‘t Hooft model on the lattice

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What? :'t Hooft model

This is just d=2 Yang-Mills theory in the large N limit. Quarks only appear as sources ($N_f/N = 0$). 't Hooft Solution:

- An infinite stable spectrum of mesons with asymptotically linearly rising mass square

$$\mu_n^2 \sim n\pi^2 m_0^2$$

where $m_0^2 = g^2 N/\pi$ is the natural unit of mass ($m_0 = 1$). States have alternating even and odd parity.

- As the quark mass m_q goes to zero the mass of the lowest state (pion) goes to zero as follows:

$$\mu_0^2 = \frac{2\pi}{\sqrt{3}} m_q + \mathcal{O}(m_q^2)$$

(The order of the limits matter).

- Higher order states do not vanish in that limit. Their masses can be determined numerically by solving a 1-dim integral eq.

't Hooft model on the lattice

Despite its attractive power very little has been done on the lattice.

- *Gross Witten 1980*:
  The partition function and rectangular Wilson loop exp. values obtained exactly for $V = N = \infty$ U(N) theory with Wilson action:

$$E \equiv \frac{1}{N}\langle \mathrm{Tr}(U(P))\rangle = 1 - \frac{1}{4b}$$

$$\log W_{R\times T} = \log(E) RT$$

- *Berruto, Giusti, Hoebling and Rebbi 2002*:
  Studied d=2 U(N) and SU(N) QCD for N=2,3,4 at rather coarse lattices but made no extrapolation.
- *Kiskis, Narayanan and Neuberger 2002*:
  First attempt towards a quantitative determination of spectrum.
  They use a **reduced model** (Very similar to the present work).

How?: Our method

We make use of **volume independence**:

$$\text{Matrix model at } N = \infty \quad \equiv \quad V = N = \infty \text{ LGT}$$

Matrix model (2d version of TEK):

$$Z = \int dU_0\, dU_1 \, \exp\{bNz\mathrm{Tr}(U_0 U_1 U_0^\dagger U_1^\dagger) + \mathrm{h.c.}\}$$

where $\mathbf{z} = e^{2\pi i \mathbf{k}/N}$ (k coprime with N) and $b = 1/\lambda_L$

Expectation values:

$$W_{R\times T} = \frac{z^{RT}}{N} \langle \mathrm{Tr}(U_0^T U_1^R U_0^{-T} U_1^{-R})\rangle$$

Finite N corrections are equivalent to finite volume for $\mathbf{L = N}$

+ only partial suppression of non-planar diagrams.

Meson correlation functions

Only assumption about quarks: They live in an $N \times (l_0 N)$ lattice.
Meson operators: $\mathbf{O}_\Gamma(x) = \bar{\Psi}(x)\Gamma\Psi(x)$ with $\Gamma = \mathbf{I}, \vec{\sigma}$.

$$C_{\Gamma\Gamma'}(t) = -\sum_x \langle \mathbf{O}_\Gamma(0)\mathbf{O}_\Gamma(t,\vec{x})\rangle$$

One gets

$$C_{\Gamma\Gamma'}(t) = \sum_{p_0} e^{ip_0 t}\,\mathrm{Tr}(\Gamma D^{-1}(p_0)\Gamma' D^{-1}(0))$$

$D(p_\mu)$ is fermion a operator (Wilson/overlap) of a single-site with
$U_\mu \longrightarrow U_\mu \otimes \Gamma^*_\mu e^{ip_\mu}$. $\quad p_0 = 2\pi n/(N l_0)$
D is an $N^2 \times N^2$ matrix
(Γ_0, Γ_1 are 't Hooft clock matrices)

Why?: Motivation

- A good testing ground for methodologies for determining the meson spectrum at large N: The exact results are known!!
- Computationally simple
- A first step towards studying other unsolved $d = 2$ theories at large N for which many conjectures and ideas are available: Veneziano limit, quarks in other representations, etc

