Form factors for moments of correlation functions

Chia Cheng Chang
Lawrence Berkeley National Lab

in collaboration with
Chris Bouchard (W&M)
Kostas Orginos (W&M/JLab)
David Richards (JLab)

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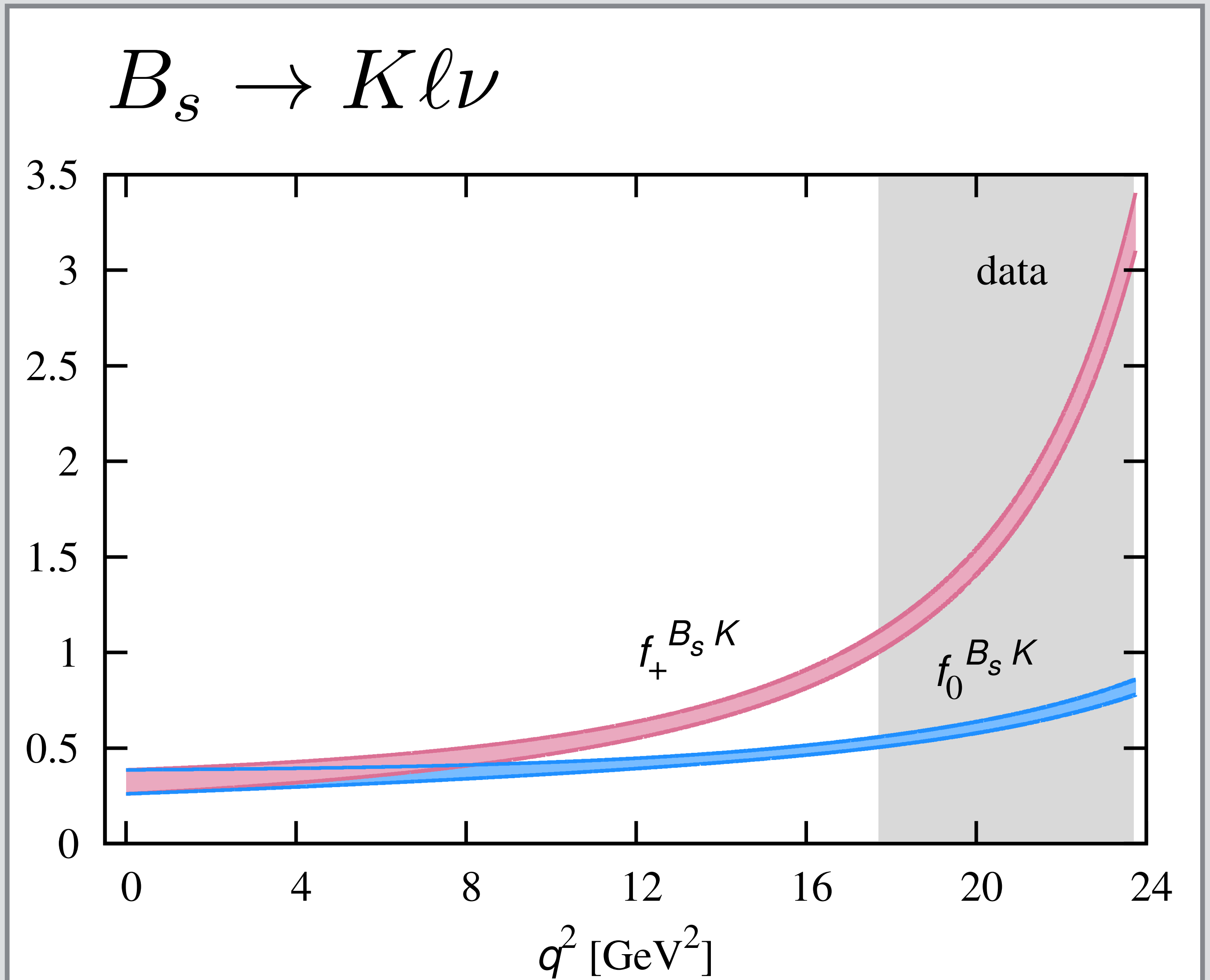
Motivation

constrain shapes of form factors in nuclear and particle physics

calculate slopes of form factors w.r.t. momenta



HPQCD [arXiv:1406.2279]

Outline

introduce formulation

apply to nucleon vector matrix element

Overview of moment methods

Issues with moment methods:

Wilcox - Moments on lattice yields wrong ground state.

