D meson semilptonic decays in lattice QCD w/ Moebius domain-wall quarks

Takashi Kaneko

Lattice 2016, July 29, 2016

introduction

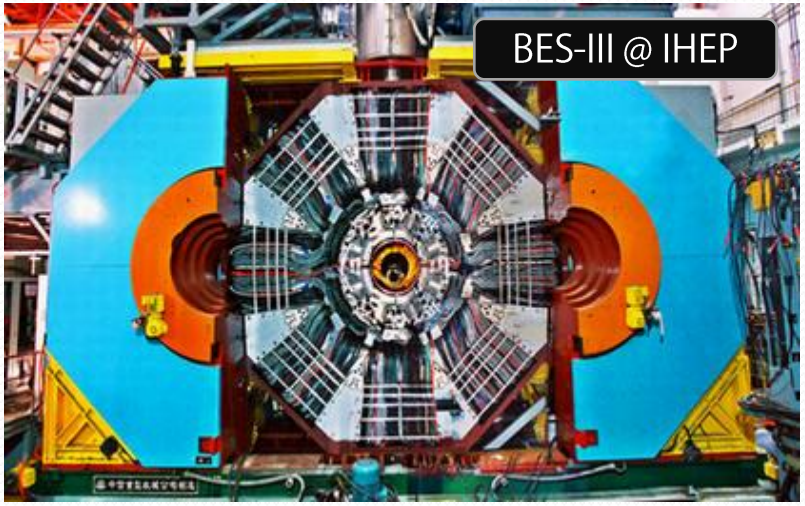
$D \to \pi l\nu, Kl\nu$ semileptonic decays and lattice QCD

- being measured at c/B factories

  $\delta\left(f_+^{D\pi(K)}(0)|V_{cd(s)}|\right) \sim 1\%$ (HFAG'14)

- determination of $|V_{cd}|$, $|V_{cs}|$

  $\Rightarrow$ over-constrain $V_{CKM} \Leftrightarrow D \to l\nu$ (2%)



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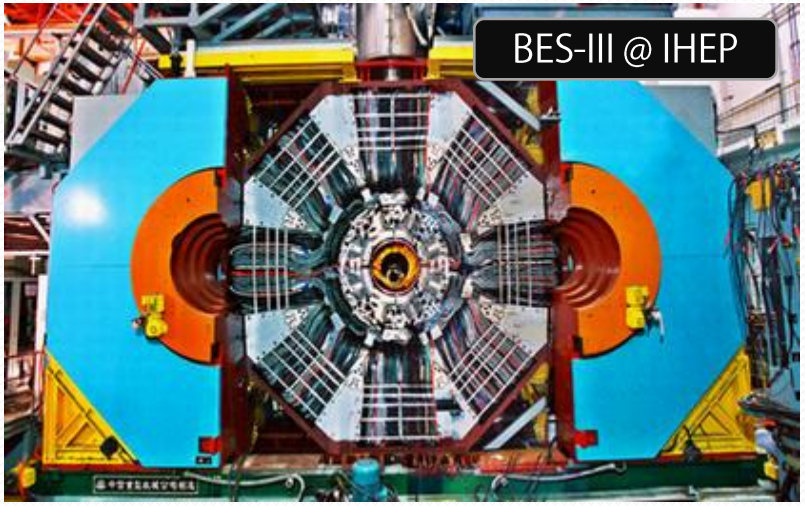

- determination of $|V_{cd}|$, $|V_{cs}|$

  $\Rightarrow$ over-constrain $V_{CKM} \Leftrightarrow D \to l\nu$ (2%)

- lattice QCD $\Rightarrow$ normalization of form factors (FFs) $f_+(0) = f_0(0)$

$$\left\langle \pi(p')\left|V_\mu\right|D(p)\right\rangle = (p+p')_\mu f_+^{D\pi}(q^2) + (p-p')_\mu f_-^{D\pi}(q^2)$$

$$f_0^{D\pi}(q^2) = f_+^{D\pi}(q^2) + \frac{q^2}{M_D^2 - M_\pi^2} f_-^{D\pi}(q^2) \quad \left(q^2 = (p-p')^2\right)$$

- independent calc. s are welcome to establish the LQCD estimate

this talk

JLQCD's calculation of $D \to \pi, K$ semileptonic FFs

- Möbius domain-wall quarks ⇒ $O(a^2)$ discretization error
- $a^{-1} = 2.5 - 4.5$ GeV

  ⇔ @ Lattice 2015

  - single cut-off $a^{-1} = 2.5$ GeV ⇒ 3 a^{-1}'s
  - improved measurement setup ⇒ better statistical accuracy
- measurements are on-going : status report + preliminary results

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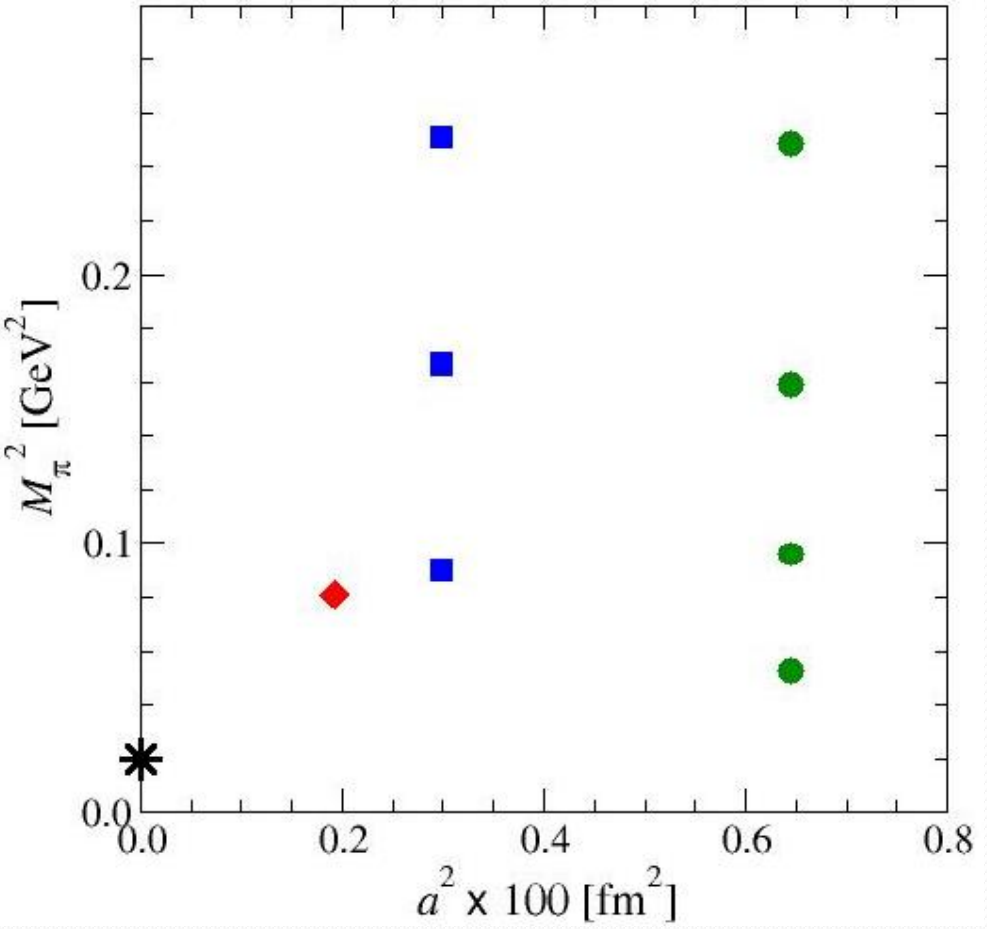
- outline
  - simulation setup : gauge ensembles, calculation of form factors
  - preliminary results for form factors : q^2, m_q, a dependences

gauge ensembles

- $N_f = 2+1$ QCD
- smeared Möbius DWF (Brower *et al.*, '05): $m_{\rm res} \lesssim O(0.1\text{-}1)$ MeV, low CPU cost
- $a^{-1} \simeq 2.5,\ 3.6,\ 4.5$ GeV
- $M_\pi \simeq 230, 300, 400, 500$ MeV



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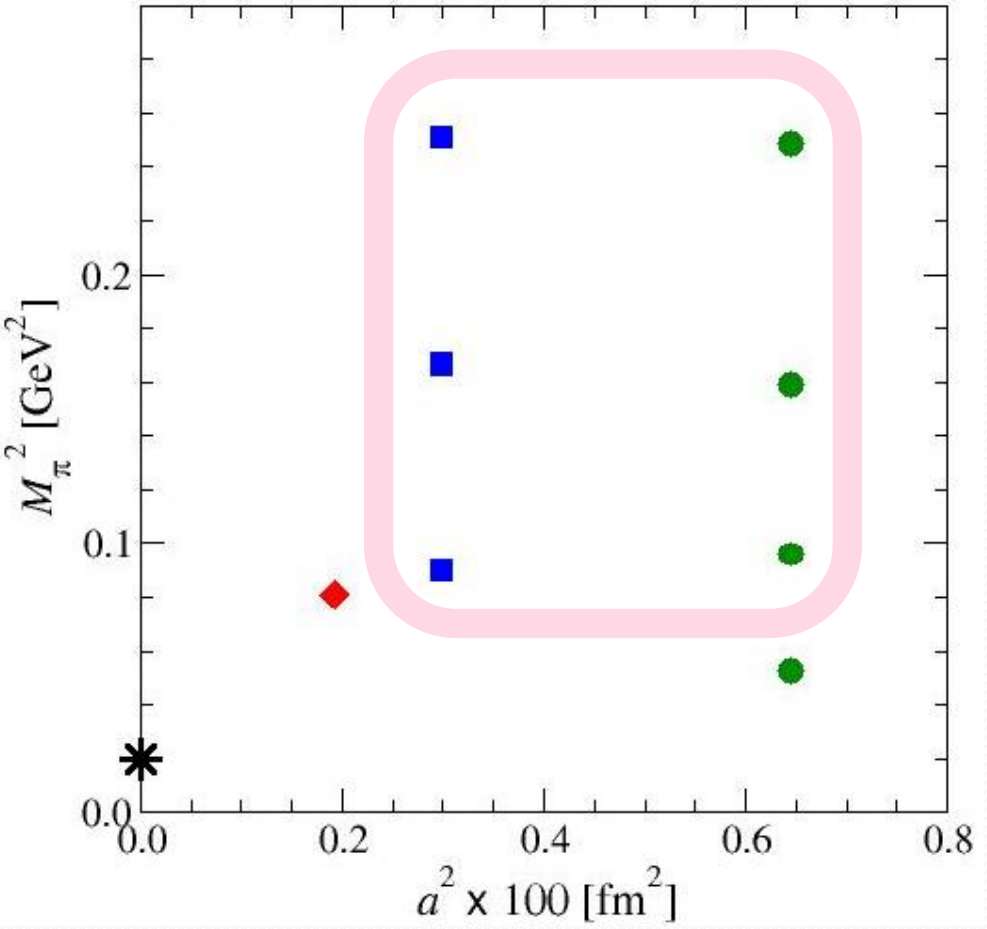

