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$\overbrace{\text{Hadronic}}^{\text{disconnected}}$ Hadronic contribution to the muon magnetic moment at the physical point

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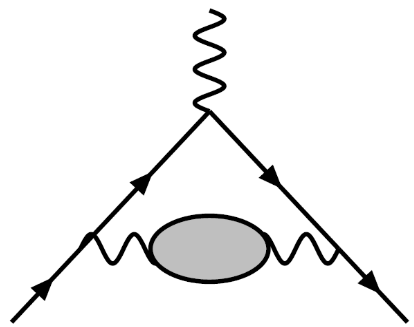
Introduction

- Muon anomalous magnetic moment a^{μ}
  - one of the most precisely determined quantities in experiment.
  - sensitive to potential new physics contributions.
  - 3σ discrepancy between theory and experiment.
- New $g-2$ experiments at FNAL and J-PARC are expected to reduce experimental error.
- Hadronic Vacuum Polarization (HVP) gives dominant theoretical error $\sim 0.6\%$.



➡ We calculate moments of HVP via LQCD simulation with $N_f = 2+1+1$ dynamical Staggered fermion at physical point with $6\mathrm{fm}$ box.

Moment of Π_1 and Π_2

- Hadronic contribution to the muon magnetic moment

$$a_\mu^{\mathrm{HVP}} = 4\pi\alpha \sum_f Q_f^2 \int_0^\infty dq^2 \boxed{F(q^2)} \boxed{\hat{\Pi}^f(q^2)}$$

Non-perturbative aspects of QCD

Well known factor

  - Integrand highly peaked near $q^2 = m_\mu{}^2/4 \sim (50\mathrm{MeV})^2$.

- Scalar polarization function $\Pi(q^2)$ can be calculated via LQCD

$$\begin{aligned} \Pi_{\mu\nu}(q^2) &= (q_\mu \nu - \delta_{\mu\nu} q^2)\Pi(q^2) = \sum e^{iqx} C_{\mu\nu} \\ C_{\mu\nu} &= \langle j_\mu(x) j_\nu(0) \rangle \end{aligned}$$

Electromagnetic quark current

$$j_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c$$

- Derivative method [HPQCD 14]

$$\boxed{\hat{\Pi}(q^2) = \sum_n q^{2n}\Pi_n, \quad \Pi_{n,\nu\mu} = (-1)^{n+1} \sum_x \frac{\hat{x}_\nu^{2(n+1)}}{\Gamma(2n+3)} C_{\mu\mu}(x)}$$

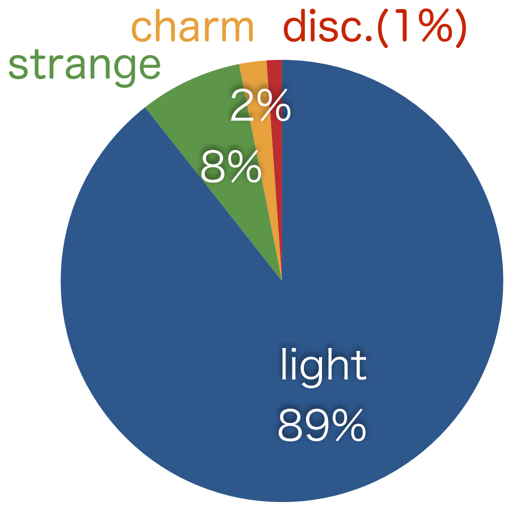
  - model-independent determination.
  - Good description with Pade-approximant.
  - $a_\mu{}^{\mathrm{HVP}}$ and its error are dominated by Π_1.

Moment of Π_1 and Π_2

- We split up the moments as

$$\Pi_n = \boxed{\frac{5}{9}\Pi_n^l + \frac{1}{9}\Pi_n^s + \frac{4}{9}\Pi_n^c} + \boxed{\frac{1}{9}\Pi_n^{\mathrm{disc}}}$$

Next talk This talk



  - This talk focus on the disconnected contribution.
  - In next talk, Kohtaroh will give details of all connected contributions.