[0204024v1]

Existing methods:

HVP - time moment current current correlator [1403.1778v2]

Rome - expand lattice operators [1208.5914v2][1407.4059]

ETMC - position space method [1605.07327v1]

Existing methods take $\partial/\partial q_j$ derivatives at $q^2 = 0$

Our method takes $\partial/\partial q^2$ generalized to all momenta

ensemble overview

2+1 flavor JLab isotropic Clover

$a \approx 0.12$ fm

$m_\pi \approx 400$ MeV

$N_x^3 \times N_t = 24^3 \times 64$

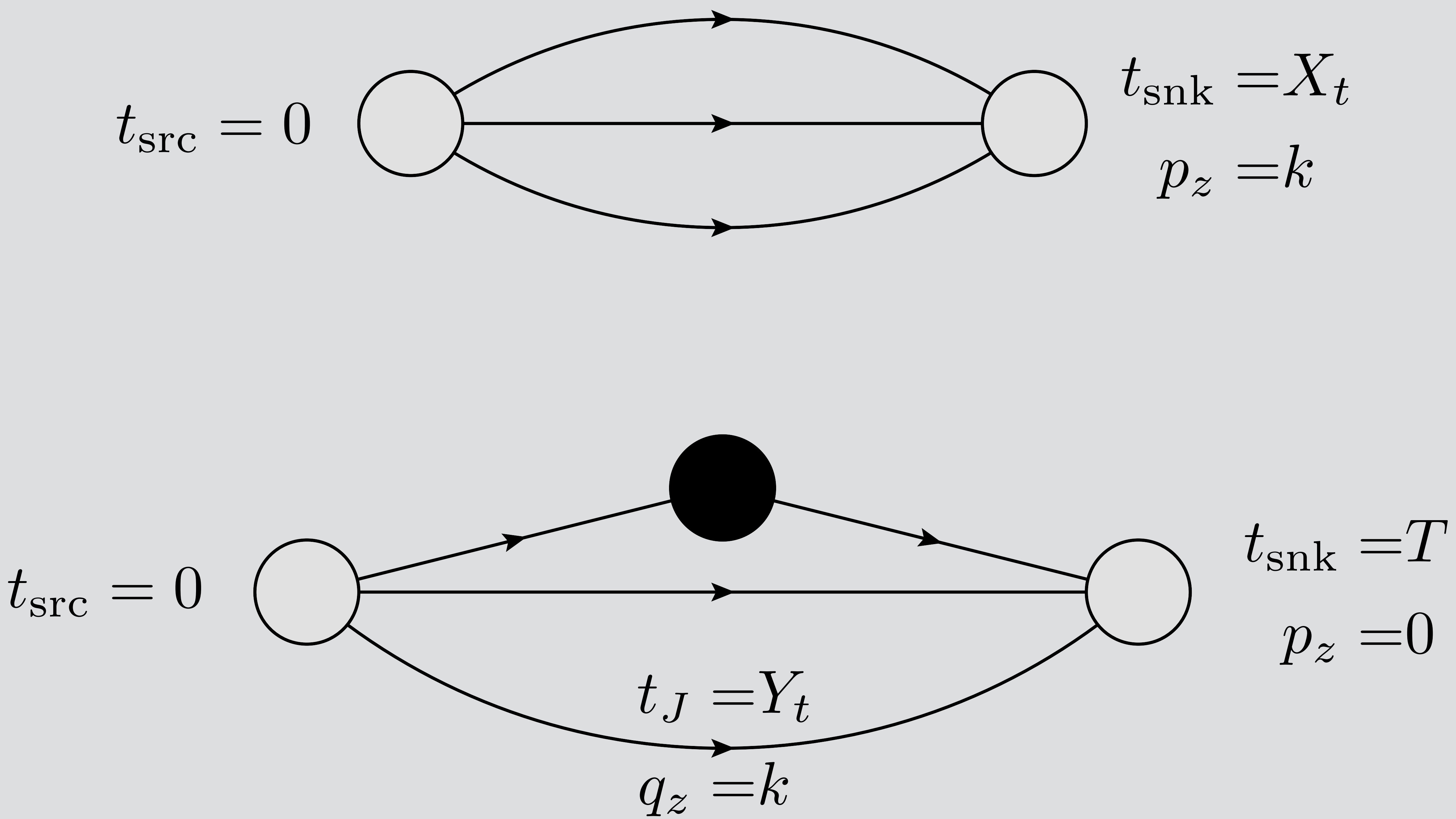
correlator overview

double z-direction: $\quad N_s^2 \times N_z \times N_t = 24^2 \times 48 \times 64$