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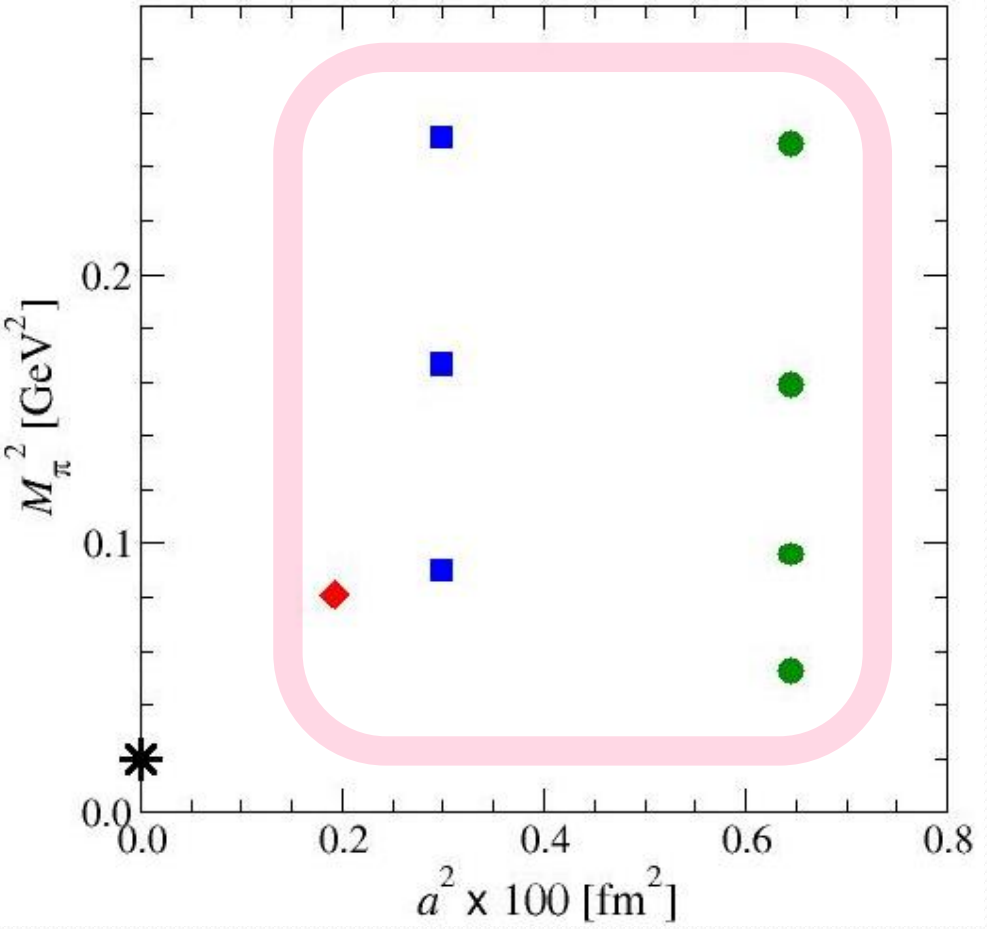
- $M_\pi L \gtrsim 4$
- 10,000 MD time at each point



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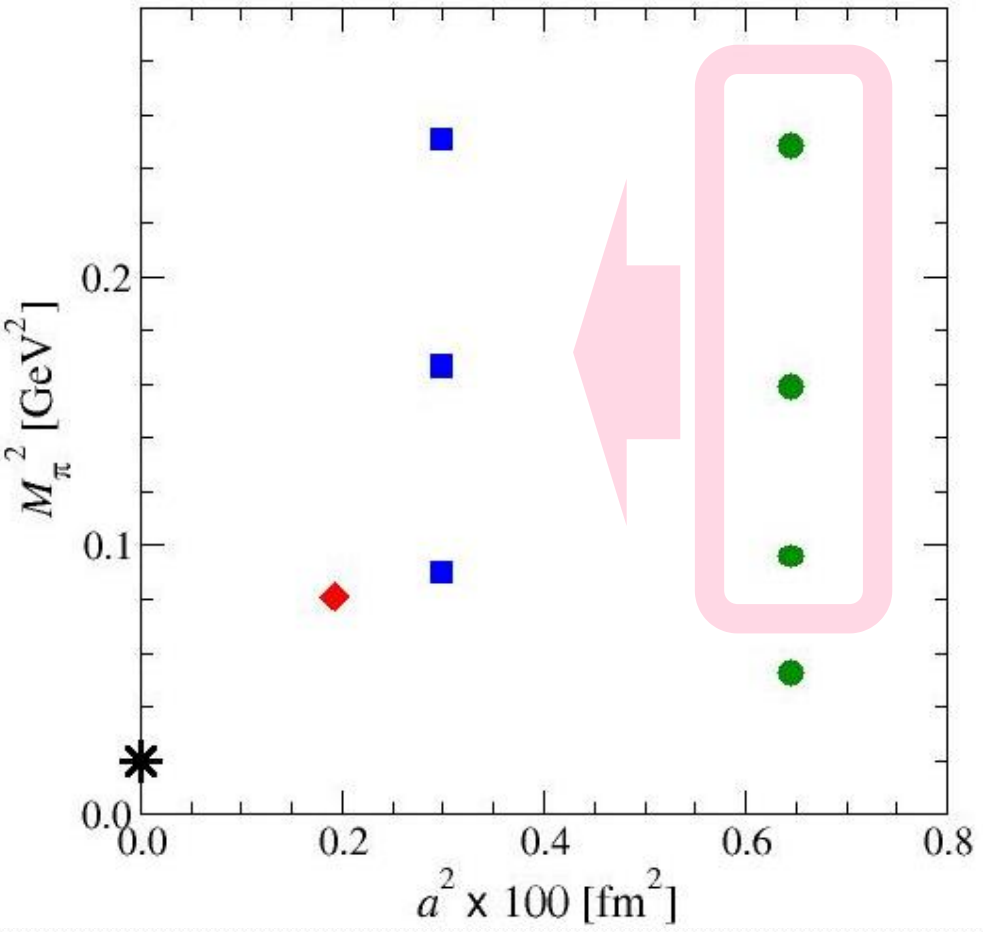


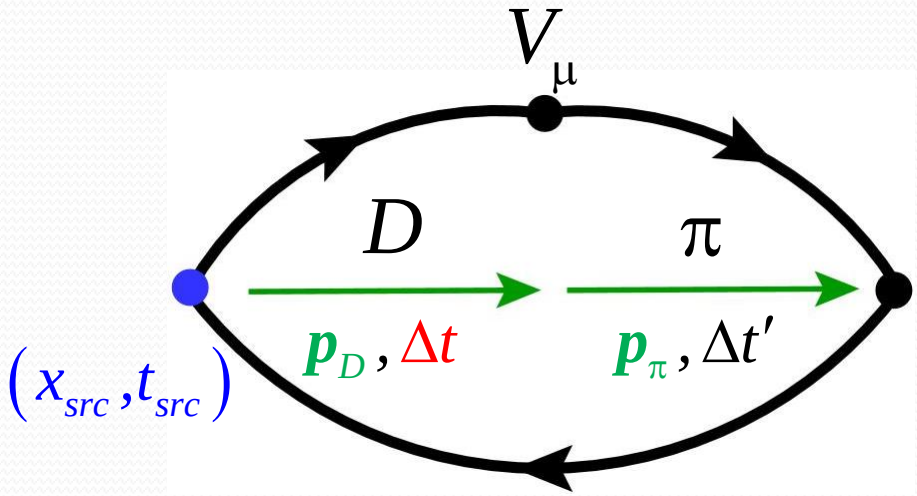
after Lattice 2015

- extending to finer lattices
- but, on-going : $\gtrsim$ 300 MeV, 1 m_s

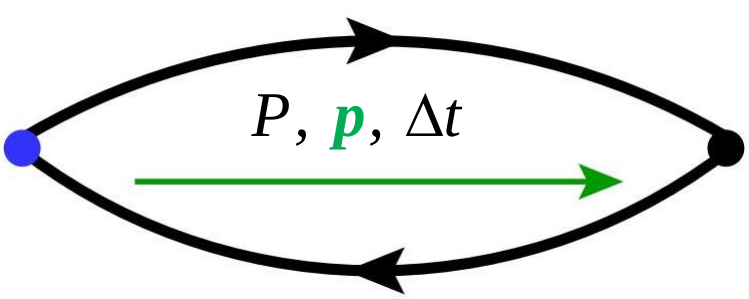
measurements

- @ $m_{c,\mathrm{phys}}$ fixed from M_D
- 3pt. : fix $\Delta t+\Delta t'$, change Δt

$$C_{V_\mu}^{D\pi}\left(p_D, p_\pi; t_{\mathrm{src}}, \Delta t, \Delta t'\right)$$



$$C^{P}\left(p; t_{\mathrm{src}}, \Delta t\right) \quad (P = \pi, D)$$



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- Gaussian smeared source & sink
- average over source points
  - x_{src} : volume src w/ Z_2 noise
  - 4 (2) t_{src} @ $M_\pi \lesssim$ (>) 300 MeV

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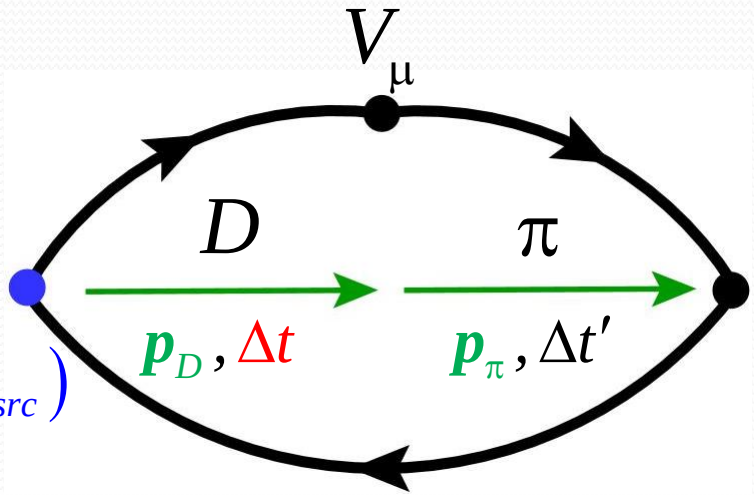