- We have three different averages $\Pi_{n,ss}$, $\Pi_{n,ts}$, $\Pi_{n,st}$ because of asymmetry $L \neq T$ (typically $T = 1.5L$).
  - invariant under spatial cubic rotation.

Disconnected contribution

- Current: $$j_\mu = \sum_f Q_f \bar{q}_f \gamma_\mu q_f = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c$$
- Correlator: $$\langle j_\mu(t) j_\nu(0)\rangle^{\text{disc.}} = \langle \sum_{\mathbf{x},f} Q_f \mathrm{Tr}\, M_f^{-1}(\mathbf{x},\mathbf{x})\gamma_\mu \times \sum_{\mathbf{x}',f'} Q_{f'} \mathrm{Tr}\, M_{f'}^{-1}(\mathbf{x}',\mathbf{x}')\gamma_\nu \rangle$$
  - Z_2 random vectors $\xi^{(r)}$: [Gülpers 14] $$\frac{1}{N}\sum_r \xi^{(r)}_{\mathbf{x},t,a} \xi^{(r)}_{\mathbf{x}',t',a'}{}^* = \delta_{\mathbf{x},t,a;\ \mathbf{x}',t',a'}$$
  - Eigen vector projection: $$J_\mu^f = \sum_{\mathbf{x}} \mathrm{Tr}\, M_f^{-1}(\mathbf{x},\mathbf{x}) = \sum_n^{1000} \frac{1}{\lambda_n} + \mathrm{tr}(M^{-1}(1 - \sum_n \psi_n \psi_n^*))$$
  - Noise reduction technique. [Blum 12] [Bali 14]

$$J_\mu^f = \frac{1}{N_s}\sum_{r=1}^{N_s} J_\mu^{f,\text{sloppy}} + \frac{1}{N-N_s}\sum_{r=N_s+1}^{N} (J_\mu^{f,\text{precise}} - J_\mu^{f,\text{sloppy}})$$

- use isospin symmetric masses: $m_l = m_u + m_d$.

$$J_\mu = \frac{2}{3}J_\mu^u - \frac{1}{3}J_\mu^d - \frac{1}{3}J_\mu^s = \frac{1}{3}(J_\mu^l - J_\mu^s)$$

  - Use same random vectors for l and s for noise reduction.
- Disconnected Charm contributions are estimated by hopping parameter expansion.

Simulation details

- Gluon: tree-level improved Symanzik gauge action.
- quark: $N_f = 2+1+1$ Staggered fermion action
  with 4 steps of ρ=0.125 stout smearing.
  - up and down quarks masses are degenerated and bracket physical point (via RHMC).
  - strange and charm quark masses via RHMC.
  - use pion and kaon masses to fix m_l and m_s.
  - charm quark mass is fixed as $m_c/m_s = 11.85$. [HPQCD 14]
- scale: set by f_π through Wilson-flow based w_0 method.

Our strategy

- Force gradient integrator for gauge generation. [Mawhinney 11] [Kennedy 11]

$$\mathcal{T}_{\text{Omelyan}} = e^{\lambda\hat{V}\tau} e^{\frac{1}{2}\hat{T}\tau} e^{(1-2\lambda)\hat{V}\tau} e^{\frac{1}{2}\hat{T}\ \tau} e^{\lambda\hat{V}\tau}.$$

$$\Rightarrow \mathcal{T}_{\text{FG}} = e^{\frac{1}{6}\hat{V}\tau} e^{\frac{1}{2}\hat{T}\tau} e^{\frac{2}{3}\hat{V}\tau - \frac{1}{72}\tau^3 \widehat{\{V,\{V,T\}\}}} e^{\frac{1}{2}\hat{T}\tau} e^{\frac{1}{6}\hat{V}\tau}.$$

  - enables total speed-up by factor 2-3 compered to Omelyan integrator.

