Isospin-0 $\pi\pi$ scattering length in twisted mass Lattice QCD

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in collaboration with
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Motivation

- Elastic $\pi\pi$ scattering process is a fundamental QCD process at low energy regime.
- $\sigma/f_0(500)$ resonance
- Lattice computations:
  - Z. Fu, Phys. Rev. D87, 074501 (2013), 1303.0517.
  - R. A. Briceno, J. J. Dudek, R. G. Edwards, D. J. Wilson, arXiv:1607.05900

Lattice setup

We use the gauge configurations generated by the ETMC with twisted mass quark action.

- $N_f = 2$ (Wilson clover twisted mass action)
  - Pion mass: *physical value* and 250 MeV.
  - Lattice spacing: 0.0931 fm
  - $M_\pi L \sim 3$
- $N_f = 2+1+1$ (Wilson twisted mass action)
  - Pion mass: 230 MeV $\sim$ 500 MeV.
  - Lattice spacing: $a_A = 0.0863$ fm, $a_B = 0.0779$ fm, $a_D = 0.0607$ fm.
  - $M_\pi L$: $3.4 \sim 5.3$

Stochastic Laplacian Heaviside quark smearing:

- See the previous talks by Markus Werner(Tue. 14:00) and Christopher Helmes(Tue. 16:30).

Twisted mass quark action

- Automatic $\mathcal{O}(a)$ improvement at maximal twist.
- Parity and *isospin* symmetry breaking.
  - Isospin-0 state not well defined.
- Mixed action: Osterwalder-Seiler(OS) action (R. Frezzotti and G. C. Rossi, JHEP 10, 070 (2004)) in the valence sector.
  - Unitarity breaking.
  - Mixing with unitary π^0.
  - Variational analysis using the isospin-0 $\pi\pi$ interpolating operator and unitary π^0 operator.

Interpolating operators

- Isospi-0 $\pi\pi$ operator:

$$\mathcal{O}_{\pi\pi,I=0}(t) = \frac{1}{\sqrt{3}}(\pi^+(t)\pi^-(t) + \pi^-(t)\pi^+(t) + \pi^0(t)\pi^0(t))$$

$$\begin{aligned}
\pi^+(t) &= \sum_{\mathbf{x}} \bar{d}' i\gamma_5 u(\mathbf{x}, t) \\
\pi^-(t) &= \sum_{\mathbf{x}} \bar{u} i\gamma_5 d'(\mathbf{x}, t) \\
\pi^0(t) &= \sum_{\mathbf{x}} \frac{1}{\sqrt{2}}(\bar{u} i\gamma_5 u - \bar{d}' i\gamma_5 d')(\mathbf{x}, t)
\end{aligned}$$

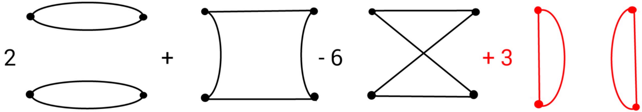
- Unitary π^0 operator:

$$\mathcal{O}_{\pi^{0,Uni}}(t) = \sum_{\mathbf{x}} \frac{1}{\sqrt{2}}(\bar{u} i\gamma_5 u - \bar{d} i\gamma_5 d)(\mathbf{x}, t)$$

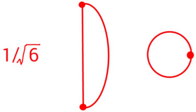
Build the matrix of correlators:

$$\mathcal{C}_{ij}(t - t_0) = \langle \mathcal{O}_i(t)\mathcal{O}_j^\dagger(t_0) \rangle$$

- $\langle \mathcal{O}_{\pi\pi,I=0}(t)\mathcal{O}^\dagger_{\pi\pi,I=0}(t_0) \rangle$



- $\langle \mathcal{O}_{\pi\pi,I=0}(t)\mathcal{O}^\dagger_{\pi^{0,Uni}}(t_0) \rangle$

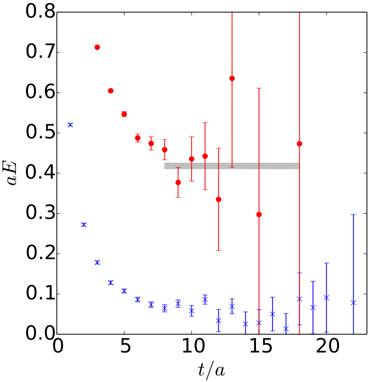


- $\langle \mathcal{O}_{\pi^{0,Uni}}(t)\mathcal{O}^\dagger_{\pi^{0,Uni}}(t_0) \rangle$