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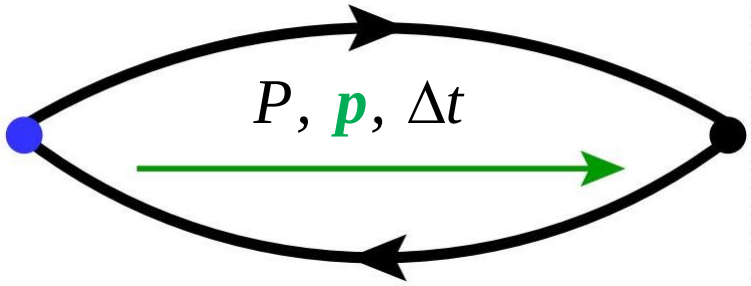

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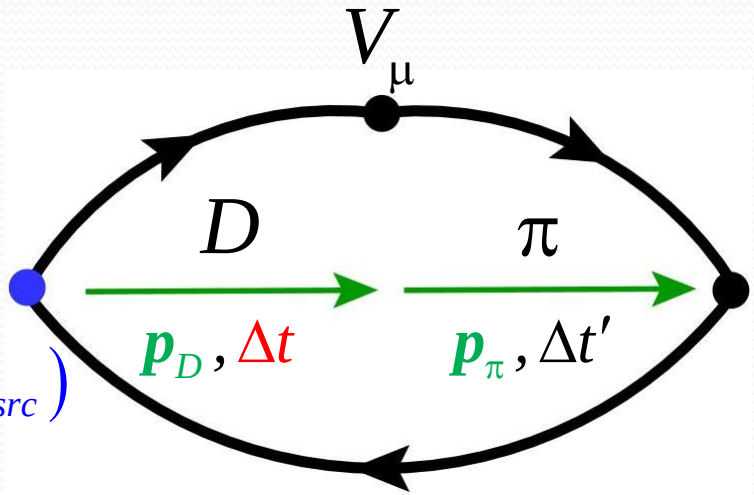
  - 4 (2) t_{src} @ $M_\pi \lesssim$ (>) 300 MeV
- D @ rest, $|\mathbf{p}|^2$=0,1,2,3 for π, K

  $\Rightarrow\ q^2_{\mathrm{min}} =$ 0.15 – 0.30 GeV^2

  $\Leftrightarrow\ q^2_{\mathrm{max}} =$ 1.8 - 2.2 GeV^2

$$C^{D\pi}_{V_\mu}\left(p_D, p_\pi; t_{\mathrm{src}}, \Delta t, \Delta t'\right)$$



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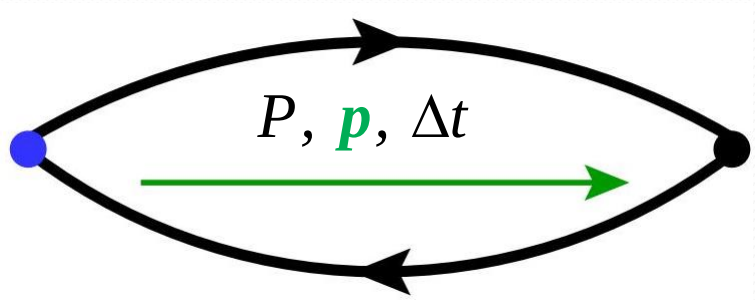

extract FFs

$$C_{V_\mu}^{D\pi}\left(p_D, p_\pi; \Delta t, \Delta t'\right) = A_{V_\mu}^{D\pi}\left(p_D, p_\pi\right) e^{-E_D \Delta t} e^{-E_\pi \Delta t'}$$

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$\langle \pi(p')|V_\mu|D(p)\rangle$

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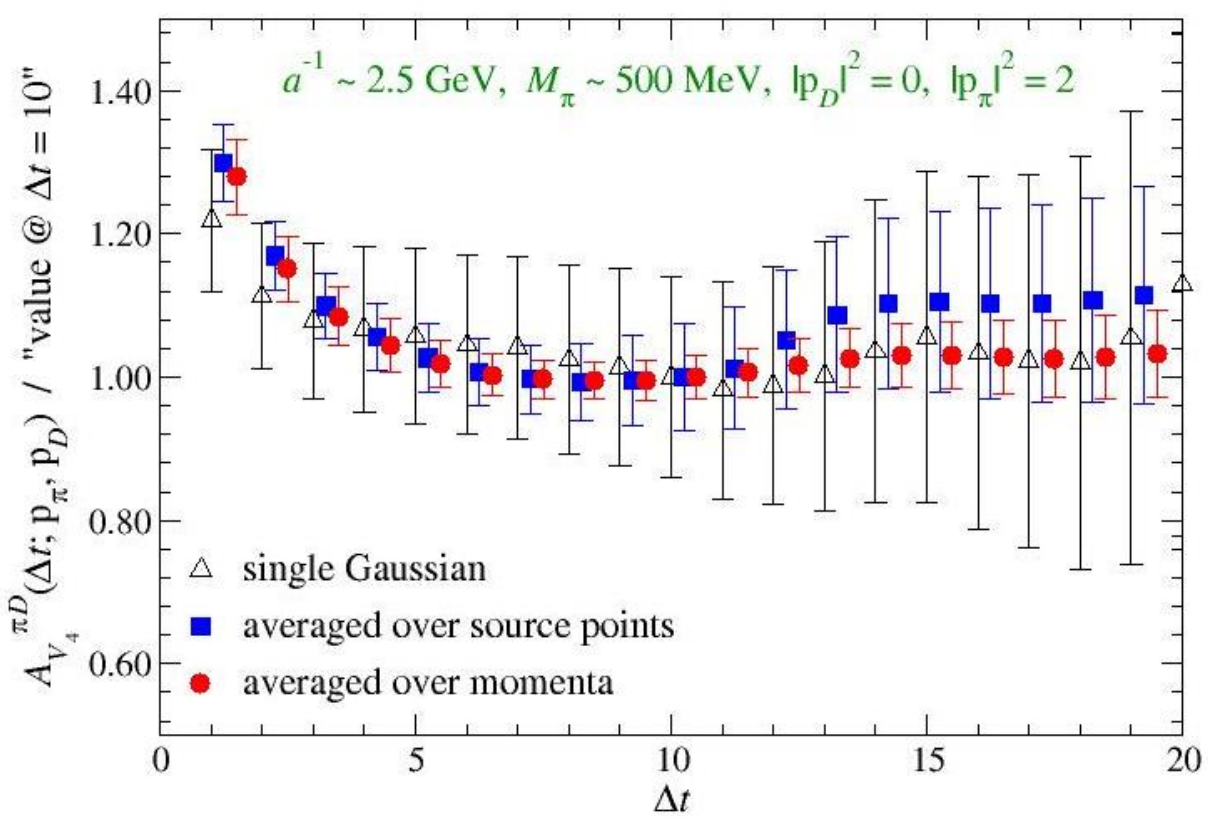
$$\langle \pi(p')|V_\mu|D(p)\rangle = Z_V \sqrt{\frac{E_D E_\pi A_{V_\mu}^{D\pi}(p_D, p_\pi)}{B^D(p_D) B^\pi(p_\pi)}} \quad \Rightarrow \quad f_{+,0}^{D\pi}(q^2)$$

- non-perturbative Z_V (JLQCD, 2016)
- $E_P \leftarrow M_P$ + dispersion relation

improvement of statistical accuracy

ex. amplitude A @ $a^{-1} \simeq 2.5$ GeV, $M_\pi \simeq 500$ MeV, $m_s \sim m_{s,\mathrm{phys}}$, $|\mathbf{p}_\pi|^2 = 2$



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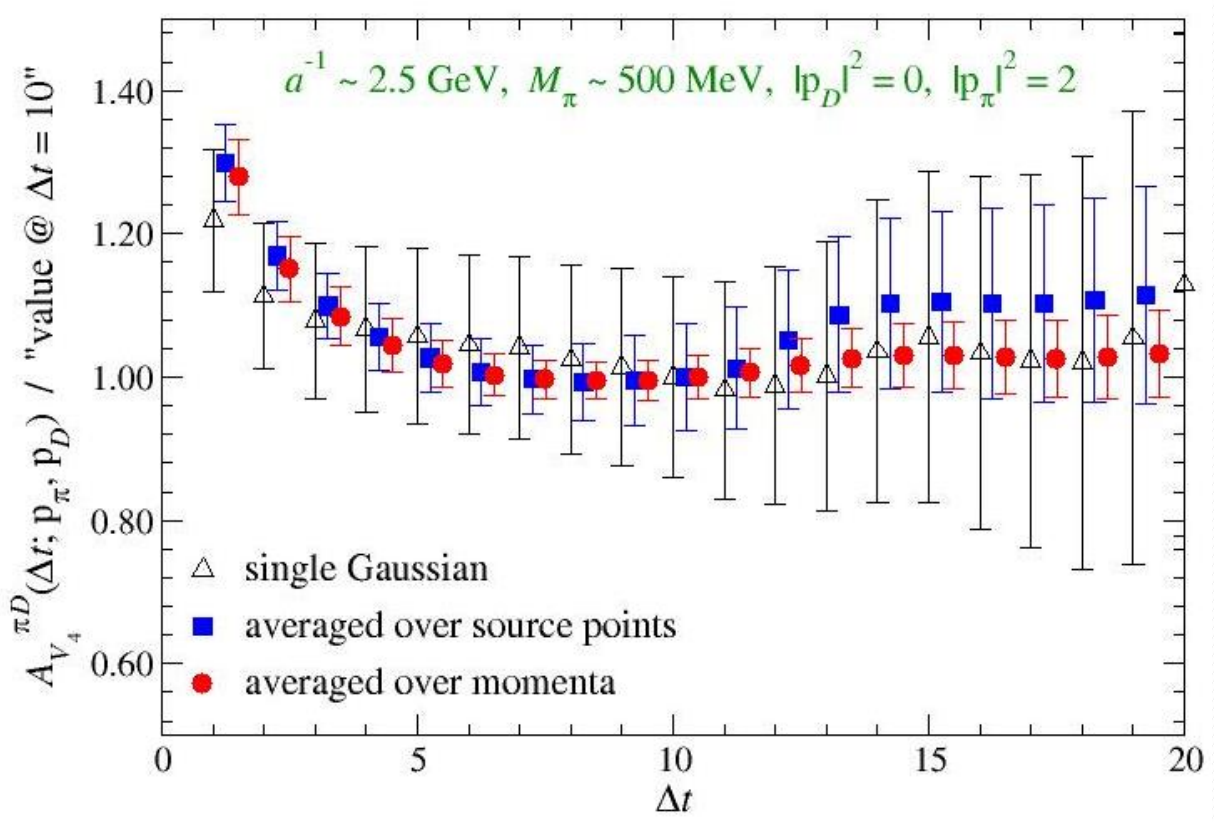


average over source pt.