    ex) # of steps for Gauge force : 1024 ➡ 64

    for RHMC force : 128 ➡ 32

- Krylov-Schur method with Chebyshev acceleration.
  - Improvement of Krylov subspace methods such as Arnoldi/Lanczos.

$$\mathcal{K}_m(A, x_1) = \text{span}\{x_1, Ax_1, A^2x_1, \ldots, A^{m-1}x_1\}$$

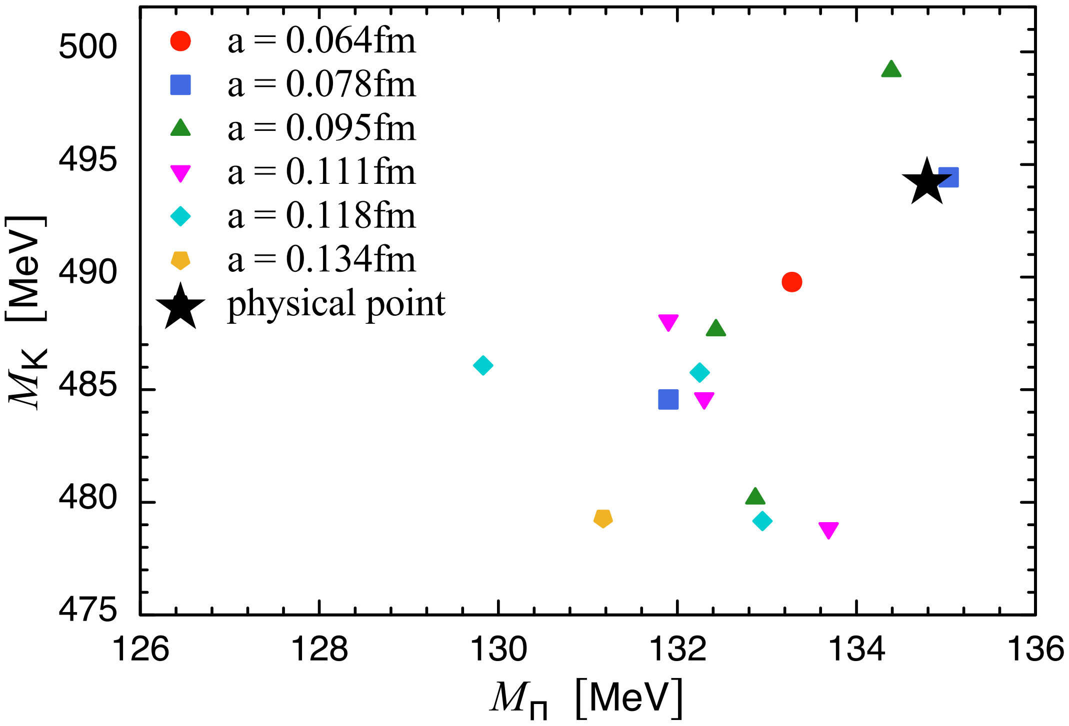
  - an Implicit restart technique in a numerically stable way.
  - use Chebyshev polynomial: $\dfrac{T_n(\lambda_{\min})}{\min(T_n(\lambda))} >> \dfrac{\lambda_{\max}}{\lambda_{\min}}$
  - compute $O(1000)$ eigen vectors used in EigCG and disconnected trace.