Solve the generalized eigenvalue problem

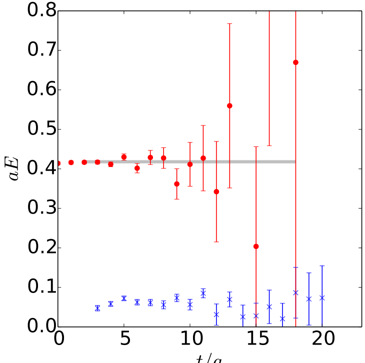
$$\tilde{C}_{ij}(t)v_j^n = \lambda_n(t)\tilde{C}_{ij}(t_0)v_j^n$$

where $\tilde{C}_{ij}(t) = C_{ij}(t) - C_{ij}(t+1)$.
Eigenvalues: $\lambda_n(t) \sim A\sinh(E_n(\frac{T}{2} - t - 0.5))$



Remove excited state contaminations

- Assumption: disconnected diagrams are only sizeable for ground states, and negligible for higher excited states. (H. Neff et. al. “Low fermionic eigenmode dominance in QCD on the lattice", Phys. Rev. D 64, 114509)
- Determine the ground state in the connected correlator and subtract the excited state contributions.
- Rebuild the matrix correlators from the subtracted connected correlators and the original disconnected parts.



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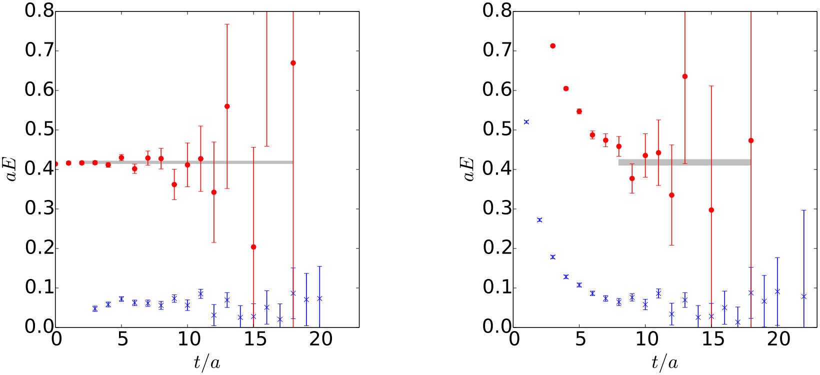
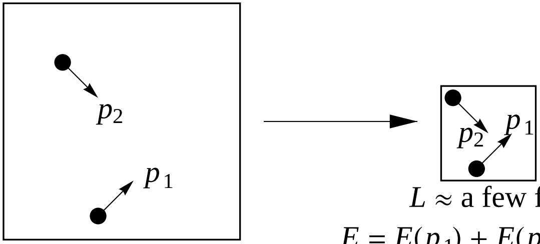


Lüscher's finite volume method



$L \gg$ a few fm

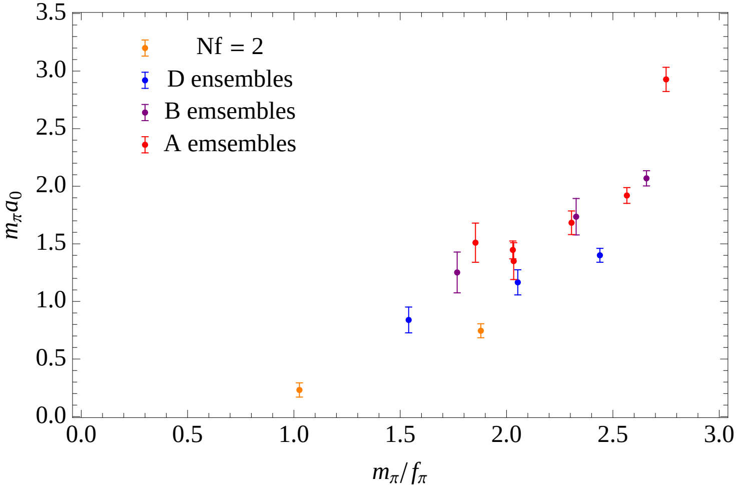
$E = E(p_1) + E(p_2)$

$L \approx$ a few fm

$E = E(p_1) + E(p_2) + \Delta E$

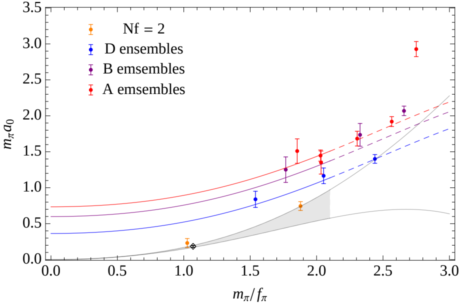
$$\Delta E = -\frac{4\pi a_0}{M_\pi L^3}\left(1 + c_1\frac{a_0}{L} + c_2\frac{a_0^2}{L^2}\right) + \mathcal{O}\left(\frac{1}{L^6}\right)$$