480 configurations $\times$ 16 sources

$m_{\text{valence}} = m_{\text{light sea}}$

kinematic setup



correlators

two-point correlator

$$C_{2\mathrm{pt}}(X_t) = \int d^3\vec{X} \left\langle A_{X_t,\vec{X}} | A_{0,\vec{0}} \right\rangle e^{-ikX_z}$$

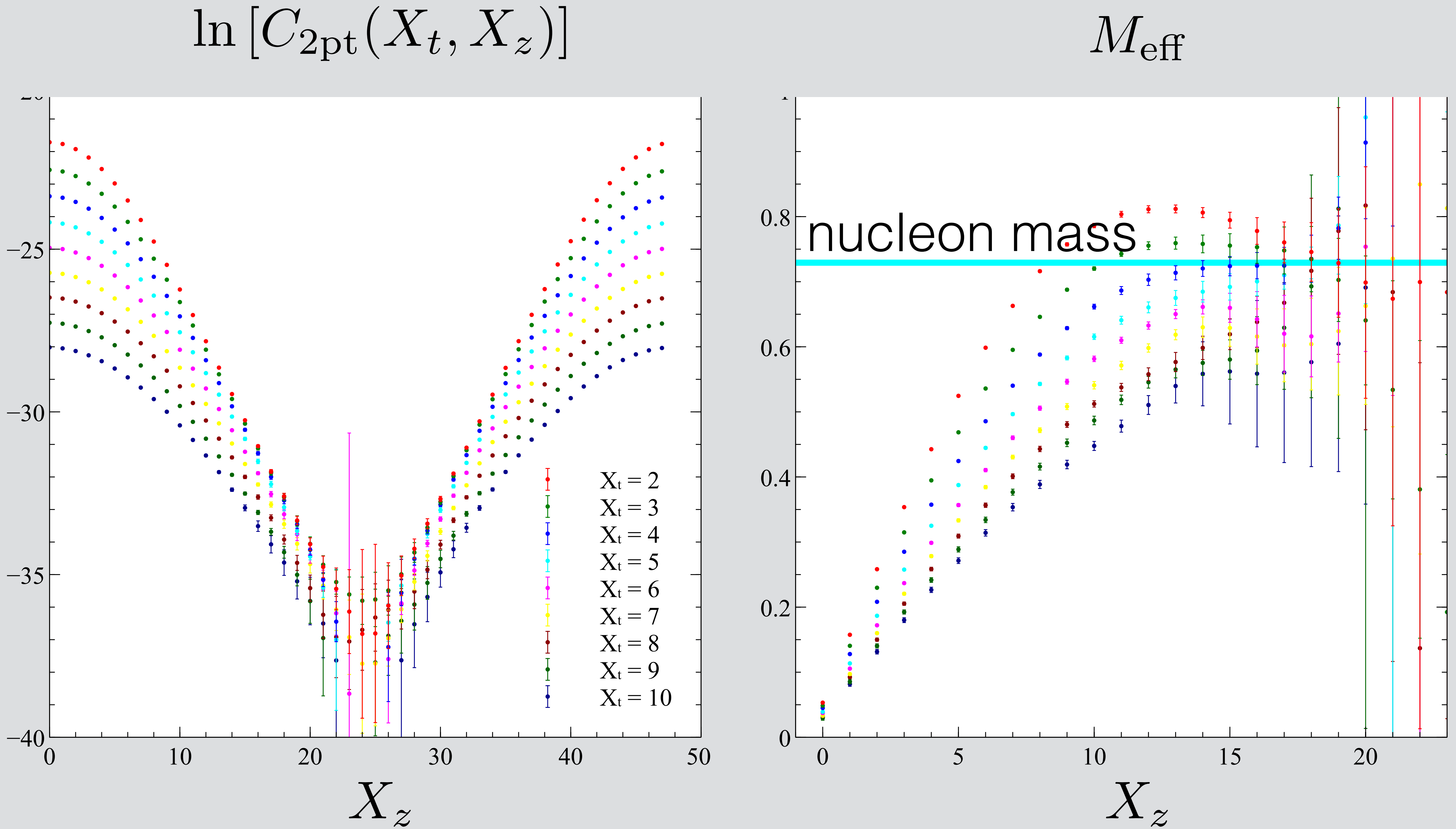
two-point moment

$$\frac{\partial}{\partial k^2} C_{2\mathrm{pt}}(X_t) = \int d^3\vec{X} \left\langle A_{X_t,\vec{X}} | A_{0,\vec{0}} \right\rangle \left(\frac{-iX_z}{2k} \right) e^{-ikX_z}$$

$$\lim_{k^2 \to 0} C'_{2\mathrm{pt}}(X_t) = \int d^3\vec{X} \left\langle A_{X_t,\vec{X}} | A_{0,\vec{0}} \right\rangle \left(\frac{-X_z^2}{2} \right)$$

only have even spatial moments

two-point z-correlator



integral of any polynomial moment converges in infinite volume

correlators cont.