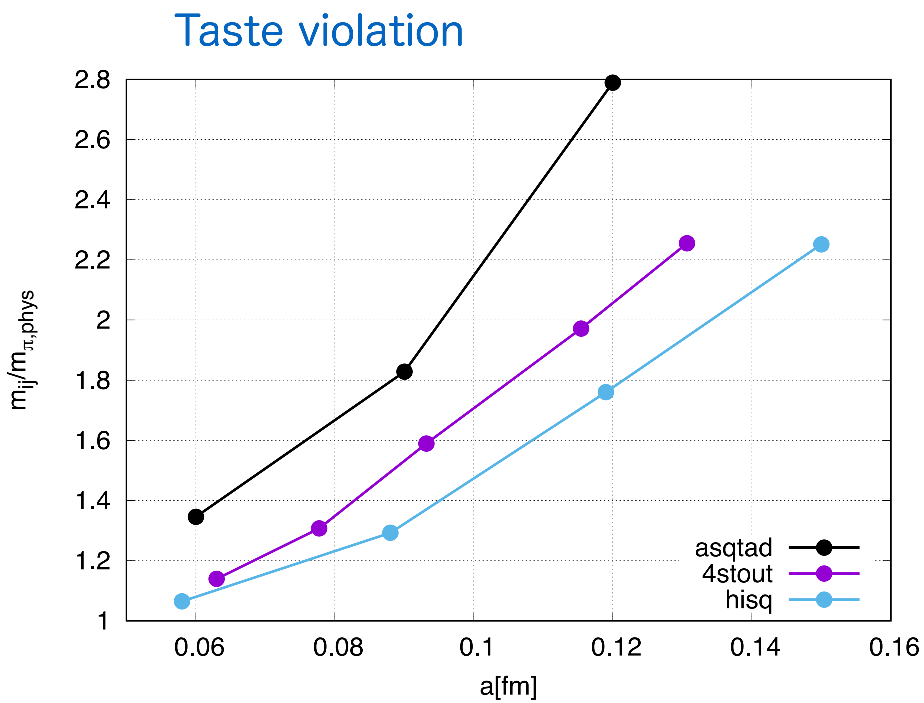
Gauge ensembles

- several β-values to allow for $a \to 0$ extrapolation.
- Lattice extent is ~6fm x 9-12fm.

β	a [fm]	a^{-1} [GeV]	T × L	# of conf. conn	# of conf. disc	#SRC (ud,s,c)
3.7000	0.134	1.47	64 × 48	1000	1000	(768, 128, 64)
3.7500	0.118	1.67	96 × 56	1500	1500	(768, 64, 64)
3.7753	0.111	1.78	84 × 56	1500	1500	(768, 64, 64)
3.8400	0.095	2.08	96 × 64	2500	1500	(768, 64, 64)
3.9200	0.078	2.53	128 × 80	3500	1000	(768, 64, 64)
4.0126	0.064	3.08	144 × 96	450		(768, 64, 64)

Pion and Kaon masses





- Our simulation sandwiches physical pion and kaon masses.
- We perform an interpolation to physical point and an extrapolation to continuum limit simultaneously.

Upper and Lower bound

- introduce a cut in time t_c to reduce statistical noise.

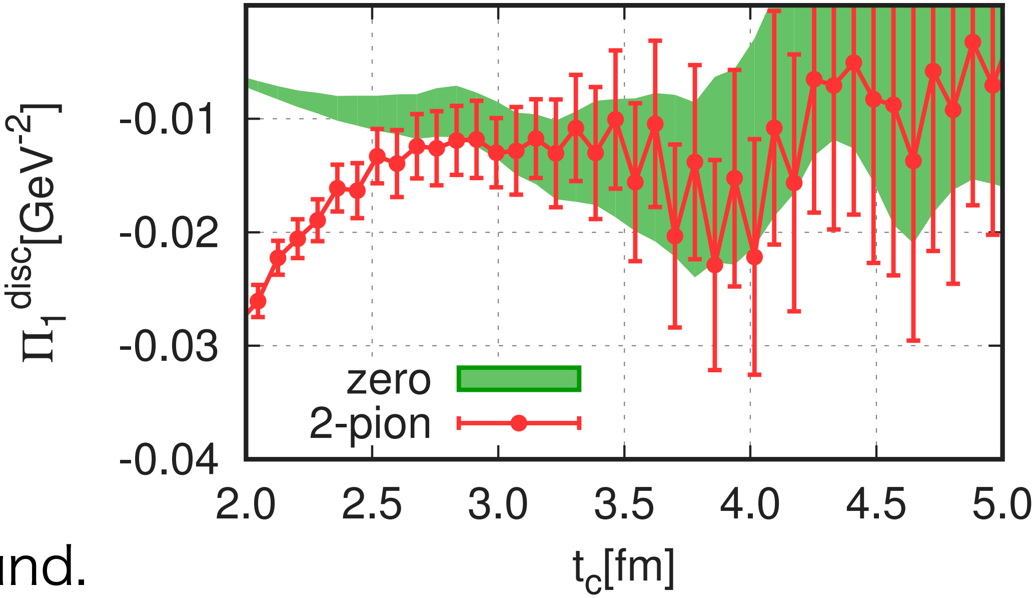
$$\Pi_{n,4i} = (-1)^{n+1} \sum_{t=0}^{t_c} \frac{t^{2(n+1)}}{\Gamma(2n+3)} C_{ii}(t)$$