Results



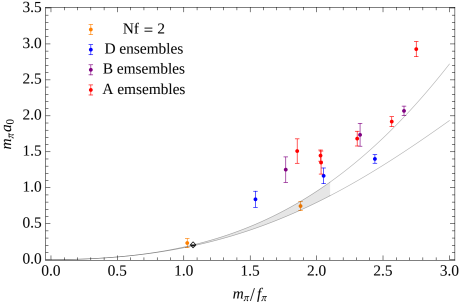
Nf = 2
D ensembles
B emsembles
A emsembles
$m_\pi a_0$
m_π / f_π
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0

$$M_\pi a_0^{\mathrm{I}=0} = \frac{7M_\pi^2}{16\pi f_\pi^2}\left[1 - \frac{M_\pi^2}{16\pi^2 f_\pi^2}\left(9\ln\frac{M_\pi^2}{f_\pi^2} - 5 - \ell_{\pi\pi}^{\mathrm{I}=0}\right)\right] + ca^2$$



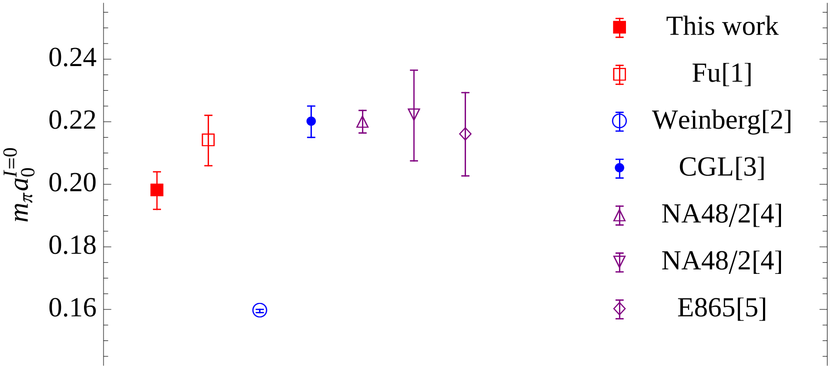
	$M_\pi/f_\pi < 2.1$
$M_\pi a_0^{\mathrm{I}=0}$(Phy.)	0.184(13)
$\ell_{\pi\pi}^{\mathrm{I}=0}$	18(11)
c [fm^{-2}]	103(26)
χ^2/dof	1.3/4

$$M_\pi a_0^{\mathrm{I=0}} = \frac{7M_\pi^2}{16\pi f_\pi^2}\left[1 - \frac{M_\pi^2}{16\pi^2 f_\pi^2}\left(9\ln\frac{M_\pi^2}{f_\pi^2} - 5 - \ell_{\pi\pi}^{\mathrm{I=0}}\right)\right]$$



	$M_\pi/f_\pi < 2.1$
$M_\pi a_0^{\mathrm{I=0}}$(Phy.)	0.198(6)
$\ell_{\pi\pi}^{\mathrm{I=0}}$	30(6)
χ^2/dof	0.7/1
$M_\pi a_0^{\mathrm{I=0}}$(Phy.Latt.)	0.23(6)

Comparison with other determinations



[1]. Z. Fu, Phys. Rev. D87, 074501 (2013), 1303.0517.
[2]. S. Weinberg, Phys. Rev. Lett. 17, 616 (1966).
[3]. G. Colangelo, J. Gasser, and H. Leutwyler, Nucl. Phys. B603, 125 (2001).
[4]. J. R. Batley et al. (NA48-2), Eur. Phys. J. C70, 635 (2010).
[5]. S. Pislak et al., Phys. Rev. D67, 072004 (2003)

Summary and outlook

- We computed the isospin-0 $\pi\pi$ scattering length in twisted mass lattice QCD with a large range of pion masses and various lattice spacings.
- Chiral and continuum extrapolations.
- Residual systematic uncertainties.
- Plan to repeat the computation with an action without isospin symmetry breaking, e.g. Wilson clover action.
- σ resonance.