three-point correlator

$$C_{3\mathrm{pt}}(X_t, Y_t) = \int d^3\vec{X} d^3\vec{Y} \left\langle A_{X_t,\vec{X}} \middle| \Gamma_{Y_t,\vec{Y}} \middle| B_{0,\vec{0}} \right\rangle e^{-ikY_z}$$

three-point moment

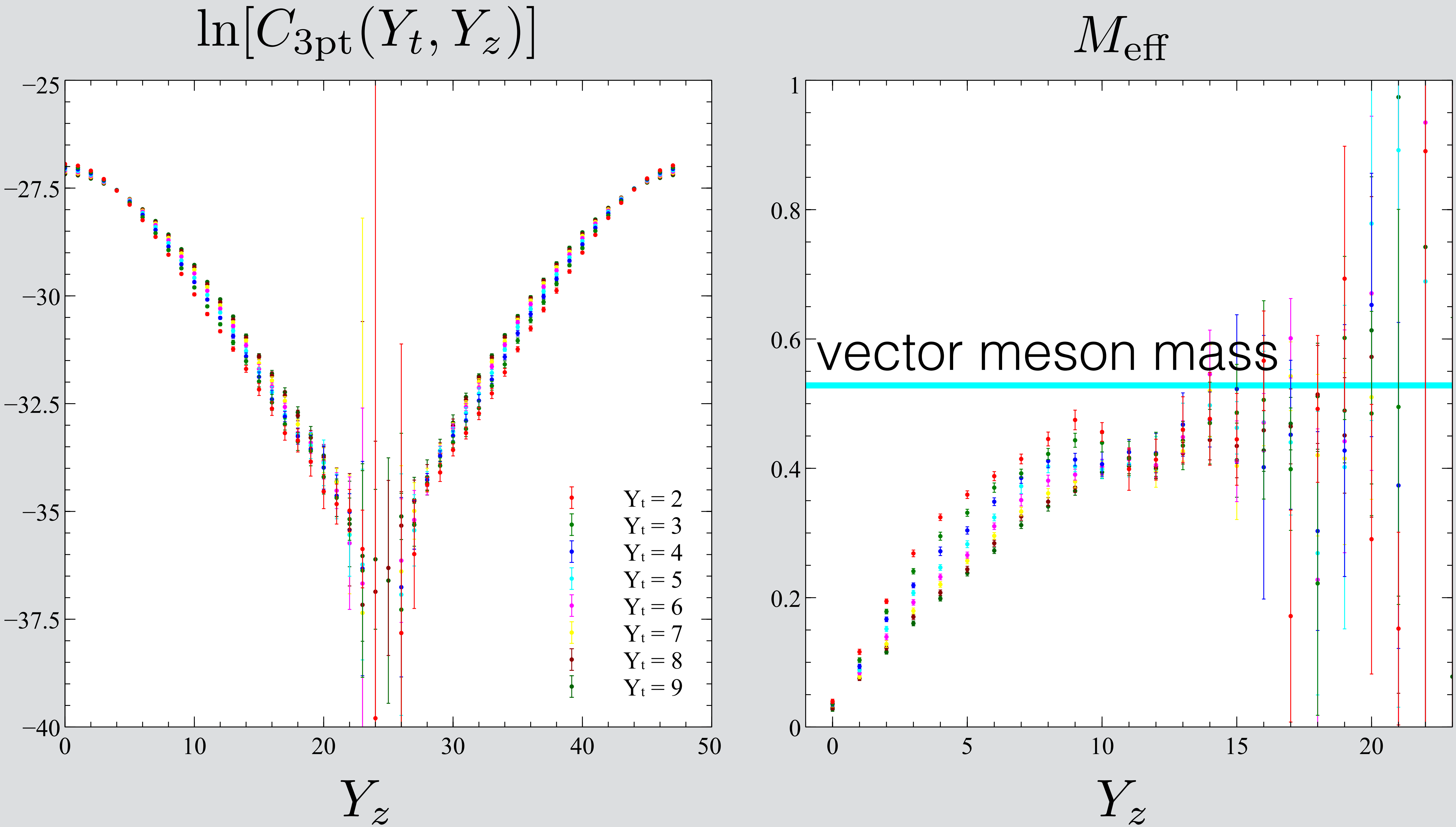
$$\frac{\partial}{\partial k^2} C_{3\mathrm{pt}} = \int d^3\vec{X} d^3\vec{Y} \left\langle A_{X_t,\vec{X}} \middle| \Gamma_{Y_t,\vec{Y}} \middle| B_{0,\vec{0}} \right\rangle \left(\frac{-iY_z}{2k} \right) e^{-ikY_z}$$