- t_c is determined by the upper and lower bound.
  - Isospin triplet/singlet, $C^{I=1} = \frac{1}{2} C^l, \quad C^{I=0} = \frac{1}{18}(C^l + 2C^2 + 8C^c + 2C^{\mathrm{disc}})$
  - bound for the connected part $0 \leq C^l(t) \leq C^l(t_c) \exp(-E_{2\pi}(t - t_c))$ [Lehner14]
  - at large t

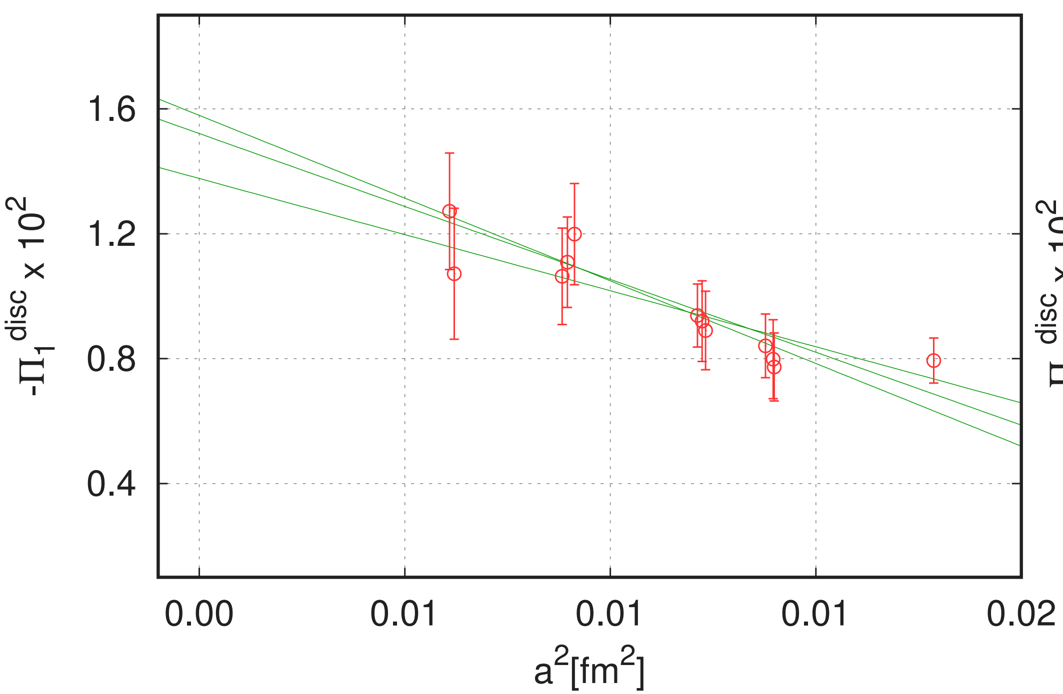
$$0 \leq C^{I=0}(t) \ll C^{I=1}(t) \leq (\text{two } \pi \text{ state})$$

