down to 0.0078 GeV^2.

$$\langle r_E^2\rangle^{\rm eff} = 6\frac{(1 - f_+(Q^2))}{Q^2}$$



Fits to $f_+(Q^2)$: $1 - \langle r^2\rangle/6Q^2 + cQ^4$, $c \approx 0$

L=48 $\langle r_E^2\rangle = 0.328(29)$ fm^2

L=64 $\langle r_E^2\rangle = 0.437(42)$ fm^2

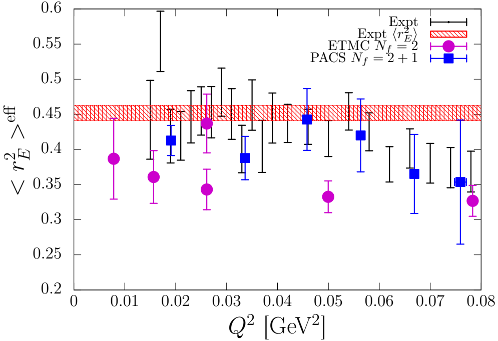
Pion electromagnetic form factors

PACS: [Kakazu,Fri,14:40] $N_f = 2+1$ NP clover, stout smeared links, $m_\pi = 145$ MeV, $a = 0.085$ fm, 40 configs $\times$ 32 measurements, $t_f - t_i$=3 fm, $Lm_\pi = 6$.

Large volume: $L \sim 8.1$ fm

$Q^2_{min} \sim 0.019$ GeV^2

$$\langle r_E^2 \rangle^{\rm eff} = 6\frac{(1 - f_+(Q^2))}{Q^2}$$

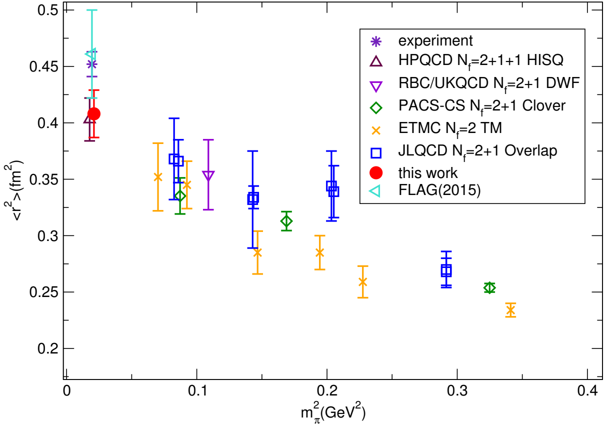


Fit NLO SU(2) ChPT expression for $f_+(Q^2) \to$ l.e.c. $l_6 = -0.01234(72)$ (consistent with FLAG 2015)

Adjust to m_π^{phys}, $\langle r_E^2 \rangle = 0.412(21) - 0.415(20)$ fm^2.

Pion electromagnetic form factors

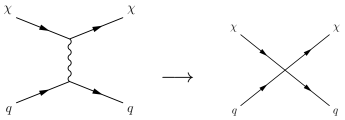
PACS: [Kakazu,Fri,14:40]



Also Mainz [Brandt,1306.2916] $\langle r_E^2 \rangle = 0.481(33)(13)\ \mathrm{fm}^2$ in chiral limit.

Sigma terms: $\sigma_q = m_q(\langle N|\bar{q}q|N\rangle - \langle 0|\bar{q}q|0\rangle)$

Scattering of DM candidates off nuclei (near zero recoil).



Spin-independent effective interaction $\sim \bar{\chi}\chi\bar{q}q$

$$\sigma_N^{SI} \propto |f_N|^2$$

$N = p, n.$

$$\frac{f_N}{m_N} = \sum_{q\in\{u,d,s,c,b,t\}} f_{T_q}^N \frac{\alpha_q}{m_q}, \qquad f_{T_q}^N = \frac{\sigma_q^N}{m_N}$$

Higgs example: $\alpha_q \propto m_q/m_W$

For heavy flavours $\langle \bar{h}h\rangle_N \propto 1/m_h$, $m_h \to \infty$.

If $\alpha_q \propto m_q$, t, b and c matter.

If α_q insensitive to m_q, only u, d, (s) play a role.