$$\lim_{k^2 \to 0} C'_{3\mathrm{pt}} = \int d^3\vec{X} d^3\vec{Y} \left\langle A_{X_t,\vec{X}} \middle| \Gamma_{Y_t,\vec{Y}} \middle| B_{0,\vec{0}} \right\rangle \left(\frac{-Y_z^2}{2} \right)$$

moments are with respect to current insertion

given correlators, moments are computationally free

three-point z-correlator



lowest lying state when current is outside nucleon operators

finite volume correction

Spatial moments push the peak of the correlator away from origin

Larger finite volume corrections compared to regular correlators

Have exponential finite volume corrections

Currently thinking about ways to implement FV corrections

fit functions

two-point fit function

$$C_{2\mathrm{pt}} = \sum_n \frac{Z_n^{A\dagger} Z_n^A}{2E_n^A} e^{-E_n^A X_t}$$

two-point moment fit function

$$C'_{2\mathrm{pt}} = \sum_n C_n^{2\mathrm{pt}} \left(\frac{2Z_n^{A\prime}}{Z_n^A} - \frac{1}{2(E_n^A)^2} - \frac{X_t}{2E_n^A} \right)$$

definitions

$$Z_n^A = \langle n | A \rangle \qquad E_n^A = \sqrt{M_A^2 + k^2} \qquad \prime = \frac{\partial}{\partial k^2}$$

expect $Z_n^{A\prime} = 0$ for point source/sink
two-point constrains all parameters except $Z_n^{A\prime}$

fit functions cont.

three-point fit function

$$C_{3\text{pt}} = \sum_{n,m} \frac{Z_n^{A\dagger}(0)\Gamma_{nm}(k^2)Z_m^B(k^2)}{4M_n^A(0)E_m^B(k^2)} e^{-M_n^A(0)(X_t - Y_t)} e^{-E_m^B(k^2)Y_t}$$

three-point moment fit function

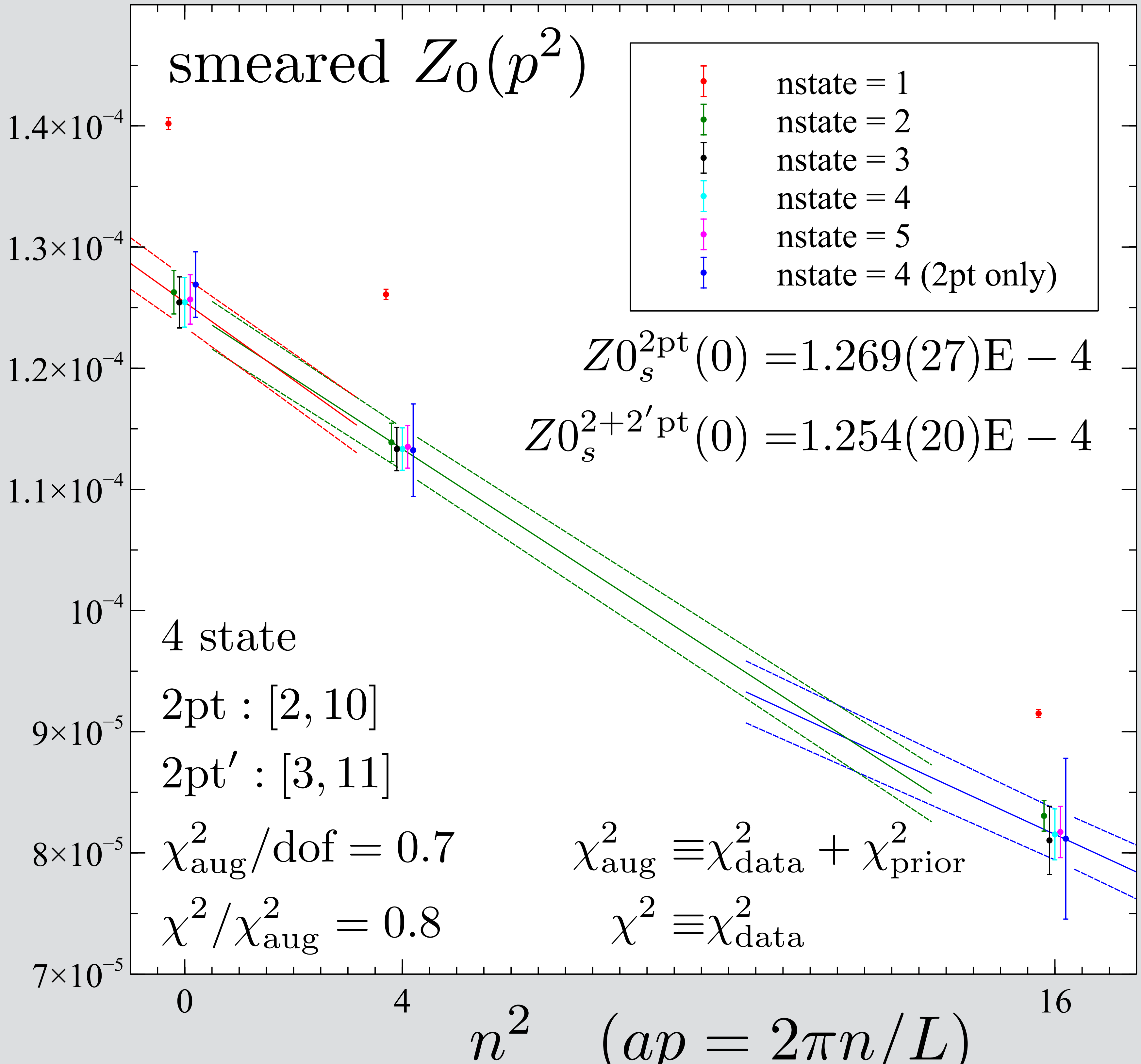
$$C'_{3\text{pt}} = \sum_{n,m} C_{nm}^{3\text{pt}} \left(\frac{\Gamma'_{nm}}{\Gamma_{nm}} + \frac{Z_m^{B\prime}}{Z_m^B} - \frac{1}{2(E_m^B)^2} - \frac{Y_t}{2E_m^B} \right)$$