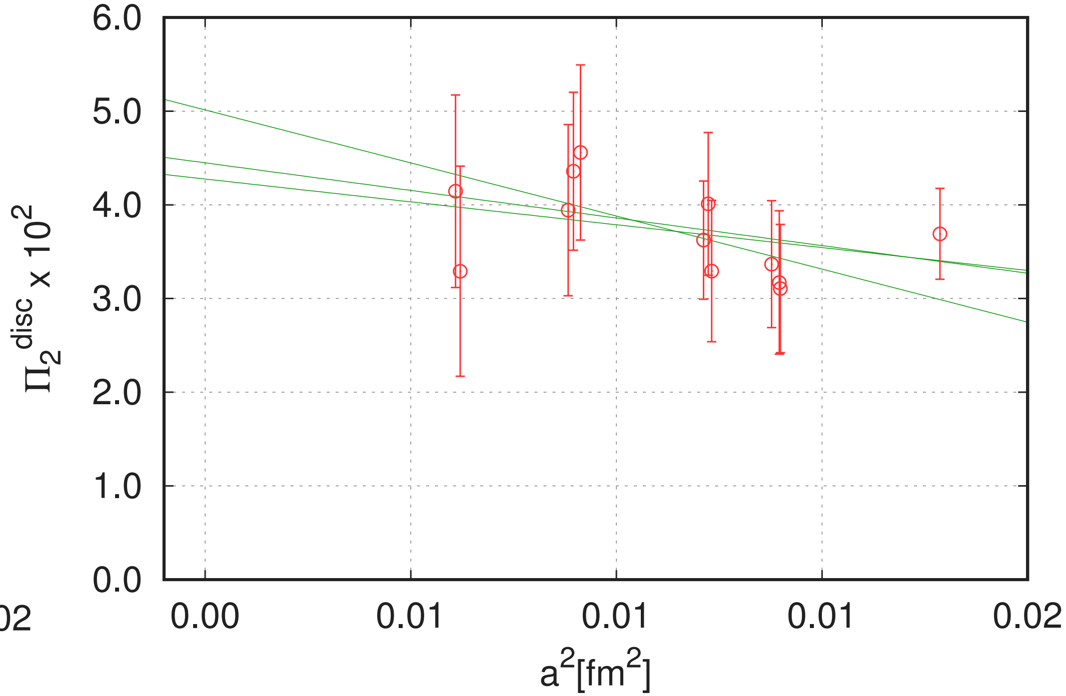
$$0 \geq 2C^s + 8C^c + 2C^{\mathrm{disc}} \geq -C^l(t_c) \exp(-E_{2\pi}(t - t_c))$$

  - We choose $t_c = 2.7\mathrm{fm}$ and average upper and lower bound.