- Z_2 noise : 30% ↓
- 2 x t_{src} 's : 30% ↓

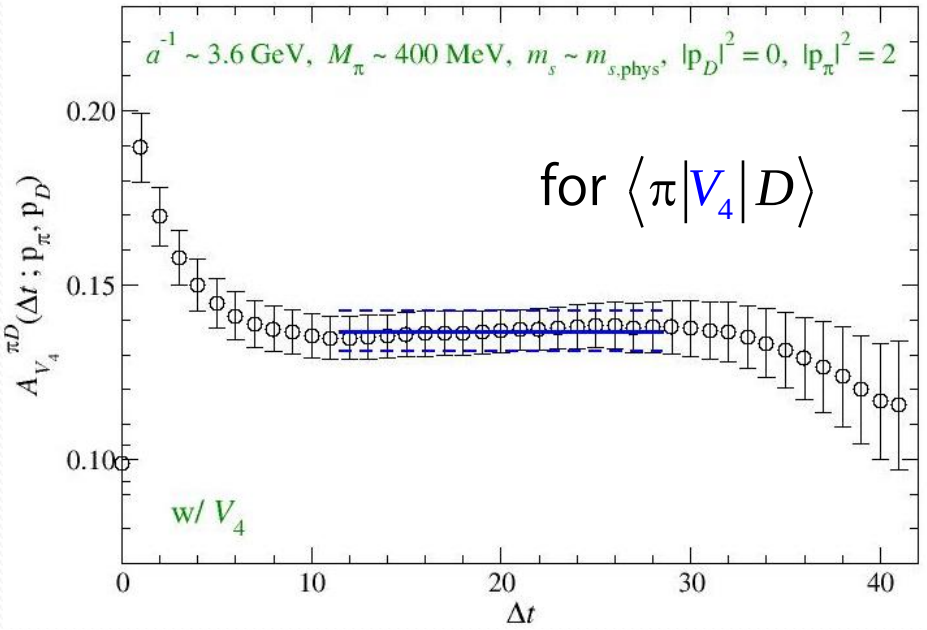
⇒ x 1/2 reduction

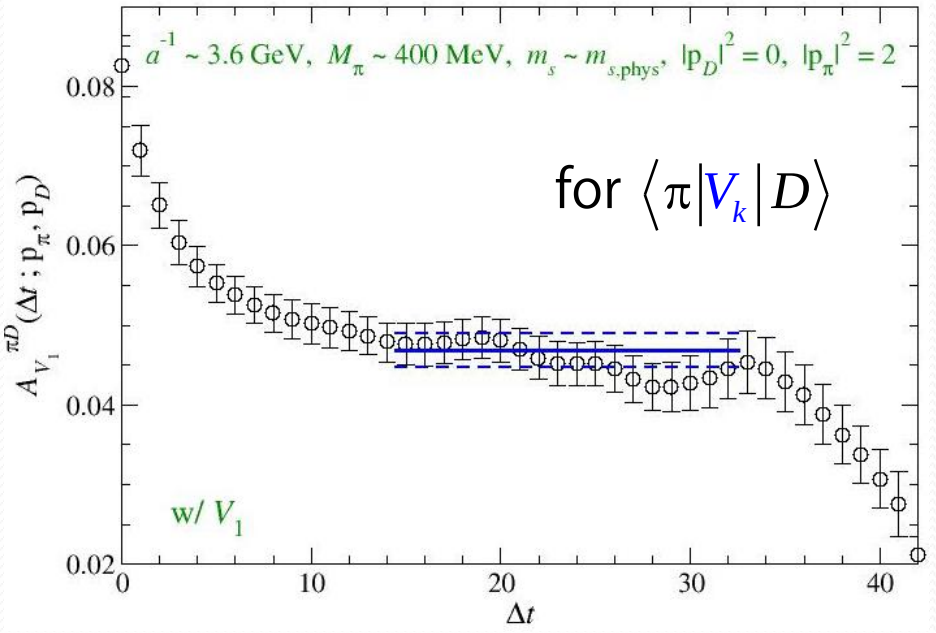
average over momenta

⇒ x 1/2 reduction

form factor at simulation points

ex. $a^{-1} \simeq 3.6$ GeV, $M_\pi \simeq 400$ MeV, $m_s \sim m_{s,\mathrm{phys}}$, $|\mathbf{p}_\pi|^2 = 2$





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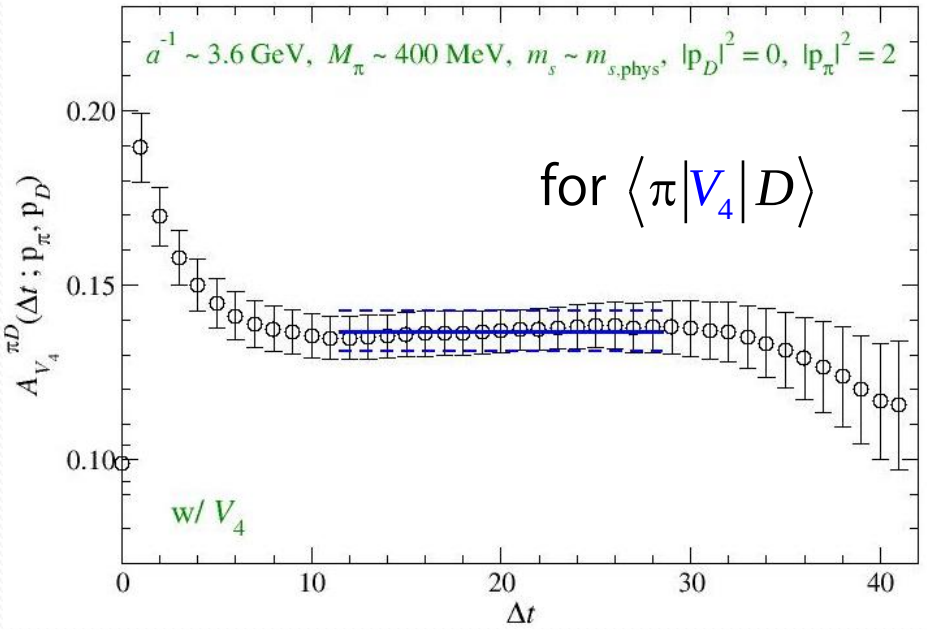
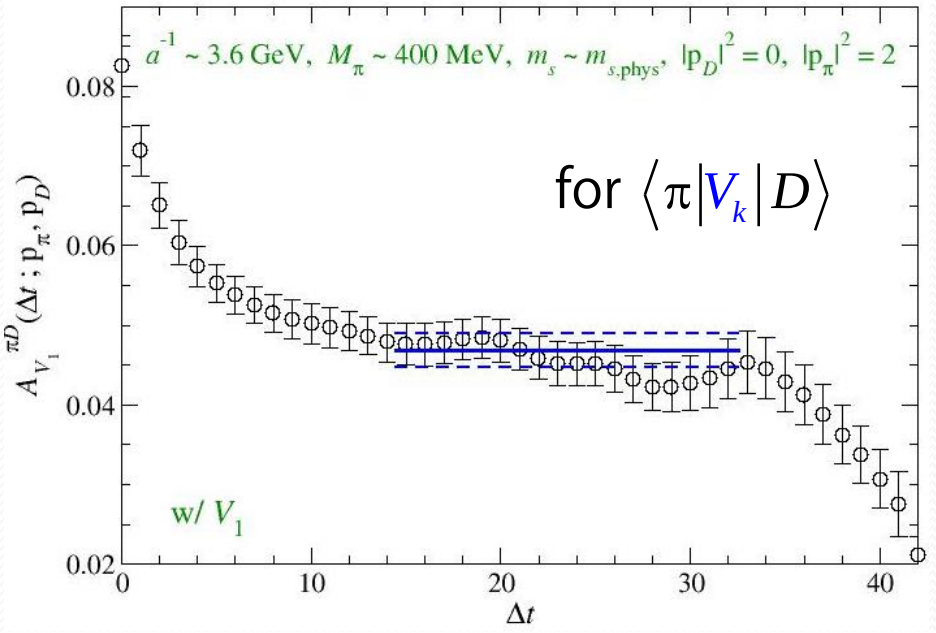




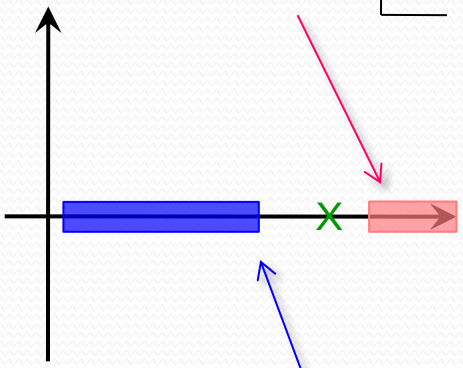
- reasonably good plateaux both for V_4 and V_k
- typical statistical accuracy of FFs

1 - 2 % @ q^2_{max}, $M_\pi \sim 500$ MeV ; 6 - 8 % @ q^2_{min}, $M_\pi \sim 300$ MeV

momentum-transfer dependence

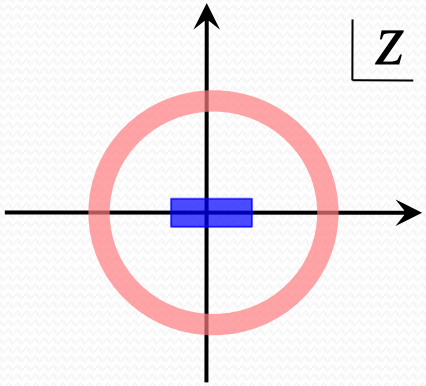
z parameter expansion (Bourrely-Machet-de Rafael, 1981)

$t_+ = (M_D + M_\pi)^2$ $\quad t = q^2$



$t_- = q^2_{max} = (M_D - M_\pi)^2$

$$z = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$



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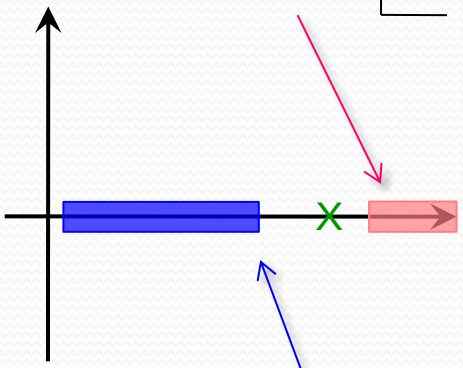