Sigma terms: experiment

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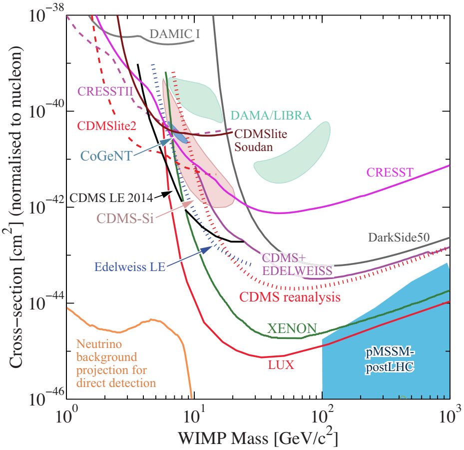
News › Science

Huge dark matter experiment finds nothing, despite burying giant machine in disused gold mine

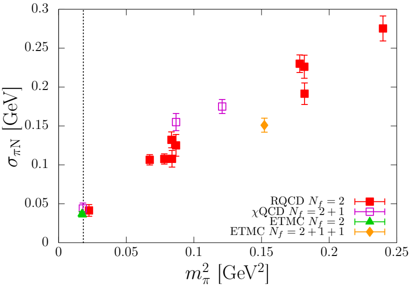
21st July 2016, referring to the Lux expt.

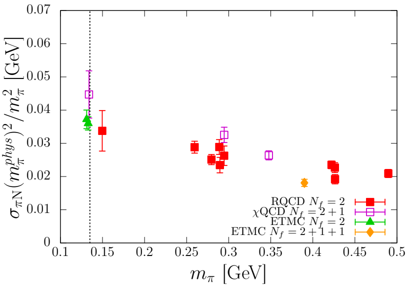
PDG 2015



Direct determinations of $\sigma_{\pi N} = \sigma_u + \sigma_d$

RQCD [Bali,1603.00827], ETMC [Abdel-Rehim,1601.01624], ETMC [Dinter,1202.1480], χQCD [Yang,1511.09089]

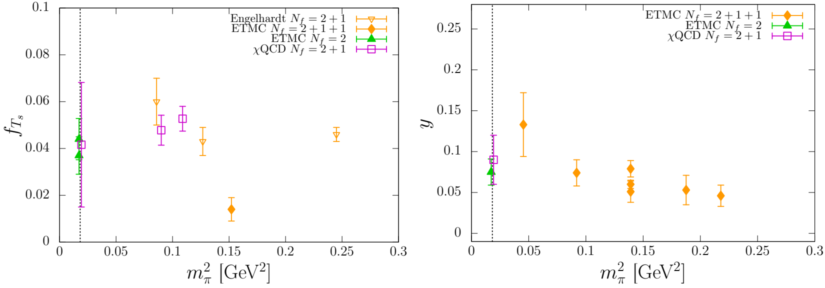




Note: $ac_{FF}\langle FF\rangle$ neglected $\rightarrow \mathcal{O}(a)$ errors for all actions.

ETMC: [Alexandrou,Mon,15:15], [Vaquero,Thu,17:30]: update with increased statistics, $\sigma_{\pi N} = 36(2)$ MeV - statistical errors only.

Strangeness: $f_{T_s} = \frac{\sigma_s}{m_N}$ and $y = \frac{2\langle \bar{s}s \rangle}{\langle \bar{u}u + \bar{d}d \rangle}$



ETMC: [Alexandrou,Mon,15:15], [Vaquero,Thu,17:30]: update with increased statistics, $\sigma_s = 37(8)$ MeV $\to f_{T_s} = 0.039(9)$ - statistical errors only.

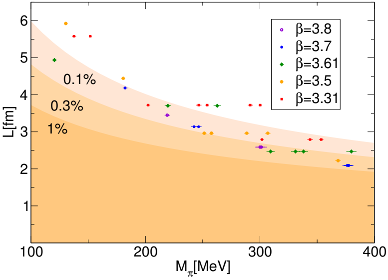
Also: JLQCD [Ohki,1208.4185] $N_f = 2+1$ overlap, $a = 0.11$ fm, $m_\pi = 300 - 540$ MeV, $f_{T_s} = 0.009(22)$.
RQCD [Bali,1603.00827] $N_f = 2$, RQCD/QCDSF NP clover, $a = 0.06 - 0.08$ fm, $f_{T_s} = 0.037(13)$
Hybrid method: [Freeman and Toussaint,1204.3866] $N_f = 2+1$ MILC Asqtad $f_{T_s} = 0.061(9)$, $N_f = 2+1+1$ MILC HISQ, $a = 0.06 - 0.15$ fm, $f_{T_s} = 0.044(1)$

Nucleon sigma terms from Feynman-Hellmann theorem

$$\sigma_{\pi N} = \sigma_u^N + \sigma_d^N = m_u \frac{\partial m_N}{\partial m_u} + m_d \frac{\partial m_N}{\partial m_d} \approx m_\pi^2 \frac{\partial m_N}{\partial m_\pi^2}, \qquad \sigma_s^N = m_s \frac{\partial m_N}{\partial m_s} \approx m_{\bar{s}s}^2 \frac{\partial m_N}{\partial m_{\bar{s}s}^2}$$

BMW-c [Dürr,1510.08013]:
$N_f = 2+1$, 2HEX-Clover,
$a = 0.054 - 0.116$ fm,
$m_\pi \geq 120$ MeV.