Simulation details

We are studying the model using the twisted reduced model at various b:

$$b = 3, 4, 5, 6, 8, \ldots ?$$

and N:

$$N = 31, 43, 53, \ldots ?$$

for various smearings and with

Wilson fermions / naive fermions / overlap fermions

and various analysis methods

variational analysis / multiexponential fits / mom. space fits

Gauge field dynamics

The reduced model captures the physics of the infinite volume system.
Twisting is not essential but helps:

$$E = 1 - \frac{1}{4b} \quad \text{for twist} + \mathcal{O}(\frac{1}{N^2}) \quad \text{for periodic} + \mathcal{O}(\frac{1}{N})$$

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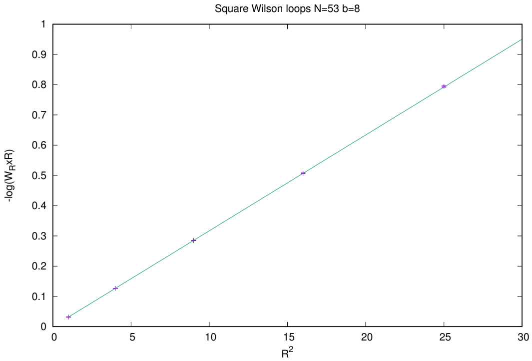

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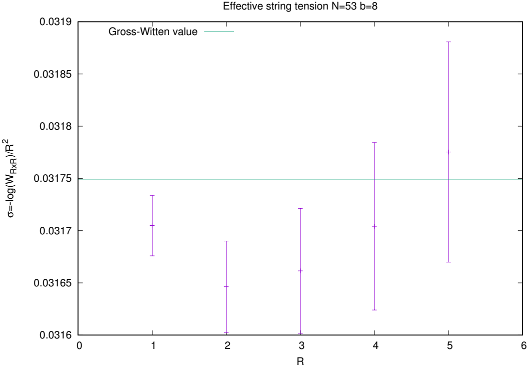
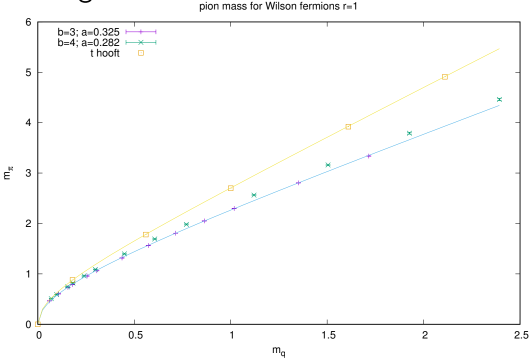


Wilson fermions r=1

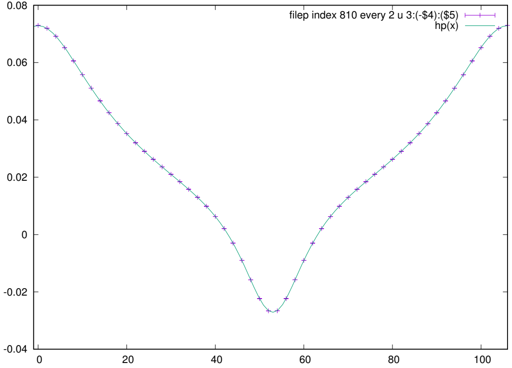
We fix the lattice spacing in m_0 units with $a = 1/\sqrt{b\pi}$.
Good behaviour for small quark masses:
$m_\pi^2 = Am_q + Bm_q^2$ with $A = 3.16$ for $b = 3$ and $A = 3.38$ for $b = 4$
('t Hooft result is $3.628 = 2\pi/\sqrt{3}$)
Large errors at large masses: Lattice artifacts ?