2pt and 3pt constraints all params. except slopes

2pt moment needed for smeared source/sink

3pt moment constrains slope of form factor preliminary

overlap factors



fit strategy

Bayesian multi-state

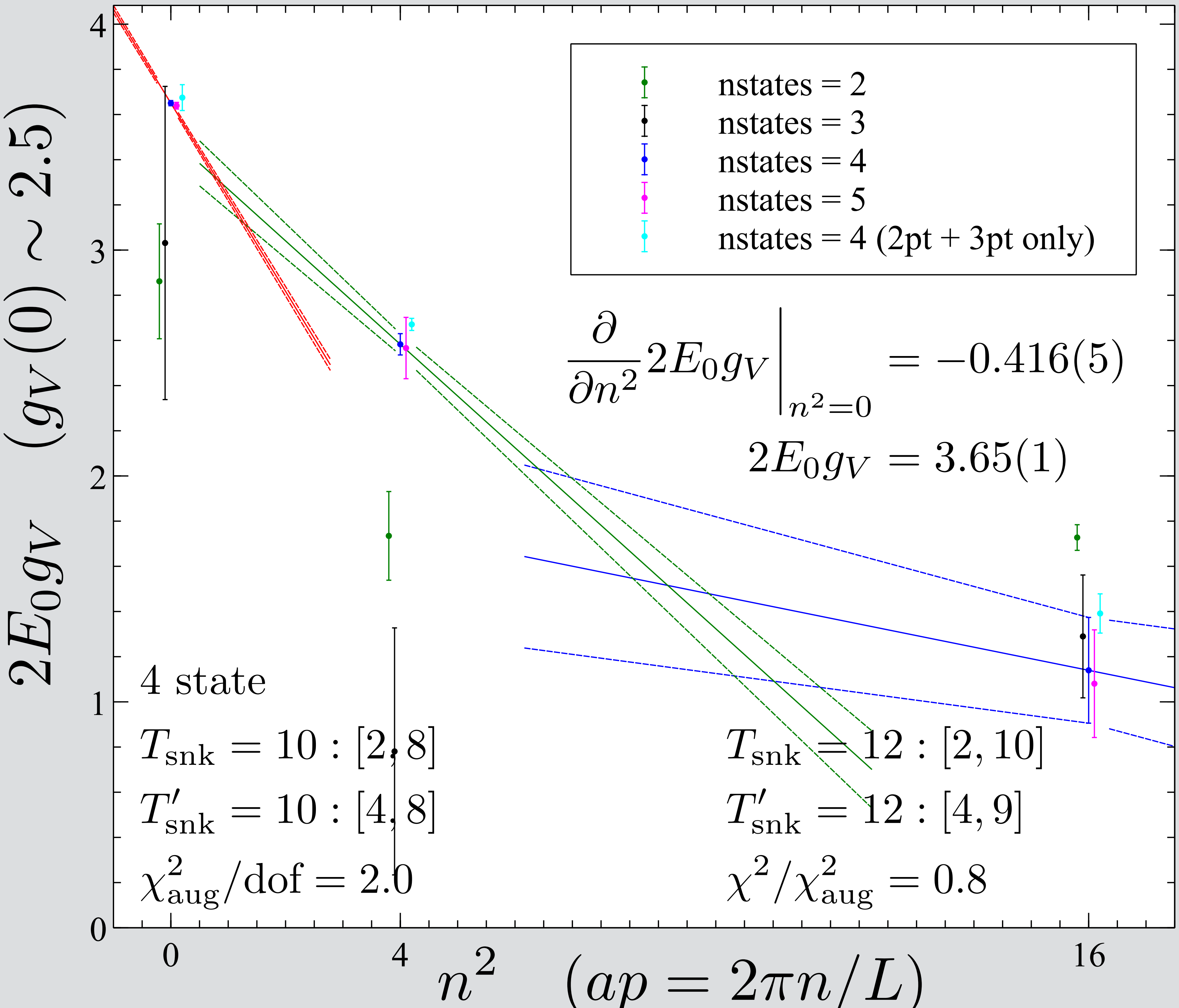
simultaneous fit to 2pt + 2pt moment @ all momenta