Figure: [Dürr,1011.2711]

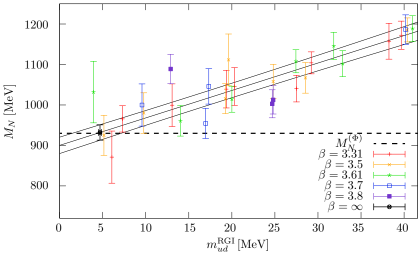


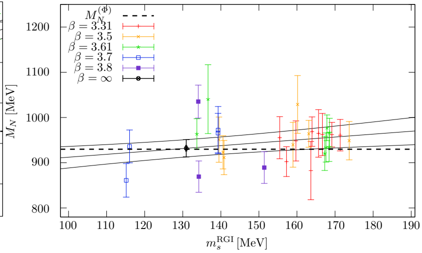
Nucleon sigma terms from Feynman-Hellmann theorem

BMW-c [Dürr,1510.08013]

$$M_N = M_N^{(\Phi)} \left(1 + g_N^{\mathrm{a}}(a)\right) \left(1 + g_N^{\mathrm{FV}}(M_\pi, L)\right) \left(1 + c_N^{\mathrm{a},ud}(a)\tilde{m}_{ud} + c_N^{\mathrm{a},s}(a)\tilde{m}_s + \ldots\right)$$

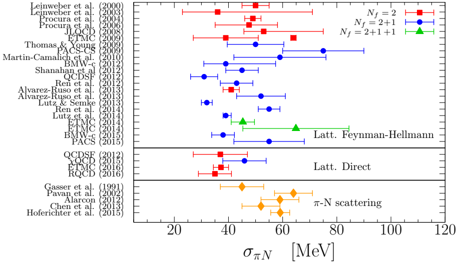
$$\boxed{\sigma_{\pi N} = 38(3)(3) \text{ MeV} \quad \sigma_s = 105(41)(37) \text{ MeV}}$$

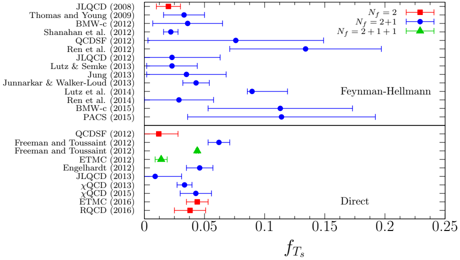




$m_{ud}^{RGI} = 40$ MeV $\rightarrow m_\pi \approx 400$ MeV. Update: [Lellouch,Thu,14:20] $N_f = 1+1+1+1$,
Also: PACS [Ishikawa,1511.09222]

Summary of sigma terms and f_{T_q}





Values from phenomenological analyses of π-N scattering tend to be higher.

[Hoferichter,1506.04142] $\sigma_{\pi N} = 59.1(3.5)$ MeV.

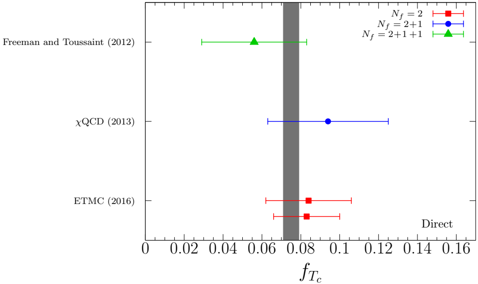
[Hoferichter,1602.07688] request for a lattice computation of πN scattering lengths.

Also interesting: $\sigma_{\pi N}(Q^2 = -2m_\pi^2) - \sigma_{\pi N}(0) \rightarrow$ scalar radius from form factor.

Heavy quark sigma terms

HQET relates $\sigma^N_{h=c,b,t}$ to $(2/27)(m_N - \sum_{q=u,d,s} \sigma^H_q)$ at leading order in $1/m_h$ and α [Shifman,1978].