Disconnected contribution Π^{disc}



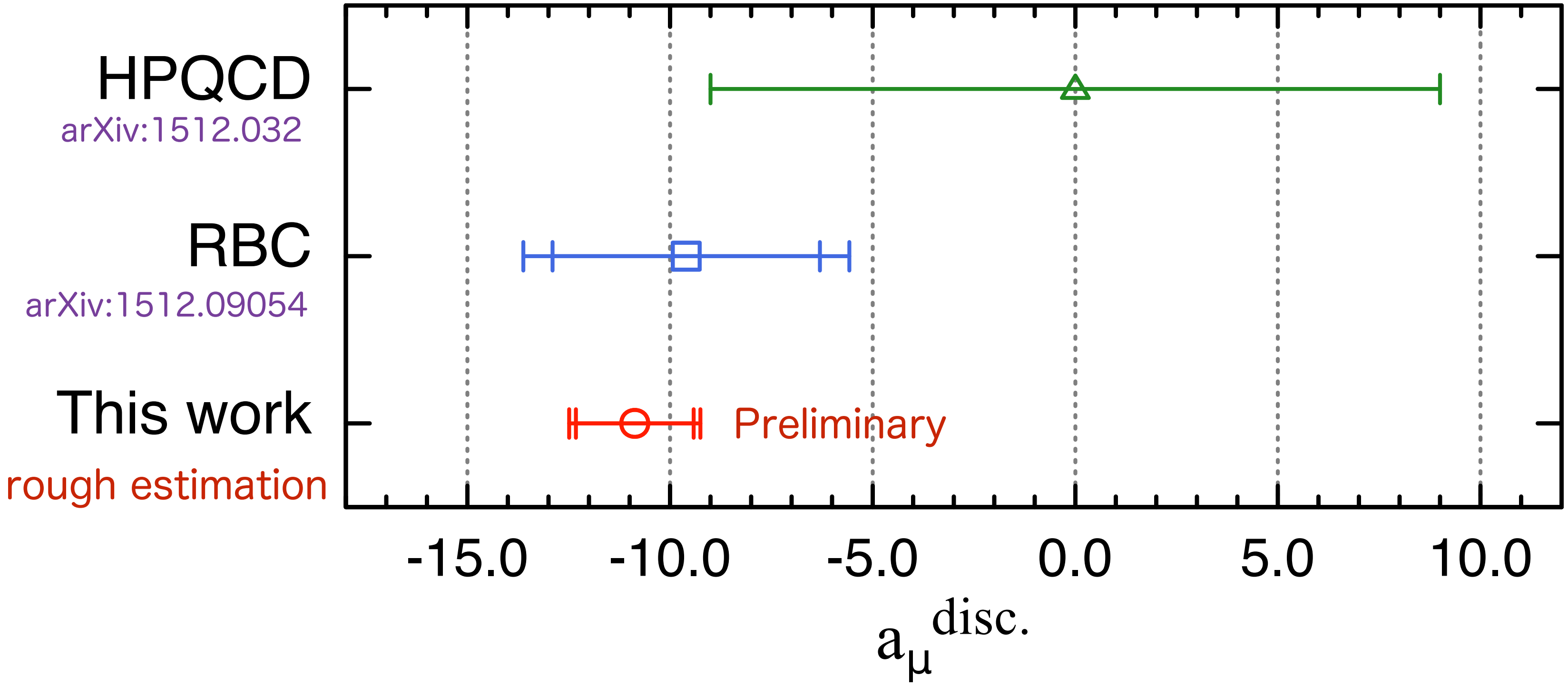


$$\Pi_1^{\text{disc}} = -1.5(2)(1) \times 10^{-3} \ \ \text{GeV}^{-2}, \qquad \Pi_2^{\text{disc}} = 4.6(1.0)(0.4) \times 10^{-4} \ \ \text{GeV}^{-4}$$

15% 23%

- We obtain reasonable fit qualities with a linear a^2 dependence.
- Systematic error due to $a \to 0$ extrapolation is estimated by imposing different cuts in the lattice spacing.
- Disconnected charm gives 0.1% contribution to total disconnected contribution.

Comparison of a_μ^{disc} with others



- We roughly estimate a_μ^{disc} by combining Π_1 and Π_2 with Pade function.

Summary

- We calculated the disconnected contributions to the moments Π_1 and Π_2 of HVP.
  - $N_f = 2+1+1$ Staggered dynamical gauge simulation at physical point.
  - Spatial box size is 6fm.
  - Upper and Lower bounds are used.
  - Taking a continuum limit.

$$\boxed{\begin{aligned} \Pi_1^{\mathrm{disc}} &= -1.5(2)(1) \times 10^{-3} \ \ \mathrm{GeV}^{-2} \\ \Pi_2^{\mathrm{disc}} &= 4.6(1.0)(0.4) \times 10^{-4} \ \ \mathrm{GeV}^{-4} \end{aligned}}$$

— Future perspective —

- We will compute $a^{\mu,\mathrm{HVP}}$ with all systematic uncertainties.
- take into account the isospin breaking.
- larger lattice than 6fm.