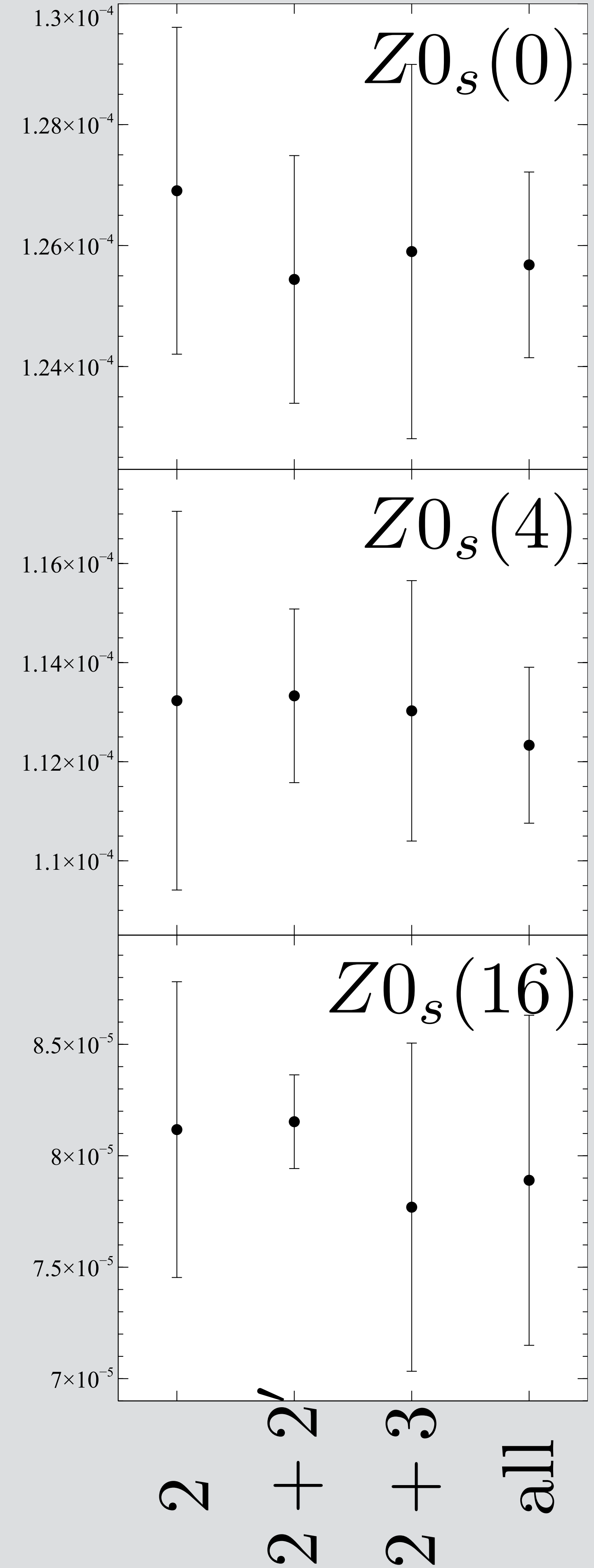
check stability

- # of states
- vs 2pt only fit
- time range (not shown)