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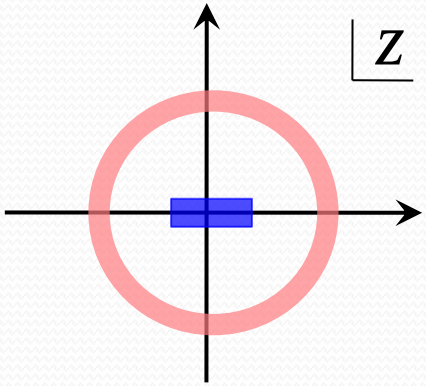
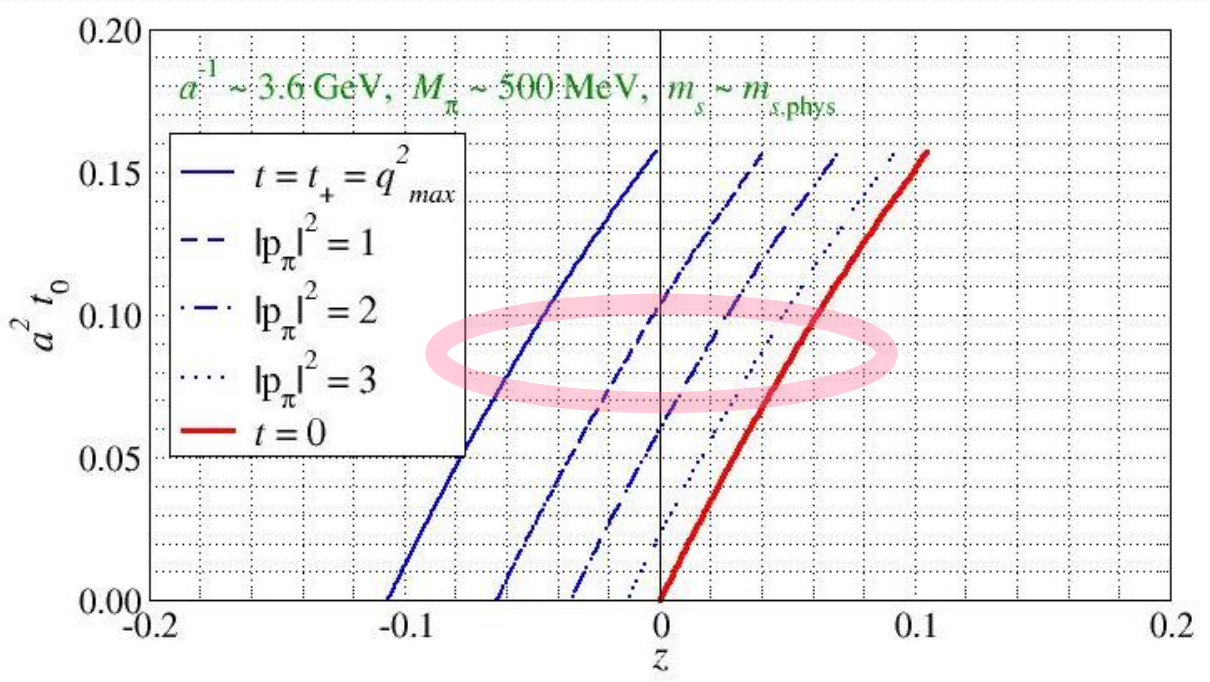
tunable parameter

$\Rightarrow$ small values of z





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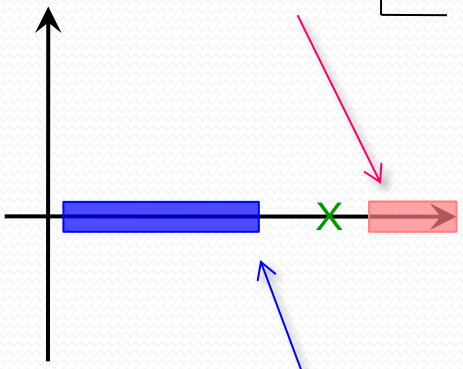

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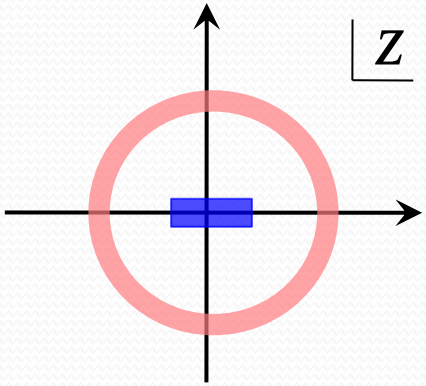
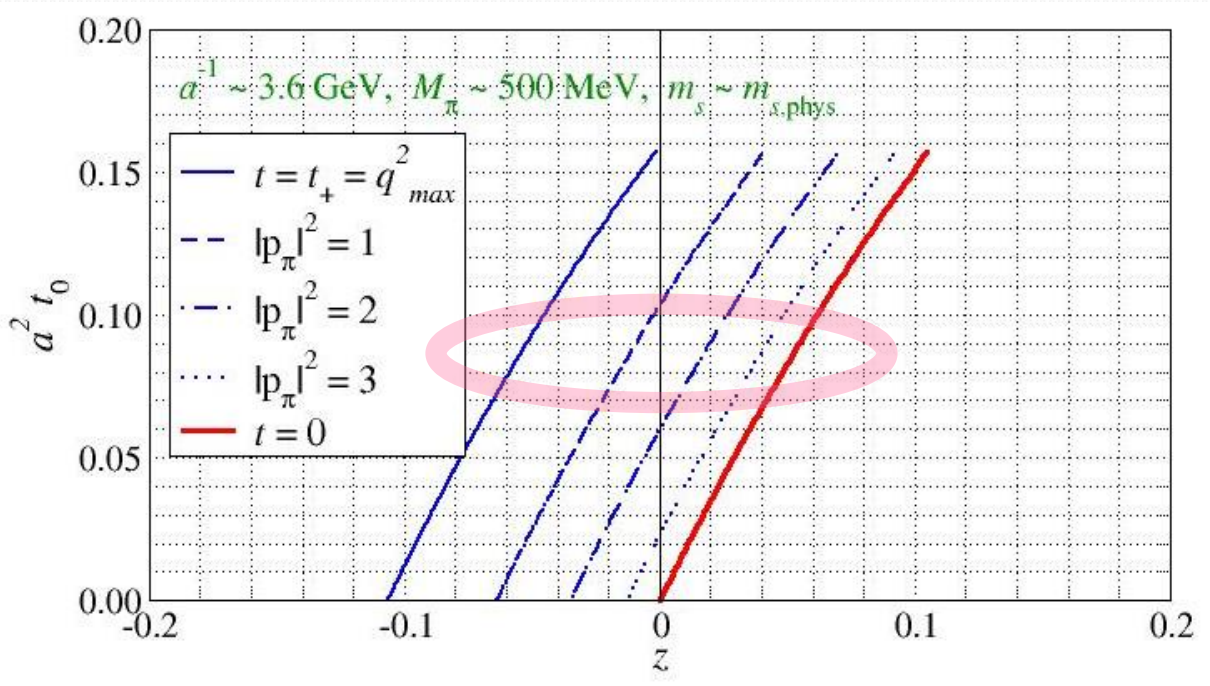
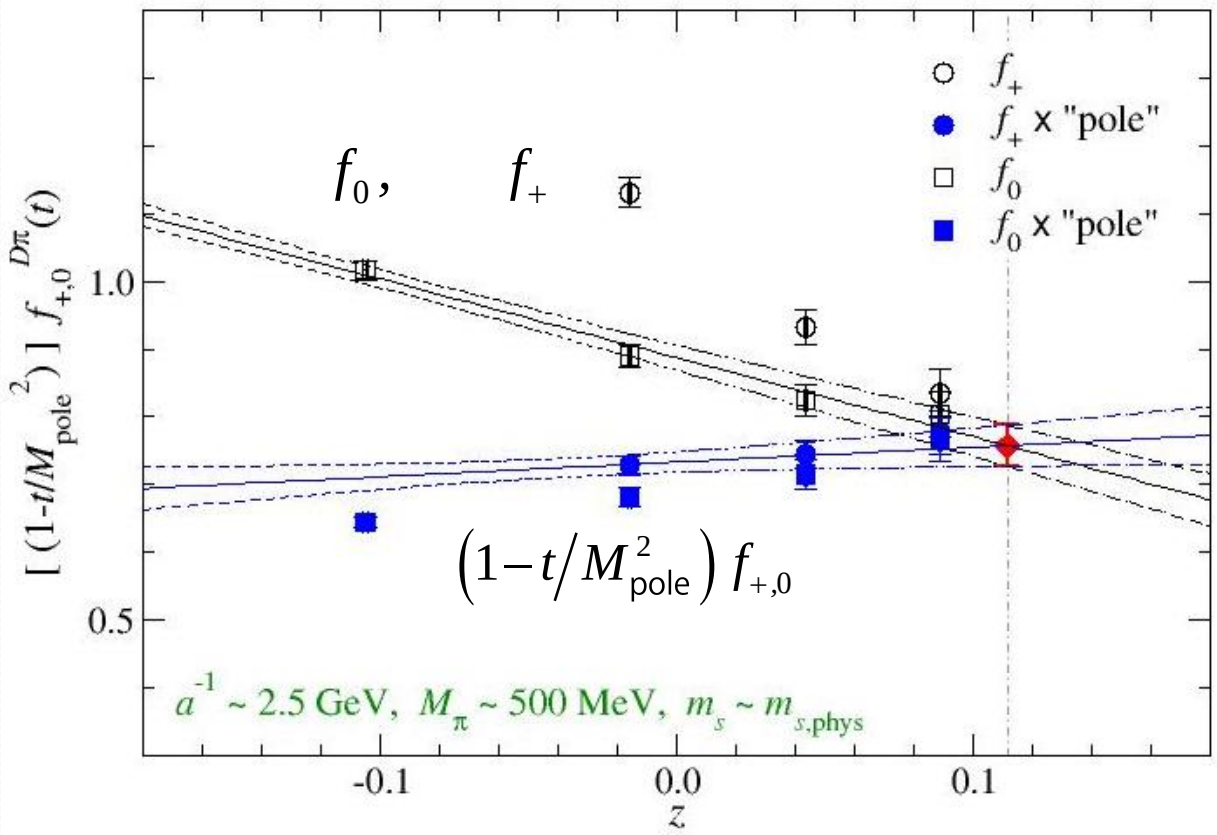