Naive fermions r=0

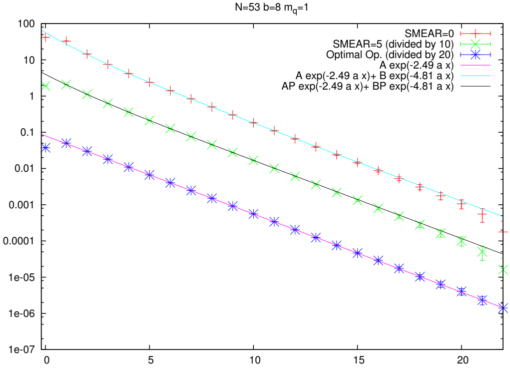
Advantages $\begin{cases} \text{Corrections are only of order } a^2 \\ M_q \text{ corresponds to } 1/(2\kappa) \end{cases}$

Disadvantages : $\begin{cases} \text{The states are (quasi)-doubly degenerate} \\ \text{The time-dep of correlators is more complex} \end{cases}$



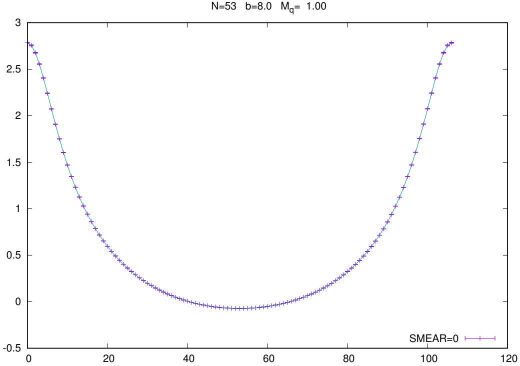
A snapshot of the quality of overlap data

Correlator in configuration space : $N = 53 \; b = 8 \; m_q = 1$ for overlap fermions



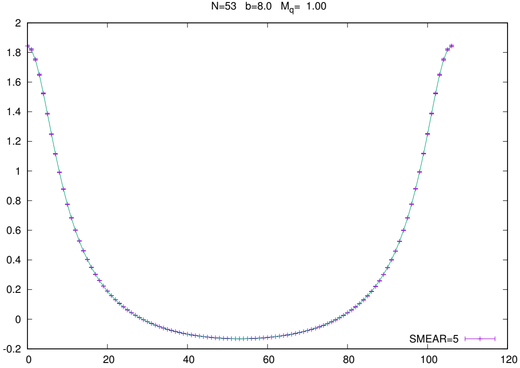
A snapshot of the quality of overlap data

Correlator in momentum space : $N = 53$ $b = 8$ $m_q = 1$ for overlap fermions

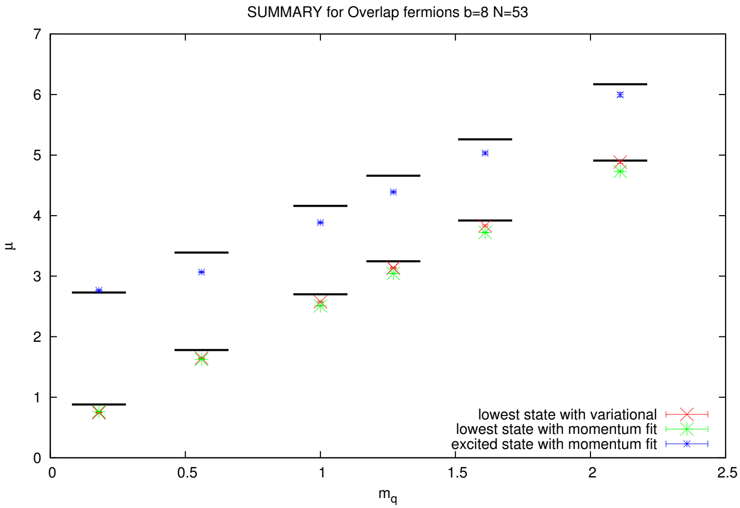


A snapshot of the quality of overlap data

Correlator in momentum space (with smearing): $N = 53$ $b = 8$ $m_q = 1$ for overlap fermions



Summary of overlap results



Conclusions and Outlook

- Meson mass results are in qualitative agreement with 't Hooft model predictions. Up to 3 excited states can be reasonably determined.
- Our meson masses are a few percent lower at equal bare quark mass. We still have to understand size of corrections: finite N, lattice artifacts, bare mass definition, etc.
- Results agree when applicable with Kiskis, Narayanan and Neuberger ones.

Many interesting large N models are within reach