preliminary

slopes of gV





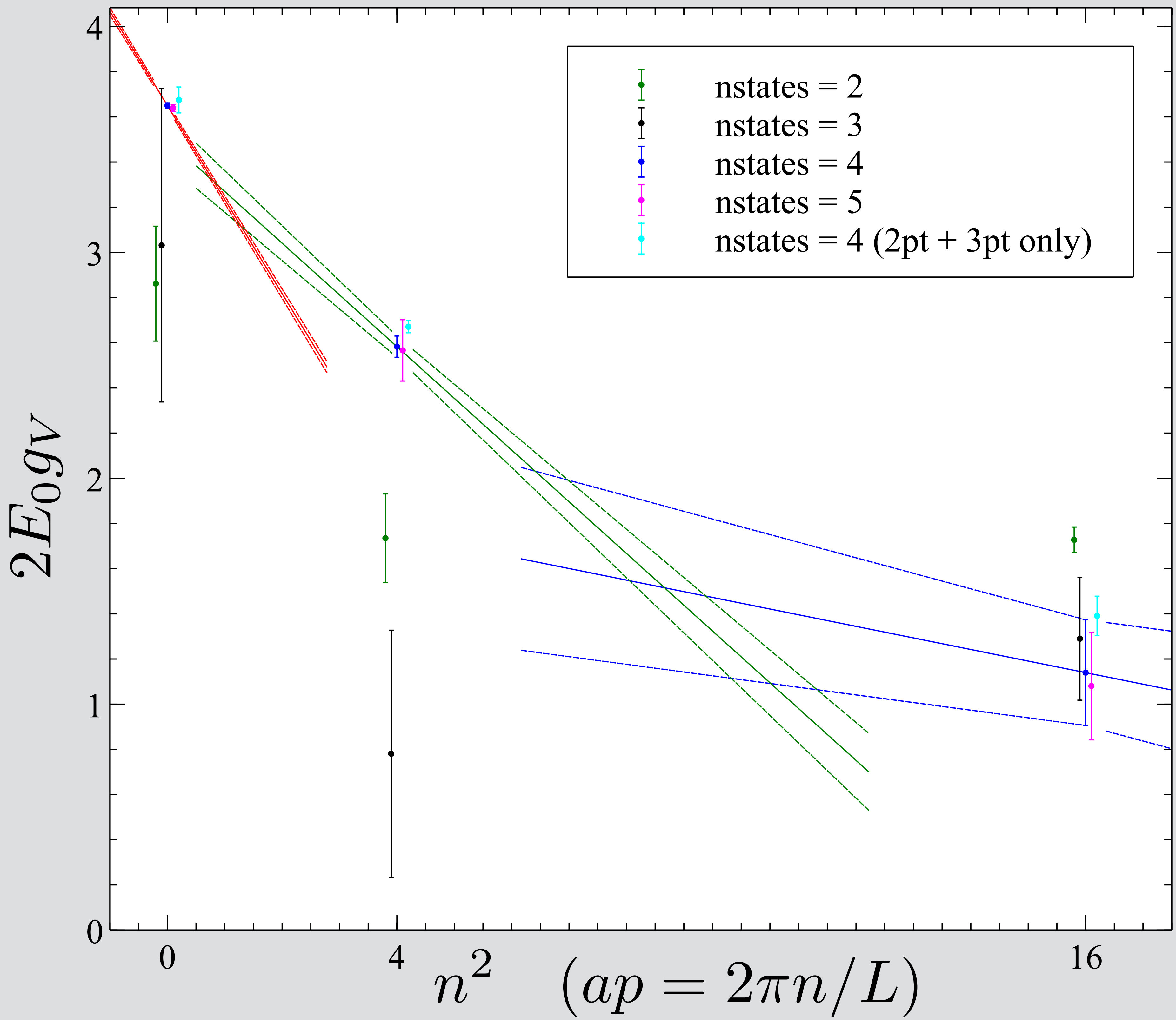
summary and outlook

requires negligible additional computation time
obtain slopes of matrix elements
increase precision of central values

run more configs
run more Tsnks
try on larger ensemble
implement FV corrections

implement with FH correlators
correlators of higher moments

thank you



nstates = 2
nstates = 3
nstates = 4
nstates = 5
nstates = 4 (2pt + 3pt only)
$2E_0 g_V$
$n^2 \quad (ap = 2\pi n/L)$
0
1
2
3
4
0
4
16