choice of t_0 $\Leftrightarrow$ small impact on extrap'ed value

momentum-transfer dependence

$$f_{+,0}^{D\pi}(t) = \frac{1}{1 - t/M_{\text{pole}}^2} \sum_k a_k z^k$$



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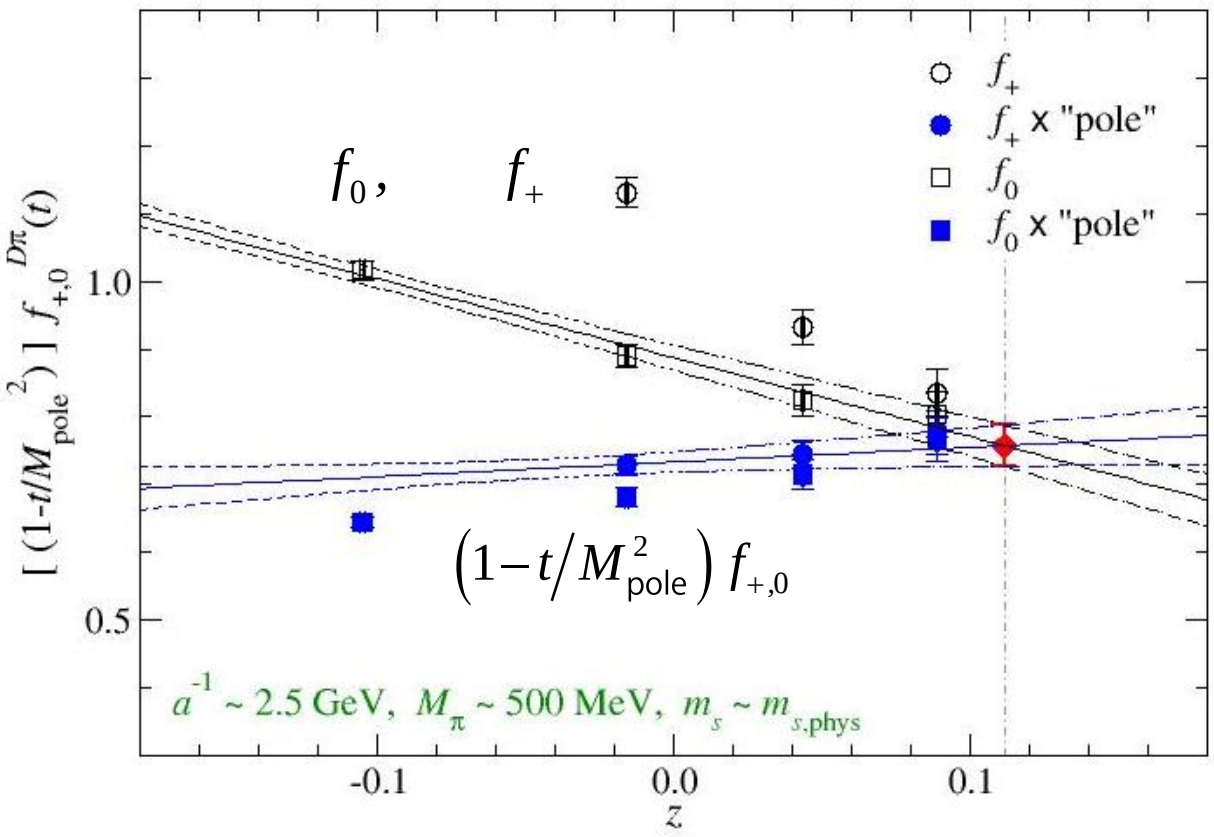

$f_+(t)$

- measured $M_{D(s)^*}$
- pole important
- linear in z w/ "pole"

  sys. err: quad. w/ "pole"

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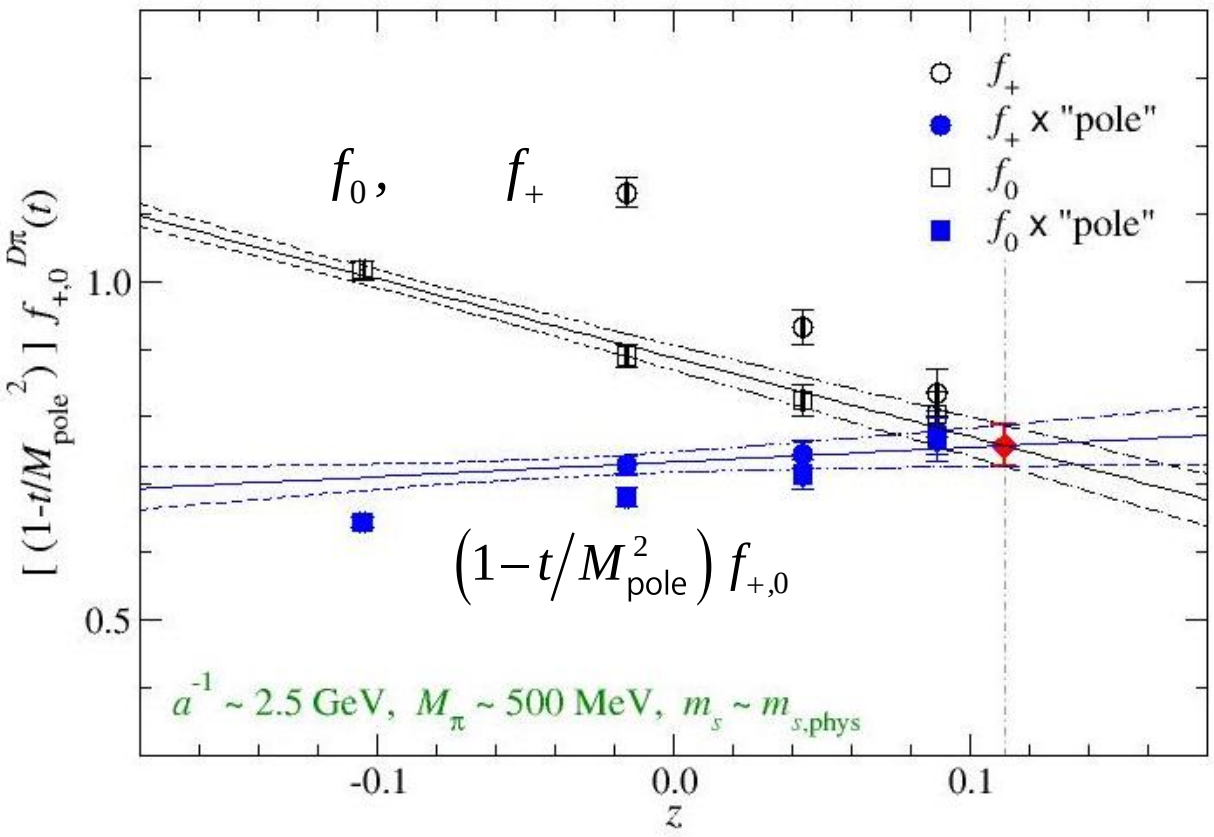

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$f_0(t)$

- $M_{D0(s)^*}$ not measured
- moderate pole factor
- linear in z w/o "pole"

  sys. err.: lin w/ "pole"

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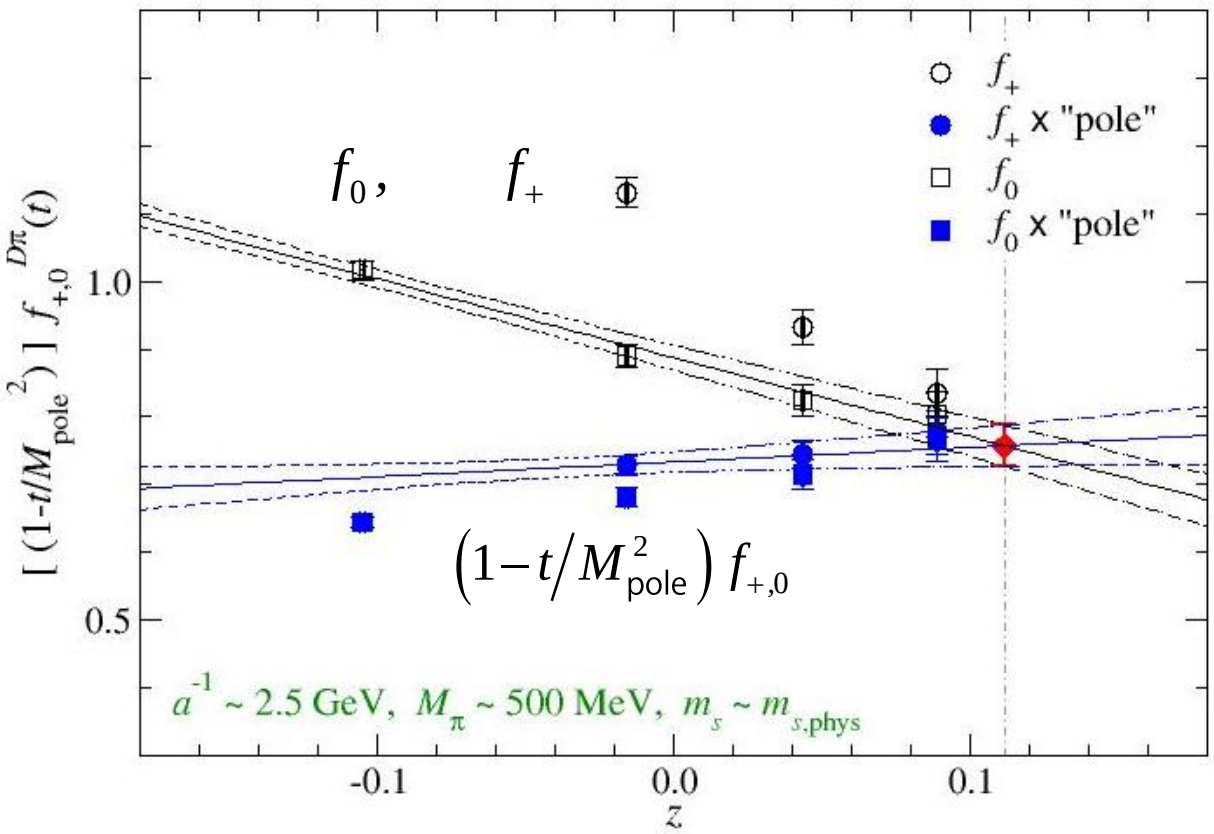