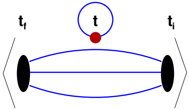
Taking:
$\sigma_{\pi N} = 35(6)$ MeV, $\sigma_s = 35(12)$ MeV
gives
$f_{T_c} = 0.075(4)$,
$f_{T_b} = 0.072(2)$,
$f_{T_t} = 0.070(1)$,
using N^{3}LO:[Chetyrkin,hep-ph/9708255]
+ others



ETMC: [Alexandrou,Mon,15:15], [Vaquero,Thu,17:30]: update with increased statistics, $\sigma_c = 87(17)$ MeV - statisical errors only.

Quark and gluon momentum fraction

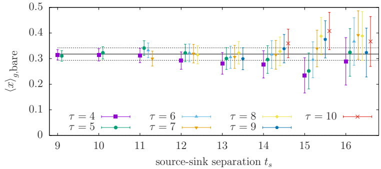
First moment of q/g parton distribution function: $\langle x \rangle_{q/g} = \int dx\, x\, F_{q/g}(x)$.



Connected insertion: u, d.
Disconnected insertion: u, d, s, g

ETMC: [Vaquero,Thu,17:30] $N_f = 2$ twisted mass with clover term,

$m_\pi = 131$ MeV, $Lm_\pi = 3$,
$a = 0.093$ fm.
Stout smearing to reduce noise.
Approx: 2000 (cfgs) $\times$ 100 (sources)



Renormalisation: mixing between $\sum_q \langle x \rangle_q$ and $\langle x \rangle_g$: 1-loop to $\overline{\text{MS}}$ at 2 GeV.

$\langle x \rangle_g^{bare} = 0.318(24) \to \langle x \rangle_g^{\overline{\text{MS}}} = 0.320(24)$, $\quad (\langle x \rangle_u + \langle x \rangle_d + \langle x \rangle_s)^{\overline{\text{MS}}} = 0.72(11)$

Momentum sum satisfied: $\sum_q \langle x \rangle_q + \langle x \rangle_g = 1.04(11)$

Consistent with χQCD quenched calculation [Deka,1312.4816].

Also computed: $g_A^{u,d}$, $g_T^{u,d}$, $g_S^{u,d}$.

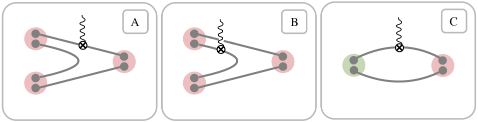
P wave $\pi\pi \to \pi\gamma$

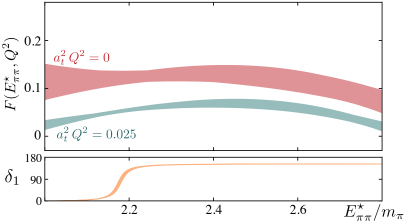
Extracting $\rho \to \pi\gamma$ form factor first step towards e.g. $N\gamma^* \to \Delta \to N\pi$

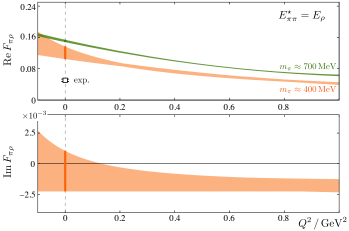
HSC: using formalism developed in [Briceño,1406.5965] for $1 \to 2$ particle transition amplitudes.

Infinite volume amplitude $A_{\pi\pi,\pi,\gamma^*}(E^*_{\pi\pi}, Q^2)$ related to finite volume $\tilde{A}(E^*_{\pi\pi}, Q^2; L)$

Anisotropic $N_f = 2{+}1$ clover fermions $a_s/a_t \approx 3.5$, $a_s \approx 0.12$ fm, $m_\pi \approx$ 400 MeV..







See also talk by David Wilson and for another study [Leskovec,Fri,17:50].

Summary

Benchmark quantities:

- First calculations with main systematics considered (continuum limit, finite V, physical point results, excited states, ...). More in progress.
- The same effort needs to be applied to $\langle x \rangle_{u-d}$.

Overall agreement on charges: g_S, g_T.
Efficient methods for achieving statistical precision + m_π^{phys} simulations.
Impact on:

- Form factor determinations, $f_+^\pi(Q^2)$, $G_{E,M}(Q^2)$, $G_A(Q^2)$.
- $\sigma_{\pi N}$.
- Systematics needs to be explored.

Progress in disconnected techniques:

- Enable calculations of strangeness form factors.
- Estimates of disconnected contributions also at the physical point $\rightarrow G_M^s(Q^2)$, $\langle x \rangle_q$, $g_{A,S,T}^q$.

Thanks for results/correspondence with:

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R. Briceño	B. Kostrzewa	T. Yamazaki
G. von Hippel	H. Perlt	C. Urbach

Apologies to those not included!