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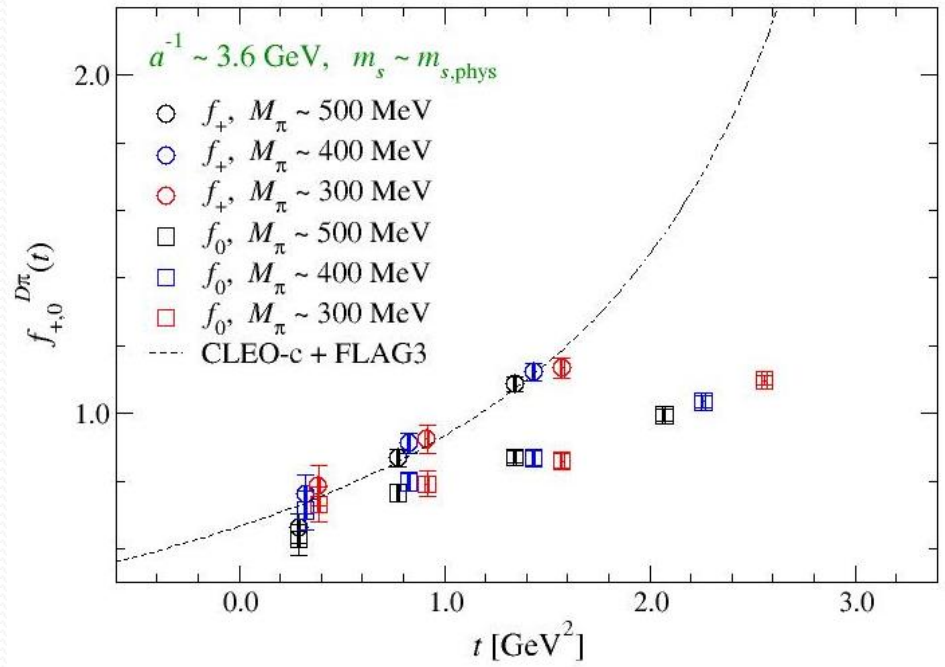
small z's + mild z-dependence $\Rightarrow$ sys. err. $\lesssim$ stat.

m_q and a dependences

@ $M_\pi \simeq 300 - 500$ MeV



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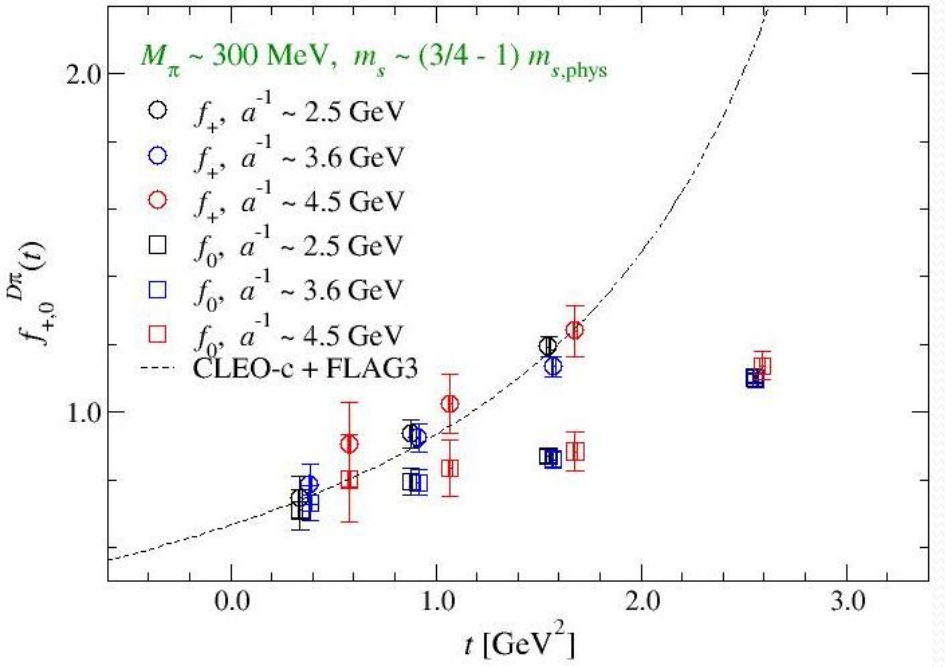

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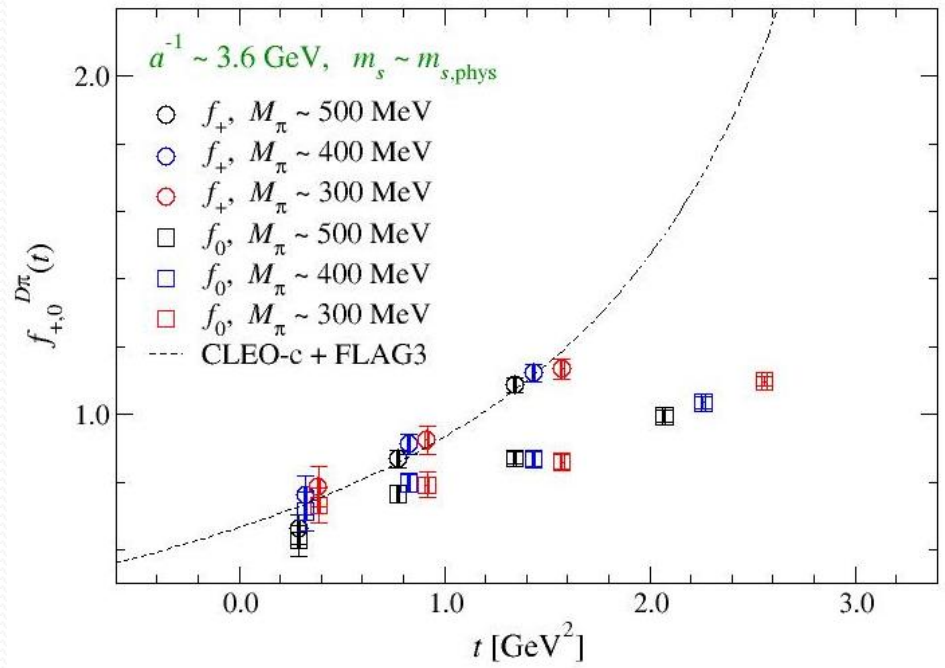
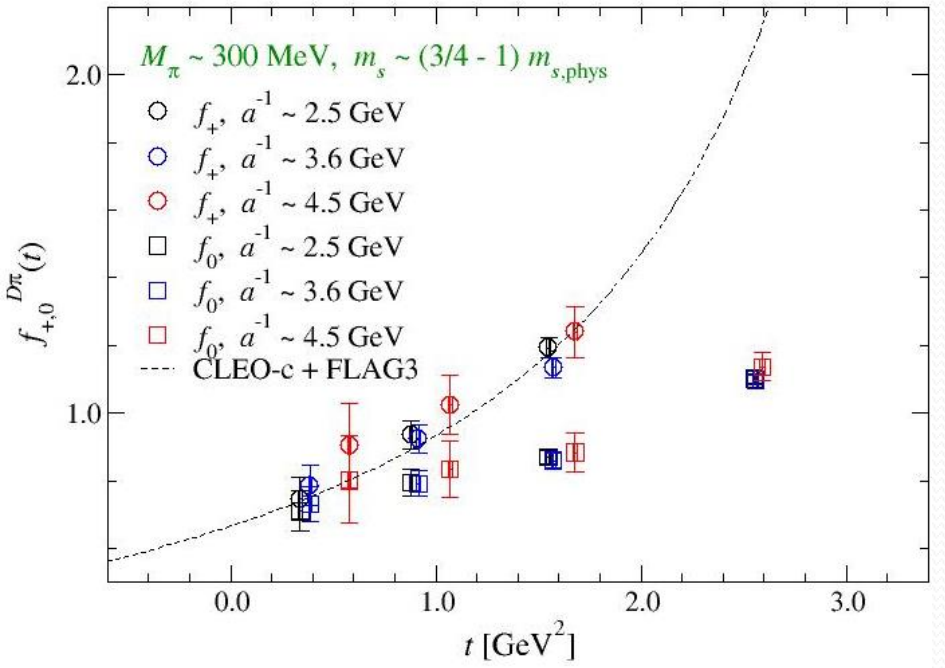


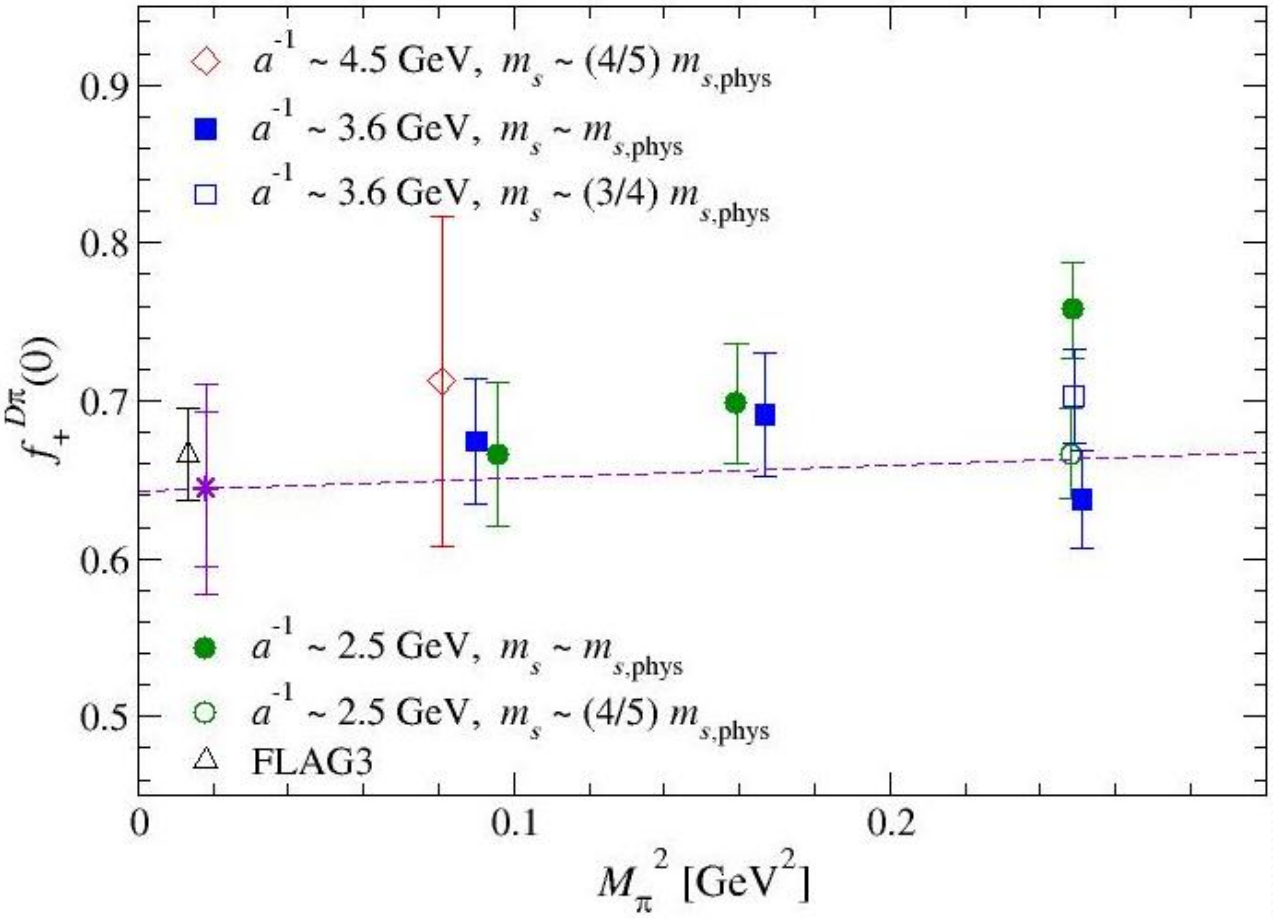
@ $a^{-1} \simeq$ 2.5 – 4.5 GeV



- reasonable agreement ⇒ mild m_{ud}, m_s, a dependences
- in this preliminary analysis ⇒ test a simple linear extrapolation

chiral+continuum extrapolation

$$f_{+,0}^{D\pi}\left(0, M_\pi^2, M_{\eta s}^2, a\right) = c_0 + c_\pi M_\pi^2 + c_{\eta s} M_{\eta s}^2 + c_a a^2 \qquad \left(M_{\eta s}^2 = 2M_K^2 - M_\pi^2\right)$$



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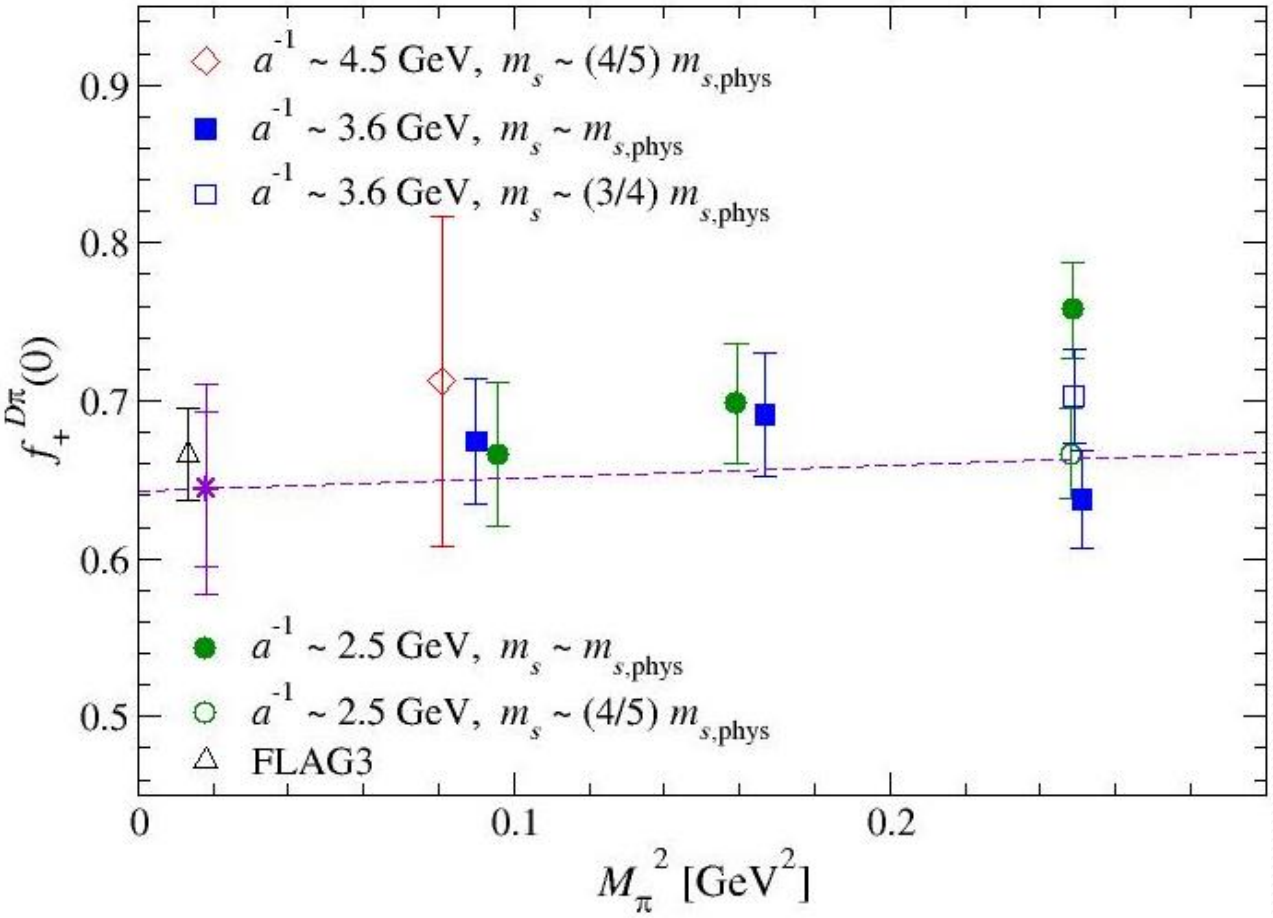

- χ^2/d.o.f $\simeq$ 1.6
- $c_{a,\pi,\eta s}$ consistent w/ 0

$\Rightarrow$ sys. err. by testing

c_π or $c_{\eta s}$ or $c_a = 0$

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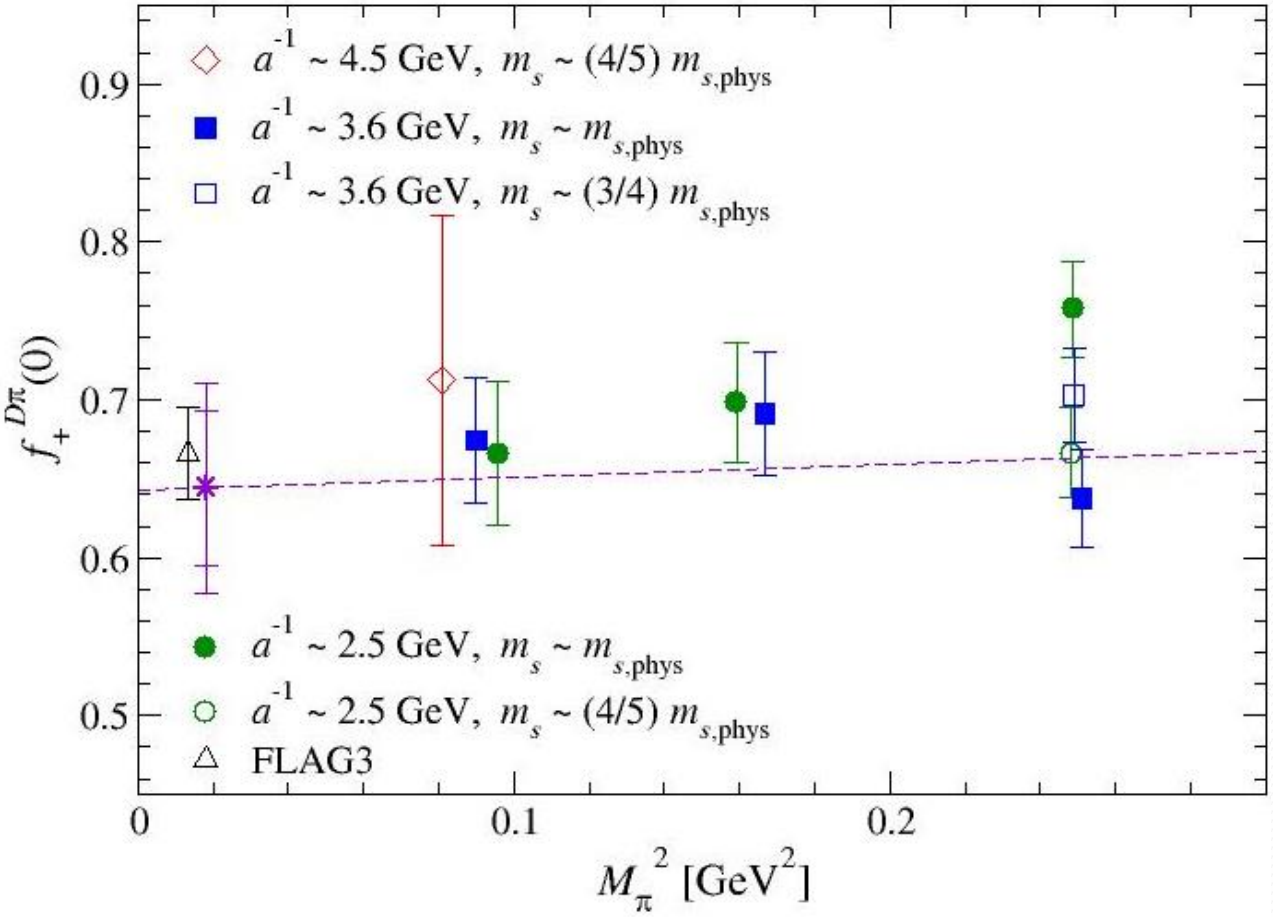

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large err. @ 4.5GeV ?

- only 1 t_{src}
- ½ reduction w/ 4 t_{src}

$f_+(0)$

preliminary estimate :

$$f_+^{D\pi}(0) = 0.644(49)_{\text{stat}}(36)_{t\to 0}(27)_{\text{cont+chiral}}$$

$$f_+^{DK}(0) = 0.701(46)_{\text{stat}}(12)_{t\to 0}(33)_{\text{cont+chiral}}$$

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$$\Rightarrow |V_{cd}| = 0.221(23)_{\text{lat}}(3)_{\text{exp}}, \quad |V_{cs}| = 1.039(86)_{\text{lat}}(7)_{\text{exp}}$$

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near-future improvement :

- $(a^{-1}, M_\pi) \sim$ (4.5GeV, 300MeV), (2.5GeV, 230MeV)
- $(a^{-1} \sim 2.4, 3.6\ \text{GeV}) \otimes (M_\pi \sim 300, 400\ \text{MeV})$ @ a smaller m_s

$\Rightarrow$ better control of extrapolation + statistical error @ extrapolated pt.

summary

JLQCD's study of D meson semileptonic decays

- independent calculation w/
  - Möbius DWF w/o $O(a)$ error
  - $a^{-1} \leq 4.5$ GeV
- 8 – 10 % total uncertainty at this preliminary stage

  expect significant reduction in the near future (within months)
- discretization error is not large ⇒ extension to B physics

  study of heavy-light decay constants

  ⇒ B. Fahy, "Hadron Spectrum", 15:20, today

continuum+chiral extrap. for D→K